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THE HYSTERESIS PHENOMENON AND BENEFIT EVALUATION FOR POLLUTION CONTROL IN AQUATIC ECOSYSTEMS

by

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I. INTRODUCTION

The genesis of this paper is the observation (see Goldman and Horne [1983]) that damaged ecosystems do not respond immediately to an abrupt cessation in pollution discharges and, when they do respond, do not exactly retrace the trajectory of their decline. Indeed, because of some irrecoverable losses from the system, they may never return to their original state. This phenomenon, which is termed "hysteresis" in the natural sciences, has important implications for policy analysis--namely, that some of the gains from pollution control are at best limited or slow to arrive, and the consequences of imposing new environmental insults are at least partly irreversible.

The purpose of this paper is to develop a benefit-evaluation framework for the control of pollution in aquatic ecosystems that takes proper account of the dynamics of recovery. An important additional consideration is that the timing and nature of the recovery are ordinarily uncertain, as are the benefits. It is, in fact, the combination of uncertainty and its behavior over time with irreversibility that has important implications for benefit evaluation. Illustrations and examples in the paper focus on aquatic ecosystems, but the concepts have application to some terrestrial systems as well. In section 2 we review relevant features of aquatic ecosystem behavior. In section 3 the stylized physical facts are used to develop an evaluation framework, and in section 4 results are extended to deal with more complicated multiperiod recovery dynamics.

A key concept that emerges from the analysis is that of option value, which turns out to be the difference between a "correct" evaluation--one that takes proper account of the hysteresis phenomenon and the behavior of uncertainty over time--and a "conventional" calculation. In section 5 we present a contrived, though plausible, example of an empirical application which suggests that option value can be substantial relative to conventionally calculated benefits.

II. LAGS, IRREVERSIBILITIES, AND UNCERTAINTIES

The hysteresis phenomenon is clearly recognized in the popular term, "dead lakes." In fact these lakes may not be dead at all but simply full of the wrong kind of life--the heavy algal growth that characterizes lakes in an advanced state of eutrophication and that inhibits growth of desirable fish species. But the point for pollution control policy is that reducing the inflow of nutrients, for example by treating sewage discharges, will reduce nutrient and algal concentrations only over a period of time, depending on lake characteristics, and beyond some point may not result in recovery of the premium species.

Long-lasting impacts can also follow the deposition in lake or stream sediments of what are commonly known as toxic substances--those that, unlike nutrients, are harmful even in small quantities. Long after the runoff that causes the deposition is controlled, the substances remain in the sediment, from which they can be released to cause potentially serious damage. Interestingly, one mechanism by which this occurs is leaching out of trace metals by another kind of pollution currently the subject of much attention and controversy--acid rain.

Retention in sediments is also one of the pathways through which nutrients and related algal growth can continue to inhibit improvements in lake water quality following construction of a treatment or diversion facility. Figure 1 shows the effect of a treatment plant that virtually removed phosphorous (the limiting nutrient) from the wastewater inflow to Shagawa Lake in northern Minnesota.[1] Because of the short hydrologic and phosphorous residence time--just eight months--in this relatively shallow lake, rapid improvement in water quality might have been expected. Yet, as shown in the Figure, peaks in phosphorous concentrations and algal growth similar to those in the pretreatment year of 1971 occurred in late 1974, nearly two years after treatment began. The apparent explanation is release late in the summer of 1974 of phosphates from lake sediments. Recharge from sediments, or internal loading, can be an important source of phosphorous--as important as annual inflow.

Recovery of the lake might, nevertheless, be anticipated over a slightly longer period although, in a sense, the recovery would not be complete if local populations of indigenous species had been lost in the meantime.[2] However, in the case of the larger and much deeper Lake Tahoe in the Sierra Nevada mountains along the California-Nevada border, recovery from additional inflows of nutrients would take so long that, on a human time scale, the damage can be considered irreversible. Because algal growth in Lake Tahoe is mainly nitrogen limited and because treatment for nitrogen removal is more difficult and more costly than for phosphorous removal [indeed, it is not (yet) technically feasible to remove sufficient nitrogen (and phosphorous) to prevent substantial degradation of the water quality of the lake] diversion, or export of wastewater from the lake basin, was the mode of control adopted. The results have not been encouraging. Since the diversion, which was begun

Figure 1

in the 1960s, primary (phytoplankton) production has continued to increase, water clarity has decreased, and shoreline algae remains visible--consistent with experimental findings on the effects of introducing even very small amounts of nutrients (Goldman and Carter [1965]).

Of course, the water quality of Lake Tahoe is still relatively good. The real difficulty would come with introduction of substantial additional amounts of nutrients, as from erosion during home and recreational site construction, and leaching from new and existing septic tanks. Because of the very great hydrologic residence time of the lake (about 700 years), impacts would be very long lasting. Further lags in recovery could come from internal loading; but, even without this, the impacts might be considered irreversible for all practical purposes.

Recovery dynamics can be illustrated with the aid of what are known as hysteresis curves. Figure 2 shows a typical curve relating primary production, or algae biomass, to lake water nutrient concentration. Time is measured along the curve. Moving to the right, concentration is increasing as is (nonlinearly) biomass. At point A, nutrient inflow is controlled, say, by diverting wastewater. Concentration may continue to increase for a while, then eventually decrease; but the curve does <u>not</u> retrace its trajectory. The level of algae remains above what it was at the same concentration earlier because the nutrient is now embodied not in the lake water but in the algae itself. The algae sinks to the bottom--perhaps into the sediment--and recycles.

A hysteresis curve for a desirable fish species, such as lake trout, is shown in Figure 3. Fish recovery typically takes longer than does algae decline and, indeed, may not occur at all owing to competitive displacement;

Figure 2

Figure 3

that is, another, less desirable, species may come in and take the trout's "niche," so that, even as nutrient concentration approaches the prepollution level, the trout population does not recover. Of course, different outcomes are possible. If the nutrient buildup is halted at a lower level, say, at point B on the Figure, the trout population eventually recovers although perhaps only after many years and perhaps not all the way as indicated by the dotted-line path. A study of the hysteresis phenomenon in aquatic ecosystems is a part of our larger project (see Horne and Harte [1985]).

From what we have said thus far it is clear that the timing and nature of aquatic ecosystem recovery are not matters of certain knowledge. This is particularly true for the higher trophic levels--the premium species at the top of the food chain. Two implications for subsequent research follow. First, it may be useful to study the role of the lower level changes, for example in primary productivity, as a kind of "leading indicator" of the behavior of populations of greater interest. Such a study forms another part of our project (see Horne [1985]).

Second, the uncertainty should be explicitly considered. Predicting changes in pollution loadings resulting from treatment, diversion, or source control; predicting changes in water quality resulting from reduced pollution; predicting changes in ecosystem structure and performance resulting from improved water quality--all of these tasks involve elements of uncertainty that compound as the analysis progresses. Because of threshold effects and nonlinearities in the response functions of natural systems, it may be misleading to ignore this compounding of errors or even simply to replace every uncertain quantity at each stage of the analysis by an estimate of its mean. Still another part of our project addresses the question of how to model the propagation of uncertainty (see Harte, Horne, and Von Hippel [1985]). Finally, note that, even if impacts on the natural environment were known with certainty, the same could not be claimed for the values society will place on them.

In the next section we begin the development of a model to evaluate pollution control in an aquatic ecosystem--a model rooted in the assumptions that the benefits of control are delayed and uncertain.

III. A FRAMEWORK FOR BENEFIT EVALUATION

We model the decision on whether or not to control pollution in an aquatic ecosystem from the point of view of an environmental authority concerned with the net present value (benefits minus costs) of control. Optimal control is defined as the choice that maximizes this value. The important constraints are those that emerge from the discussion of the preceding section: (1) Benefits of control undertaken today are not realized until some future date; (2) beyond some point, failure to control is irreversible, i.e., system recovery and the associated benefits are unobtainable at any date; and (3) the timing and nature of the recovery are uncertain, as are the benefits.

As our hysteresis curve suggests, recovery is a continuous process; yet, we may capture some of the essential features for purposes of decision analysis with a simple discrete-time framework. In the simplest case, only two periods--present and future--are needed. Let us see how some of the structure of the problem, including the three constraints, can be set out analytically in this framework.

The assumption that control undertaken in the present does not lead to benefits in the present is represented as

$$B_1(1) = B_1(0),$$

where B_1 is the present, or first-period, benefit and the number in parentheses (1 or 0) is the control decision (1 equals control and 0 equals no control). Note that we are considering a binary choice, neglecting intermediate levels of control. Letting X_1 represent the first-period control decision, $X_1 = 1$ corresponds to building a treatment plant, a wastewater diversion, or whatever the appropriate control mode may be; $X_1 = 0$ corresponds to not building. The results we obtain can be extended to the case of continuous control, but this is somewhat beside the point and comes at a substantial cost in complexity.[3]

If first-period control does not lead to any first-period benefits but does entail costs, why do it? To answer this question, we must introduce a future, or second, period in which benefits of first-period control are realized. We assume, therefore, that

$$B_2(0, 0) < B_2(1, 0) < B_2(1, 1),$$

where B_2 is the second-period benefit and the numbers in parentheses are X_1 and X_2, the levels of first- and second-period control, respectively. What this says is that building a treatment plant, say, does produce benefits [$B_2(1, 0) > B_2(0, 0)$] and that these benefits are still greater [$B_2(1, 1) > B_2(1, 0)$] if the plant is maintained and operated in the second period. Of course, whether or not construction and continued operation of a treatment plant are optimal will depend, also, on the costs.

There is one other possible sequence of controls in our terminology: $B_2(0, 1)$, i.e., no control in the first period followed by control in the second. Given the lag in recovery (control undertaken in a given period does not result in benefits in that period), we have

$$B_2(0, 1) = B_2(0, 0).$$

Equally clearly, benefits of second-period control might be realized in a later period just as benefits of first-period control were realized in the second period.

This is a convenient spot at which to introduce the second constraint: Beyond some point, failure to control is irreversible. If we simply cut off at two periods, we say, in effect, that failure to control in the first period is irreversible. (Of course, control can be undertaken in the second period; but since it yields no benefit and it carries a cost, it cannot be optimal.) This is not an unreasonable formulation. The system may be in a fairly advanced state of degradation in the first planning period, so that further neglect is fatal; or we may interpret the period as lasting for several years as in a five-year plan. For that matter, the system may be irreversibly damaged regardless of what one does in the first period. In this case, however, the decision problem is trivial. For the remainder of this section, we assume that first-period control does yield benefits in the second period and that failure to control in the first period is irreversible. In the next section we consider how the results we obtain are affected if failure to control in the first period is not irreversible.

The third constraint or assumption is that the benefits of ecosystem recovery are uncertain due to a lack of knowledge about behavior of the system itself or about the willingness of individuals to pay for the goods and services it can produce. Because the major uncertainty is about future (second-period) benefits, for simplicity we will assume that first-period benefits (which are, in any event, not affected by control) and costs are known; therefore, the only new notation we need is that for second-period benefits which we represent as

$$B_2(X_1, X_2, \theta),$$

where θ is a random variable. Clearly, a major problem in moving toward an empirical application is to characterize the uncertainty. How is the random variable, θ, distributed; what are the expected value, dispersion, etc.? (As indicated earlier, we are studying these questions elsewhere as a part of the larger project on economic valuation of aquatic ecosystems.)

Although we have nothing further to add at this stage about the propagation of uncertainty through different stages of ecosystem measurement and analysis, we do need to address two aspects of the economic modeling of decisions under uncertainty. First, what about attitudes toward risk? Following an established tradition (see, for example, Samuelson [1964] and Arrow and Lind [1970]), we will assume a neutral attitude toward risk on the part of the social decision-maker--the environmental authority. There is another reason for our choice: The results we obtain will look like results that would normally depend on risk aversion. The point we wish to emphasize here is that in our analysis they do not depend on risk aversion.

A second aspect of the modeling of uncertainty in a dynamic setting, which was mentioned earlier, is its behavior over time. Uncertainty means a lack of information; yet, it is likely that this situation changes--that information is acquired over time. Our model is largely concerned with the consequences of a failure on the part of the decision-maker to take this prospect into account; thus, we will consider how benefits are evaluated and a control decision is made when it is assumed that nothing further will be learned about the value of θ by period two and, by comparison, when it is assumed that information about the value of θ will become available.

The information can be partial in the sense that the decision-maker combines it with his prior distribution for θ in a Bayesian manner to obtain a posterior distribution for θ; or it can be perfect in the sense that the posterior distribution collapses to a single point. Because the case of perfect information is easier to represent, we consider it first. We also show that the qualitative results generalize to the more realistic setting of partial information. One final point: We assume that the acquisition of information does not depend on the choice of first-period control. This turns out to be important, and we will have more to say later in defense of this point.

Now let us write expressions for the value over both periods in each information structure. Where no information is forthcoming by the second period, we have either

$$V^*(0) = B_1(0) + \beta_2 E[B_2(0, 0, \theta)], \tag{1}$$

where $V^*(0)$ is the value over both periods if no control is undertaken in the first period, β_2 is a discount factor ($\beta_1 \equiv 1$), and $E[\cdot]$ is the expected value of the expression in brackets; or

$$V^*(1) = B_1(0) - C_1 + \beta_2 \max \{E[B_2(1, 0, \theta)], E[B_2(1, 1, \theta) - C_2]\}, \tag{2}$$

where $V^*(1)$ is the (maximum) value over both periods if control is undertaken in the first period and C_1 and C_2 are first- and second-period costs of control. Notice that the uncertain second-period values, $B_2(0, 0, \theta)$, $B_2(1, 0, \theta)$, and $B_2(1, 1, \theta)$ are simply replaced by their expected values. Note, further, that no choice is indicated for second-period control in the event that the first-period choice was no control [equation (1)]. This follows from our assumption $B_2(0, 1, \theta) = B_2(0, 0, \theta)$, i.e., that second-period control yields no benefit (but, of course, carries a cost) following a failure to control in the first period. Finally, note that first-period net benefits of control are $B_1(0) - C_1$ from our assumption $B_1(1) = B_1(0)$, i.e., that recovery is delayed.

Combining equations (1) and (2), we have

$$V^*(1) = V^*(0) = \begin{cases} -C_1 + \beta_2 E[B_2(1, 0, \theta)] - \beta_2 E[B_2(0, 0, \theta)] \\ \qquad\qquad\qquad \text{if } E[B_2(1, 0, \theta)] > E[B_2(1, 1, \theta) - C_2] \\ -C_1 + \beta_2 E[B_2(1, 1, \theta) - C_2] - \beta_2 E[B_2(0, 0, \theta)] \quad \text{otherwise} \end{cases} \tag{3}$$

and

$$X_1^* = \begin{cases} 1 & \text{if } V^*(1) - V^*(0) > 0 \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

where X_1^* is the level of first-period control that maximizes value over both periods on the assumption that no information about second-period benefits will be available to affect the choice.

Now suppose that (perfect) information about second-period benefits is forthcoming; then we can write

$$\hat{V}(0) = B_1(0) + \beta_2 E[B_2(0, 0, \theta)] \tag{5}$$

and

$$\hat{V}(1) = B_1(0) - C_1 + \beta_2 E[\max\{B_2(1, 0, \theta), B_2(1, 1, \theta) - C_2\}]. \tag{6}$$

Notice that, in this case, second-period benefits are <u>not</u> replaced by their expected values. Instead, the decision-maker is assumed to learn which of the two control options, $X_2 = 0$ or $X_2 = 1$, will yield greater net benefits and to choose that one. Of course, at the start of the first period, when X_1 must be chosen, he has only an <u>expectation</u> of their maximum value. Notice also that $V^*(0) = \hat{V}(0)$. With no control in the first period, value over both periods must be the same in either information setting since it is too late to control in the second period regardless of what is learned about the random variable, θ, in the first period.

In any event, we have

$$\hat{V}(1) - \hat{V}(0) = -C_1 + \beta_2 E[\max\{B_2(1, 0, \theta), B_2(1, 1, \theta) - C_2\}] - \beta_2 E[B_2(0, 0, \theta)] \tag{7}$$

and

$$\hat{X}_1 = \begin{cases} 1 & \text{if } \hat{V}(1) - \hat{V}(0) > 0 \\ 0 & \text{otherwise,} \end{cases} \tag{8}$$

where X_1 is the level of first-period control that maximizes value over both periods on the assumption that information about second-period benefits will be available by the start of the second period.

What is the relationship of $\hat{X}_1$ to X_1^*? A natural conjecture is that first-period control is more likely to be optimal in the case where information about second-period benefits will be available, i.e., that prob $(\hat{X}_1 = 1) \geq$

prob ($X_1^* = 1$). This result follows immediately from the convexity of the maximum operator and Jensen's Inequality. Thus,

$$\hat{V}(1) - V^*(1) = \beta_2 E[\max\{B_2(1, 0, \theta), B_2(1, 1, \theta) - C_2\}] - \beta_2 \max\{E[B_2(1, 0, \theta)], E[B_2(1, 1, \theta) - C_2]\} \geq 0. \tag{9}$$

Since $\hat{V}(0) = V^*(0)$, $[\hat{V}(1) - \hat{V}(0)] \geq [V^*(1) - V^*(0)]$ and first-period control ($X_1 = 1$) is more likely to be optimal when it is possible to learn about the benefits of second-period control.

The evaluation of benefits and the resulting decision on pollution control are affected by what one assumes about the behavior of uncertainty over time. If, as seems plausible, information about the benefits of ecosystem recovery will become available, a first-period evaluation of benefits (over both periods) that fails to take this prospect into account will be biased downward. We can show this with the aid of a concept of option value introduced independently by Arrow and Fisher [1974] and by Henry [1974].[4]

Option Value

Suppose that the decision-maker ignores the prospect of new information: He simply replaces random variables with their expected values, i.e., he compares $V^*(1)$ and $V^*(0)$. A correct decision--one that takes account of learning--can be induced by a penalty for failing to control in the first period. The optimal penalty is one that leads the decision-maker to compare $\hat{V}(1)$ and $\hat{V}(0)$. To solve for the penalty, Z, write

$$V^*(1) - [V^*(0) - Z] = \hat{V}(1) - \hat{V}(0) \tag{10}$$

so that

$$Z = [\hat{V}(1) - \hat{V}(0)] - [V*(1) - V*(0)]. \tag{11}$$

The quantity on the right-hand side of equation (11) can be interpreted as the difference between the advantage of first-period control (over no control) when one takes into account the prospect of new information and when one does not. This is just the Arrow-Fisher-Henry option value, OV. Since $\hat{V}(0) = V*(0)$, we have

$$OV = [\hat{V}(1) - \hat{V}(0)] - [V*(1) - V*(0)] = \hat{V}(1) - V*(1) \geq 0; \tag{12}$$

therefore, option value can be considered a "correction factor" to a conventional evaluation--one that does not take into account the prospect of new information.

It is tempting to identify this concept of option value with another one that is familiar in decision theory: the value of information or, more precisely, the expected value of perfect information.[5] However, the identification is not quite correct. Option value in this interpretation is a conditional value of information, conditional on first-period control.

The unconditional value of information is $\hat{V}(\hat{X}_1) - V*(X_1^*)$, the gain from being able to learn about future benefits provided that X_1 is optimally chosen in each case. This may or may not correspond to $\hat{X}_1 = X_1^* = 1$. Two other outcomes are possible: $\hat{X}_1 = X_1^* = 0$ and $\hat{X}_1 = 1$, $X_1^* = 0$. (Note that $\hat{X}_1 = 0$, $X_1^* = 1$ is ruled out.) If $\hat{X}_1 = X_1^* = 0$, the value of information is $\hat{V}(0) - V*(0) = 0$, whereas option value is still $\hat{V}(1) - V*(1) \geq 0$. If $\hat{X}_1 = 1$, $X_1^* = 0$, the value of information is $\hat{V}(1) - V*(0)$. Option value is once again greater than the value of information since $\hat{V}(1) > \hat{V}(0) = V*(0) \geq V*(1)$.

We stated earlier that these results on comparative benefit evaluation can be extended to the case in which the information that becomes available is only partial information. We now show this.

Partial Information

Suppose that at the end of the first period, instead of obtaining perfect information, the decision-maker obtains partial information about the consequences of development which he combines with his prior distribution for θ in a Bayesian manner to obtain a posterior distribution for θ. Specifically, suppose that at the end of the first period he observes the value of some random variable, ζ, with which he updates his distribution for θ; expectations with respect to this posterior distribution will be denoted by $E_{\theta\zeta}[\cdot]$. At the beginning of the first period before he observes ζ, he has a joint distribution over θ and ζ. The marginal distribution for θ is his prior distribution. Expectations with respect to the marginal distribution for ζ will be denoted by $E_{\zeta}[\cdot]$.

At the beginning of the first period, expected benefits over both periods, as a function of the initial level of control (X_1), are given for $X_1 = 1$ [for $X_1 = 0$, they are given by equation (1)] by

$$V^{+}(1) = B_1(0) - C_1 + \beta_2 E_{\zeta}\Big[\max\ \{E_{\theta\zeta}[B_2(1, 0, \theta)],\ E_{\theta\zeta}[B_2(1, 1, \theta) - C_2]\}\Big]. \quad (13)$$

Comparing this to $V^*(1)$ in equation (2), we have from the convexity of the maximum operator and Jensen's Inequality

$$V^{+}(1) - V^{*}(1) \geq 0. \quad (14)$$

This is exactly analogous to equation (9), which gives the result in the case of perfect information.

Similarly, we can write an expression for option value that is exactly analogous to equation (12):

$$\begin{aligned} OV^+ &= [V^+(1) - V^+(0)] - [V^*(1) - V^*(0)] \\ &= V^+(1) - V^*(1) \geq 0, \end{aligned} \qquad (15)$$

where OV^+ is option value with partial information; therefore, the assumption of perfect information is not crucial to our analysis. The key result--that a first-period benefit evaluation which fails to take into account the prospect of new information will be biased downward--holds for partial information.

IV. MULTIPERIOD RECOVERY DYNAMICS

We have been considering a situation in which failure to act immediately--or at least sometime over the course of a first planning period--to control pollution leads to irreversible change in an aquatic ecosystem. This is consistent with the constraint that beyond some point the system cannot recover. However, suppose that the change is not triggered by failure to control in the first period. Instead, several periods of disruption are required. This, also, is consistent with the constraint and may be appropriate in some situations. The evaluation framework of the preceding section is readily extended from two to several periods. We will show that an option-value correction to a conventional benefit estimation is still indicated, but it diminishes in importance as the number of periods of "permitted" disruption increases.

To prove this, we need some additional assumptions about the benefits of different control sequences. It will be sufficient to consider just one more period to establish a result that holds for any number of periods. Suppose that two periods of uncontrolled disruption (rather than one) will trigger an irreversible change. Control in the third period is too late in the sense that

$$B_3(0, 0, 1, \theta) = B_3(0, 0, 0, \theta),$$

where B_3 is the third-period benefit and the numbers in parentheses are first-, second-, and third-period control choices, X_1, X_2, and X_3, respectively.

Furthermore, we assume that

$$B_3(0, 0, 1, \theta) < B_3(0, 1, 0, \theta) < B_3(0, 1, 1, \theta),$$

i.e., failure to control in the first period is not fatal; second-period control does yield a third-period benefit which is still greater if the control is maintained in the third period. To assure that it is not the calendar period (in this case the third period) that is crucial rather than the number of prior periods of failure to control (in this case, two periods), we assume that

$$B_3(1, 0, 0, \theta) < B_3(1, 0, 1, \theta).$$

Third-period control does yield a benefit as long as prior failure persists for only one period. In other words, once the treatment plant is built, failure to operate it for a period is not fatal.

In a three-period model, the expression for option value is

$$OV_3 = [\hat{V}(0, 1) - \hat{V}(0, 0)] - [V^*(0, 1) - V^*(0, 0)], \tag{16}$$

where OV_3 refers to third-period benefits conditional on control choices in the first two periods. Our strategy will be to show, first, that $\hat{V}(0, 0) = V^*(0, 0)$ and then that $\hat{V}(0, 1) - V^*(0, 1) \geq 0$.

The expression for value over all three periods in the event that no control is undertaken in the first two periods and no information is anticipated by the third period is

$$\begin{aligned} V^*(0, 0) = B_1(0) + \beta_2 E[B_2(0, 0, \theta)] + \beta_3 \max \{&E[B_3(0, 0, 0, \theta)], \\ &E[B_3(0, 0, 1, \theta) - C_3]\}, \end{aligned} \tag{17}$$

where β_3 is a discount factor, $\beta_3 < \beta_2$. As in the two-period framework, random variables are simply replaced by their expected values. Bearing in mind that $B_3(0, 0, 1, \theta) = B_3(0, 0, 0, \theta)$, the right-hand side of equation (17) can be simplified to yield

$$V^*(0, 0) = B_1(0) + \beta_2 E[B_2(0, 0, \theta)] + \beta_3 E[B_3(0, 0, 0, \theta)]. \tag{18}$$

Maximum value over all three periods in the event no control is undertaken in the first period but is undertaken in the second period is

$$\begin{aligned} V^*(0, 1) = B_1(0) + \beta_2 E[B_2(0, 0, \theta) - C_2] + \beta_3 \max \{&E[B_3(0, 1, 0, \theta)], \\ &E[B_3(0, 1, 1, \theta) - C_3]\}. \end{aligned} \tag{19}$$

Now suppose that information about third-period benefits is anticipated. If no control is undertaken in the first two periods, the value over all three periods is

$$\hat{V}(0, 0) = B_1(0) + \beta_2 E[B_2(0, 0, \theta)] + \beta_3 E[\max \{B_3(0, 0, 0, \theta), B_3(0, 0, 1, \theta) - C_3\}]. \tag{20}$$

Again, bearing in mind that $B_3(0, 0, 1, \theta) = B_3(0, 0, 0, \theta)$, the right-hand side of the equation can be simplified to the right-hand side of equation (18) so that $\hat{V}(0, 0) = V^*(0, 0)$.

Finally, (maximum) value over all three periods in the event that no control is undertaken in the first but is undertaken in the second and information is anticipated is

$$\hat{V}(0, 1) = B_1(0) + \beta_2 E[B_2(0, 0, \theta) - C_2] + \beta_3 E[\max \{B_3(0, 1, 0, \theta), B_3(0, 1, 1, \theta) - C_3\}]. \tag{21}$$

Since $\hat{V}(0, 0) = V^*(0, 0)$, the expression for option value in equation (16) can be simplified to

$$\begin{aligned} OV_3 &= \hat{V}(0, 1) - V^*(0, 1) \\ &= \beta_3 E[\max \{B_3(0, 1, 0, \theta), B_3(0, 1, 1, \theta) - C_3\}] \\ &\quad - \beta_3 \max \{E[B_3(0, 1, 0, \theta)], E[B_3(0, 1, 1, \theta) - C_3]\}. \end{aligned} \tag{22}$$

Once again, the resulting expression is in the form $E[\max \{\cdot\}] - \max \{E[\cdot]\}$, and application of Jensen's Inequality yields

$$OV_3 \geq 0. \tag{23}$$

This looks like the result in the two-period case, but there is a difference. In that case, the E[max {•}] - max {E[•]} expression is weighted by the discount factor, β_2. Now it is weighted by β_3, where $\beta_3 < \beta_2$; thus, for given (undiscounted) terminal-period benefits, option value is diminished as the terminal period is pushed back. This makes sense. The longer it is possible to pollute without triggering an irreversible change in the system, the less is the importance of the option-value correction to a benefit calculation.

FOOTNOTES

[1]Our discussion of this case and subsequent discussion of another, that of Lake Tahoe, is based on material in Goldman and Horne [1983].

[2]A well-known example is the mayfly in the western basin of Lake Erie (________).

[3]The continuous case yields analogous results if the benefit functions discussed below are concave (see Freixas and Laffont [1984] and Jones and Ostroy [1984]).

[4]The concept was called "quasi-option value" by Arrow and Fisher [1974] to distinguish it from another earlier concept of option value. In a separate report we provide a comparative analysis of the different concepts of option value that have been proposed in the economic and financial literature (see Hanemann and Fisher [1985]).

[5]This has been suggested by Conrad [1980].

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Arrow and Fisher (1974)

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Freixas and Laffont (1984)

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Mayfly example (footnote No. 2)

FIGURE LEGENDS

Figure 1.

Figure 2.

Figure 3.