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### Authors

Tu, Ming-Yen  
Tsai, Frank T-C.  
Yeh, William W-G.

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**Modeling and Optimization of Water Quality in a Large-Scale Regional  
Water Supply System**

**By**  
**Ming-Yen Tu, Frank T-C. Tsai, and William W-G. Yeh**  
**Department of Civil and Environmental Engineering**  
**University of California, Los Angeles**  
**Los Angeles, CA 90095**

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## **ABSTRACT**

This paper develops a multicommodity flow model to optimize water distribution and water quality in a regional water supply system. Waters from different sources with different qualities are considered as distinct commodities, which concurrently share a single water distribution system. Volumetric water blend is used to represent water quality in the model. The model can accommodate two-way flow pipes, represented by undirected arcs, and the perfect mixing condition. Additionally, blending requirements are specified at certain control nodes within the system to ensure that downstream users receive the desired water quality. The optimization model is highly nonlinear and solved by a hybrid genetic algorithm (GA). We first use GA to globally search for the directions of all undirected arcs. We then use a generalized reduced gradient (GRG) algorithm, which is embedded in GA, to optimize the objective function for fitness evaluation. The proposed methodology was first tested and verified on a simplified hypothetical system and then applied to the regional water distribution system of the Metropolitan Water District of Southern California (MWD). The results obtained indicate that the optimization model can efficiently allocate waters from different sources with different qualities to satisfy the blending requirements, perfect mixing and two-way flow conditions.

## TABLE OF CONTENTS

INTRODUCTION	1
MULTICOMMODITY FLOW MODEL AND TWO-WAY FLOW CONDITION	2
SOLUTION METHODOLOGY	5
CASE STUDY	7
CONCLUSION	12
ACKNOWLEDGMENTS	13
REFERENCES	14

## LIST OF TABLES

Table 1.	Data Used in the Simplified System	16
Table 2.	Water Supply Sources (Units)	17
Table 3.	Optimal Water Supply (Units) to Node D3 with Blending Requirements in the Simplified System	18

## LIST OF FIGURES

Figure 1.	Two-way Flow Pipe	19
Figure 2.	A Chromosome Representing the Two-way Flow Solution	20
Figure 3.	Classification of Hybrid Global-local Methods: (A) Sequential Global-local and (B) Embedded Global-local	21
Figure 4.	Flow Diagram of the Proposed Hybrid GA	22
Figure 5.	Simplified Water Distribution System	23
Figure 6.	Minimized Objective Function Values (Simplified System)	24
Figure 7.	Convergence Performance of Hybrid GA (Simplified System)	25
Figure 8.	MWD's Regional Water Supply System	26
Figure 9.	Network Representation of MWD System	27
Figure 10.	Sensitivity of Objective Function Value to Blending Requirement	28
Figure 11.	Sensitivity of Total Shortage to Blending Requirement	29
Figure 12.	Sensitivity of Objective Function Value to Blending Requirement at Control Point 1	30
Figure 13.	Sensitivity of Objective Function Value to Blending Requirement at Control Point 2	31
Figure 14.	Sensitivity of Objective Function Value to Blending Requirement at Control Point 3	32
Figure 15.	Sensitivity of Total Shortage to Blending Requirement at Control Point 1	33
Figure 16.	Sensitivity of Total Shortage to Blending Requirement at Control Point 2	34
Figure 17.	Sensitivity of Total Shortage to Blending Requirement at Control Point 3	35

## INTRODUCTION

A regional water distribution system is often represented as a network, consisting of nodes and arcs. In general, inflow sources, water demands, reservoirs and hydraulic diversion structures are represented by nodes. Rivers, canals and pipes are represented by arcs. When flow directions in the arcs are known beforehand, a directed network can be used to characterize such a system. Diba et al. (1995) used a directed graph algorithm as a preprocessor to produce a network representation of a large-scale water distribution system and created an input file for linear programming. Recognizing that the linear program for water distribution optimization is an embedded generalized network problem, Sun et al. (1995) introduced a fast network algorithm, EMNET, which solves linear programming problems with network substructures to improve the computational efficiency. They applied EMNET to the regional water distribution system associated with the Metropolitan Water District of Southern California (MWD) and the results show that EMNET is several orders of magnitude faster than standard linear programming. Based on the concept of source-demand connectivity, Yang et al. (1996a) developed a method for reliability analysis and applied it to the MWD water distribution system. To accommodate stochastic component failures in the MWD water distribution system, Yang et al. (1996b), used stochastic simulation to evaluate the impacts of such failures on meeting demand at a certain quantity level. In Yang et al. (1996a,b), the MWD water distribution system was represented by a directed network. However, if the network involves undirected arcs, e.g., two-way flow pipes, the system configuration becomes partially undirected. As a result, flow directions must be treated as unknown variables themselves and optimized along with all other decision variables in the optimization model. An undirected network is generally not amiable to standard optimization.

Two-way flows are common occurrence in the field of transportation (see, for example, Brown 1981; Griffith et al. 1984; Carruthers and Hamilton 1994). For example, two opposite directional flows, which represent the pedestrians or vehicles, can simultaneously exist on one crosswalk or road at a given time. However, in a water distribution system, the definition of two-way flow associated with a two-way flow pipe is different from transportation in that, at a given time, an undirected arc allows water to flow only in one direction. The situation in which water flows in both directions at the same time is not allowed. Therefore, in the optimization model, one must take into consideration the flow directions in two-way arcs as decision variables.

Yang et al. (2000) presented a methodology for optimizing a regional water distribution system with blending requirements. In their study, a nonlinear multicommodity flow model was proposed to accommodate the blending requirements and perfect mixing conditions. Blending requirements are specified at certain control nodes within the system to ensure that downstream users receive the desired water quality. The perfect mixing condition assumes that all incoming waters of different qualities are mixed at the merging junction and that all outgoing waters from the junction have the same blend. In their model, water

quality was specified in terms of the volumetric ratio of waters from different sources. The monthly optimization model was steady state and hence the perfect mixing assumption was justified. The numerical results showed that their proposed methodology could efficiently control the water blend in a complex regional water distribution system. However, two-way flow condition was not considered in their study.

In the literature, a few studies on multiple-quality water supply involving two-way flows have been reported. Ostfeld and Shamir (1993a) reported a steady state, nonlinear optimization model, in which, undirected pipes, if necessary, were incorporated in the optimization model by using the absolute value of the flow. The solute concentration of water in such an undirected flow pipe depended on the final direction of flow and was modeled by a smoothing approximation equation. The steady-state optimization model was extended by Ostfeld and Shamir (1993b) to incorporate the unsteady state solute transport in the distribution system. Ostfeld and Shamir (1996) presented a methodology for the optimal design and reliability of a multiquality water-supply system. This methodology, which was an iterative procedure, incorporated a decomposition approach to solve the resulting nonconvex, nonsmooth problem. The flow direction of an undirected pipe was allowed to change in each iteration. In the above-mentioned studies, the directions of undirected arcs were determined along with all other decision variables. As a result, iteration and approximation are generally required for convergence.

In the present study, a hybrid global-local method is proposed to model an undirected network system. First, the multicommodity flow model developed by Yang et al. (2000) is modified to include two-way flow arcs. Second, a hybrid genetic algorithm (GA) is developed to optimize the model. Specifically, we implement a GA to globally search for the optimal directions of all undirected flows using evolutionary operators (crossover and mutation). With the directions of flows in all undirected arcs determined by the GA, we then optimize the resulting multicommodity flow model by a nonlinear programming technique, namely the generalized reduced gradient (GRG) method, and the solution is fed back to GA for fitness evaluation. Then GA updates the flow directions and returns to GRG. Convergence is achieved by iterations between GA and GRG. A key advantage of the proposed approach is that it converts an undirected network to a directed one, which can be readily preprocessed to produce a directional network representation for standard optimization and, at the same time, it separates the highly nonlinear two-way flow condition from the original nonlinear model.

## **MULTICOMMODITY FLOW MODEL AND TWO-WAY FLOW CONDITION**

### ***Multicommodity Flow Model***



Ahuja et al. (1993) proposed a minimum-cost, multicommodity flow model in which  $K$  commodities share common facilities. The following formulation is a revised version of their model to include multiple time periods ( $T$ ):

$$\text{Min.} \quad \sum_{t=1}^T \sum_{k=1}^K c_{ij,t}^k x_{ij,t}^k \quad \text{for } (i,j) \in \mathbf{A} \quad (1)$$

$$\text{s.t.} \quad \sum_{j:(i,j) \in \mathbf{A}} x_{ij,t}^k - \sum_{j:(j,i) \in \mathbf{A}} x_{ji,t}^k = b_{i,t}^k \quad \text{for } i \in \mathbf{N}, k = 1, 2, \dots, K, \text{ and } t = 1, 2, \dots, T \quad (2)$$

$$l_{ij} \leq \sum_{k=1}^K x_{ij,t}^k \leq u_{ij} \quad \text{for } (i,j) \in \mathbf{A}, \text{ and } t = 1, 2, \dots, T \quad (3)$$

$$0 \leq x_{ij,t}^k \leq u_{ij} \quad \text{for } (i,j) \in \mathbf{A}, k = 1, 2, \dots, K, \text{ and } t = 1, 2, \dots, T \quad (4)$$

where

$k$  : commodity index;

$t$  : time index;

$\mathbf{A}$  : arc set of the system;

$\mathbf{N}$  : node set of the system;

$x_{ij,t}^k$  : nonnegative flow of commodity  $k$  in arc  $(i,j)$  from node  $i$  to node  $j$ ;

$l_{ij}$  : lower bound of the total flow in arc  $(i,j)$ ;

$u_{ij}$  : upper bound of the total flow in arc  $(i,j)$ ;

$b_{i,t}^k$  : sink/source term of commodity  $k$  at node  $i$ ;

$c_{ij,t}^k$  : cost (weighting factor) for unit commodity  $k$  transported through arc  $(i,j)$ .

In the above model, the dominating constraints are the continuity equations, Eq. (2), which consider the mass balance of all commodities at each one of the individual nodes. Eq. (3) is the bundle constraint for multiple flows sharing an arc. Eq. (4) specifies the flow bound of each commodity in each arc. The assumptions of this model are: (1) no loss occurs during the flow transshipment; (2) all commodities are homogeneous, i.e. every unit flow of each commodity uses one unit of capacity of each arc; (3) no congestion occurs in commodity flows.

In a water distribution system, waters from different sources are considered as distinct commodities representing different water qualities. To apply the multicommodity flow model to a real-world water distribution system, Yang et al. (2000) suggested adding the following two additional constraints :

$$(1 - \beta) \sum_{i:(i,j) \in \mathbf{A}} x_{ij,t}^m - \beta \sum_{\substack{k=1 \\ k \neq m}}^K \sum_{i:(i,j) \in \mathbf{A}} x_{ij,t}^k \geq 0, \quad 0 \leq \beta \leq 1, \text{ and } t = 1, 2, \dots, T \quad (5)$$

$$\frac{x_{ji,t}^k}{\sum_{m=1}^K x_{ji,t}^m} = \frac{x_{jl,t}^k}{\sum_{m=1}^K x_{jl,t}^m} \quad \text{for } (j,i), (j,l) \in \mathbf{A}, k = 1, 2, \dots, K, \text{ and } t = 1, 2, \dots, T \quad (6)$$

Eq. (5) is the blending requirement constraint, which is linear. This constraint states that at least the fraction  $\beta$  of the total water transported to node  $j$  must be from source  $m$ . Eq. (6) represents the perfect mixing condition, which is nonlinear. This constraint assumes that all incoming waters of different qualities are perfectly mixed at the merging junction. Hence, all outgoing waters from that merging junction have the same blend. Another assumption is that no chemical reactions occur between different waters.

### ***Two-way Flow Condition***

In the present paper we further explore the two-way flow condition, which frequently occurs in a real-world water distributed system that has undirected pipes. Undirected pipes do not restrain the flow direction. In other words, they allow water to flow in either direction, but not in both directions at a given time. Undirected pipes provide high flexibility in water delivery and enhance the reliability in water supply.

Fig. 1(a) shows the configuration of an undirected arc connecting two nodes. To consider the two-way flow, the undirected arc is represented by two artificial arcs with opposite directions as shown in Fig. 1(b). During the system optimization, only one of these two artificial arcs with the corresponding flow amount is active at a given time. In other words, a two-way flow condition requires that these two artificial arcs be mutually exclusive. To ensure that only one of the two artificial arcs is active, we impose the following nonlinear constraint where the sum of flows on the inactive arc must be zero:

$$\left( \sum_{k=1}^K x_{ij,t}^k \right) \cdot \left( \sum_{k=1}^K x_{ji,t}^k \right) = 0 \quad \text{for } (i,j), (j,i) \in A, \text{ and } t = 1, 2, \dots, T \quad (7)$$

Note that Eq. (7) is theoretically satisfied if there is no flow in the undirected pipe. This situation is unusual, implying that the undirected pipe is not utilized to carry the water. In a real-world water distribution system, this situation almost never occurs. Eq. (7) is also applicable to the situation where two pipelines are physically installed between two nodes, but only one of the two pipes is allowed to deliver the water.

The multicommodity flow model, represented by Eqs. (1)-(4), is linear. Yang et al. (2000) modified the basic model by introducing the blending requirement and perfect mixing constraints and applied it to a real-world water distribution system. The present paper further considers the two-way flow condition. Accordingly, we consider both the linear and nonlinear constraints in the optimization model.

The perfect mixing and two-way flow constraints are nonlinear and mostly in the form of the multiplication. In addition, several decision variables may be simultaneously involved in both nonlinear constraints. This creates a high degree of nonlinearity in the model. For such a nonlinearly constrained problem, the

traditional gradient-based algorithms do not perform well and are often trapped in a local optimum. To increase the likelihood of reaching the global optimum, we develop a hybrid global-local method below.

## SOLUTION METHODOLOGY

We propose a hybrid global-local methodology that incorporates a genetic algorithm (GA) and a gradient-based nonlinear programming technique for solving the formulated nonlinear multicommodity flow model. GA is first used to globally search for the directions of all two-way flows, thus converting an undirected network to a directed network. For each generation in the GA, a gradient-based nonlinear programming solver evaluates the fitness by optimizing the objective function of the multicommodity flow model.

### *Genetic Algorithms*

A genetic algorithm (GA) searches for a global optimal solution using a set of analogous bio-evolution process and operators: selection, crossover, mutation, reproduction, and replacement. A population of starting chromosomes that are randomly generated to represent the problem's initial solutions goes through the process of GA. This procedure, generation by generation, improves the objective function value. GA approaches the global optimum by evolving chromosomes without the necessity of gradient evaluations. With such a characteristic, GA is capable of jumping out of local optima and achieving the global optimum. For a better understanding of GA and its application, readers are referred to Goldberg (1989) and Michalewicz (1994). In many applications, GA has shown to have the ability to find the global or near-global optimum for complicated problems. GA is especially promising for solving discontinuous, integer, and combinatorial optimization problems.

In our study, the GA operators are primarily used to search for the directions of the two-way arcs. In Fig. 2, a representative chromosome is designed to represent a two-way flow solution set that includes  $m$  two-way flow arcs in  $n$  time periods. Schematically, a chromosome includes  $n$  blocks, where each block represents a set of two-way flow solutions for a specific time period.

In each block (one time period), each undirected arc has two possible direction variables labeled as 0 or 1. If an arc is labeled as 0, the flow is in one direction; and if an arc is labeled as 1, the flow is in an opposite direction. Consequently, a binary sequence represents one possible directed network for each time period. This representation is consistent with the chromosomes that are coded into a binary sequence (Fig. 2) and optimized with GA basic operators. The total length of each chromosome is  $n \times m$  bits.

## *Nonlinear Programming Solver*

A commercially available optimization solver, LINGO (2001), is used as the basic nonlinear solver for the multicommodity flow model. LINGO is capable of solving linear, nonlinear, and integer programming problems. LINGO implements the generalized reduced gradient (GRG) algorithm, which is a gradient-based algorithm that is frequently used to solve large-scale nonlinear problems.

In the GRG algorithm, the original nonlinear problem is converted to the so-called “reduced problem”, with a “reduced objective function”. GRG solves the original nonlinear problem by iteratively solving a sequence of reduced problems. In each major iteration the search direction is evaluated using the gradient of the reduced objective function. A particular characteristic of GRG is that it attempts to satisfy all the constraints, i.e., to move within the feasible region, in all iterations. Details of the GRG method can be found in Lasdon and Waren (1978), and Drud (1994).

## *Hybrid GA*

In general, a global optimization algorithm can effectively reach a near-global solution, which then slowly converges to the global optimum. On the other hand, given an initial estimate, a local optimization algorithm generally converges quickly to a local optimum. To take advantage of each algorithm, global optimization has been combined with local optimization for solving nonlinear and non-convex problems. In literature, several efficient hybrid global-local optimization algorithms have already been developed for solving complex optimization problems.

How to combine a local optimizer with a global optimizer to construct a hybrid global-local algorithm depends on the mathematical characteristic of the particular optimization problem under consideration. Hybrid global-local methods can be generally classified into the following two types:

1. Sequential global-local method – This approach generally involves two stages. In the first stage, a global optimizer is implemented to provide a near-global solution. The near-global solution provides a good initial estimate for the local optimizer. In the second stage, with the known initial estimate from the global optimizer, the local optimizer searches for the global optimum in the neighborhood of the near-global solution. Fig. (3a) shows the concept. This type of hybrid algorithm can be found in different fields (see, for example, Heidari and Ranjithan, 1998; Crain et al., 2000; Potty et al., 2000; Duffy and McNelis, 2001; Lee et al., 2002; Tsai et al., 2002a,b).
2. Embedded global-local method - The basic idea is to embed a local optimizer in a global algorithm. It is an iteration procedure between the search operators of the global optimizer and the local optimizer. Fig 3(b) shows this concept.

Detailed discussions of this type of hybrid algorithm can be found in different fields (see, for example, Musil et al., 1999; Urdaneta et al., 1999; Sabatini, 2000; Cai et al., 2001; Dosso et al., 2001; Park, 2001; and Tsai et al., 2002a).

In the present study, we use the embedding approach. We first use GA to globally search for the optimal flow directions of all two-way flow arcs. With the directions of the two-way flows determined by GA, LINGO optimizes the objective function of the multicommodity flow model for fitness evaluation. The iteration search procedure is stopped when GA meets the stopping criteria, e.g., meeting the maximum number of generation runs. Fig. 4 shows the flowchart of the proposed hybrid GA.

The specific steps of the proposed hybrid GA are:

- Step 1: Randomly generate an initial population with  $N$  two-way flow solutions, i.e. a sequence of binary solutions. Set  $i = 1$ .
- Step 2: Solve Eqs. (1)~(6) with the  $i^{\text{th}}$  two-way flow solution by LINGO and obtain the objective function value. Let  $i = i + 1$ .
- Step 3: If  $i = N$ , then go to step 4 with  $N$  objective function values. If  $i \neq N$ , go to step 2.
- Step 4: Use GA operators to evaluate and update the population of  $N$  two-way flow solutions.
- Step 5: Check the stopping criterion. If met, then the best solution obtained so far is the optimal solution. If not, let  $i = 1$  and go to step 2 with the updated population of  $N$  two-way flow solutions.

In this study, a GA solver developed by Carroll (1996) is modified and linked with LINGO to iteratively solve the proposed nonlinear optimization model.

## CASE STUDY

We first test and verify the proposed methodology on a simplified, but realistic, water distribution system. The proposed hybrid GA methodology is implemented to solve this nonlinear model for the simplified system. In addition, a sensitivity analysis of the objective function value with respect to the blending requirement is performed to examine the impact of the imposed requirement. Then the methodology is applied to a real-world, large-scale, regional water distribution system of the Metropolitan Water District of Southern California.

### *Simplified System*

Fig. 5 shows the simplified water distribution system in which there are two water supply sources (S1 and S2), four demand nodes (D1 ~ D4), eight junction nodes (J1 ~ J8), three surface reservoirs (R1 ~ R3), two ground-water reservoirs (G1 and G2) and 25 arcs (P1 ~ P25). The two water supply sources directly deliver water to the surface reservoirs, R1 and R2, respectively.

According to the system configuration, demand nodes D1 and D2 can receive waters from R1, R2, and G1, but not from R3 and G2. However, demand nodes D3 and D4 can receive waters from all surface and ground-water reservoirs. In addition, four arcs (P5, P10, P12 and P19) are undirected.

To link all the time periods in the planning horizon, the storage of a reservoir is represented by a reservoir carryover arc, and this arc connects two adjacent time periods to construct a large-scale water distribution network structure. A ground-water basin is treated as a reservoir node, which also has storage carryover effect.

The physical capacities of reservoirs and arcs, and water demands are listed in Table 1. A blending requirement is specified at node D3 to require that at least a fraction  $\beta$  of the total water supplied to node D3 be from source 2. Table 2 lists the two supply sources in a planning horizon of five time periods.

In the network structure, each demand node has a corresponding arc that terminates at the demand node. The flow in the corresponding arc is the water supply to the demand node. Hence, the water supply variable, which is a decision variable in the optimization model, represents the amount of water that flows into a demand node.

The following two objectives are considered for this simplified system: (1) minimization of the total shortage in water supply; and (2) minimization of the deviations of reservoir storage from the preset target, which is set at the maximum storage. The multiple objective function can be formulated by the weighting method and written as follows:

$$\text{Min. } Z = w_1 f_1 + w_2 f_2 \quad (8)$$

where

$$f_1 = \sum_{t=1}^T \sum_{j \in N_D} \left( d_{j,t} - \sum_{k=1}^K x_{ij,t}^k \right) \quad (9)$$

$$f_2 = \sum_{t=1}^T \sum_{(i,j) \in A_R} \left( T_{ij,t} - \sum_{k=1}^K x_{ij,t}^k \right)^2 \quad (10)$$

$d_{j,t}$ : planned demand at node  $j$ ;

$N_D$ : set of demand nodes;

$A_R$ : set of reservoir arcs;

$T_{ij,t}$ : target of reservoir arc  $(i,j)$ ;

$w_1, w_2$ : weighting coefficient.

The shortage at a demand node is the difference between the planned demand and the actual water supply to that demand node, as represented by Eq. (9). Eq. (10) represents the squared deviations of reservoir storage from the target storage. For the simplified system, it is important to note that we set the reservoir target storage at its maximum. Hence, the difference between target storage and the actual reservoir storage is always nonnegative. However, in practice, it is possible that the reservoir preset target storage is any value between the minimum storage and maximum storage, and the actual reservoir storage may be greater than the target storage. The proposed quadratic objective function penalizes storage deviations from the target storage in either direction; thus, the optimization model encourages the solution to be close to the target storage.

In the optimization model, the first objective function is linear and the second is quadratic. The decision variable is the flow of each water quality in each arc in the network structure. The weighting coefficients are respectively assumed to be  $w_1 = 1$  and  $w_2 = 0.001$ . This implies that water supply has a higher priority than meeting the reservoir storage target.

For planning purposes, we have assumed that water supply is equal to or less than the minimum of the planned demand and the capacity of the supply arc, i.e.,

$$\sum_{k=1}^K x_{ij,t}^k \leq \min(d_{j,t}, CAP_{j,t}), \quad j \in N_D, \text{ and } t = 1, 2, \dots, T \quad (11)$$

where

$CAP_{j,t}$ : the capacity of the supply arc to demand node  $j$ .

Eq. (11) has the following two implications:

1. If the capacity of the supply arc is greater than the planned demand, the arc capacity will not restrict water supply. On the other hand, if the capacity of the supply arc is less than the planned demand, the capacity restriction causes water shortage to occur even though the system as a whole could potentially supply more water to satisfy the planned demand.
2. The assumption that water supply is smaller than or equal to the planned demand has the effect of equalizing the shortage distribution throughout the entire planning horizon. For planning purposes, it is customary to assume that the total planned demand for the entire planning horizon is greater than the total available water and, hence, a major objective is to minimize the total shortage in water supply over the entire planning horizon.

For the simplified system, the optimization problem, Eq. (8) subject to Eqs. (2)~(7) and (11), has 300 decision variables and 430 constraints, including 330 linear constraints and 100 nonlinear constraints. However, at each iteration of the hybrid GA, when the flow directions in all undirected pipes are pre-determined by the GA operators, the number of nonlinear constraints is slightly reduced.

Table 3 shows the optimization results. The  $\beta$  value was systematically increased from 0.0 to 0.5 with an increment of 0.1. Further increase in  $\beta$  beyond 0.5 resulted in infeasibility; that is, for the given set of conditions, the system is unable to supply D3 with more than 50% of the water from source 2.

As can be seen from Table 3, all blending requirements are satisfied up to  $\beta=0.5$ . The minimized objective function values for each of the specified  $\beta$  values are plotted in Fig. 6. We note that there is no significant increase in the objective function value when  $\beta$  is increased, indicating that the two-way pipes and system redundancy have provided the system with the needed flexibility in distributing the supply sources to the demand nodes and that the optimization model is working properly for the simplified system.

Fig. 7 shows the rate of convergence of the hybrid GA for three blending requirements:  $\beta = 0.0, 0.2, \text{ and } 0.5$ . Examining this figure, it is evident that there is no mathematical relationship between the rate of convergence and the  $\beta$  value. The model performance depends on the system characteristics and the input data

set. In addition, the results show that numerous iterations are required for the hybrid GA to converge.

### *Metropolitan Water District of Southern California System*

The Metropolitan Water District of Southern California (MWD) serves a population of approximately 17 million. The total service area is about 13,462 km<sup>2</sup> (5,200 mi<sup>2</sup>) and extends into Ventura, Los Angeles, Orange, Riverside, San Bernardino, and San Diego counties (Fig. 8). The total length of pipeline is about 1,246 km (775 mi). The Colorado River Aqueduct (CRA) and the State Water Project (SWP) are the two major sources of imported waters that have different water qualities. When these waters merge at a junction node, they are completely mixed, and the outgoing water has a new quality (commodity). Thus, MWD is considered to be a multiple-source, multiple-quality regional water distribution system.

MWD is responsible for delivering water to its member agencies at the designated demand locations. For a system of such size and complexity, it presents a challenge to the system operators. To control water quality in the distribution system, several control points are set up to ensure the water quality downstream of the control points. A control point represents either a treatment or infiltration plant in the system, which serves to mix and convey the multiple sources of water supply. In general, the quality of the SWP water is much better than that of the CRA water. For planning purposes, MWD prefers to use the volumetric ratio of SWP and CRA waters to represent the water quality in the system.

In the MWD network configuration (Fig. 9), there are two source nodes, nine surface reservoir nodes, 18 ground-water basins, 83 junction nodes, 38 demand nodes, and 190 arcs. Additionally, there are three blending control points and eight two-way flow arcs.

A monthly time period is used in the optimization model with a six-month planning horizon. All the input data were provided by MWD. We now use the hybrid GA to solve the multicommodity nonlinear flow model: Eq. (8), subject to constraints (2)~(6) and (11). In the MWD system, the target storages of certain reservoirs are not set at their maximum and may vary from one time period to another. The weighting factors ( $w_1 = 10$ ,  $w_2 = 0.001$ ) show that MWD puts less emphasis on the reservoir target storage and more on the shortage in water supply.

For the MWD system, if Eqs. (2)~(7) and (11) are considered simultaneously, there are 2,496 decision variables and 3,114 constraints (including 2598 linear constraints and 516 nonlinear constraints). However, when the directions of all undirected flows are first determined by the GA operators, the number of nonlinear constraints is reduced and changes in each iteration. Again, the decision variables are the flows and their corresponding water qualities in the arcs of the network.

Let  $\beta_1$  be the required percentage of the SWP water at control point 1,  $\beta_2$  at control point 2 and  $\beta_3$  at control point 3. For planning purposes, we wish to



minimize the composite objective function ( $w_1f_1 + w_2f_2$ ) as represented by Eq. (8). We are also interested in the total shortage in water supply. We therefore perform sensitivity analysis on blending to the composite objective function (referred to as the objective function in the rest of the paper) as well as the total shortage,  $f_1$ . To perform sensitivity analysis,  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$  are simultaneously increased from 0 to 1 with an increment of 0.2. The optimized objective function values with different values of  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$  are shown in Fig. 10. The corresponding total shortages in water supply are shown in Fig. 11. Generally speaking, the larger the  $\beta$  value, the higher the objective function value and more shortage in water supply will occur.

We further examine the sensitivity of the objective function and total shortage to a control point of interest while fixing the blending requirements at the other two control points. Fig. 12 shows the sensitivity of the objective function to  $\beta_1$  while  $\beta_2$  and  $\beta_3$  are fixed at 0.6. Similarly, Figs. 13 and 14 show the sensitivities of the objective function to  $\beta_2$  and  $\beta_3$  while fixing the  $\beta$  values at the other two control points at 0.6. Fig. 15 shows the sensitivity of total shortage to  $\beta_1$  while  $\beta_2$  and  $\beta_3$  are fixed at 0.6. Similarly, Figs. 16 and 17 show the sensitivities of total shortage to  $\beta_2$  and  $\beta_3$  while the  $\beta$  values at the other two control points are fixed at 0.6.

Examining the sensitivity of blending to the objective function value, we note that, in Fig. 12, when  $\beta_1 \geq 0.4$ , there is an appreciable increase in the objective function value. This implies that high blending requirement ( $\beta_1 \geq 0.4$ ) reduces the flexibility in system operation and results in a higher objective function value. In Figs. 13 and 14, if  $\beta_1$  is fixed at 0.6, there is no significant difference in the objective function value when  $\beta_2$  and  $\beta_3$  are varied from 0 to 1. Similar results are observed with regard to the sensitivity of blending to total shortage, Figs. 15-17.

In this planning horizon, CRA delivers more water to the MWD system than SWP. To satisfy the objectives, the results indicate that the model distributes more SWP water to control points 2 and 3 than that to control point 1. As expected, when  $\beta_1$  is increased to a higher value ( $\geq 0.4$ ), the flexibility in system operation is greatly reduced. Sensitivity analyses provide tradeoffs between water supply shortage and water quality requirement. However, we note that the sensitivity analysis results obtained are predicated on the specified values of demand and storage target, system configuration, input data, as well as the planning horizon.

In a monthly model for a large-scale multiple-source system, different waters completely mix and the mixed water can be considered as a new commodity. In the present study, we have assumed that no chemical reactions occur among different waters. Therefore, if necessary, in each new commodity on the arc, the actual amount of water from SWP or CRA can be traced and investigated in the optimization model.

Examining the system components (Fig. 9), it is evident that the operation of certain key nodes in the upstream region control the water conveyance and water quality in the downstream region. There are two types of key nodes in the system: (1) a storage facility node (reservoir), which stores water and controls the water release, and (2) a junction node, which mixes the incoming waters. The following types of mixing in the junction nodes are possible: (1) SWP water

mixes with CRA water and vice versa, (2) a mixed SWP and CRA water mixes with the SWP water, (3) a mixed SWP and CRA water mixes with the CRA water.

The developed model can be used to assist in decision-making. For example, the model results indicate that the operation of control points 1 and 3 influence the blending in control point 2. Furthermore, the junction nodes  $Ja$ ,  $Jb$ ,  $Jc$ , and reservoir nodes  $Ra$ ,  $Rb$ ,  $Rc$  have a direct bearing on the blending in control point 1. The junction nodes  $Jd$ ,  $Je$ ,  $Jf$ , and reservoir node  $Ra$ , and ground-water basin  $Ga$  control the blending in control point 3.

For such a large and complex system with a high degree of redundancy, we have found that the solution is non-unique; that is, different water allocation and two-way flow solutions yield the same objective function value. On the other hand, system redundancy provides a high degree of flexibility for the system operators to optimize the water distribution and meeting the water quality requirements.

## CONCLUSION

In a multiple-source, multiple-quality regional water distribution system, water agencies often find that it is necessary to impose blending requirements at certain control points in the system in order to secure the desired water quality downstream of the control points. A multicommodity flow model has been developed to optimize water distribution in a large-scale water distribution system with blending requirements, perfect mixing and two-way flow conditions. For planning purposes, we have used the volumetric water blend to represent the water quality.

A hybrid GA has been developed to solve the formulated optimization model. We first use GA with multiple starting points to globally search for the directions of all two-way flow arcs. With the directions of all two-way flows determined by the GA operators, we then use GRG to optimize the objective function of the multicommodity model for fitness evaluation and chromosome evolution. Convergence is achieved by iterations. This approach has the following three distinctive advantages: (1) It converts an undirected network to a directed network that is amiable to standard optimization, (2) It separates the highly nonlinear two-way flow constraints from the gradient-based algorithm, (3) GA with multiple starting points increases the likelihood of reaching a global optimum.

The proposed model was first tested and verified on a simplified, but realistic water distribution system. The model was then applied to the MWD system. Additionally, a series of sensitivity analyses was performed to analyze the impact of blending requirements. The results demonstrate the applicability of the proposed model to a real-world, large-scale regional water distribution system.

## **ACKNOWLEDGMENTS**

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Table 1. Data Used in the Simplified System

Data	Units
Maximum storage of each reservoir	50
Maximum storage of each ground-water basin	50
Demand at each demand node	50
Maximum capacity of each arc	300

Note: At the beginning, R2 stores 25 units of source 2, and all other reservoirs and ground-water basins have 25 units of source 1 in storage.

Table 2. Water Supply Sources (Units)

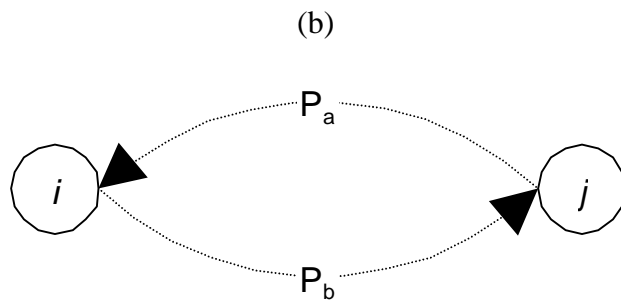
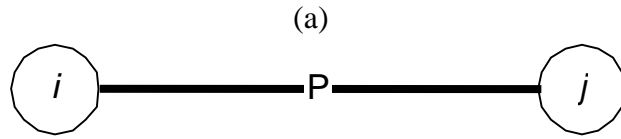
Source	Time period				
	1	2	3	4	5
S1	80	60	80	60	20
S2	80	60	80	60	20

Table 3. Optimal Water Supply (Units) to Node D3 with Blending Requirements in the Simplified System

$\beta$	<b>Time period</b>									
	1		2		3		4		5	
	Units	%	Units	%	Units	%	Units	%	Units	%
0.0	0.00	-	50.00	0.00	50.00	0.00	50.00	0.00	50.00	0.00
0.1	35.00	58.86	20.00	73.60	20.00	14.15	50.00	11.10	50.00	11.08
0.2	0.00	-	50.00	52.00	50.00	20.00	50.00	21.02	50.00	20.46
0.3	35.01	58.93	50.00	34.22	50.00	30.00	50.00	35.68	50.00	30.00
0.4	40.69	57.70	50.00	64.82	50.00	40.00	50.00	40.48	50.00	40.00
0.5	39.85	70.01	50.00	64.86	50.00	50.00	50.00	50.00	50.00	50.00



Figure 1. Two-way Flow Pipe



————— Two-way flow pipe

.....▶ Artificial pipe, representing one of two directions of the original two-way flow pipe. Only one of  $P_a$  and  $P_b$  is active.

Figure 2. A Chromosome Representing the Two-way Flow Solution

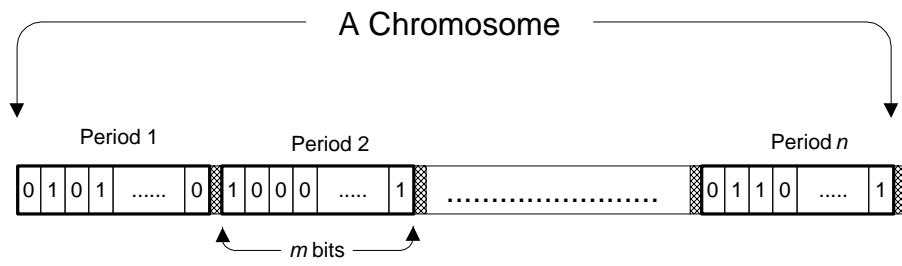


Figure 3. Classification of Hybrid Global-local Methods: (A) Sequential Global-local and (B) Embedded Global-local

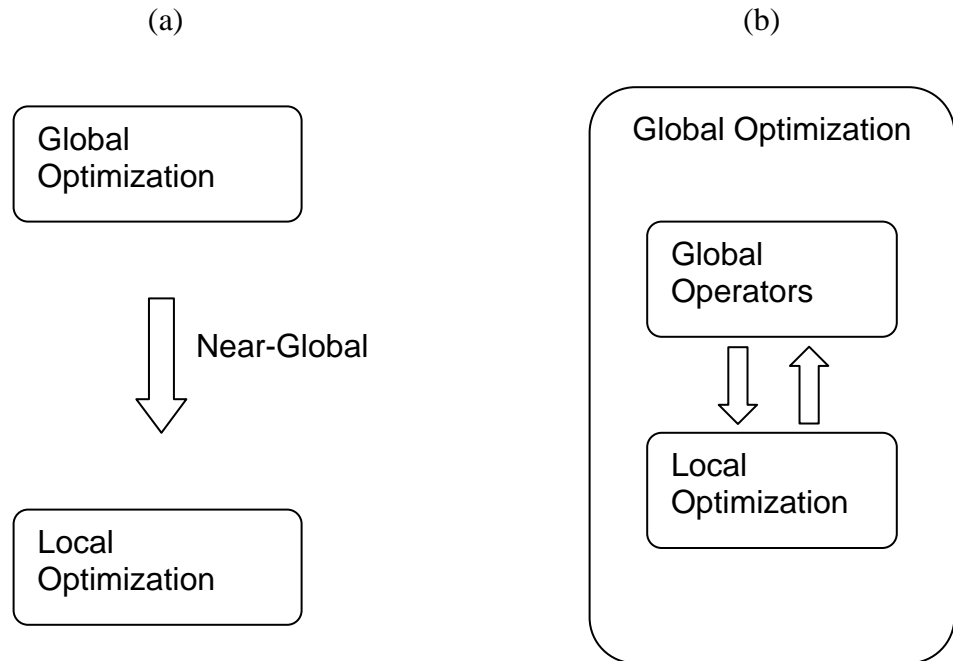


Figure 4. Flow Diagram of the Proposed Hybrid GA

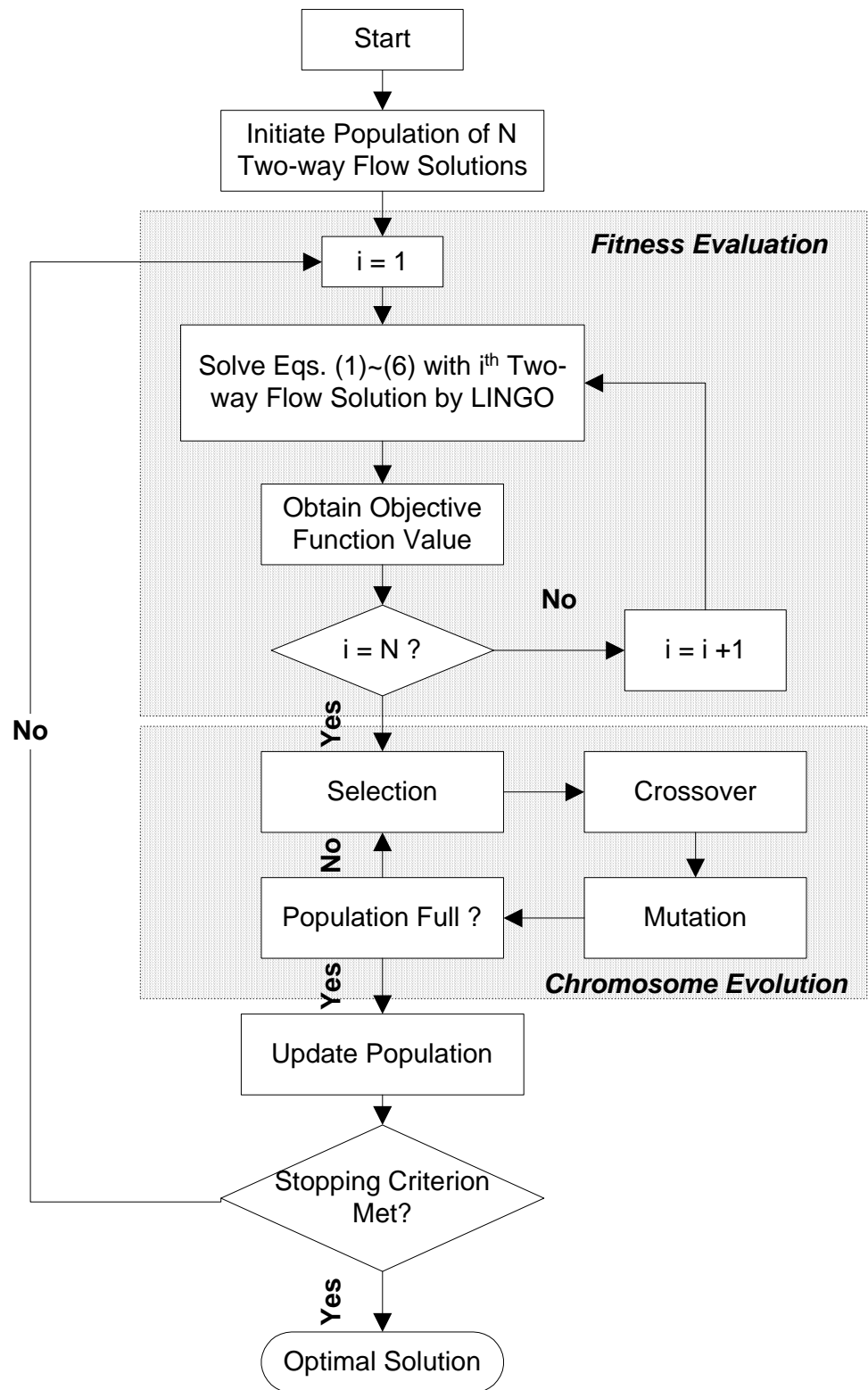


Figure 5. Simplified Water Distribution System

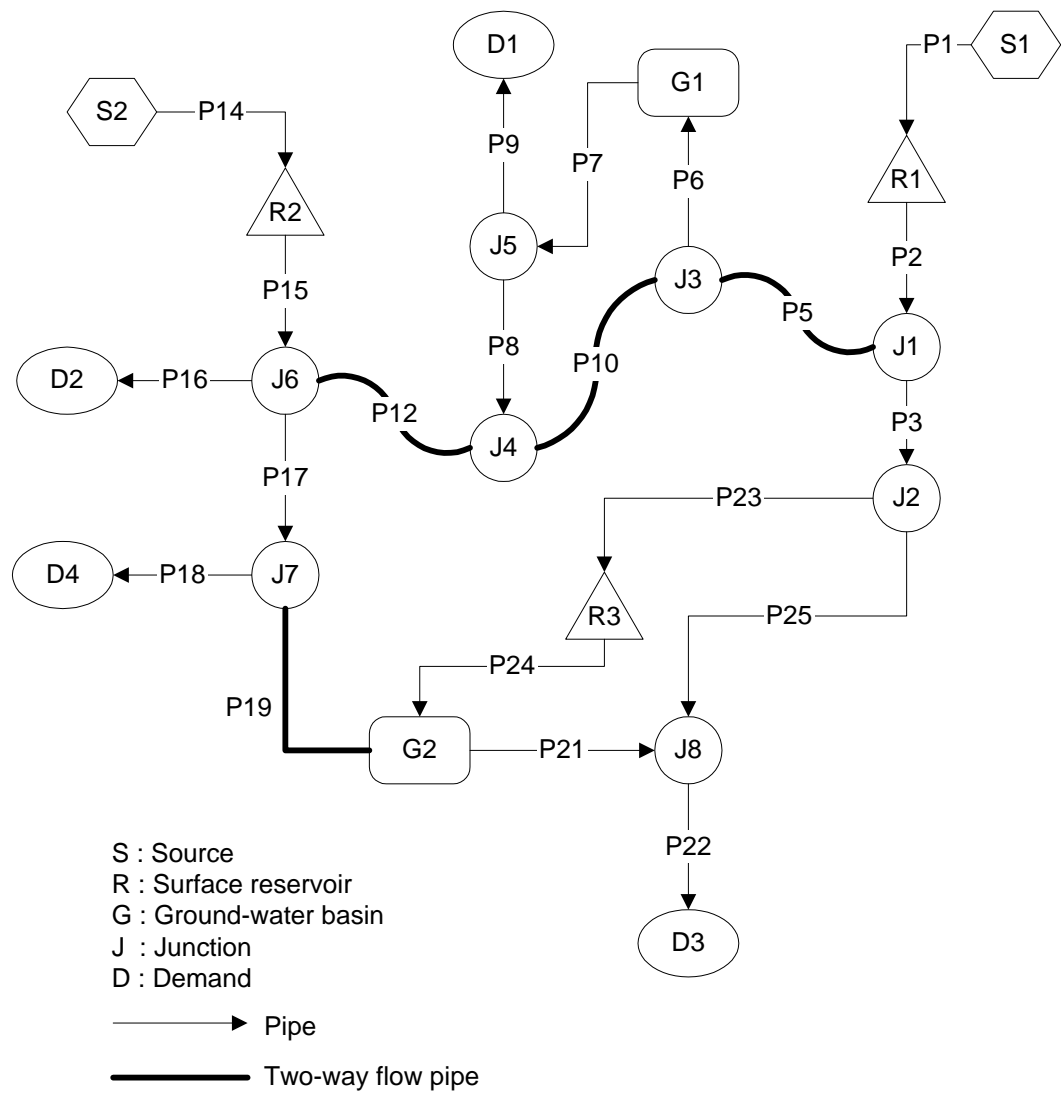


Figure 6. Minimized Objective Function Values (Simplified System)

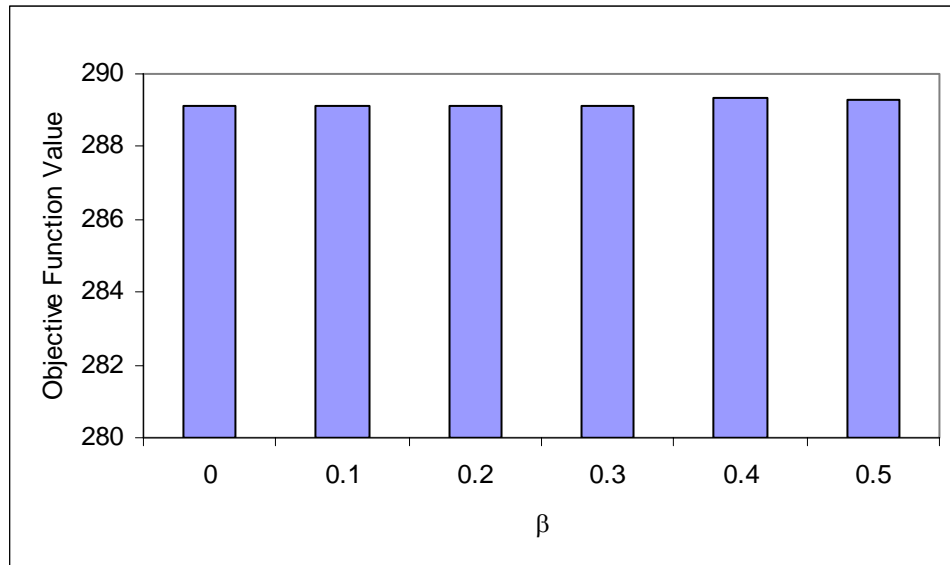


Figure 7. Convergence Performance of Hybrid GA (Simplified System)

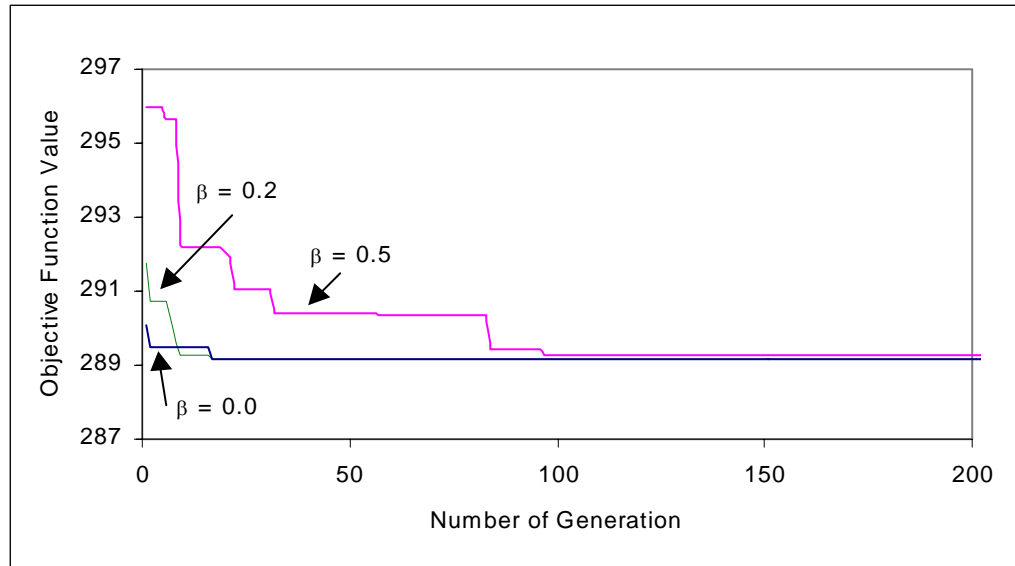


Figure 8. MWD's Regional Water Supply System

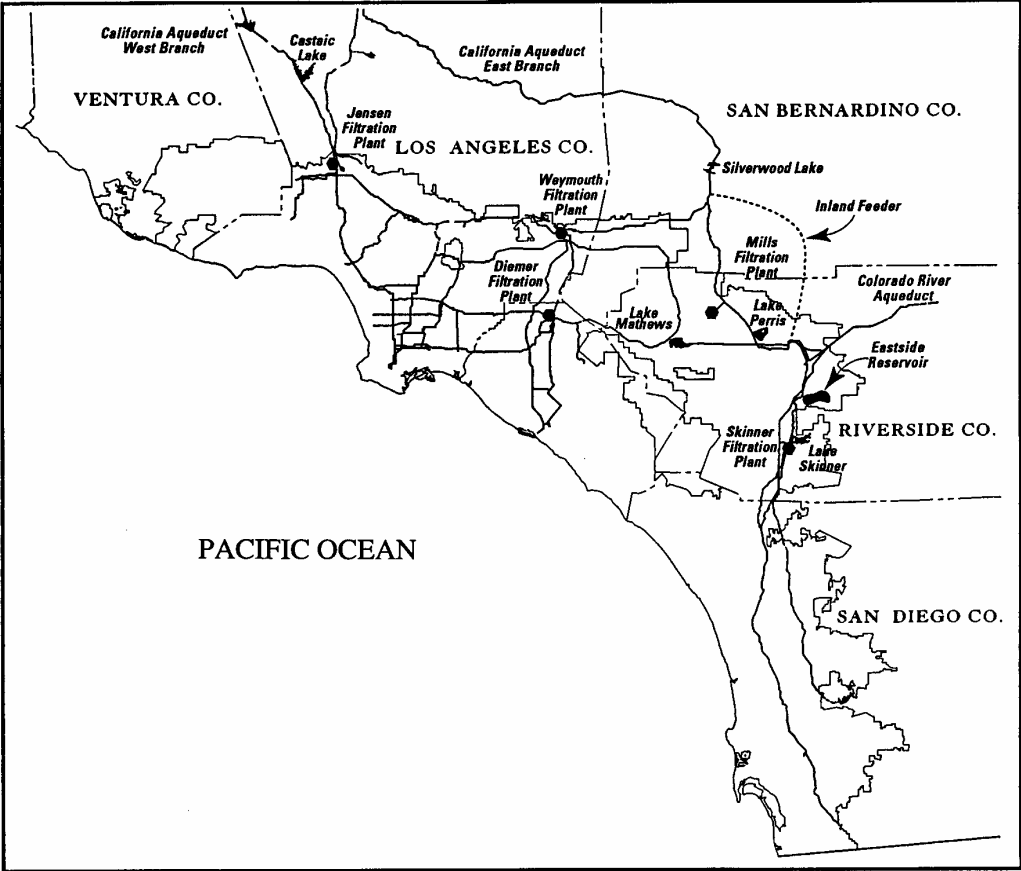




Figure 9. Network Representation of MWD System

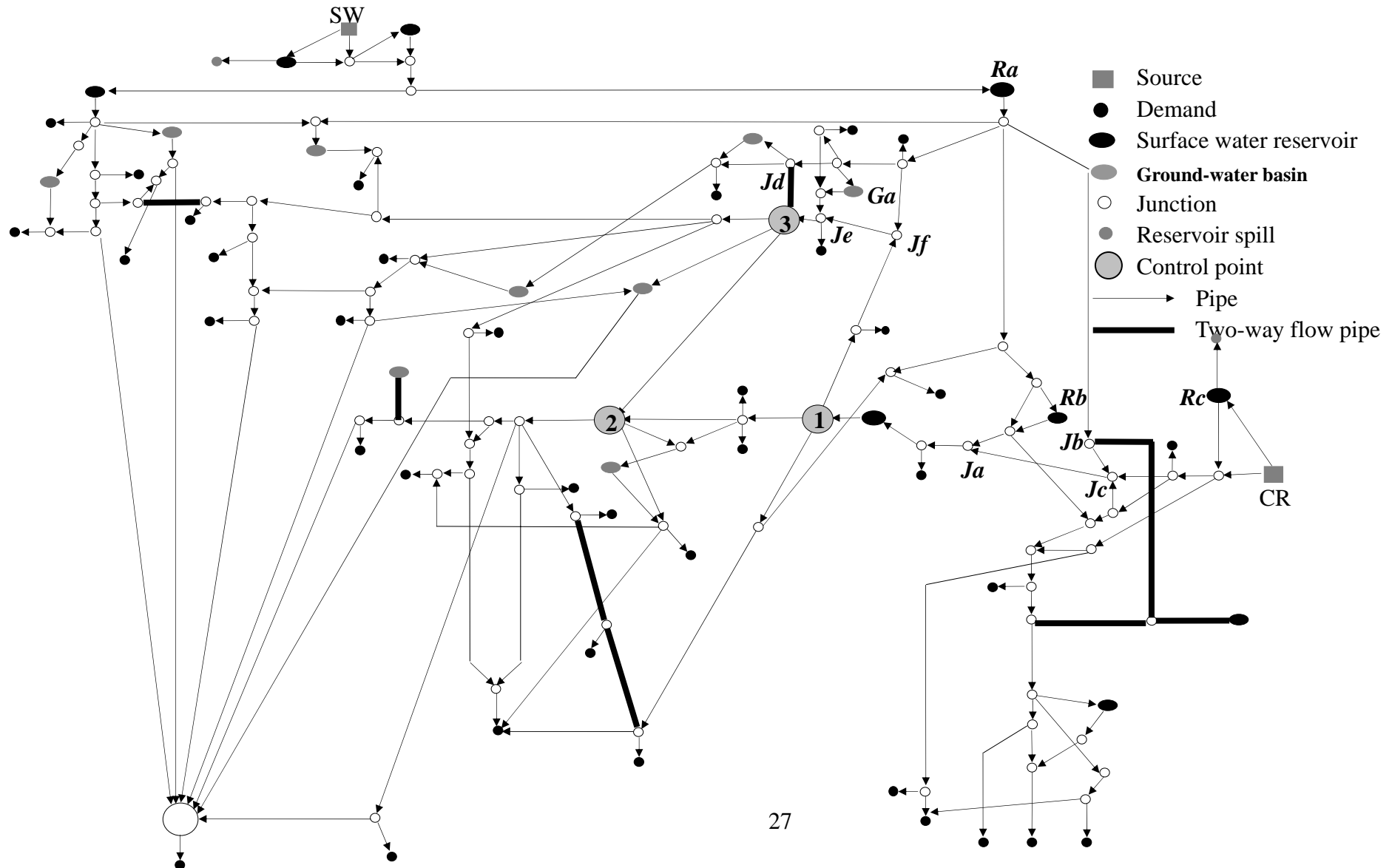


Figure 10. Sensitivity of Objective Function Value to Blending Requirement

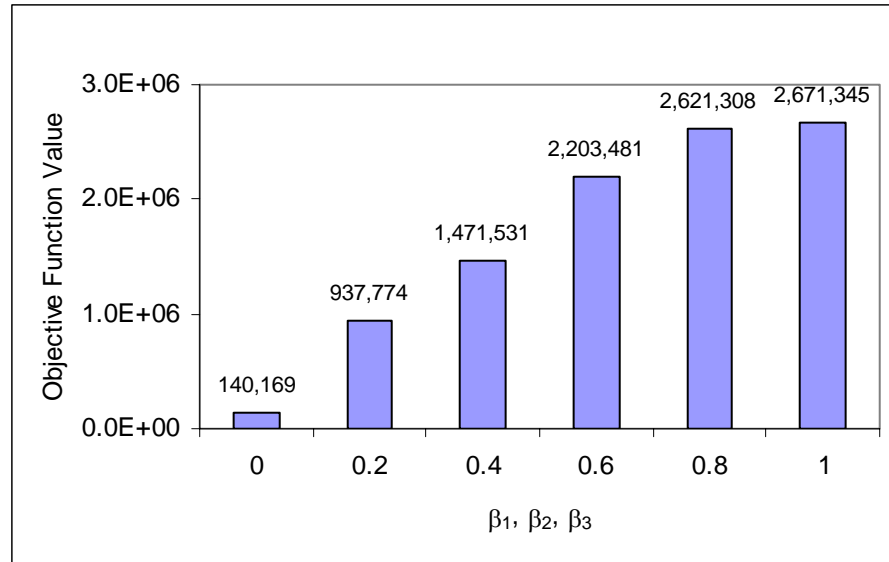


Figure 11. Sensitivity of Total Shortage to Blending Requirement

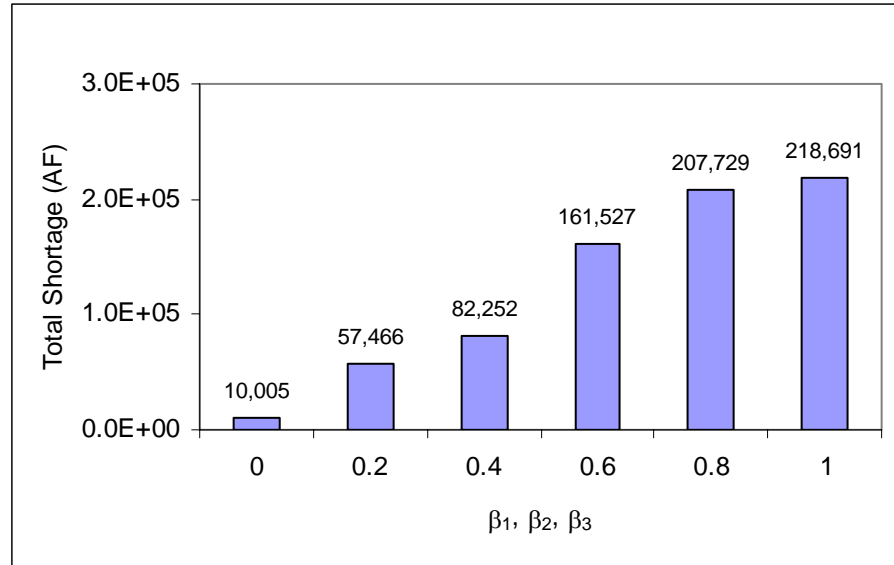


Figure 12. Sensitivity of Objective Function Value to Blending Requirement at Control Point 1

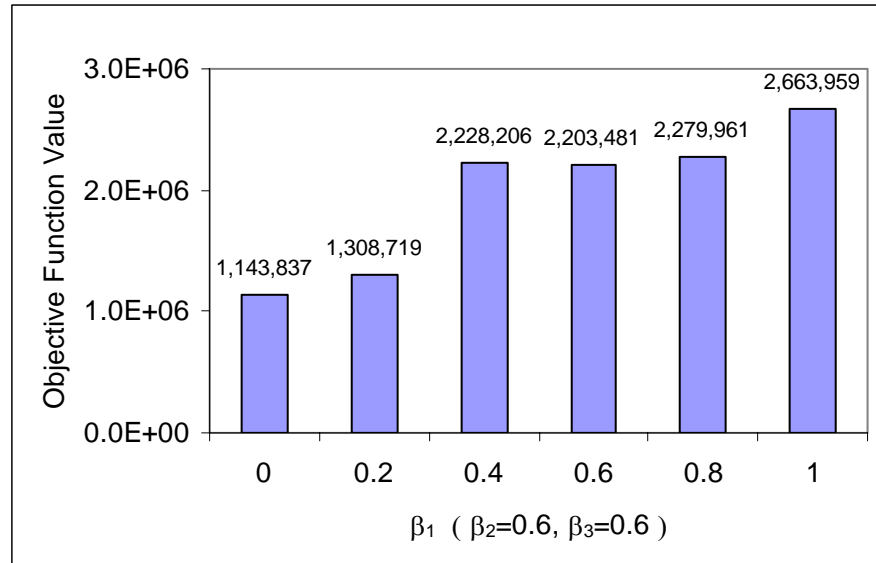


Figure 13. Sensitivity of Objective Function Value to Blending Requirement at Control Point 2

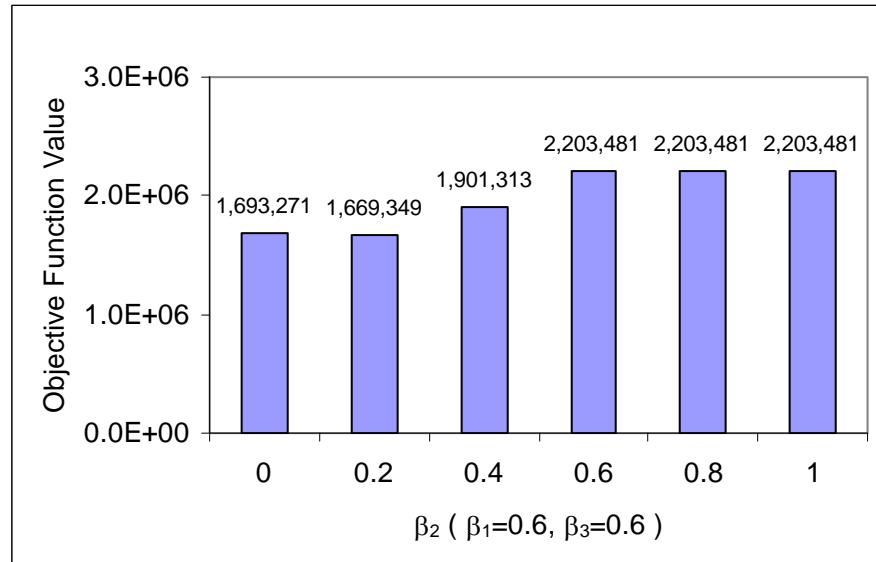


Figure 14. Sensitivity of Objective Function Value to Blending Requirement at Control Point 3

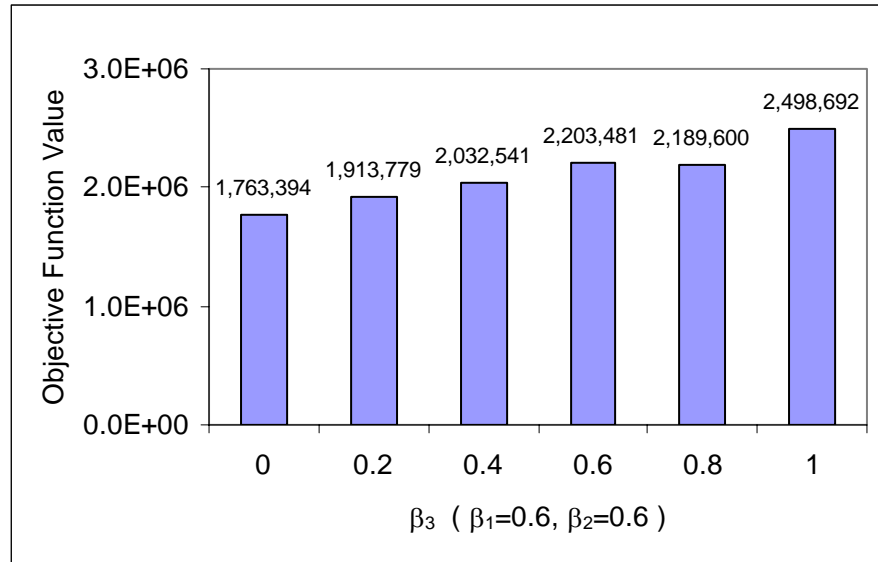


Figure 15. Sensitivity of Total Shortage to Blending Requirement at Control Point 1

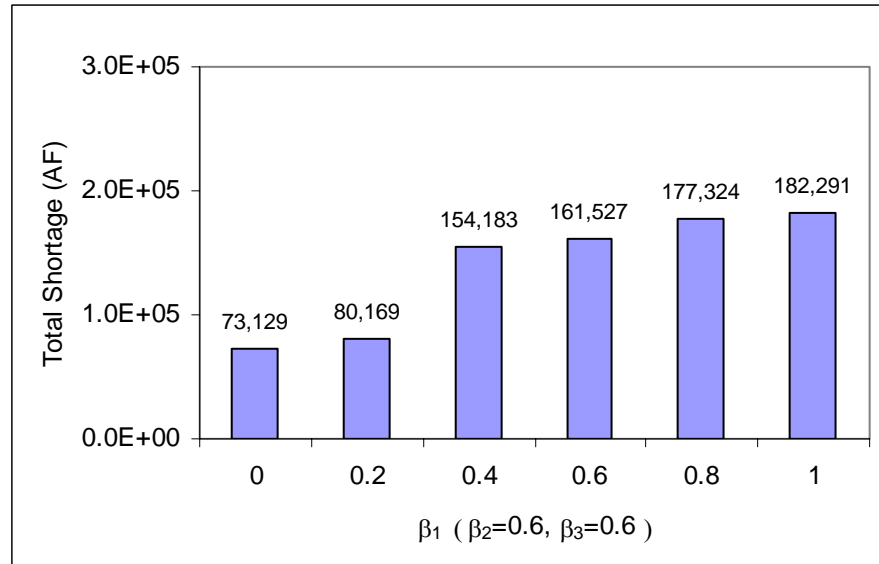


Figure 16. Sensitivity of Total Shortage to Blending Requirement at Control Point 2

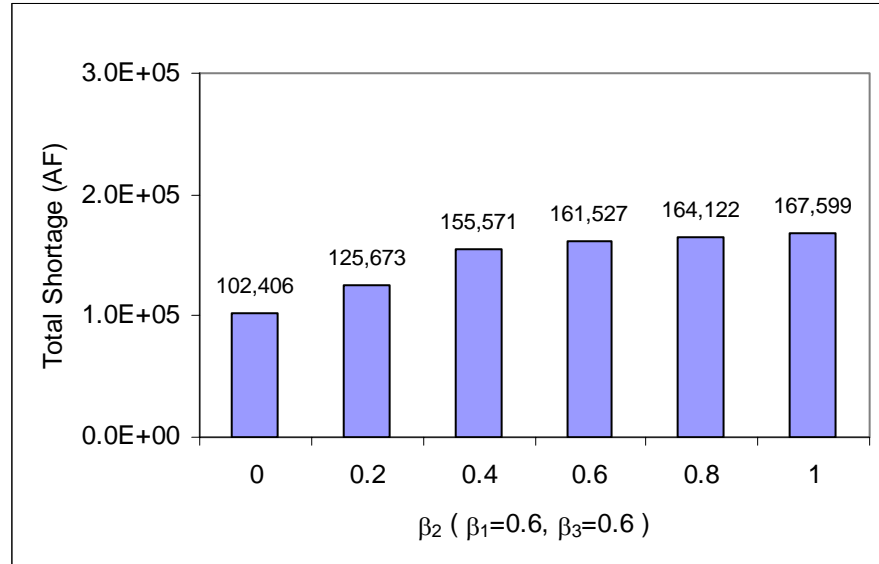




Figure 17. Sensitivity of Total Shortage to Blending Requirement at Control Point 3

