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by

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A STRONG TEST OF THE VON LIEBIG HYPOTHESIS

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An implication of the von Liebig hypothesis is that crop production functions have square isoquants. This paper presents a nonparametric test for square isoquants. The procedure is used to test experimental data on corn and wheat.

Key words: production function, von Liebig

A STRONG TEST OF THE VON LIEBIG HYPOTHESIS

In the 1840s von Liebig, an agricultural chemist, hypothesized his "law of the minimum" for describing crop response to inputs. An implication of this hypothesis is that crop production functions exhibit square isoquants. His hypothesis has received critical re-evaluation by Perrin; Paris; Berck and Helfand; Llewelyn and Featherstone; and Chambers and Lictenberg.[1] Perrin compared optimal fertilizer recommendations derived from von Liebig and standard production theory. Paris used nonnested hypothesis tests to test among five alternative production functions. His conclusion was that the von Liebig model with Mitscherlich regimes best interpreted the experimental data. Berck and Helfand showed how a micro-level von Liebig response could give rise to an aggregated smooth production function. Llewelyn and Featherstone used the CERES-maize simulator to produce data to evaluate production functions and find evidence for both a Mitscherlich-Baule formulation and a nonlinear von Liebig. Chambers and Lichtenberg took a nonparametric approach, constructing an outer and inner approximation to the data and then testing for the presence of yield plateaus and nonsubstitutability of inputs. This paper presents a nonparametric estimation of von Liebig response functions and a test of the von Liebig nonsubstitution hypothesis. The estimation does not require approximation and is free of assumptions about functional form.

The von Liebig hypothesis is testable against the composite alternate hypothesis of any other production function because any production function consistent with the von Liebig hypothesis has right-angle (also called square) isoquants. Agronomic experiments for nutrient response use incomplete block designs. These experimental designs include many observations that differ in only the amount of one nutrient applied. Given the position of the von Liebig expansion path relative to input-level combinations, the experiments allow direct observation of multiple points on the same isoquant. It is reasonable to assume that such points will differ only by experimental error.[2] Thus, the incomplete block designs of the agronomic experiments make it possible to directly test the von Liebig hypothesis.

There are four sections of this paper of which this is the first. In the next section, the data and the experimental design that led to that data are described. In the third section, the estimation and test methods are discussed. The fourth section includes the results of estimation and testing and is the conclusion.

A Nonparametric Test: The Theory

The von Liebig hypothesis, when applied to experimental data from an incomplete block design, can be tested with a set of restrictions on a regression of grain yield on dummy variables. These tests depend upon the details of the construction of incomplete block design experiments, which are described in the next subsection. They also depend upon the nonsubstitution or square isoquant property of the von Liebig described next. Putting together, in the fourth subsection, the nature of the experiments and the square isoquant hypothesis, it is possible to characterize all possible orderings of predicted yields that are consistent with theory. These orderings are finite, and the succeeding section shows how to place an upper bound on the number of them. Finally, the testing methodology is described.

Experimental Design

The agronomic experiments in question—and many others as well—were conducted using an incomplete block design. A crop was grown on a number of experimental plots using different combinations of inputs. In the designs considered here, there were two inputs—water and nitrogen—that were applied in up to five different levels for a total of 25 different possible combinations of treatments. More generally, there would be two inputs applied at n different levels for up to a total of n^2 different possible combinations. In input space those n^2 points would make a regular grid pattern—n columns of n points each or n rows of n points each. When all points in the grid represent input levels used in the experiment, the experimental design is said to be complete. If some input combinations are not included in the data, the design is called incomplete. The agronomic experiments that provide the data for the estimations described below utilize an incomplete block design with only 13 different treatments. That is, 13 combinations of two nutrients were applied to experimental plots and the yields from those plots were measured.

Figure 1 illustrates a grid for the case of an incomplete block design where the 13 solid squares represent the included data points. Quantities of the two inputs—water and nitrogen—are on the axes. A mean output level—yield of wheat for instance—also corresponds to each of these points but is not shown. The data used for this paper are from incomplete designs, so the case of complete designs will be treated only parenthetically.

Application to Wheat and Corn Production Experiments

The data for this paper come from the appendix of Hexem and Heady and reflect experiments in several Western states to measure crop response to water and nitrogen over a variety of soil and climate conditions. Among the crops they chose, we have limited ourselves to those that are determinate in their flowering—wheat and corn—since it is these crops that are believed to follow von Liebig's law of the minimum. An experiment recorded either one or two years of production data on a particular type of crop and place (e.g., corn in Mesa, Arizona, in 1971). Generally, 44 plots were part of each experiment allowing for some input combinations to be replicated twice and others four times.

Null hypothesis

The null hypothesis in this study is that yields are determined by input levels and a normal error term. Estimation under the null hypothesis is achieved by simply regressing the observed yields on 13 dummy variables—one for each of the 13 different input combinations. The error term from this regression is the experimental error and should be normally distributed. The von Liebig hypothesis will be shown in the next section to be a restriction on the values that the dummy variables may take.

The von Liebig Hypothesis

Von Liebig hypothesized that agricultural production would be determined by a limiting nutrient. This hypothesis, treated in historical detail by Paris, implies that the agricultural production function should have isoquants with square corners located on the expansion path. With inputs x_1 and x_2, plateau level P, and output Y, the formal representation of the generalized von Liebig production function (Paris and Knapp) is given by

$$(1) \qquad Y = \min(f_1(x_1), f_2(x_2), P) + u,$$

where u is an error term and f is an increasing continuous function. Consider an isoquant through an input combination point, $x^* = (x_1^*, x_2^*)$, where $Y = f_1(x_1^*) = f_2(x_2^*) \leq P$.[3] From equation (1), it is immediate that output is unchanged by adding x_1 or x_2 (but not both). Any point "to the right of" or "above" x^* has the same output as x^* and is on the same isoquant. Thus, a representative isoquant consists of a horizontal leg to the right of and a vertical leg above a right-angle corner at x^*. After adding sufficient quantities of both inputs, the plateau is eventually reached (which could be interpreted as a right-angle isoquant in a higher dimension). Once the plateau is reached, increasing either input leaves output unchanged.

The expansion path is the set of cost-minimizing input combinations. Since all isoquants have "corners," any isoquant will intersect an isocost line (with strictly positive prices) at the corner of the isoquant. The expansion path consists of the set of all such corners. By the assumption that production is nondecreasing in inputs, the expansion path must not slope downward. There is no reason, in general, for the expansion path to be linear though, of course, it is for the special case where both $f_1(\,)$ and $f_2(\,)$ are linear. In figure 1, the expansion path is the dark line beginning at the origin and going through points A and B and ending in an arrow at the top right of the graph. The lighter vertical and horizontal lines are isoquants or parts of isoquants. On the expansion path and nearest the origin are square isoquants with a corner at (1/3, 1) and then at (1, 2). A little farther up the expansion path is the horizontal leg of the isoquant intersecting the expansion path at (2, 3). Three further parts of isoquants are shown higher up the expansion path. The path shown is a stylized representation of the collection of all expansion paths that pass between the data points in the order depicted. The limited number of input-level combinations offers no information about the exact position of an expansion path between data points.

An upward-sloping expansion path and its associated isoquants contain all of the restrictions imposed by the von Liebig hypothesis. In terms of the data points, the predicted output level at a point is the same as at another point if they are on the same isoquant and higher if on a higher isoquant. Thus, drawing an expansion path is equivalent to ordering the predicted grain yields of the experimental treatments. For instance, in figure 1, the input combinations (3, 5) and (5, 5) have equal output and output greater than the input combination (4, 4). To each possible expansion path, there is an ordering of the predicted outputs. The least restrictive statistical model that preserves the ordering is a restricted regression of yield on dummy variables. There is one dummy for each level of output, and the regression is run with restrictions on the dummy variables that assure the ordering is preserved. In the example given above, there would be one dummy variable for the yield level achieved by points (3, 5) and (5, 5) and a second dummy variable restricted to have a lower value for the point (4, 4). When two expansion paths lead to the same ordering and, hence, to the same restrictions on the dummy variables, the paths cannot be distinguished by the existing data and are observationally equivalent. Below, it is shown that there is a small and finite number of orderings induced by the expansion paths.

Orderings and Expansion Paths

A finite set of stylized paths (the paths through the circles in figure 1), along with the implied rules for dummy assignment, generate the set of all possible restrictions on the dummy variables consistent with the von Liebig hypothesis. The steps are (1) defining the set of stylized paths, (2) showing the correspondence between stylized paths and the division of observed input combinations into those neither below nor above the expansion path, and (3) deriving the dummy-variable restrictions from the sets of input combinations neither below nor above the paths.

In our graphical representation, figure 1, a stylized expansion path is symbolically depicted as a series of five steps from the origin. Each step is either up or to the right or both and takes the path to a higher dashed diagonal line. The paths connect the circles that are between, above, or below data points represented by black squares. A stylized path divides the observed input combinations into those that are not above the expansion path (lying on a horizontal leg of an isoquant) and those that are not below the expansion path (lying on a vertical leg of an isoquant).

The set of stylized paths leads to all the possible divisions of observed input combinations consistent with the theory. To see this, consider an arbitrary expansion path that does not exactly coincide with any data points. A path through the open circles nearest the subject path on each diagonal that it crosses produces the same separation of the data points into points not above or below the expansion path as the subject path would. An expansion path through an observed input combination is treated as producing two classifications of the observed input combinations: The point on the path is treated alternatively as not above or below. The division of the points produced by treating the path as not above is the same as is produced by the stylized path through the open circle below the data point. Treating the point as not below produces the division that is the same as the path that goes through the circle above. Thus, the set of stylized paths gives rise to the complete set of divisions of observed input combinations into those not above or below some expansion path.

Now consider a specific division of points into those not above or below a stylized expansion path. Those above are on vertical arms of an isoquant and those below, on horizontal arms. Where two input combinations are on the same vertical (or horizontal arm), they are restricted to have the same value for their dummy variable. In the example in figure 1, such groups of input combinations are in the same rectangle. For instance, the points (1, 3) and (1, 5) have the same predicted output because they are on the same vertical leg. Where an input combination lies on an isoquant that intersects the stylized expansion path above a diagonal and another combination lies on an isoquant that intersects that expansion path below that diagonal, the dummy value for the upper input combination is restricted to be greater than or equal to the value for the lower path. In figure 1, the group labeled 1 has its dummy restricted to a value less than the groups labeled 2/3 and 3/2 for this reason. Groups of input combinations, such as those labeled 2/3 and 3/2, that have the same intersection with the stylized path can have either ordering of the dummy variables. A slight perturbation of the path would result in group 3 having a higher output than two, and a perturbation in the other direction would reverse the inequality. Groups 4 and 5 show the case where a perturbation of the path makes the dummy associated with group 5 greater than or equal to that of group 4. Since we cannot know a priori which is the case, both dummy level orderings are feasible for the classification shown.

Finally, we comment on the case where a path goes directly through a data point and there are points both above and to the right of that point. In that case, treating the path as below the point will rank the horizontal leg of the isoquant as having a greater or equal dummy variable to the vertical leg. Similarly, treating the path as above the point will restrict the vertical legs dummy level weakly above that of the horizontal leg. Thus, the use of weak inequality constraints on the dummy variables accounts for the possibility of equality of the two legs of an isoquant when a path goes directly through a data point.

The set of all these stylized paths (those that go through the open circles on the diagonals), together with the dummy assignment rules just enumerated, generates the set of all the dummy-variable restrictions which are consistent with the maintained hypothesis of a von Liebig production function.

Bounding the Number of Feasible Dummy Configurations

An upper bound on the number of dummy configurations that are observationally equivalent to some path with square isoquants can be obtained by the method of backwards induction, illustrated in figure 2. The induction relies on the principle that, from any circular node in the tree, the number of possible dummy-variable assignments from that point forward in the tree depends only upon classification of data points above and to the right of the circle in question.

The initial step in the induction is a move forward at Step 5 from one of the two circular nodes. In either case, a forward move has no impact on the assignment of dummy levels since there are no more data points to classify above and to the right of these circles. Hence, there is only a single (null) dummy-variable assignment forward from either of these nodes.

Now assume we are at an arbitrary circular node before Step 5 and, by inductive hypothesis, suppose that the upper bounds on the number of forward dummy assignments are valid at all successor nodes. Each circle has at most three successors, which may be vertically up, diagonally up to the right, or horizontally right from the current circle. So a bound on the number of dummy configurations forward from the current node is the sum of the bounds for successor nodes. This is subject to the proviso that the bound for a diagonal move that splits horizontal and vertical isoquants is adjusted by doubling the count of possible forward dummy configurations in that direction.

First, consider a move up from the current node. Any dummy assignments to input-level combinations below or to the left of the current node are unaffected by this move. All input-level combinations lying to the right of the segment between the current node and its vertical successor are grouped into horizontal legs of square isoquants. This grouping is unique for a vertical move. Further, the assignment of dummy-variable levels to the horizontal isoquant groupings is unambiguously increasing. Thus, the upper bound on the number of dummy configurations going forward from the current node to its vertical successor equals the bound on the number of possible dummy configurations forward from the vertical successor.

Second, consider a move directly to the right of the current node. Analogous to the vertical successor case, the effect of such a move is to group all input-level combinations that lie on vertical lines above the segment of the path between the current node and its horizontal successor into vertical legs of square isoquants. Again, the assignment of dummy levels to the isoquant groupings is unambiguously increasing along the portion of the path from the current node to the horizontal successor. So the upper bound on the number of dummy configurations for a move from the current node to its horizontal successor is the bound on the number of possible dummy configurations forward from the horizontal successor.

The final case is a diagonal move up to the right from the current node. If this move does not terminate on a circle diagonally between two input-level combinations in the incomplete block design, the same line of reasoning employed above applies: The upper bound on the number of forward dummy configurations is given by the bound at the diagonal successor. In this case, the data point below the terminal circle and all data points on a horizontal ray to its right are grouped into a horizontal leg of a square isoquant. Similarly, the data point above the terminal circle is grouped with any data points vertically above it into a vertical leg of a square isoquant.

If a diagonal move terminates at a circle between two input-level points, both possible orderings of the monotonicity constraint need to be considered. The grouping of points into horizontal and vertical isoquant legs is unique, but the assignment of dummy levels to these isoquant legs is not. Since either the horizontal or the vertical leg may be assigned the higher dummy level, there is a doubling of the bound on the number of possible forward dummy configurations in this case, achieved by doubling the forward bound on the diagonal successor node.

Adding upper bounds over horizontal, vertical, and diagonal successors, including the doubling for the diagonal case where warranted, completes the inductive step for the current node. For illustration, consider the lower node at Step 3, labeled 3/7. The label reflects that this node has 3 successor nodes, and there is an upper bound of 7 possible dummy assignments forward from this node which are observationally equivalent to some von Liebig expansion path. The three successors respectively have upper bounds of 1, 2 and 2 possible paths forward. In calculating bound on possible dummy configurations at the current node, we sum 1 + 2*2 + 2 = 7 possibilities. The multiplication by 2 reflects two possible orderings of dummy levels over the horizontal and vertical isoquants depending on the curvature of the path between the lower node labeled 3/7 and its diagonal successor labeled 2/2.

In contrast the lower node labeled 2/2 requires no such adjustment for splitting a path since a diagonal move to the lower node labeled 1/1 does not create any ambiguity in assigning dummy levels (i.e., the node at nitrogen and water level 5 is unambiguously assigned a higher dummy level).

Applying this counting principle across the entire incomplete block design establishes an upper bound of 172 on the number of feasible dummy configurations as verified by the counts given in Figure 2 (with the final count of 172 given in the lower left corner of the figure). Thus, any expansion path leads to the same inequality and equality constraints on the observations as does one of the 172 constructed dummy configurations.

As illustrated in figure 3, it is possible for several different paths to map to the same dummy-variable configurations. Paths one through four (labeled P1 through P4) are identical through the third step (circle) along the expansion path up to the right from the origin. Beyond this point, the four paths diverge (so, for instance, paths one and two branch diagonally up to the right in the fourth step while paths three and four move up vertically). It is clear that the implied assignments of dummy levels 6 through 8 (as shown in the figure) are unchanged over these four different paths. For this reason, 172 is an upper bound on the number of dummy configurations rather than an exact count.

Starting with the method to place an upper bound on the number of dummy variables, an algorithm was developed based on the backwards induction principle to produce an exhaustive list of the 172 dummy configurations, which are observationally equivalent to some von Liebig expansion path. Duplicate dummy configurations were then pared from the exhaustive list to obtain a unique set of 82.

Testing Methodology

The first step in our testing procedure was to perform unrestricted regressions using dummy-variable assignments to each of the 13 input levels in the incomplete block design for each of the 12 experimental data sets tested. Regardless of the form of the underlying production function, the residuals at this stage are only due to experimental error.

By the constructive argument presented previously, any expansion path for a von Liebig production function is observationally equivalent to one of the 82 dummy-variable configurations in our set. If the von Liebig hypothesis is valid, then one of these 82 dummy configurations must fit the data within experimental error. Subject to the assumption that observed residuals from the unrestricted regressions were distributed

$N(0, \sigma^2)$, we used a quadratic programming approach to perform restricted regressions for each of the 82 possible dummy-variable configurations consistent with the von Liebig model for each of the 12 experimental data sets.

Statistical tests were then performed comparing the unrestricted and restricted models under each of the 82 possible dummy configurations. The tests employed were the t-test for difference in levels of output along an isoquant and the likelihood ratio test. The t-test may be construed as a measure of squareness, as it directly reflects relative output levels implied by the von Liebig structure under any particular dummy configuration. The likelihood ratio test can be interpreted as a test of fit, measuring the loss of fit when von Liebig restrictions are added to the unrestricted regression of output on input levels.

The t-test for any particular dummy configuration compares output levels at adjacent input combinations along the leg of an isoquant. Differences are computed between the levels of output at pairs of input combinations that lie on the same isoquant. Under an assumption of normally distributed error term, if the (null) von Liebig hypothesis is valid, the expected mean difference in output for such pairs is zero. A standard t-test is employed to calculate this measure of squareness.

The likelihood ratio test compares the magnitude of the sum of squared residuals under the unrestricted and restricted cases. A large increase in residuals when restrictions are imposed is indicative that the von Liebig hypothesis is not valid for the dummy configuration in question. As suggested by Varian, rejection of the null hypothesis for a particular dummy configuration implies that the perturbation of observed levels of output required to satisfy the von Liebig alternative is not statistically plausible. In other words, the von Liebig hypothesis is too strong an assumption for the data to fit the posited dummy configuration. If some of the 82 results do not lead to rejection of the null hypothesis, there are candidate dummy configurations that are consistent with the von Liebig hypothesis, from the standpoint of fit.

In defense of employing two distinct testing approaches, note that the tests are correlated but not equivalent. For instance, it is possible to conceive of a case where the model fits well, resulting in a small increase in residual sum of squares for the restricted model, but a slight upward slope in output along the hypothesized von Liebig isoquants results in too many positive signs. Similarly, it is possible to have a large loss of fit when the von Liebig restrictions are imposed without having a lopsided sign distribution. Of course, if a particular dummy configuration results in large increases in residuals across the legs of isoquants under the restricted model, significant loss of fit and squareness will both be reflected in test results.

Conclusion

A representative summary of results is shown in figure 4. Points in the figure represent the value of the likelihood ratio statistic, shown on the horizontal scale, and the calculated t-statistics for squareness, on the vertical scale.

Label A in the figure illustrates points that lie outside the rejection regions for the fit and squareness hypotheses. The corresponding dummy configurations are consistent with both fit and squareness. Label B illustrates points that satisfy squareness but not fit. Label C points correspond to dummy configurations that do not satisfy fit or squareness.

Since the number of independent pairs of points lying along the same isoquant varies over dummy configuration, the range of such pairs employed in calculating t-statistics varies from 10 to 20. The critical boundaries in the figure represent a two-tailed 5% rejection limit assuming 9 degrees of freedom, resulting in cutoffs of ± 2.262 for the t-statistic. These values are conservative with respect to the significance level of the test as t-statistics using more data points will be tested at a lower significance level. With this critical region, in all 12 experiments, there were at least some dummy configurations that satisfied the squareness criterion.

The LR statistic has a Chi square distribution with degrees of freedom given by the number of restrictions in the restricted regression. This number varies from 5 to 12 over the 82 dummy configurations employed, and the cutoff value of 21.03 shown in figure 4 represents the 5% Chi square tail value assuming 12 degrees of freedom. Again, the 5% significance level is conservative. In 4 of the 12 experiments, the fit measure was consistent with the von Liebig hypothesis while, in the remaining 8 experiments, none of the 82 von Liebig dummy configurations resulted in values of the LR statistic that satisfied the fit criterion.

Table 1 reports the smallest values of the likelihood ratio for those dummy configurations that satisfy the squareness criterion for each of the 12 experiments. In no case was the squareness hypothesis rejected because of a significantly large positive or negative value of the mean difference in output levels along an isoquant.

The evidence presented here suggests that, at the level of aggregation represented by Hexem and Heady's experimental plots, the von Liebig hypothesis does not consistently explain the results. While four of the experiments showed results for which there was some dummy configuration consistent with the von Liebig hypothesis, in the remaining eight cases, we reject the hypothesis due to lack of fit.

Footnotes

*Peter Berck is professor of and Stephen Stohs is a graduate student in Agricultural and Resource Economics and Policy at the University of California at Berkeley; Jacqueline Geoghegan is assistant professor of Economics at Clark University.

[1]There are many users of von Liebig production functions including Cate and Nelson; Feinerman, Bresler, and Dagan; Lanzer and Paris; Letey and Dinar; Seginer; Wang and Lowenberg-DeBoer; Warrick and Gardner; and Paris and Knapp.

[2]Varian explores the general construction of isoquants when there is observational error. His work does not take advantage of the special nature of right-angle isoquants and block experimental designs.

[3]Berck and Helfand use $Y = \min(a_0 + a_1x_1 + u_1, b_0 + b_1x_2 + u_2, P + u_3)$, where u is the error term.

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Table 1. Minimum Likelihood Ratio Statistic for Path with Square Isoquant

Table Number	Likelihood Ratio
6.06	14.4
6.13	7.0
6.16	28.1*
6.19	63.6*
6.22	85.8*
7.01	37.7*
7.03	10.8
7.05	57.5*
7.07	3.1
7.09	33.0*
7.11	64.7*
7.13	21.5*

Note: Table Number is the number of the table in the Appendix to Hexem and Heady that presents the data for the experiment. Likelihood Ratio is computed. See text. The asterisk indicates rejection for poor fit.

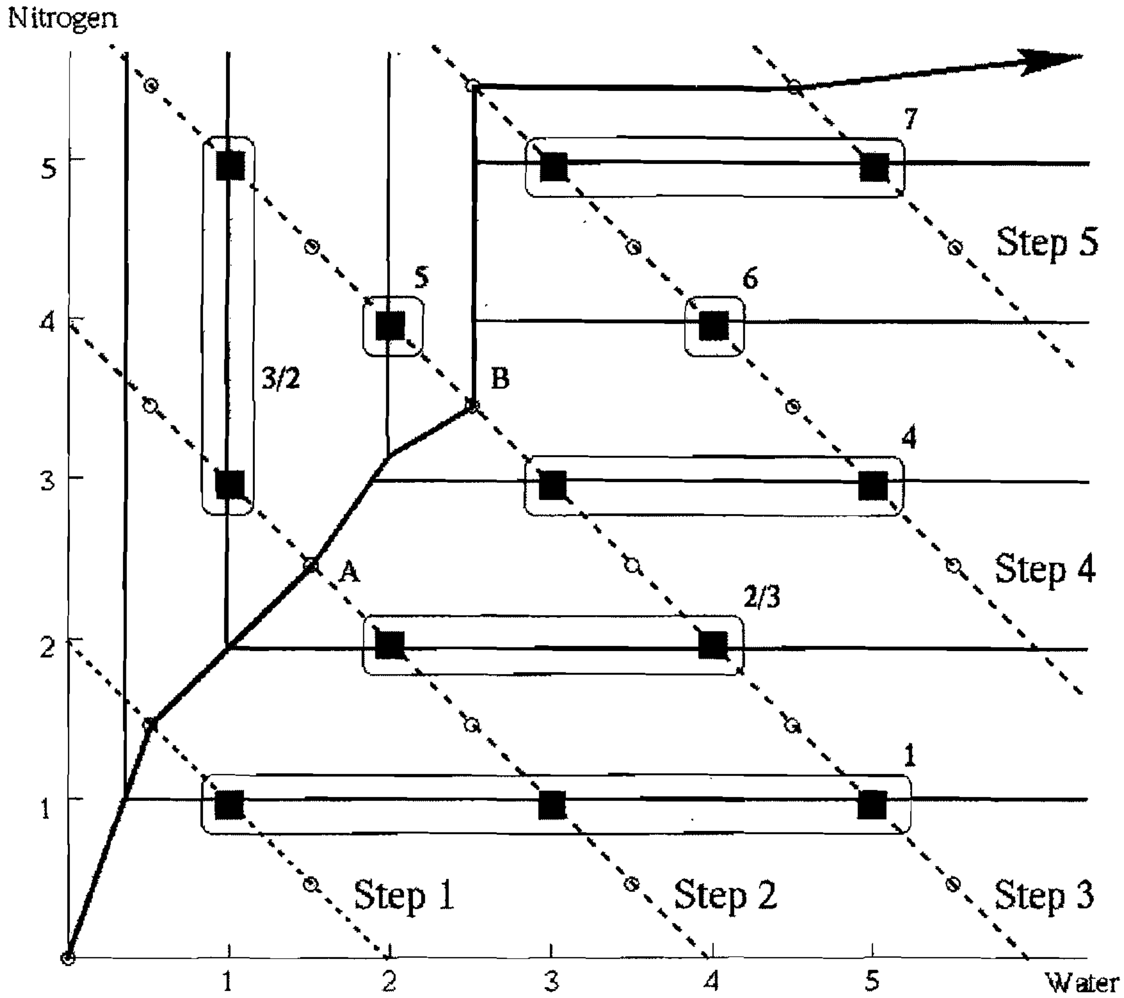

Figure 1. A feasible production expansion path

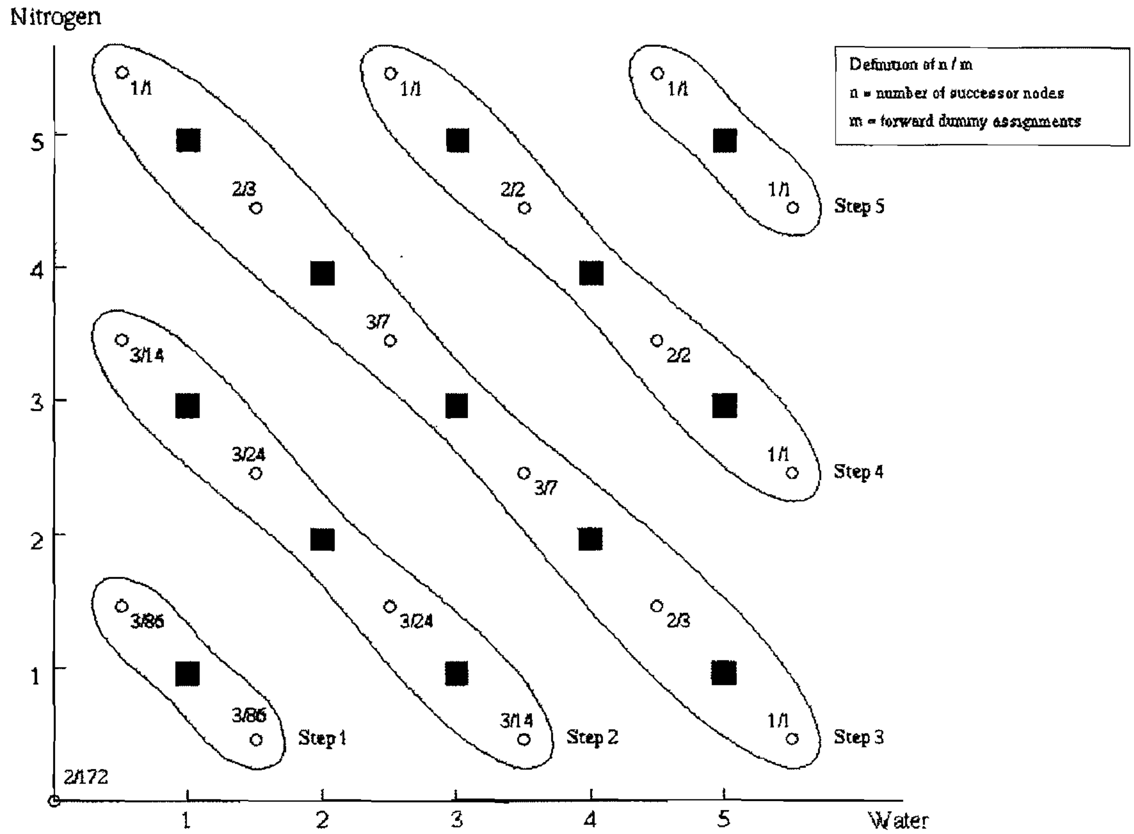

Figure 2. Bounding the number of dummy configurations by backwards induction

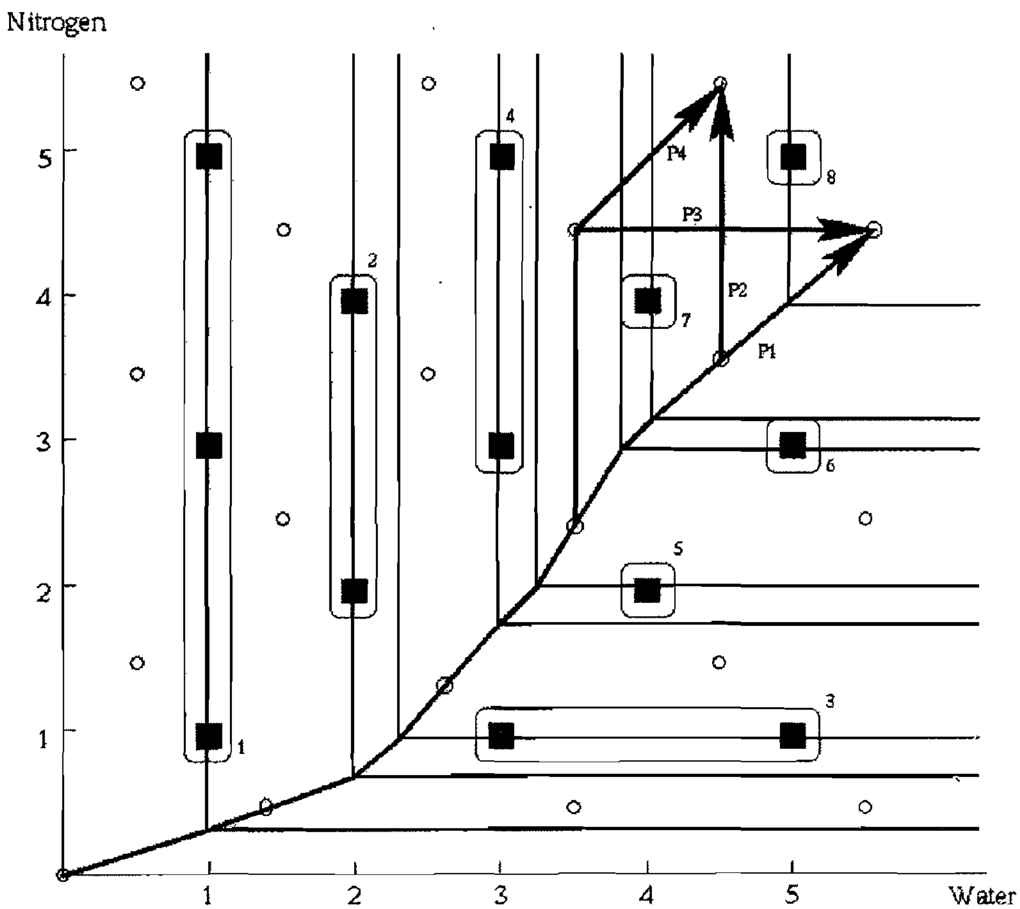

Figure 3. Multiple expansion paths with a common dummy configuration

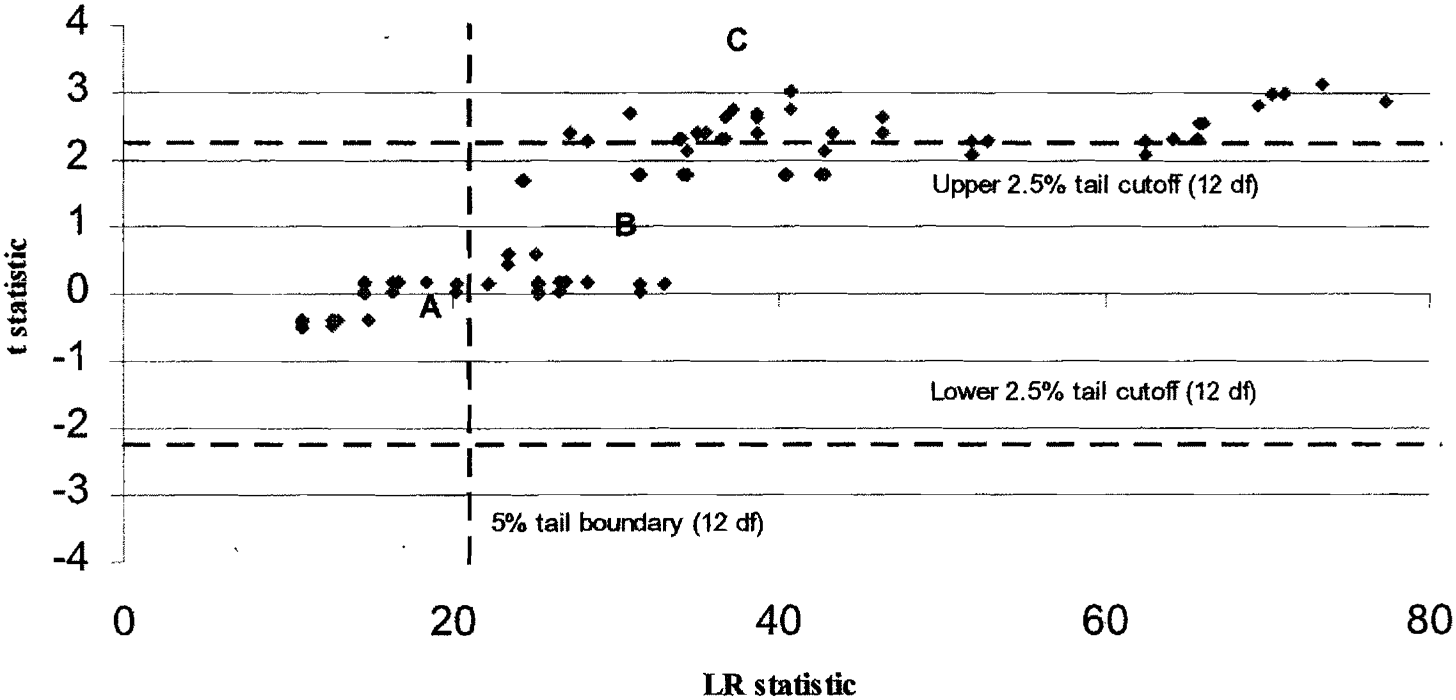

Figure 4. LR fit statistic versus t-statistic for Tb7.03