Title
Sustainability, Limited Substitutability and Non-Constant Social Discount Rates

Author
Traeger, Christian P.

Publication Date
2008-11-01

University of California, Berkeley
Department of Agricultural &
Resource Economics

CUDARE Working Papers

Year 2008 *Paper 1045R*
(Year2004 *Paper 1045)*

Sustainability, Limited Substitutability, and Non-constant Social Discount Rates

Christian P. Traeger

Sustainability, Limited Substitutability, and Non-constant Social Discount Rates[1]

Christian P. Traeger

Department of Agricultural and Resource Economics, UC Berkeley

This Version: November 2008

First Version: January 2004

Correspondence:

Mail:	Christian Traeger
	Department of Agricultural & Resource Economics
	207 Giannini Hall #3310
	University of California
	Berkeley, CA 94720-3310
E-mail:	traeger@berkeley.edu

[1] I thank Larry Karp, Hans Gersbach, Ralph Winkler, Grischa Perino and Tony Fisher for substantial support. I also owe thanks to the participants of the EEA 2004 and the EAERE 2004 meetings, in particular to Ben Groom. The usual disclaimer applies.

Sustainability, Limited Substitutability, and Non-constant Social Discount Rates

Abstract: The paper explores the consequences of limited substitutability in welfare between environmental and produced goods for long-term evaluation. I show how the magnitude and time development of optimal social discount rates depend on the substitutability between the different classes of goods. I relate the degree of substitutability to the notions of weak and strong sustainability in a way suggested in the literature. I show that a strong notion of sustainability results in lower weights given to long-run service and consumption streams compared to a weak notion of sustainability. The paper develops an alternative definition of weak and strong sustainability preferences that incorporates the intertemporal concern of sustainability.

JEL Classification: D 61, D 90, H 43, Q 01, Q 20, Q 51

Keywords: environmental discount rate, hyperbolic, limited substitutability, non-constant discounting, project evaluation, propagator of marginal utility, social discount factor, social discount rate, strict sustainability, strong sustainability, time preference, weak sustainability

1 Introduction

I show how limited substitutability in consumption between different classes of goods affects the magnitude and time development of social discount rates. I relate substitutability between environmental and produced goods to the paradigms of weak and strong sustainability. For welfare specifications corresponding to the weak sustainability paradigm the substitutability effect implies falling discount rates. For welfare specifications corresponding to the strong sustainability paradigm the substitutability effect implies growing discount rates. The established time behavior contradicts to what has previously been suggested in the literature on the basis of less rigorous analysis. I show how the derived substitutability effect interacts with the absolute growth effect, which is the standard contribution of marginal utility to the social discount rates in the one-commodity Ramsey equation. Drawing on my findings on the relation between social discount rates and substitutability, I suggest a modified translation of the weak and the strong sustainability paradigm into the welfare function. This definition acknowledges weight distribution over time.

The paper formalizes and reviews a reasoning put forth by Neumayer [16] in the context of climate change evaluation. He argues that limited substitutability is more critical to long-term evaluation than the effects generally discussed in social discounting, i.e. the rate of pure time preference and decreasing marginal utility under growth. I restate the substitutability effect as a third contribution to the social discount rate. While Neumayer [16] argues verbally that a stronger limitation of substitutability would increase the attention paid to the long term, the opposite holds true in a model presented here.

The notion that relative scarcity influences discount rates is already formulated in Krutilla [13] and translated into a formal setting by Fisher et al. [5] and Fisher and

Krutilla [4]. While the latter formalization implants an exogenous scarcity term into the discount rate, this paper derives the discount rate (and it's time behavior) from the underlying welfare function. During the evolution of this paper, a couple of related and very interesting articles have been published.[2] Guesnerie [11] derives the limit of long run-discount rates in the case of an isoelastic welfare function that aggregates over goods employing a constant elasticity of substitution (CES) function and aggregates over time employing a constant intertemporal elasticity of substitution function (CIES). His focus is on the (infinitely) long run in a setting with uncertainty that does not resolve over time. This combination of assumption yields quite different conclusion with regard to the effect of substitutability on long-run welfare. The present paper extends Guesnerie's [11] proposition on long-run discount rates to a setting where growth rates of both, produced and environmental goods, vary over time and corrects a minor mistake in one of the limits. More importantly, I focus on the time behavior of the discount rates from the present into the future. Weikard and Zhu [27] is the first paper published in English to analytically derive a magnitude effect on the social discount rate from explicitly introducing limited substitutability and in the welfare function and translating it into the discount rates. The authors do not analyze the time behavior of discount rates as it is the focus of this paper.

Closest to the analysis carried out here are Hoel and Sterner [12] and Sterner and Persson [22]. Hoel and Sterner [12] use the same model as Guesnerie [11] and point out how limited substitutability in consumption can cause relative prices for the environmental goods to increase and social discount rates to be non-constant over time. Sterner and Persson [22] numerically apply the model to analyze the quantitative im-

[2] The first version of this paper was circulated and presented at the 2004 EAERE and EEA meetings under the name "Marginal Utility Propagation, Prices and the Rate of Discount. Should environmental goods be discounted hyperbolically?".

portance of such falling discount rates for climate change evaluation. For this purpose, they adapt Nordhaus's [18] integrated assessment model 'DICE' and find significant changes in the optimal policies when compared to the standard evaluation with a single aggregate consumption good (corresponding to a situation of perfect substitutability). Both of these papers restrict their analysis of the time behavior of social discount rats to numerical examples. I give a complete specification of the parameter combinations that lead to falling, constant, and to increasing social discount rates. Moreover, I show that this time behavior always coincides for both, produced and environmental services and consumption streams. Hoel and Sterner [12] moreover propose that the social discount rates are always lower, the lower is the degree of substitutability between environmental and produced goods. This proposition would support Neumayer's [16] argument that a stronger limitation of substitutability would increase the attention paid to the long term. However, the calculation underlying the statement neglects an important dependence of value shares and the social discount rate on substitutability. I explain the various qualitatively different scenarios that can arise by decomposing the social discount rate in the isoelastic model employed by Hoel and Sterner [12] into an absolute growth effect and a real substitutability effect. The absolute growth effect is a value share weighted mean of all growth rates, while the substitutability effect depends on substitutability and growth differences. Alternatively, I show how the social discount rate can be decomposed into an Eigengrowth effect, proportional only to the good under observation, and a utility substitutability effect. The explanation of the derived results relates to a finding by Gerlagh and van der Zwaan [7] who analyze the time development of value shares in a comparable growth scenario.

More generally my model relates to a broad field of literature that motivates and works with non-constant discount rates. Groom et al. [10, 7 et seqq.] present an excellent review of (other) theoretical arguments causing social discount rates to decline. Frederick et al. [6, 378] survey experiments showing that a falling (hyperbolic) discount rate describes behavior better than constant discounting. In 2003 hyperbolic discount rates made their way into applied policy, when the British Green Book started to prescribe hyperbolic discount rates for the evaluation of long-term projects [24, 97 et sqq.]. While some of the models describing falling discount rates yield time inconsistent policy recommendations,[3] the present model derives the non-constancy from real changes of relative scarcity in the economy and, thus, does not cause time inconsistencies.

The paper is structured as follows. Section 2 gives a rigorous derivation of social discount rates and factors in the multi-commodity setting. Moreover, it relates the concepts of weak and strong sustainability to the degree of substitutability between man-made and environmental goods. Section 3 isolates and discusses the substitutability effect in a scenario where growth rates for produced goods exceed those of environmental service streams. It points out how the substitutability effect generally implies that the welfare specification identified with a strong sustainability preference gives less weight to future consumption and service streams than the one identified with a weak sustainability preference. Section 4 discusses the interaction of the substitutability effect and the absolute growth effect, using the same welfare specification as employed by Guesnerie [11] and Hoel and Sterner [12]. Building on the results derived for social discount rates the section revisits and refines the formal characterization of the concepts of weak and strong sustainability. Section 5 concludes. Calculations and proofs are gathered in an online appendix.

[3] That is a continual revision of the (formerly) optimal plan, even if not receiving new information.

2 Social Discount Rates and the Strength of Sustainability

2.1 Social Discount Rates and Factors

This section derives social discount rates from the trajectory of marginal utility. Consumption quantities of two goods at time t are characterized by positive real numbers, denoted $x_1(t)$ and $x_2(t)$. The time argument will generally be omitted. With x: $[0,\infty) \rightarrow \mathbb{R}^2_+$[4] I denote the consumption path of the two goods from the present $t=0$ to the infinite time horizon. Welfare is

$$\mathcal{U} = \int_0^\infty U(x_1,x_2,t)\,dt \ , \tag{1}$$

with a twice differentiable (instantaneous) utility function $U(x_1,x_2,t)$. I define the *good specific social discount factor* between time t_0 and time t for a given consumption path x by[5]

$$D_i^{\mathrm{x}}(t,t_0) \equiv \frac{\frac{\partial U(x_1,x_2,t)}{\partial x_i}}{\frac{\partial U(x_1,x_2,t_0)}{\partial x_i}} \quad \Leftrightarrow \quad \frac{\partial U(x_1,x_2,t)}{\partial x_i} = D_i^{\mathrm{x}}(t,t_0)\,\frac{\partial U(x_1,x_2,t_0)}{\partial x_i} \ ,$$

$i \in \{1,2\}$. The discount factors $D_i^{\mathrm{x}}(t,t_0)$ capture the value development over time, relating the value of an additional unit of consumption good x_i at time t to the value of an additional unit at time t_0. The $D_i^{\mathrm{x}}(t,t_0)$ are *time propagators of marginal utility.*[6]

[4] $\mathbb{R}_+ = \{x \in \mathbb{R} | x \geq 0\}$ and $\mathbb{R}_{++} = \{x \in \mathbb{R} | x > 0\}$.

[5] For a given consumption path x, $U(t) \equiv U(x_1(t),x_2(t),t)$ and its derivative are evaluated at the implied consumption levels $x_1(t)$ and $x_2(t)$.

[6] The name is based on a general concept in physics and group theory, see footnote 7 for reference. Malinvaud [15, 234] uses these discount factors in a discrete time setting in a general equilibrium context. The $D_i^{\mathrm{x}}(t,t_0)$ can be calculated even if pure time dependence of instantaneous utility is not multiplicatively separable.

The discount rates corresponding to the discount factors are

$$\delta_i(t) = -\frac{\frac{d}{dt} D_i^{\chi}(t,t_0)}{D_i^{\chi}(t,t_0)} = -\frac{\frac{d}{dt}\frac{\partial U(x_1,x_2,t)}{\partial x_i}}{\frac{\partial U(x_1,x_2,t)}{\partial x_i}} = -\frac{\frac{\partial^2 U}{\partial t \partial x_i}(t) + \frac{\partial^2 U}{\partial x_i^2}(t)\dot{x}_i + \frac{\partial^2 U}{\partial x_j \partial x_i}(t)\dot{x}_j}{\frac{\partial U}{\partial x_i}(t)} \tag{2}$$

for $i, j \in \{1, 2\}$ with $i \neq j$. The $\delta_i(t)$ are the *generators* of the propagators $D_i^{\chi}(t,t_0)$ and generate the value development of an additional unit of good x_i in the future.[7] The discount factor is recovered from a discount rate by:

$$D_i^{\chi}(t,t_0) = \exp\left(-\int_{t_0}^{t} \delta_i(x(t'), \dot{x}(t'), t')\, dt'\right) . \tag{3}$$

2.2 Review of the One Commodity Special Case

In models with a single (aggregate) consumption good, $\delta_i(t)$ is known as the (instantaneous) *social discount rate*. This fact stands out more clearly if instantaneous utility is specified as $U(x_1, x_2, t) = u(x_1, x_2)e^{-\rho t}$. Neglect the second commodity by setting it constant. Then, the discount rate $\delta \equiv \delta_1$ becomes

$$\delta(t) = \rho - \frac{\frac{\partial^2 u}{\partial x_1^2}}{\frac{\partial u}{\partial x_1}}\dot{x}_1 = \rho - \frac{\partial \frac{\partial u}{\partial x_1}}{\partial x_1}\frac{x_1}{\frac{\partial u}{\partial x_1}}\frac{\dot{x}_1}{x_1} = \rho + \theta_{(x(t))}\ \hat{x}_{1(x_1(t),\dot{x}_1(t))} . \tag{4}$$

This expression for the social discount rate is well known in the literature, see e.g. Arrow et al. [2, 136] or Groom et al. [10]. The constant ρ is called the *pure rate of time preference*. The term θ is the (absolute value of the) *elasticity of marginal utility of consumption*, which is the inverse of the *intertemporal elasticity of substitution*. Finally, $\hat{x}_1$ denotes the *growth rate* of the consumption commodity. Equation (4) states that the value development of an additional unit of good x_i is generated by the pure rate of time preference as well as a term proportional to the growth rate of consumption and

[7] Precisely, the negative of the *dis*count rate $\delta_i(t)$ would be called the generator. See Sakurai [21, 46 et sqq.,71 et sq.] or Goldstein [8, chapter 9] for this view on classical and quantum mechanics (e.g. momentum being the generator of translation).

the elasticity of marginal utility. To gain intuition for the second term, assume that consumption is growing over time. Then, an individual with a decreasing marginal valuation of consumption values an additional unit of consumption in the future less than in the present. Therefore, growth increases the rate at which he discounts future consumption. In most macroeconomic models the function u is assumed to exhibit constant elasticity of intertemporal substitution (CIES). The CIES assumption implies that in a steady state, where growth rates are constant, the term $\theta\,\hat{x}_1$ and, thus, the social discount rate $\bar{\delta} = \rho + \theta\hat{x}_1$ are constant. A constant rate of discount implies by equation (3) a social discount factor $D^{\mathsf{x}}(t, t_0) = e^{-\bar{\delta}(t-t_0)}$ and, thus, exponential discounting of future consumption.

In general, expression (4) need not be constant. A non-constant $\theta\,\hat{x}_1$ can lead to hyperbolic discounting. A discount function is said to be *hyperbolic* if it is characterized by a falling instantaneous discount rate [14, 450]. Dasgupta [3, 183 et sqq.] points out that in the face of global climate change, a decline in consumption growth $\hat{x}_1$ would imply a falling social discount rate. This effect is inversely proportional to the intertemporal elasticity of substitution (θ^{-1}). For a given decline in growth, a *lower* intertemporal elasticity of substitution (θ^{-1}) induces a stronger decrease of the social discount rate and, thus, a relatively *higher* weight given to future consumption. Finally, Gollier [9] derives conditions under which the term $\theta\,\hat{x}_1$ leads to a falling discount rate in a model with uncertainty.

2.3 Limited Substitutability in Consumption

Returning to equation (2), I analyze how equation (4) changes in the multi-commodity setting. From now on, good x_1 is interpreted as a flow of environmental goods and services, while x_2 represents an aggregate of produced consumption. To assure *time* *consistency* of the planning functional (1), I assume $U(x_1, x_2, t) = u(x_1, x_2)e^{-\rho t}$ implying a constant rate of pure time preference ρ. Then, the discount rate corresponding to the social discount factor $D_1^{\chi}(t, t_0)$ becomes

$$\delta_1(t) = \rho - \frac{\frac{\partial^2 u}{\partial x_1^2}}{\frac{\partial u}{\partial x_1}} \dot{x}_1 - \frac{\frac{\partial^2 u}{\partial x_1 \partial x_2}}{\frac{\partial u}{\partial x_1}} \dot{x}_2 \,. \tag{5}$$

It comprises an additional term that depends on the substitutability $\frac{\partial^2 u}{\partial x_1 \partial x_2}$ between the two classes of goods. Equation (5) has independently been derived by Weikard and Zhu [27] who also comment on the magnitude effects (see below) but do not analyze time behavior of the discount rates. To bring out the influence of substitutability in welfare on the social discount rate and its evolvement over time, I take instantaneous utility to be

$$u(x_1, x_2) = A \left[a_1 u_1(x_1)^s + a_2 u_2(x_2)^s\right]^{1/s} \tag{6}$$

with $s \in \mathbb{R}, a_1, a_2 \in \mathbb{R}_{++}, a_1 + a_2 = 1, A \neq 0$ and $u_1, u_2 \geq 0$.[8] This step separates good-specific utility $u_i(x_i)$ from substitutability effects parameterized in a simple form by s. As derived in appendix A, such a welfare specification yields the social discount rate for the environmental service stream

$$\delta_1(t) = \rho - \frac{\frac{\partial^2 u_1}{\partial x_1^2}}{\frac{\partial u_1}{\partial x_1}} \dot{x}_1 - (1-s) \frac{a_2 u_2(x_2)^s}{a_1 u_1(x_1)^s + a_2 u_2(x_2)^s} \left(\frac{\frac{\partial u_2}{\partial x_2}(x_2)}{u_2(x_2)} \dot{x}_2 - \frac{\frac{\partial u_1}{\partial x_1}(x_1)}{u_1(x_1)} \dot{x}_1 \right) . \tag{7}$$

The first and the second term in equation (7) resemble the widely used equation (4). This paper focuses on the analysis of the third term that depends on the substitutability parameter s. Labeling the two terms that add to the pure rate of time preference, I suggest calling the second term an "*Eigengrowth effect*", because it only depends on

[8] For $s = 0$ the function is defined by the limit $s \to 0$ yielding $u(x_1, x_2) = u_1(x_1)^{a_1} u_2(x_2)^{a_2}$. For $s \to -\infty, \infty$ the limit functions are $\min\{u_1(x_1), u_2(x_2)\}$ and $\max\{u_1(x_1), u_2(x_2)\}$ respectively. $u_i \geq 0$ abbreviates $u_i(x_i) \geq 0$ for all $x_i \in \mathbb{R}_+$.

the absolute growth of good x_1 and the curvature of u_1. I suggest calling the third term a "*relative growth*" or "*utility substitutability effect*", because it depends on the difference in growth of x_1 and x_2 and on the degree of substitutability between the good-specific utility u_1 derived from x_1 and u_2 derived from x_2.

2.4 A Preference for Weak versus Strong Sustainability

This subsection relates the substitutability parameter s to the concepts of weak and strong sustainability. These concepts suggest differing implementations of a sustainable development, i.e. a "development that meets the needs of the present without compromising the ability of future generations to meet their own needs" [26]. The paradigm of *weak sustainability* translates the latter definition into the demand that overall welfare should not decline over time. To this end, its proponents allow for a substitution between environmental and man-made capital. On the other hand, the advocates of the *strong sustainability* paradigm demand that natural capital (or its service flows) by itself should not decline.[9] They do not believe in substitutability between the different types of capital.

Traditionally, the economic analysis of sustainability mostly focuses on capital and its substitutability *in production*. For a list of environmental assets that are considered non-substitutable by man-made capital see Pearce et al. [19, 37] or Neumayer [16, 39]. The claim of non-substitutability of these assets comes down to pointing out that the corresponding service flows cannot be replaced by those of man-made capital. This

[9] Opinions whether natural capital should be non-declining in value or in physical terms differ. Moreover, natural capital is often broken down further into different classes, each of which should be kept non-declining. Often, strong-sustainability is additionally associated with an intrinsic value of nature. The latter can be mapped into 'existence service flows', e.g. proportional to the amount of existing capital. For an overview over the more detailed differences between weak and strong sustainability as well as further differentiations of sustainability demands consult e.g. Neumayer [17] and van den Bergh and Hofkes [25].

claim is defensible if we are concerned with a perfect replication of service streams. Take for instance the ozone layer with its UV-protection function. Opponents to the non-substitutabilty assumption would argue that, at least at the margin, the ozone in the stratosphere can be replaced by sunscreen lotion or shelter under glass, both of which protect to some degree from ultraviolet radiation. However, such an argument already involves the *welfare judgment* that taking a sun bath with or without sunscreen are perfect substitutes, or that a glass roof is a substitute for the open air. Assuming a non-perfect replicability of natural capital, I consider the degree of *substitutability in welfare* between man-made and environmental goods and service streams to be the most important difference between the weak and the strong sustainability paradigm.

Neumayer [16] introduces a similar reasoning into the debate on climate change evaluation. His essay argues that an appropriate characterization of sustainability and limited substitutability would be more critical to long-term evaluation than pure time preference and the growth effect reviewed in equation (4). As I have shown, the substitutability effect can also be translated into the social discount rate. Neumayer [16] claims that the consideration of strongly limited substitutability would result in a higher weight given to the needs of future generations. The next sections formally analyze this claim.

Neumayer's [16] verbal discussion identifies weak sustainability with perfect substitutability and strong sustainability with 'close to lexicographic preferences'. The model in this paper captures a continuum of different degrees of substitutability. I restrict attention to preferences with convex better sets. Thereby I eliminate preferences for extreme consumption bundles, i.e. for which only consuming man-made goods or only consuming environmental goods and services is preferred to consuming a mixture of the two. I assign preferences to a weak sustainability paradigm whenever it is possible to extract an arbitrary welfare level from *only* consuming man-made goods and service streams. I assign preferences to the strong sustainability paradigm whenever a constraint on environmental goods and services limits[10] the achievable overall welfare (no matter how much of the aggregate produced good is consumed). This straight-forward characterization is simple and serves for the analysis in section 3 where welfare is characterized by a constant elasticity of substitution (CES) function. The definition is close to the one given by Gerlagh and van der Zwaan [7] and coincides with their definition for the CES scenario. A more profound reflection will be the content of section 4.3.

3 The Substitutability Effect

3.1 A Stylized Growth Model

This section analyzes how the weights for future consumption streams evolve in a scenario where produced consumption grows at a faster rate than consumption of environmental services. The underlying assumption is that technological progress increases the availability of produced consumption at a faster rate than the availability of environmental service and consumption streams can be increased. When thinking about essential life-support services that most advocates of a notion of strong sustainability are concerned about (e.g. climate regulation functions), it is hard to think of a long-term positive growth rate of environmental services at all. When considering environmental goods like those defined in Fisher and Krutilla [4, 360] as goods that are "generally consumed on site, with little or no transformation by ordinary productive processes", including e.g. scenic views, then by definition these goods are not affected

[10] The corresponding formal definition is given in definition 2 on page 33.

by technological progress in production.[11] The appreciation of biodiversity and its existence value is another example where the growth rate of the corresponding existence service flow is negative and a serious growth within a human planning horizon is hard to imagine. Against this background I introduce

Assumption 1: There exists $\epsilon > 0$ such that $\hat{x}_1(t) < \hat{x}_2(t) - \epsilon$ for all t.

The assumption allows for a decline in environmental goods and services. It also allows for a scenario, which is sometimes put forth in relation to climate change, where production and environment decline together and environmental service flows decline at a higher rate. In general, Assumption 1 contains the kind of scenarios that most advocates of a strong sustainability concept are concerned about.[12]

Under this stylized growth assumption, I analyze how different degrees of substitutability between the two classes of goods and services affect the weights given to future consumption. I focus on the effect resulting from the *difference in growth* rates and the limited *substitutability*. Focusing on this objective, I simplify the utility function in equation (6) by setting $u_1(x_1) = x_1$ and $u_2(x_2) = x_2$, which leads to the standard CES utility function

Assumption 2: Welfare is representable in the functional form[13]

$$\mathcal{U} = \int_0^\infty [a_1 x_1^s + a_2 x_2^s]^{1/s} e^{-\rho t}\, dt \quad \text{with } a_1, a_2 \in \mathbb{R}_{++}, a_1 + a_2 = 1 \text{ and } s \in \mathbb{R}, s \leq 1.$$

[11]One can think of several cases where technological progress helps accessing or enjoying environmental goods. However, such a complementarity between produced and environmental goods and services is captured in the welfare function, i.e. in the parametrization of substitutability.

[12]Part *i*), *ii*) and *iii*) of Proposition 1 as well as Corollary 1 and Proposition 2 also hold under the slightly weaker assumption that there exist $\epsilon > 0$ and $t^* \in [0, \infty)$ such that $\hat{x}_1(t) < \hat{x}_2(t) - \epsilon$ for all $t \geq t^*$.

[13]For $s = 0$ the integrand is defined by limit, yielding the well known Cobb-Douglas specification: $\lim_{s \to 0}[a_1 x_1^s + a_2 x_2^s]^{1/s} = x_1^{a_1} x_2^{a_2}$ [1, 231]. In the range $s \in (1, \infty)$ extreme choices are generally preferred to mixtures. Such an assumption does neither seem reasonable when analyzing environmental and produced consumption and service streams, nor does it correspond to any notion of sustainability.

CES functions exhibit a constant elasticity of substitution σ that relates to the substitutability index s by the formula $\sigma = \frac{1}{1-s}$ [1]. As CES functions are homogeneous of degree one, proportional overall growth does not change marginal utility (which is homogeneous of degree zero). Therefore, the chosen functional form is well suited to focus on the new effect, due to limited substitutability and relative difference in growth, filtering out the well-known overall growth effect discussed in section 2.2 in connection to equation (4). This step leads to the discount rate

$$\delta_1(t) = \rho - (1-s) \underbrace{\frac{a_2 x_2^s}{a_1 x_1^s + a_2 x_2^s}}_{\equiv V_2{}^s(x_1, x_2)} (\hat{x}_2 - \hat{x}_1) \ . \tag{8}$$

The first determinant in the social discount rate for the environmental service stream in equation (8) is the pure rate of time preference ρ. It is reduced by a second term which comprises three different components. The first component $(1-s) = \sigma^{-1}$ *is a measure for the limitedness in substitutability* between the two classes of goods. The second component depicts the value share of the produced consumption stream

$$V_2{}^s(x_1, x_2) = \frac{\frac{\partial u}{\partial x_2} x_2}{\frac{\partial u}{\partial x_1} x_1 + \frac{\partial u}{\partial x_2} x_2} = \frac{a_2 x_2^s}{a_1 x_1^s + a_2 x_2^s} \ . \tag{9}$$

It depends on the ratio $\frac{x_1}{x_2}$ between the environmental services and the produced goods consumed,[14] the utility weights a_1 and a_2, and the substitutability parameter s. The last component in equation (8) is the difference in growth rates between produced and environmental consumption and service streams. Altogether the second term on the right hand side of equation (8) can be summarized as follows. The difference in growth rates is weighted with the value share of produced consumption. This expression is then

[14] That $V_2{}^s$ only depends on the ratio is easily observed by multiplying numerator and denominator on the right hand side of equation (9) with x_2^{-s}.

weighted with the limitedness in substitutability between produced and environmental service streams and subtracted from the pure rate of time preference. Section 3 analyzes the expression for different degrees of substitutability.

Similarly, the social discount rate for produced consumption and service streams is

$$\delta_2(t) = \rho + (1-s)\,\underbrace{\frac{a_1 x_1^s}{a_2 x_2^s + a_1 x_1^s}}_{\equiv V_1^s(x_1,x_2)}\,(\hat{x}_2 - \hat{x}_1)\ . \tag{10}$$

The interpretation is analogous to that of equation (8). However, depicting the difference in relative growth the same way as in equation (8) implies a sign switch. Therefore, the additional effect, which is weighted with the value share of the environmental services

$$V_1^s(x_1,x_2) = \frac{\frac{\partial u_1}{\partial x_1} x_1}{\frac{\partial u_1}{\partial x_1} x_1 + \frac{\partial u_2}{\partial x_2} x_2} = \frac{a_1 x_1^s}{a_1 x_1^s + a_2 x_2^s}\ ,$$

enters the social discount rate for produced consumption positively.

Assumptions 1 and 2 yield an easily tractable model fleshing out the relation between substitutability and long-term consumption weights. It is straight forward to show that for the welfare specification in assumption 2 preferences of a weak sustainability proponent are identified with parameters $1 > s > 0$ and an elasticity of substitution $\sigma > 1$.[15] Preferences of a strong sustainability proponent translate into the parameter range $s < 0$ and $0 < \sigma < 1$. The welfare specification dividing weak and strong sustainability is represented by Cobb-Douglas preferences ($s = 0$, $\sigma = 1$). Here, a limit to welfare is only implied if the environmental service stream is constraint to zero.

[15]For a formal derivation see proof of corollary 3 which implies the claim.

3.2 Results

There are four qualitatively different scenarios for the social discount rates. They correspond to the welfare specifications $s = 1$ (perfect substitutability, $\sigma = \infty$), $s = 0$ (Cobb-Douglas preferences, $\sigma = 1$), $s \in (0, 1)$ (moderate substitutability, $\sigma > 1$) and $s < 0$ (strongly limited substitutability, $0 < \sigma < 1$). The interpretation of the social discount rates derived for the different welfare specifications is the following. Take as given an underlying growth scenario that satisfies Assumption 1. A decision-maker or social planner is asked to evaluate a small[16] project that affects environmental service streams and produced consumption streams over some period of time. Then, the social discount rates and factors specify the weight that a planner, subscribing to a particular welfare specification, gives to the corresponding future consumption streams. In Traeger [23] I present a formal setup of such a project evaluation.

The case of *perfect substitutability* in consumption between environmental service flows and produced consumption is characterized by the substitutability parameter $s = 1$ ($\sigma = \infty$). It implies additivity in welfare between the different classes of goods $u(x_1, x_2) = a_1 x_1 + a_2 x_2$. As there is no limit to substitutability ($1 - s = \sigma^{-1} = 0$), equations (8) and (10) show that the social discount rates for both classes of goods coincide with the pure rate of time preference: $\delta_1 = \delta_2 = \rho$. This result holds by construction (and reduction) of the welfare function carried out in section 3.1 to focus on the substitutability effect and disregard other growth effects.

In the case of limited substitutability the following result obtains. Recall that I use the term steady state for a scenario where growth rates are constant.

Proposition 1: Let Assumptions 1 and 2 hold with $s < 1$.

[16]Smallness of the project assumes that changes brought about by the project do not affect the overall growth scenario.

Then, the social discount rates are given by equations (8) and (10).

The *social discount rate for the environmental service stream is reduced* proportional to the difference in growth rates, the value share of the produced consumption stream and the limitedness in substitutability expressed by $(1-s)$.

The *social discount rate for the produced consumption stream is increased* proportional to the difference in growth rates, the value share given to the environmental consumption stream and the limitedness in substitutability expressed by $(1-s)$.

Moreover, for

i) $\sigma = 1$, $s = 0$: In a steady state, *both social discount rates are constant.* In general, the discount rates are $\delta_1(t) = \rho - a_2\left(\hat{x}_2(t) - \hat{x}_1(t)\right)$ and $\delta_2(t) = \rho + a_1\left(\hat{x}_2(t) - \hat{x}_1(t)\right)$.

ii) $\sigma \in (1, \infty)$, $s \in (0,1)$: In a steady state, *both social discount rates fall over time.* In general, the long-run discount rates approach the form $\delta_1(t) = \rho - (1-s)\left(\hat{x}_2(t) - \hat{x}_1(t)\right)$ and $\delta_2(t) = \rho$.

iii) $\sigma \in (0,1)$, $s < 0$: In a steady state, *both social discount rates grow over time.* In general, the long-run discount rates approach the form $\delta_1(t) = \rho$ and $\delta_2(t) = \rho + (1-s)\left(\hat{x}_2(t) - \hat{x}_1(t)\right)$.

The slower growing environmental consumption good becomes relatively more scarce as time evolves. Expressing its value development over time the social discount rate is reduced, resulting in a higher weight given to future environmental service streams. On the other hand, the produced good becomes relatively more abundant and, therefore, its social discount rate is increased. The reduction/increase is proportional to the limitedness in substitutability $1-s$ $(= \sigma^{-1})$ and the difference in growth rates. Moreover, it is proportional to the value share of the other good, characterizing the importance of the relative abundance/scarcity with respect to that good.[17] For Cobb-Douglas preferences in case i) the value share of a commodity x_i corresponds to its utility weight a_i and is independent of the consumption levels. Then, in a steady state, the social discount rate is constant and discounting stays exponential. In general however, the value share ${V_i}^s$ depends on consumption, implying non-constant social discount rates.

For weak sustainability preferences, where $s \in (0,1)$ and $\sigma > 1$, statement ii) specifies the time behavior. The change of value shares over time causes both social discount rates to fall. Outside of a steady state, however, a strong fluctuation in the difference in growth rates can counteract this effect and cause the social discount rates to be constant or growing for some period. The discount rate for the environmental service stream x_1 will eventually become negative if there exists t^* such that $(1-s)(\hat{x}_2(t)-\hat{x}_1(t)) > \rho \; \forall \; t > t^*$. That is, if the difference in the growth rates between the two classes of services, weighted with the limitedness in substitutability, dominates the rate of pure time preference ρ.[18] For a strong sustainability preference, where $s < 0$ and $\sigma \in (0,1)$, the change of value shares over time causes both social discount rates to grow (statement iii). Again, outside of a steady state a strong fluctuation in the difference in growth rates can counteract this effect and cause the social discount rates to be constant or falling for some period.

3.3 Implications

For preferences identified with the paradigm of weak sustainability, the optimal social discount rates fall over time (Proposition 1ii). The result matches the intuition

[17]If the other good is important for welfare, relative scarcity is important, too. However, if the other good is of no importance to welfare, the relative scarcity or abundance with respect to that good becomes insignificant as well.

[18]This relation determines only the instantaneous discount rate, in addition it can happen that the social discount factor $D_i^{\mathbf{X}}(t,t_0)$ grows bigger than 1.

expressed e.g. in Groom et al.'s [10, 2] survey on declining discount rates that "It is immediately obvious that using a declining discount rate would make an important contribution towards meeting the goal of sustainable development". Pezzey [20] even defines sustainable discount rates as falling discount rates.

However, for a strong sustainability preference with strongly limited substitutability between the two classes of goods, part *iii*) of Proposition 1 no longer supports this intuition. Here, optimal social discount rates are growing. This result seems to be even more surprising in the light of Neumayer's [16] claim that the strong sustainability paradigm, by implying strongly limited substitutability, would make evaluation models pay more attention to long-run environmental service streams. The following corollary to Proposition 1 fleshes out the relation between the optimal social discount rates in the two scenarios.

Corollary 1: Evaluating the social discount rates for a given growth scenario under Assumptions 1 and 2 the following assertion holds.

There exists $\bar{t} \in [0, \infty)$ such that $\delta_i^{s<0}(t) > \delta_i^{0<s<1}(t)$ for all $t > \bar{t}$ and $i \in \{1, 2\}$.

The *long-term social discount rates* corresponding to a *strong sustainability preference* ($s < 0$) are *higher* than those implied by a *weak sustainability preference* ($0 < s < 1$). Corollary 1 contradicts Hoel and Sterner's [12, 272,273] statement that the social discount rate for the environmental good would always increase in the elasticity of substitution.[19]

[19]Their statement is based on an unnumbered equation on page 278 (third equation). The derivative of 'R' (which corresponds to δ_1 in the setting of this paper) with respect to the elasticity 'σ' neglects that what the authors defined as 'γ^*' (corresponding to a transformation of the value share) is itself a function of 'σ'.

Note that Hoel and Sterner's [12, 272,273] 'R' simultaneously captures the substitutability effect and the absolute growth effect, and the functional form corresponds to that discussed in the next section. However, the special case discussed here points right to the culprit why Hoel and Sterner's [12, 272,273] statement is not true in general, i.e. the substitutability effect. See also proposition 3 in the next section.

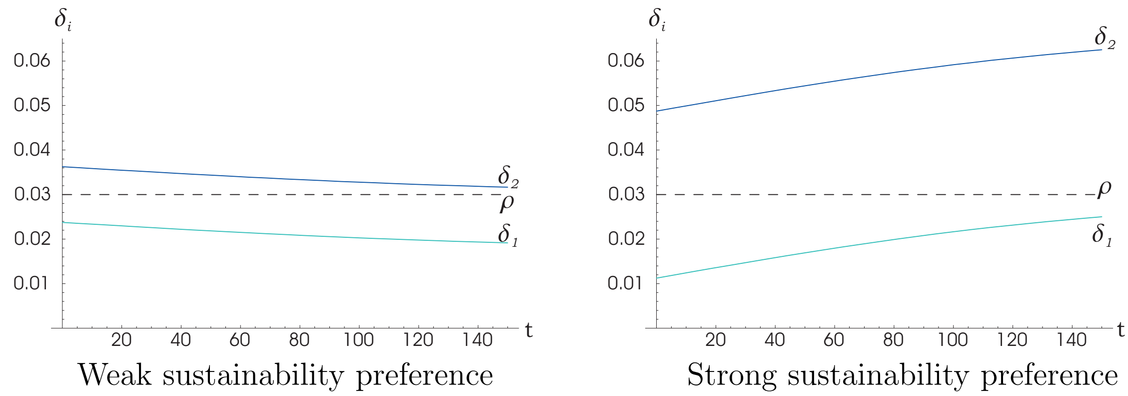


Figure 1 : Numerical example for the time development of social discount rates over time in years. The upper line represents the social discount rate δ_2 for the produced consumption stream, the lower line represents the discount rate δ_1 for the environmental service stream. The dashed line reflects the pure rate of time preference ρ, corresponding to the common discount rate if perfect substitutability in consumption is assumed. In the left diagram the substitutability parameter is chosen to be $s = .5$, on the right it is $s = -.5$. The other parameters coincide for both scenarios and are $\rho = 3\%$, $\hat{x}_2 - \hat{x}_1 = 2.5\%$ and $a_1 = a_2 = .5$.

A numerical example of the time evolvement of the social discount rates for the two different scenarios is drawn in Figure 1. In the left diagram the substitutability parameter is chosen to be $s = .5$ corresponding to moderate substitutability and a weak sustainability preference. In the right diagram the substitutability parameter is chosen to be $s = -.5$, corresponding to strongly limited substitutability and a strong sustainability preference. The other parameters are chosen equally for both scenarios as $\rho = 3$, $\hat{x}_2 - \hat{x}_1 = 2.5\%$ and $a_1 = a_2 = .5$.[20] As the model is constructed to only depend on the relative growth difference, this scenario depicts equally well a situation where both growth rates of consumption are positive (e.g. $\hat{x}_2 = 3\%$ and $\hat{x}_1 = .5\%$), a scenario where produced consumption grows and environmental services decline (e.g. $\hat{x}_2 = 1.5\%$ and $\hat{x}_2 = -1\%$), or one where both forms of cosumptions are subject to a decrease over time. The reduction/increase with respect to pure time preference (complete substitutability) as well as the time behavior pointed out in proposition 1 are clearly

[20]The initial values in the example are $x_1(0) = x_2(0) = 1$.

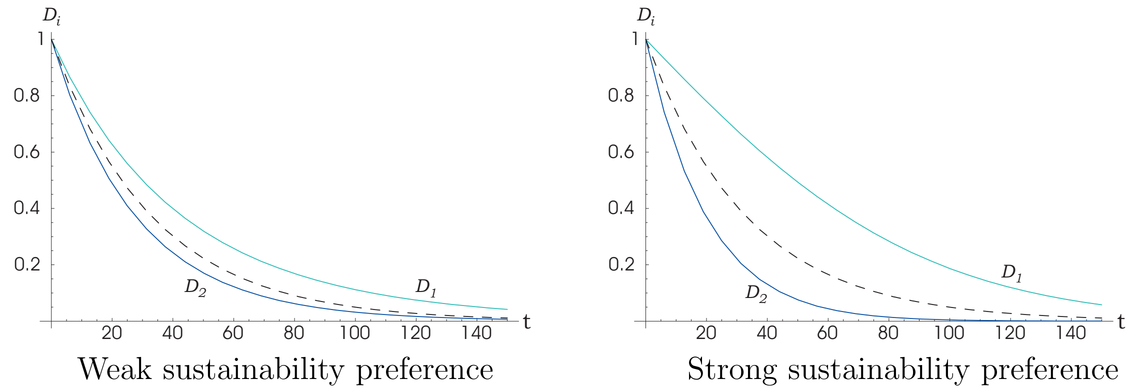


Figure 2 : Numerical example continued (same specifications as for Figure 1). Drawn are the social discount factors for the environmental (upper line, D_1) and the produced (lower line, D_2) good. The dashed line reflects exponential discounting corresponding to the pure rate of time preference. In the depicted scenario, D_1 for the strong sustainability scenario falls below D_1 for the weak sustainability scenario after $t = 195$ years.

observed. Moreover, after $t = 88$ years, the (instantaneous) discount rate for the environmental service stream grows bigger in the strong sustainability scenario than in the weak sustainability scenario. Note that the latter does not immediately imply that the weight given to the environmental service stream is lower with a strong sustainability preference. As derived in Section 2.1, the evaluation of an extra unit of environmental services is captured by the corresponding discount factor. Figure 2 depicts the discount factors for the same scenario specifications as in Figure 1. By equation (3), the discount factor relates to the rate as $D_i^{\mathbf{x}}(t, t_0) = \exp\left(-\int_{t_0}^{t} \delta_i(x(t'), \dot{x}(t'), t')dt'\right)$. Hence, a small discount rate at the beginning is 'memorized' in the discount factor for all times and, therefore, raises the weight given to the future not only at early times, but also in the long run. Therefore, the second figure matches the intuition better than the first that environmental goods, which in relative terms become increasingly scarce over time, should be valued higher in the long term in a setting with strong sustainability preferences than in a setting with weak sustainability preference. However, the following proposition shows that, in the long run, the development of the discount factors does not agree with this intuition, either.

Proposition 2: Evaluating the social discount rates for a given growth scenario under Assumptions 1 and 2 the following assertion holds.

For any $t_0 \in [0, \infty)$ there exists $\bar{\bar{t}} \in [0, \infty)$ such that $D_i^{\chi^{s<0}}(t, t_0) < D_i^{\chi^{0<s<1}}(t, t_0)$ for all $t > \bar{\bar{t}}$ and $i \in \{1, 2\}$.

The proposition implies that a strong sustainability decision-maker gives less weight to long-run environmental service streams than does a weak-sustainability decision-maker.[21] It opposes the statement put forth in Neumayer [16] that strongly limited substitutability between environmental good and service streams and produced consumption, as associated with a strong sustainability paradigm, would increase the weight given to the long run in a growth scenario as analyzed here.[22]

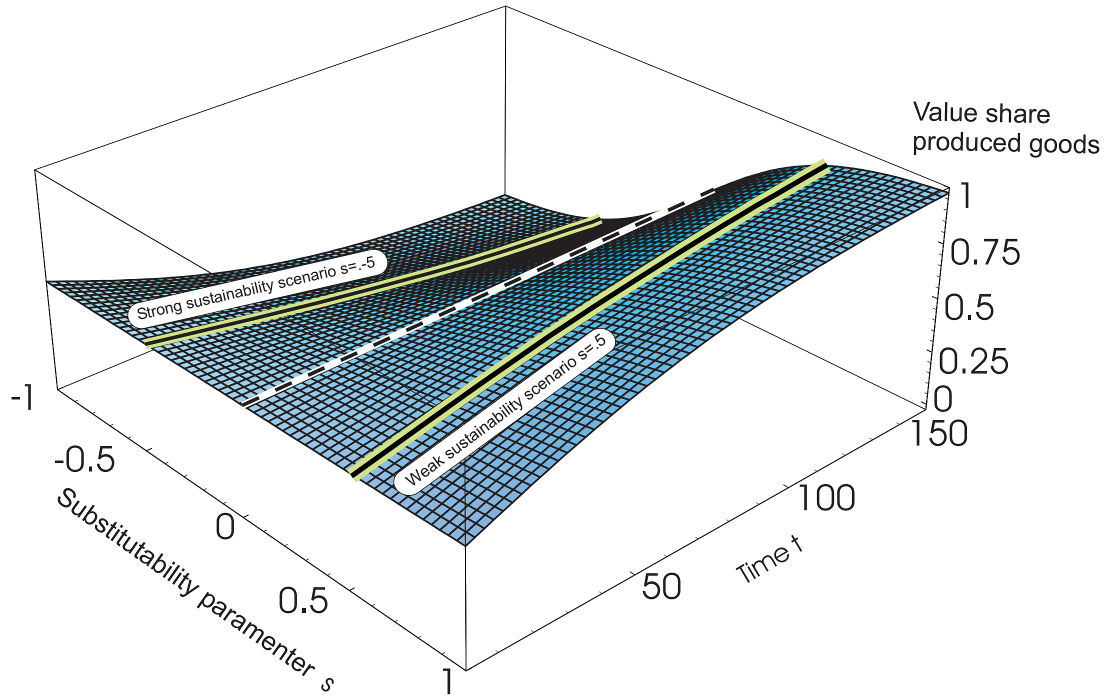
3.4 Explanation and Value Share

The key to the puzzle fleshed out in proposition 2 lies in the time development of value share and relates closely to an observation by Gerlagh and van der Zwaan [7]. The authors find in a comparable growth scenario that for *strongly limited substitutability* between the two classes of commodities, the *value share of man-made consumption goes to zero in the long run.*[23] Figure 3 depicts how the value share of the produced

[21]The proof even shows that in the long run $\frac{D_i^{\chi^{s<0}}(t,t_0)}{D_i^{\chi^{0<s<1}}(t,t_0)} \to 0$.

[22]Neumayer [16, 39] acknowledges Fisher and Krutilla's [4] approach of an (exogenous) correction of the discount rates for the produced and environmental goods to incorporate relative value development into the discount rate. However, Neumayer [16, 39] criticizes their approach for not getting at the heart of the *strong* sustainability paradigm. In a framework where discount rates are derived from the underlying welfare function, Fisher and Krutilla's [4] approach - featuring constant (positive/negative) corrections for the discount rates of the (produced/environmental) service and consumption streams - would correspond closest to the Cobb Douglas scenario at the border between the weak and the strong sustainability paradigm (see proposition 1).

[23]Precisely, Gerlagh and van der Zwaan [7] assume that produced consumption grows to infinity while environmental service streams are bounded. My analysis implies the same result building only



Development of the value share of produced consumption over time

Figure 3: Numerical example continued (same specifications as for figure 1). Drawn is the value share of the produced consumption stream. The thick lines correspond to the substitutability parameters used for the weak and strong sustainability preference scenario drawn in figures 1 and 2.

consumption stream evolves in the scenario underlying Figures 1 and 2. The value share of produced consumption grows for a weak sustainability scenario and falls for a strong sustainability scenario. Only for a welfare specification at the border of the two different regions ($s = 0$) does the value share stay constant over time (proposition 1i).

The value share is a combination of the amount consumed and its evaluation. In the analyzed growth scenario, the environmental service stream grows relatively scarce over time while produced consumption becomes relatively more abundant. At the same time, the limited substitutability causes a unit of environmental services to be increasingly more valuable than a unit of produced consumption. For weak sustainability

on the difference in growth rates (see proof of Proposition 1).

preferences (moderate substitutability), the relative physical scarcity of the environmental service stream dominates its value share development. Thus, the value share of environmental services declines, while that of produced consumption grows. For strong sustainability preferences (strongly limited substitutability), however, the increase in unit value dominates the relative physical scarcity in determining the value share of the environmental service stream. Therefore, the total amount of environmental services consumed becomes more valuable than the total amount of produced goods consumed. The value share of produced consumption declines to zero.

In the analyzed CES model, the substitutability effect in the social discount rate for the environmental good - i.e. the influence of x_2 on the value development of x_1 - is proportional to the value share of x_2. The lower the value share of produced consumption, the less influence has an increase in relative scarcity of environmental services with respect to produced consumption. In the limit of vanishing influence, where the decision maker only pays attention to the environmental good itself, the valuation for an extra unit of environmental services is *solely generated by the pure rate of time preference* ($\delta_1 = \rho$). The interaction between the two goods causing the substitutability effect and lowering the discount rate for the environmental service stream vanishes. On the other hand, in the weak sustainability scenario, produced consumption stays important for (marginal) welfare. Then, an increase in relative scarcity lowers the social discount rate proportional to the limitedness in substitutability.

4 Substitutability Effect and Absolute Growth Effect Interaction

4.1 The CES-CIES Setting

The preceding section has analyzed the substitutability effect in isolation. However, equation (7) has pointed out that, in general, the substitutability effect is a relative growth effect that works hand in hand with an absolute growth effect in augmenting or diminishing time preference. This section combines the CES specification for aggregation over goods (assumption 2) with the widespread CIES aggregation over time to bring the absolute growth effect back into play. Such a welfare specification is also used by Guesnerie [11] and Hoel and Sterner [12]. I show two different ways to decompose the resulting discount rate into a substitutability effect and an absolute growth effect. This new decomposition shows how the substitutability effect and its derived implications pointed out in the previous section carry over to the more general setting. Moreover, the decomposition is key for deriving and understanding a complete characterization of the parameter domains that lead to falling, constant, and increasing social rates of discount.

The welfare specification (6) can be transformed into a CES-CIES form by assuming that the good specific utility functions are coinciding power functions $u_1(z) = u_2(z) = z^{1-\alpha}$. In such an isoelastic setting, the order of aggregation specifying substitutability and growth response can be exchanged in the following sense. Define $s^* = (1-\alpha)s$ and let $A = \frac{1}{1-\alpha}$ in equation (6) to find that

$$U(x_1, x_2, t) = A\Big[a_1 x_1^{(1-\alpha)s} + a_2 x_2^{(1-\alpha)s}\Big]^{1/s} e^{-\rho t} = \frac{1}{1-\alpha}\left[a_1 x_1^{s^*} + a_2 x_2^{s^*}\right]^{\frac{1-\alpha}{s^*}} e^{-\rho t}. \quad (11)$$

While the parameter s characterizes substitutability between the two goods in (pergood-) *utility* terms, the parameter s^* (or the corresponding elasticity $\sigma^* = \frac{1}{1-s^*}$) characterizes substitutability between x_1 and x_2 in *real* terms. The special case of section 3 corresponds to $\alpha = 0$ so that s and s^* coincide.

Assumption 3: Welfare is representable in the functional form[24]

$$\mathcal{U} = \int_0^\infty \frac{[a_1 x_1^{s^*} + a_2 x_2^{s^*}]^{\frac{1-\alpha}{s^*}} - 1}{1-\alpha} e^{-\rho t}\, dt \tag{12}$$

with $a_1, a_2 \in \mathbb{R}_{++}, a_1 + a_2 = 1$, $s^* \in \mathbb{R}, s^* < 1$, and $\alpha \in \mathbb{R}_+$.

For this welfare specification equation (7) delivers the following discount rate for the environmental good (see appendix C)

$$\delta_1(t) = \rho + \alpha \hat{x}_1 - (1-s)(1-\alpha) V_2^{(1-\alpha)s} (\hat{x}_2 - \hat{x}_1) \tag{13}$$

$$= \rho + \alpha \left(V_1^{s^*} \hat{x}_1 + V_2^{s^*} \hat{x}_2 \right) - (1-s^*) V_2^{s^*} (\hat{x}_2 - \hat{x}_1) \tag{14}$$

and similarly for the produced consumption stream

$$\delta_2(t) = \rho + \alpha \hat{x}_2 + (1-s)(1-\alpha) V_1^{(1-\alpha)s} (\hat{x}_2 - \hat{x}_1) \tag{15}$$

$$= \rho + \alpha \left(V_1^{s^*} \hat{x}_1 + V_2^{s^*} \hat{x}_2 \right) + (1-s^*) V_1^{s^*} (\hat{x}_2 - \hat{x}_1) \,. \tag{16}$$

Equations (13) and (15) decompose the discount rates into the '*Eigengrowth effect*'

[24]As before for $s^* = 0$ the integrand is defined by limit using $\lim_{s^* \to 0} [a_1 x_1^{s^*} + a_2 x_2^{s^*}]^{1/s^*} = x_1^{a_1} x_2^{a_2}$. For $\alpha = 1$ the integrand is also defined by limit, yielding a logarithmic aggregation over time: $\lim_{\alpha \to 1} \frac{1}{1-\alpha} ([a_1 x_1^{s^*} + a_2 x_2^{s^*}]^{(1-\alpha)/s^*} - 1) = \frac{1}{s^*} \ln [a_1 x_1^{s^*} + a_2 x_2^{s^*}]$. The limit taking in α is made possible by the ordinal transformation $-\int_0^\infty \ln \frac{1}{1-\alpha} e^{-\rho t}\, dt$ of the original overall welfare function implied by equations (1) and (6) respectively (11). This transformation corresponds to the term '-1' in equation (12). Thus, whenever looking at equation (12) as a special case of the setting spelled out in equations (1) and (6) the value $\alpha = 1$ has to be excluded. For $s^* = 0$ and $\alpha = 0$ find $U(x_1, x_2, t) = [a_1 \ln x_1 + a_2 \ln x_2] e^{-\rho t}$. Welfare for $x_1 = 0$ or $x_2 = 0$ is defined by limit, using $\lim_{x_i \to 0} [a_i x_i^{s^*} + a_j x_j^{s^*}]^{\frac{1-\alpha}{s^*}} = \lim_{x_i \to 0} [a_i + a_j \frac{x_i}{x_j}^{-s^*}]^{\frac{1-\alpha}{s^*}} x_i^{1-\alpha} = \lim_{x_i \to 0} a_i^{\frac{1-\alpha}{s^*}} x_i^{1-\alpha}$ for $s < 0$.

and a relative growth or '*utility substitutability effect*' as pointed out in section 2.3. In the special case of a CES-CIES combination adopted here, I can transform the utility substitutability effect into a real substitutability effect by changing the order of aggregation as pointed out above. Then, equations (14) and (16) describe the decomposition of the social discount rate. Here, the first term can be interpreted as an '*overall growth effect*' and the second term as a relative growth or '*real substitutability effect*'. The overall growth effect is a value share weighted mean of the growth rates of the two goods. The real substitutability effect coincides with the one discussed in the previous section and is proportional to the value share of the other good and limitedness in substitutability.[25][26]

4.2 Results

Signing the utility substitutability effect in equations (13) and (15) and the real substitutability effect in equations (14) and (16) follows immediately from the above discussion.

Proposition 3: Let Assumptions 1 and 3 hold. Then, the social discount rates are given by equations (13) to (16).

1. i) For $\sigma^* < \sigma^{\text{int}}$ the *utility substitutability effect* $(1-s)(1-\alpha){V_i}^{(1-\alpha)s}(\hat{x}_2-\hat{x}_1)$ reduces the social discount rate for the environmental service stream and

[25]Note that for preference specification (12) the value share of good i is characterized in general by ${V_i}^{s^*} = \frac{a_i x_i^{s^*}}{a_1 x_1^{s^*} + a_2 x_2^{s^*}}$ (not by ${V_i}^s$). Only in the previous preference specification corresponding to assumption 2 it was $\alpha = 0$ and, thus, $s = s^*$.

[26]Observe that the 'utility substitutability effect' $(1-s)(1-\alpha){V_2}^{(1-\alpha)s}\big(\hat{x}_2 - \hat{x}_1\big)$ does not only depend on substitutability between the two good-specific utilities, but also on the propensity to smooth consumption over time characterized by α. From that perspective the decomposition into the 'real substitutability effect' and 'overall growth effect' in equations (14) and (16) yields a cleaner disentanglement of the two different effects generating social discounting.

increases the social discount rate for the produced consumption and service stream.[27]

ii) For $\sigma^* > \sigma^{\text{int}}$ the *utility substitutability effect* $(1-s)(1-\alpha)V_i^{(1-\alpha)s}(\hat{x}_2-\hat{x}_1)$ increases the social discount rate for the environmental service stream and reduces the social discount rate for the produced consumption and service stream.[28]

2. The *real substitutability effect* $(1-s^*)V_i^{s^*}(\hat{x}_2-\hat{x}_1)$ always reduces the social discount rate for the environmental service stream and always increases the social discount rate for the produced good.

Note that the form of the real substitutability effect is same as in the simplified setting of section 3. The following proposition analyzes the time behavior of the social discount rates.

Proposition 4: Let Assumptions 1 and 3 hold. The following time evolvement of the social discount rates prevails:

i) For $\sigma^* = 1$ resp. $s^* = 0$: In a steady state, both discount rates are constant over time.
In general, the discount rates are $\delta_1(t) = \rho + \alpha\Big(a_1\hat{x}_1(t) + a_2\hat{x}_2(t)\Big) - a_2\Big(\hat{x}_2(t)-\hat{x}_1(t)\Big)$ and $\delta_2(t) = \rho+\alpha\Big(a_1\hat{x}_1(t)+a_2\hat{x}_2(t)\Big)+a_1\Big(\hat{x}_2(t)-\hat{x}_1(t)\Big)$.

ii) For $\sigma^* \in (1,\infty)$ resp. $s^* \in (0,1)$: In a steady state, the real substitutability effect and the overall growth effect in equations (14) and (16) *grow* over

[27] In 'utility substitutability' terms s respectively σ rather than 'real substitutability' σ^* the condition $\sigma^* < \sigma^{\text{int}}$ translates into $(1-s)(1-\alpha) > 0$ or, equivalently, $(\sigma^{\text{int}}-1)\sigma > 0$. Note that while s^* is always positive σ can be negative.

[28] In 'utility substitutability' terms s respectively σ rather than 'real substitutability' σ^* the condition $\sigma^* > \sigma^{\text{int}}$ translates into $(1-s)(1-\alpha) < 0$ or, equivalently, $(\sigma^{\text{int}}-1)\sigma < 0$.

time in absolute terms. For

– $\sigma^* < \sigma^{\text{int}}$ both social discount rates fall over time.

– $\sigma^* = \sigma^{\text{int}}$ both social discount rates are constant over time.

– $\sigma^* > \sigma^{\text{int}}$ both social discount rates grow over time.

In general, the long-run discount rates approach the form

$\delta_1(t) = \rho + \alpha\,\hat{x}_2(t) - (1-s^*)\Big(\hat{x}_2(t) - \hat{x}_1(t)\Big)$ and $\delta_2(t) = \rho + \alpha\hat{x}_2(t)$.

iii) for $\sigma^* < 1$ resp. $s^* < 0$: In a steady state, the real substitutability effect and the overall growth effect *fall* over time in absolute terms. For

– $\sigma^* < \sigma^{\text{int}}$ both social discount rates grow over time.

– $\sigma^* = \sigma^{\text{int}}$ both social discount rates are constant over time.

– $\sigma^* > \sigma^{\text{int}}$ both social discount rates fall over time.

In general, the long-run discount rates approach the form

$\delta_1(t) = \rho + \alpha\,\hat{x}_1(t)$ and $\delta_2(t) = \rho + \alpha\,\hat{x}_1(t) + (1-s^*)\Big(\hat{x}_2(t) - \hat{x}_1(t)\Big)$.

The following intuition explains the time behavior of the discount rates. If $\sigma^* < \sigma^{\text{int}}$, i.e. between-good substitutability is more limited that intertemporal substitutability, then the real substitutability effect dominates the overall growth effect and, qualitatively, everything in scenarios *i*)-*iii*) is as discussed in the previous section. However, if $\sigma^{\text{int}} < \sigma^*$ the overall growth effect takes over. The overall growth effect is a value share weighted mean of the two growth rates. In the weak sustainability scenario *ii*) it grows as the produced good takes over the value share (approaching $\alpha\hat{x}_2$). Thus, the social discount rates grow over time. In the strong sustainability scenario *iii*) the overall growth effect declines as the environmental good takes over the value share (approaching $\alpha\hat{x_1}$). Therefore, both discount rates fall over time. For $\sigma^* = \sigma^{\text{int}}$ substitutability and growth effect balance each other and the discount rates simply become $\delta_1 = \rho + \hat{x}_1$

and $\delta_2 = \rho + \hat{x}_2$.

When only interested in classifying the regimes in which the discount rates grow respectively fall over time, a case discrimination in terms of utility substitutability turns out to be simpler.

Corollary 2: Let Assumptions 1 and 3 hold with $\alpha \neq 1$. In a steady state, for

– $\sigma > 1$ resp. $0 < s < 1$ both social discount rates *fall* over time.

– $\sigma = 1$ or $\sigma = \infty$ resp. $s = 0$ or $s = 1$ both social discount rates are *constant* over time.

– $\sigma < 1$ resp. $s < 0$ or $s > 1$ both social discount rates *grow* over time.

For $\sigma > 0$ coinciding with $s < 1$ the corollary is equivalent to proposition 1 with good-specific utility replacing real measurement units of the good. Thus, thinking of substitutability as happening between good-specific utility, the reasoning carried out in section 3 carries over to the more general setting of this section. The absolute growth effect is then 'neutralized' by measuring in terms of good-specific utility rather than real units. In addition, however, utility substitutability s can be greater than unity if $\alpha > 1$ ($\Leftrightarrow \sigma^{\text{int}} < 1$). Then, the overall welfare function becomes negative with (positive) good-specific utility decreasing in real consumption. In consequence, the CES function aggregates "disutility" and therefore the parameter s extends to a range that was not economically interesting for aggregating real consumption. In this $s > 1$ scenario social discount rates grow over time.

The utility substitutability specification (11) of welfare used in corollary 2 is not defined for $\alpha = 1$ (which corresponds to $s = \frac{s^*}{(1-\alpha)}$ going off to infinity). Introducing an ordinal transformation in equation (11) that allows for taking the according limit[29]

[29]The functional form $U(x_1, x_2, t) = \frac{1}{1-\alpha}\left[a_1 x_1^{(1-\alpha)s} + a_2 x_2^{(1-\alpha)s}\right]^{1/s} e^{-\rho t}$ does not converge for

yields $u(x_1, x_2) = a_1 \ln x_1 + a_2 \ln x_2$. Thus, the utility substitutability dependence drops out. In the real substitutability specification this case corresponds to $\alpha = 1$ and $s^* = 0$ implying $\sigma^* = \sigma^{\text{int}}$ and yielding the constant discount rates $\delta_1 = \rho + \hat{x}_1$ and $\delta_2 = \rho + \hat{x}_2$.

Relating proposition 4 to the literature, note that Guesnerie [11] has derived the long-run social discount rate limits in scenarios *ii*) and *iii*) for the special case where the environmental consumption and service stream is constant ($\hat{x}_1 = 0$) and the long run growth rate of the produced good is some positive constant ($\lim_{t\to\infty} \hat{x}_2(t) = \hat{x}_2^\infty$).[30] In a growth scenario where both classes of goods and their growth rates can change over time Hoel and Sterner [12, 272] show that the long-run limit of the social discount rate for the environmental good in the strong sustainability scenario *iii*) is $\delta_1 = \alpha\hat{x}_1$ whenever $\hat{x}_2 > \hat{x}_1$. Let me point out that the latter assumption is not quite enough for the result. While under $\hat{x}_2 > \hat{x}_1$ it can happen that the value share of environmental consumption converges to unity yielding the cited resuld, the condition is not sufficient. A sufficient condition is the ϵ difference between the two growth rates in assumption 1. This nuance would probably not be worth mentioning if the ambiguous scenario where $\hat{x}_2(t) > \hat{x}_1(t)$ but $\nexists \epsilon > 0$ such that $\hat{x}_2(t) > \hat{x}_1(t) + \epsilon\, \forall t$ would not be common to growth models where the growth rates of different goods end up converging to a common steady state growth rate.[31]

$\alpha \to 1$. See footnote 24 for the necessary ordinal transformation '$\frac{-1}{1-\alpha}$'.

[30] In his proposition 4 Guesnerie [11] extends his result on the long run social discount rate for the environmental good to account for environmental degradation. He does this without further calculation and gets the strong sustainability scenario *iii*) which he calls 'présence de blocage écologique' wrong. Instead of $\delta_1 = \rho + \alpha\hat{x}_1$ he finds $\delta_1 = \rho + (1 - s^*)\hat{x}_1$. To compare his setting with mine note that Guesnerie [11] assumes a zero rate of pure time preference and my $(\alpha, \sigma^*, \hat{x}_1, \hat{x}_2)$ correspond to his $(\sigma', \sigma, g^*, -g')$.

[31] From $\hat{x}_2(t) > \hat{x}_1(t)$ alone it does not follow that $\lim_{t\to\infty} \int_0^t \hat{x}_2(t') - \hat{x}_1(t')\, dt'$ goes to infinity. If $\lim_{t'\to\infty} \hat{x}_2(t') - \hat{x}_1(t') \to 0$ the integral *can* converge to a finite value. Then, equation (20) in the appendix shows that the value share does not converge to unity respectively zero and the long-run limits in proposition 4, including the one pointed out by Hoel and Sterner [12], do not hold. This possibility has been overlooked in the corresponding proof in Hoel and Sterner [12, Appendix C].

4.3 The Two Sustainability Paradigms Revisited

At a *given point of time* the social discount rates for the different scenarios reflect the different concepts of sustainability. Here, the *difference in evaluation* between an extra unit of environmental services and an extra unit of produced consumption *increases in* the *relative scarcity* of the environmental service as well as in the *limitedness in substitutability.* This fact is observed by taking the difference between equations (14) and (16) yielding

$$\delta_2(t) - \delta_1(t) = (1 - s^*)\left(\hat{x}_2(t) - \hat{x}_1(t)\right) . \tag{17}$$

The difference in social discount rates in equation (17) generates a relative difference in weights given to the consumption streams at time t corresponding to

$$\begin{aligned}\frac{D_1^{\mathrm{X}}(t,t_0)}{D_2^{\mathrm{X}}(t,t_0)} &= \exp\left(-\int_{t_0}^{t} \delta_1(t') - \delta_2(t')\,dt'\right) \\ &= \exp\left(\int_{t_0}^{t} (1-s^*)\left(\hat{x}_2(t') - \hat{x}_1(t')\right)\,dt'\right) .\end{aligned} \tag{18}$$

A *stronger notion of sustainability* corresponds to a reduced substitutability in the welfare function. As equations (17) and (18) show, such an increase in $(1 - s)$ implies an *increase in the weight given to environmental services as opposed to produced consumption.* Moreover, the difference in weight is monotonically growing in the time distance $t - t_0$ as relative scarcity of the environmental service stream increases.

However, the two preceding sections have derived a second implication of differentiating weak and strong sustainability through a parametrization of the substitutability between environmental services and produced consumption streams. A *stronger notion of sustainability* can result in a *reduced weight given to the future* as opposed to the present. Whenever environmental services or both consumption streams are declining over time ($\hat{x}_1 < \min\{\hat{x}_2, 0\}$), such a reduced attention paid to future service and consumption streams seems to oppose the fundamental objective of a sustainable development as expressed in the Brundtland report (see Section 2.4). I offer two alternative perspectives on the notion of weak versus strong sustainability in the light of these findings.

From the first or *'temporal' perspective*, the notions of *strong* versus *weak* sustainability only relate to the substitutability between the different classes of goods and services. Thus, the concepts are concerned *only* with distributing weight between produced and environmental goods at a given *point of time*. Then, a strong sustainability preference attaches relatively more weight to environmental as opposed to produced goods than does a weak sustainability preference. Attaching a relatively higher weight to the scarcer environmental goods through an according specification of substitutability can come at the cost of shifting weight from the future to the present. Then, a 'weak sustainability preference' in a growth scenario satisfying assumption 1 can correspond to a stronger sustainability demand than a 'strong sustainability preference' in the intertemporal sense that a higher weight is given to long-run future consumption and service streams.

From the second or *'intertemporal' perspective*, strong sustainability should not only give more relative weight to a scarce environmental good, but also pay more (or at least as much) attention to long-run service and consumption streams than does a weak sustainability preference. In the following I provide the basis for a corresponding concept of strong and weak sustainability. In a first step I give a formal characterization of the two sustainability notions employed so far. In a second step I add the requirement that for any sustainability preference the social discount rates should not increase over time. For such a refined definition I then compare the weights that weak and strong sustainability give to long-run environmental service streams.

The formal characterization of weak and strong sustainability makes use of

Definition 1: Let a preference be expressed by a welfare function of form (1).

1) The good x_i [strongly] limits welfare if and only if for all $t \in [0, \infty)$ there exist $\bar{x}_i, x_i^*, x_j^* \in \mathbb{R}_+$ [$\mathbb{R}_{++}$] such that $U(\bar{x}_i, x_j, t) \leq U(x_i^*, x_j^*, t) \forall x_j \in \mathbb{R}$.[32]

2) Mixed consumption is preferred if and only if the upper contour sets ('better sets') are strictly convex for all $x_1, x_2 > 0$ and $t \in [0, \infty)$.[33]

If a good or service stream limits welfare, there exists a level $\bar{x}_i$ that restricts the maximal obtainable welfare no matter how much of the other good will be consumed. This criteria lies at the heart of the strong sustainability concern. If the limiting consumption level $\bar{x}_i$ is strictly positive I say that good x_i strongly limits welfare. If the only consumption level limiting welfare in the above sense is $\bar{x}_i = 0$ the good limits welfare but does not strongly limit welfare. I use this distinction to tell apart the Cobb-Douglas scenario from the other sustainability scenarios. The second part of the definition is the standard formalization of a preference for mixtures (a quasiconcave utility function).

Definition 2: 1) A preference is called a *strong sustainability preference* if and only if the environmental service stream x_1 strongly limits welfare.

2) A preference is called a *weak sustainability preference* if and only if the environmental service stream x_1 does not limit welfare and mixed consumption is preferred.

[32] In the context of a stationary utility function as used below it does not matter whether the requirement is 'for all $t \in [0, \infty)$' or 'for some t'.

[33] I.e. for U twice differentiable $\frac{d^2 x_2}{dx_1^2}\big|_{\bar{U}} > 0$. Then it becomes increasingly less pleasant to replace a unit of x_1 by x_2 the smaller is x_1.

3) A (weak/strong) sustainability preference is called a *strict* (weak/strong) *sustainability preference* if and only if in any steady state growth scenario satisfying assumption 1 the social discount rates for the scarce environmental good are non-increasing over time.

The characterization of a strong and a weak sustainability preference in parts 1) and 2) of the definition is tailored to the isoelastic setting. Note that the condition that mixed consumption is preferred could be added to the definition of strong sustainability, however, in the isoelastic context it is already implied by the condition that x_1 limits welfare. It is part 3) of the definition on which I would like to focus. It adds the intertemporal perspective to the definitions of a weak and a strong sustainability preference. In this context note that Pezzey [20] tags falling discount rates as 'sustainable discount rates'. The following corollary first shows that definitions 2-1) and 2-2) confirm (or match) my earlier use of weak and strong sustainability and then points out an immediate consequence of proposition 4 with respect to strict sustainability.

Corollary 3: Let a preference be represented in a form satisfying assumption 3 with $s^* \in \mathbb{R}$.

1. It is a weak sustainability preference if and only if $1 < \sigma^*$ resp. $1 > s^* > 0$ and a strong sustainability preference if and only if $0 < \sigma^* < 1$ resp. $s^* < 0$.
2. It is a strict weak sustainability preference if and only if $1 < \sigma^* < \sigma^{\text{int}}$ resp. $1 > s^* > \max\{0, 1 - \alpha\}$.
3. It is a strict strong sustainability preference if and only if $\sigma^{\text{int}} < \sigma^* < 1$ resp. $s^* < \min\{0, 1 - \alpha\}$.

The 'Cobb-Douglas' preference scenario where $\sigma^* = 1$ could be attributed to any of

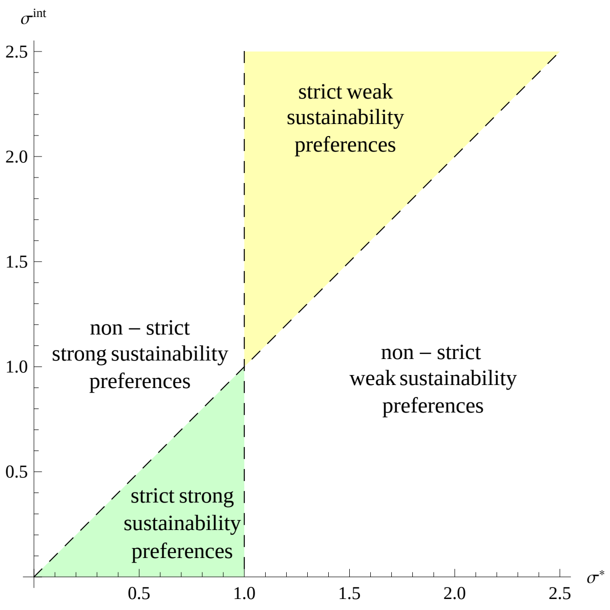


Figure 4: Sustainability Regions in the s^*-σ^{int} Plane

the weak, the strong, and the strict scenarios by marginal changes in definition 2.[34] Figure 4 reflects the set of strict sustainability preferences as shaded triangles in the σ^*-σ^{int} plane. The lower left triangle (dark shade resp. green) characterizes the strict strong sustainability preferences while the upper right triangle (light shade resp. yellow) characterizes the strict weak sustainability preferences. These shaded areas are also the only regions where the social discount rates fall over time.

The intuition for the result builds on proposition 4. For a weak sustainability preference social discount rates are only falling if the substitutability effect dominates. Otherwise, the overall growth effect causes discount rates to increase as the value share shifts toward the aggregate of produced goods. In the strong sustainability scenario

[34]Changing 'strongly limiting welfare' to 'limiting welfare' in definition 2-1) would add the $\sigma^* = 1$ case to the (strict) strong sustainability scenario. Changing 'does not limit welfare' to 'does not strongly limit welfare' in definition 2-2) would add the $\sigma^* = 1$ case to the (strict) weak sustainability scenario. Requiring the social discount rates to decrease strictly over time in definition 2-3) would eliminate the $\sigma = 1$ case from any of the strict sustainability regions.

the discount rates only fall if the overall growth effect dominates. Here the value share of the environmental goods increases over time. Thus, the overall growth effect pays increasing attention to the slower growing environmental good and makes discount rates decline. On the other hand, a dominating substitutability effect would cause the discount rates to increase as discussed in section 3.

Proposition 5: Let assumptions 1 and 3 hold. Pick an arbitrary strict weak sustainability preference ($1 < \sigma^* < \sigma^{\text{int}}$) and an arbitrary strict strong sustainability preference ($\sigma^{\text{int}} < \sigma^* < 1$).

i) For $\hat{x}_1 = 0$ the strict weak sustainability preference gives more weight to environmental services at every point in time in the future, that is $D_1^{x\text{strict weak}}(t, t_0) > D_1^{x\text{strict strong}}(t, t_0)$.

ii) For $\hat{x}_1, \hat{x}_2 < 0$ the strict strong sustainability preference gives more weight to long-run environmental services, that is for any t_0 there exists $\bar{t}$ such that $D_1^{x\text{strict strong}}(t, t_0) > D_1^{x\text{strict weak}}(t, t_0)\ \forall\, t > \bar{t}$.

iii) Let $\hat{x}_1 < 0$ and $\hat{x}_2 > 0$.

For $\hat{x}_1$ and $\hat{x}_2$ small enough (i.e. $|\hat{x}_1|$ large enough), the strict strong sustainability preference gives more weight to long-run environmental services, that is for any t_0 and any bounded $\hat{x}_2 > 0$ [$\hat{x}_1 < 0$] there exist $\bar{t}$ and $\bar{\hat{x}}_1 > 0$ [$\bar{\hat{x}}_2 < 0$] such that for growth rates satisfying $\hat{x}_1(t) < \bar{\hat{x}}_1$ [$\hat{x}_2(t) < \bar{\hat{x}}_2$] for all t it holds $D_1^{x\text{strict strong}}(t, t_0) > D_1^{x\text{strict weak}}(t, t_0)\ \forall\, t > \bar{t}$.

For $\hat{x}_1$ and $\hat{x}_2$ large enough (i.e. $|\hat{x}_1|$ small enough), the strict weak sustainability preference gives more weight to long-run environmental services, that is for any t_0 and any bounded $\hat{x}_2 > 0$ [$\hat{x}_1 < 0$] there exist $\bar{t}$ and $\bar{\hat{x}}_1 > 0$ [$\bar{\hat{x}}_2 < 0$] such that for growth rates satisfying $\hat{x}_1(t) > \bar{\hat{x}}_1$ [$\hat{x}_2(t) > \bar{\hat{x}}_2$] for all t it holds $D_1^{x\text{strict weak}}(t, t_0) > D_1^{x\text{strict strong}}(t, t_0)\ \forall\, t > \bar{t}$.

For a constant flow of environmental services preferences in the strict weak sustainability domain will generally give more weight to future environmental service streams than preferences in the strict strong sustainability domain (case i). However, advocates of a strong sustainability paradigm are more likely concerned about a scenario where environmental goods and services decline. Case ii) of proposition 5 analyzes an overall decline where the environment decays at a faster rate than produced consumption. For such a (negative) growth scenario there is a point of time $\bar{t}$, corresponding to some relative scarcity level of environmental services, where preferences in the strict strong sustainability domain start giving higher weights to environmental service streams than preferences in the strict weak sustainability domain. Case iii) of proposition 5 analyzes the case where the environmental good and service stream declines, while produced consumption keeps increasing over time. In such a scenario there is no general relation between the weights given to future environmental service streams in the two differing preference regions. However, the proposition states that for any given pair of preferences there exists a rate of decline of environmental services such that the strict strong sustainability preference gives more weight to long-run environmental services than the strict weak sustainability preferences. The same holds true if produced consumption grows sufficiently slow. The statement expresses that whenever the scenario gets sufficiently bad (that is either of the growth rates gets sufficiently low) there will be a point in time $\bar{t}$, corresponding to some relative scarcity level of environmental services, from which on the strict strong sustainability preferences pay more attention to long-run environmental service streams.

The confinement to strict sustainability preferences yields non-increasing social discount rates. Moreover, proposition 5 shows that this confinement yields some reasonable results for the relation between weak and strong sustainability preferences and the long-run weights given to environmental service streams. It is left to the reader to decide whether – or under which circumstances – these relation are enough to capture and distinguish preferences of the advocates of a weak sustainability paradigm from those of a strong sustainability paradigm. The intention of this paper has been to flesh out the consequences for intertemporal weight distribution that are caused by distinguishing weak and strong sustainability on the basis of between-good substitutability.

5 Conclusions

A characterization of value development over time is necessary to evaluate long-term projects. Such a characterization is given by social discount rates that allow the economist to think in rates and elasticities, and lay out different contributions that generate value development in a convenient additive form. This study elaborates one such contribution to value development over time that emerges in a multi-commodity world with limited substitutability between different forms of consumption. I decomposed the resulting determinants of the social discount rates into a utility substitutability effect and an Eigengrowth effect or, alternatively, a real substitutability effect and an overall growth effect. I signed the effect of the different contributions and analyzed the resulting development of social discount rates over time.

For the cases of perfect substitutability, Cobb-Douglas type preferences, or coinciding elasticities of intertemporal and between-good substitution the social discount rates are constant in a steady state. For all other isoelastic preference specifications the social discount rates are non-constant, even in a steady state (defined by constant but differing growth rates). I gave a complete classification of the parameter domains yielding qualitatively different time behavior of the social discount rates. In particular, if the real substitutability effect dominates the overall growth effect, a decrease in between-good substitutability can cause an increase in long-run discount rates. This finding opposes a belief held in the literature that a decrease in between-good substitutability always increases the weight given to long-run environmental service streams (in growth scenarios where environmental service flows decline over time in absolute and/or relative terms). An immediate implication of the finding is that a stronger notion of sustainability, in the sense of a lower substitutability in welfare between environmental and produced goods, can shift weight away from future generations into the present.

I introduce a notion of weak and strong sustainability that adds an intertemporal component to the 'intratemporal' concern of between-good substitutability. The corresponding definition requires that social discount rates for the environmental service stream must not increase over time in growth scenarios that exhibit a relative decline of environmental services. An according characterization for isoelastic preferences shows that, for this purpose, in the strong sustainability domain intertemporal substitutability has to be more limited than between-good substitutability. In the weak sustainability domain between-good substitutability has to be more limited than intertemporal substitutability. I show that such a refinement of the definitions of weak and strong sustainability partly recovers the desideratum that long-run weights for declining environmental consumption streams should not be lower in a strong sustainability setting than in a weak sustainability setting.

.

References

[1] Arrow, K., Chenery, H., Minhas, B. and Solow, R. [1961], 'Capital–labor substitution and economic efficiency', *The Review of Economics and Statistics* **43**(3), 225–

248.

[2] Arrow, K., Cline, W., Maler, K.-G., Munasinghe, M., Squitieri, R. and Stiglitz, J. [1995], Intertemporal equity, discounting, and economic efficiency, *in* J. Bruce, H. Lee and E. Haites, eds, 'Climate Change - Economic and Social Dimensions of Climate Change', Cambridge University Press, Cambridge, chapter 4, pp. 128–44.

[3] Dasgupta, P. [2001], *Human Well-Being and the Natural Environment*, Oxford University Press, New York.

[4] Fisher, A. C. and Krutilla, J. V. [1975], 'Resource conservation, environmental preservation, and the rate of discount', *Quarterly Journal of Economics* **89**(3), 358–370.

[5] Fisher, A. C., Krutilla, J. V. and Cicchetti, C. J. [1972], 'The economics of environmental preservation: A theoretical and empirical analysis', *The American Economic Review* **62**(4), 605–619.

[6] Frederick, S., Loewenstein, G. and O'Donoghue, T. [2002], 'Time discounting and time preference: A critical review', *Journal of Economic Literature* **40**, 351–401.

[7] Gerlagh, R. and van der Zwaan, B. [2002], 'Long-term substitutability between environmental and man-made goods', *Journal of Environmental Economics and Management* **44**, 2002.

[8] Goldstein, H. [1980], *Classical Mechanics*, 2 edn, Addison-Wesley, Reading.

[9] Gollier, C. [2002], 'Time horizon and the discount rate', *Journal of Economic Theory* **75**, 229–253.

[10] Groom, B., Hepburn, C., Koundouri, P. and Pearce, D. [2005], 'Discounting the future: the long and the short of it', *Environmental and Resource Economics* **31**(1), 445–493.

[11] Guesnerie, R. [2004], 'Calcul économique et développement durable', *Revue économique* **55**(3), 363–382.

[12] Hoel, M. and Sterner, T. [2007], 'Discounting and relative prices', *Climatic Change* **84**, 265–280.

[13] Krutilla, J. V. [1967], 'Conservation reconsidered', *The American Economic Review* **57**(4), 777–786.

[14] Laibson, D. [1997], 'Golden eggs and hyperbolic discounting', *The Quaterly Journal of Economics* **112**(2), 443–477.

[15] Malinvaud, E. [1974], *Lectures on Microeconomic Theory*, 3 edn, North-Holland, Amsterdam.

[16] Neumayer, E. [1999*a*], 'Global warming: discounting is not the issue, but substitutability is', *Energy Policy* **27**, 33–43.

[17] Neumayer, E. [1999*b*], *Weak versus Strong Sustainability - Exploring the Limits of Two Opposing Paradigms*, Edward Elgar, Cheltenham.

[18] Nordhaus, W. D. [1994], *Managing the Global Commons: The Economics of the Greenhouse Effect*, MIT Press, Cambridge.

[19] Pearce, D., Markandya, A. and Barbier, E. B. [1997], *Blueprint for a Green Economy*, Earthscan Publications, London.

[20] Pezzey [2006], Reconsidering reconsidered: Why sustainable discounting need not be inconsistent over time, *in* D. Pannell and S. Schilizzi, eds, 'Discounting and Discount Rates in Theory and Practice', Edward Elgar, Cheltenham, chapter 6. Forthcoming.

[21] Sakurai, J. [1985], *Modern quantum mechanics*, Addison-Wesley, Reading.

[22] Sterner, T. and Persson, M. [2008], 'An even sterner review: Introducing relative prices into the discounting debate', *Review of Environmental Economics and Policy* **2**(1), 61–76.

[23] Traeger, C. P. [2007], 'Sustainability, limited substitutability and non-constant social discount rates', *Department of Agricultural and Resource Economics, UCB. CUDARE Working Paper 1045* . Section 4, which had to be cut for the present publication.

[24] Treasury, H. [2003], *The Green Book – Appraisal and Evaluation in Central Government*, HM Treasury, London.

[25] van den Bergh, J. C. and Hofkes, M. W. [1998], A survey of economic modelling of sustainable development, *in* J. C. van den Bergh and M. W. Hofkes, eds, 'Theory and Implementation of Economic Models for Sustainable Development', Kluwer, Dordrecht, chapter 2, pp. 11–38.

[26] WCED [1987], 'Our common future', *UN General Assembly Document* **A/42/427**.

[27] Weikard, H.-P. and Zhu, X. [2005], 'Discounting and environmental quality: When should dual rates be used?', *Economic Modelling* **22**, 868–878.

Online Appendix

A Calculations for Section 2

Calculation of the social discount rate for

$U(x_1, x_2, t) = A[a_1 u_1(x_1)^s + a_2 u_2(x_2)^s]^{\frac{1}{s}} e^{-\rho t}$ **:**

The derivatives needed for the computation of δ_1 are for $s \notin \{0, 1\}$:

$$\begin{aligned}
\frac{\partial U}{\partial x_1} &= A a_1 u_1(x_1)^{s-1} u_1'(x_1) \Big[a_1 u_1(x_1)^s + a_2 u_2(x_2)^s\Big]^{\frac{1}{s}-1} e^{-\rho t} \,, \\
\frac{\partial^2 U}{\partial x_1^2} &= A\Big(a_1 u_1(x_1)^{s-1} u_1''(x_1) - (1-s) a_1 u_1(x_1)^{s-2} u_1'(x_1)^2\Big) \\
&\quad \cdot \Big[a_1 u_1(x_1)^s + a_2 u_2(x_2)^s\Big]^{\frac{1}{s}-1} \cdot e^{-\rho t} \\
&\quad + (1-s)\Big(a_1 u_1(x_1)^{s-1}\Big)^2 u_1'(x_1)^2 \Big[a_1 u_1(x_1)^s + a_2 u_2(x_2)^s\Big]^{\frac{1}{s}-2} e^{-\rho t} \quad \text{and} \\
\frac{\partial^2 U}{\partial x_1 \partial x_2} &= A(1-s)\Big(a_1 u_1(x_1) a_2 u_2(x_2)\Big)^{s-1} u_1'(x_1) u_2'(x_2) \\
&\quad \cdot \Big[a_1 u_1(x_1)^s + a_2 u_2(x_2)^s\Big]^{\frac{1}{s}-2} e^{-\rho t} \,.
\end{aligned}$$

Inserting these into equation (5) yields:

$$\begin{aligned}
\delta_1(t) &= \rho - \frac{(a_1 u_1(x_1)^{s-1} u_1''(x_1) - (1-s) a_1 u_1(x_1)^{s-2} u_1'(x_1)^2)\, [a_1 u_1(x_1)^s + a_2 u_2(x_2)^s]^{\frac{1-s}{s}}}{a_1 u_1(x_1)^{s-1} u_1'(x_1) [a_1 u_1(x_1)^s + a_2 u_2(x_2)^s]^{\frac{1}{s}-1}} \cdot \dot{x}_1 \\
&\quad - \frac{(1-s)(a_1 u_1(x_1)^{s-1})^2 u_1'(x_1)^2 [a_1 u_1(x_1)^s + a_2 u_2(x_2)^s]^{\frac{1}{s}-2}}{a_1 u_1(x_1)^{s-1} u_1'(x_1) [a_1 u_1(x_1)^s + a_2 u_2(x_2)^s]^{\frac{1}{s}-1}} \,\dot{x}_1 \\
&\quad - \frac{(1-s)\,(a_1 u_1(x_1) a_2 u_2(x_2))^{s-1}\, u_1'(x_1) u_2'(x_2) [a_1 u_1(x_1)^s + a_2 u_2(x_2)^s]^{\frac{1}{s}-2}}{a_1 u_1(x_1)^{s-1} u_1'(x_1) [a_1 u_1(x_1)^s + a_2 u_2(x_2)^s]^{\frac{1}{s}-1}} \,\dot{x}_2 \\
&= \rho - \frac{u_1''(x_1)}{u_1'(x_1)} \,\dot{x}_1 + (1-s) u_1(x_1)^{-1} u_1'(x_1) \,\dot{x}_1 \\
&\quad - (1-s) \frac{a_1 u_1(x_1)^{s-1} u_1'(x_1)}{a_1 u_1(x_1)^s + a_2 u_2(x_2)^s} \,\dot{x}_1 - (1-s) \frac{a_2 u_2(x_2)^{s-1} u_2'(x_2)}{a_1 u_1(x_1)^s + a_2 u_2(x_2)^s} \,\dot{x}_2
\end{aligned}$$

$$
\begin{aligned}
&= \rho - \frac{u_1''(x_1)}{u_1'(x_1)}\,\dot{x}_1 \\
&\quad +(1-s)\frac{u_1(x_1)^{-1}u_1'(x_1)(a_1u_1(x_1)^s + a_2u_2(x_2)^s) - a_1u_1(x_1)^{s-1}u_1'(x_1)}{a_1u_1(x_1)^s + a_2u_2(x_2)^s}\,\dot{x}_1 \\
&\quad -(1-s)\frac{a_2u_2(x_2)^{s-1}u_2'(x_2)}{a_1u_1(x_1)^s + a_2u_2(x_2)^s}\,\dot{x}_2 \\
&= \rho - \frac{u_1''(x_1)}{u_1'(x_1)}\,\dot{x}_1 + (1-s)\frac{a_2u_2(x_2)^s}{a_1u_1(x_1)^s + a_2u_2(x_2)^s}\,\frac{u_1'(x_1)}{u_1(x_1)}\,\dot{x}_1 \\
&\quad -(1-s)\frac{a_2u_2(x_2)^s}{a_1u_1(x_1)^s + a_2u_2(x_2)^s}\,\frac{u_2'(x_2)}{u_2(x_2)}\,\dot{x}_2\ .
\end{aligned}
$$

Which brings about equation (7):

$$
\delta_1(t) = \rho - \frac{u_1''(x_1)}{u_1'(x_1)}\dot{x}_1 - (1-s)\frac{a_2u_2(x_2)^s}{a_1u_1(x_1)^s + a_2u_2(x_2)^s}\left(\frac{u_2'(x_2)}{u_2(x_2)}\dot{x}_2 - \frac{u_1'(x_1)}{u_1(x_1)}\dot{x}_1\right).
$$

For $s = 1$ with $\frac{\partial^2 U}{\partial x_1 \partial x_2} = 0$ and $1 - s = 0$ it is easily observed that the same equation has to hold. For the case $s = 0$ it is $u(x_1, x_2) = u(x_1)^{a_1}u(x_2)^{a_2}$. The derivatives needed for the computation of δ_1 are

$$
\frac{\partial U}{\partial x_1} = Aa_1u_1(x_1)^{a_1-1}u_1'(x_1)u_2(x_2)^{a_2}e^{-\rho t}\ ,
$$

$$
\begin{aligned}
\frac{\partial^2 U}{\partial x_1^2} &= Aa_1(a_1-1)u_1(x_1)^{a_1-2}u_1'(x_1)^2u_2(x_2)^{a_2}e^{-\rho t} \\
&\quad +a_1u_1(x_1)^{a_1-1}u_1''(x_1)u_2(x_2)^{a_2}e^{-\rho t} \qquad \text{and} \\
\frac{\partial^2 U}{\partial x_1 \partial x_2} &= Aa_1u_1(x_1)^{a_1-2}u_1'(x_1)a_2u_2(x_2)^{a_2-1}u_2'(x_2)e^{-\rho t}\ .
\end{aligned}
$$

These derivatives deliver the social discount rate

$$
\delta_1(t) = \rho - \frac{u_1''(x_1)}{u_1'(x_1)}\dot{x}_1 - a_2\left(\frac{u_2'(x_2)}{a_2u_2(x_2)}\dot{x}_2 - \frac{u_1'(x_1)}{a_1u_1(x_1)}\dot{x}_1\right)
$$

which coincides with equation (7) for $s = 0$ as $a_1 + a_2 = 1$.

B Proofs for Section 3

Proof of proposition 1: By Assumtpion 1, all of the terms in equations (8) and (10) are positive. Therefore, the verbal statements in the proposition concerning the reduction/increase and its proportionality merely summarize the equations.[35]

For cases *ii*) and *iii*), the proof makes use of a transformation of the value share V_2^s. This transformation employs the relation

$$\begin{aligned} \frac{d \ln x_i(t)}{dt} &= \frac{\dot{x}_i(t)}{x_i(t)} \\ \Rightarrow \quad \ln x_i(t) &= \int_0^t \hat{x}_i(t')\, dt' + c \\ \Rightarrow \quad x_i(t) &= x_i(0) e^{\int_0^t \hat{x}_i(t')\, dt'} \\ \Rightarrow \quad x_i(t)^s &= x_i(0)^s e^{s \int_0^t \hat{x}_i(t')\, dt'} \,. \end{aligned} \tag{19}$$

Using equation (19) the value share V_2^s can be transformed to

$$\begin{aligned} V_2^s(x_1, x_2) &= \frac{a_2 x_2(0)^s e^{s \int_0^t \hat{x}_2(t')\, dt'}}{a_1 x_1(0)^s e^{s \int_0^t \hat{x}_1(t')\, dt'} + a_2 x_2(0)^s e^{s \int_0^t \hat{x}_2(t')\, dt'}} \\ &= \frac{1}{\frac{a_1 x_1(0)^s}{a_2 x_2(0)^s} \frac{e^{s \int_0^t \hat{x}_1(t')\, dt'}}{e^{s \int_0^t \hat{x}_2(t')\, dt'}} + 1} = \frac{1}{\frac{a_1 x_1(0)^s}{a_2 x_2(0)^s} e^{-s \int_0^t \hat{x}_2(t') - \hat{x}_1(t')\, dt'} + 1} \,. \end{aligned} \tag{20}$$

Case i: Given that the utility weights a_1 and a_2 sum to unity, it is $V_2^0 = a_2$ and $V_1^0 = a_1$, which yields the equations stated in the proposition. In a steady state, also $\hat{x}_1$ and $\hat{x}_2$ are constant over time and, thus, the social discount rates are constant.

Case ii: *First*, I show that V_2^s is strictly increasing. By Assumption 1, it is $\hat{x}_2(t) - \hat{x}_1(t) > 0 \,\forall\, t$. As derived above, equation (20) holds. For $s > 0$ the expression

[35] This is the only statement in Section 3 which does not necessarily hold true if Assumption 1 is relaxed to the form stated in footnote 12. Then, for $t < t^*$ the term $(\hat{x}_2(t) - \hat{x}_1(t))$ can flip sign.

$\frac{a_1 x_1(0)^s}{a_2 x_2(0)^s} e^{-s \int_0^t \hat{x}_2(t') - \hat{x}_1(t')\, dt'}$ is strictly falling in time. Therefore, the value share of the produced consumption stream $V_2{}^s$ is strictly increasing over time.

Second, in a steady state, such an increasing $V_2{}^s$ causes the second term in the social discount rate for the environmental service stream $(1-s)V_2{}^s\,(x_1(t), x_2(t))\,(\hat{x}_2 - \hat{x}_1)$ to increase over time. As this term is subtracted from the constant rate of pure time preference, the social discount rate for the first commodity class $\delta_1(t)$ *declines* in a steady state.

Third, a strictly increasing term $V_2{}^s$ implies a strictly decreasing value share of the environmental service stream $V_1{}^s = 1 - V_2{}^s$. In a steady state, such a strictly decreasing term $V_1{}^s$ implies that the expression $(1-s)V_1{}^s\,(x_1(t), x_2(t))\,(\hat{x}_2 - \hat{x}_1)$ strictly decreases. This expression is added to the constant rate of pure time preference to yield the social discount rate for the produced consumption stream. Thus, the social discount rate $\delta_2(t)$ *declines* as well in a steady state.

Finally, by Assumption 1 there exists $\epsilon > 0$ such that $\hat{x}_1(t) < \hat{x}_2(t) - \epsilon\, \forall\, t$. In consequence, the expression $\lim_{t\to\infty} \frac{a_1 x_1(0)^s}{a_2 x_2(0)^s} e^{-s \int_0^t \hat{x}_2(t') - \hat{x}_1(t')\, dt'}$ falls to zero and the value share $V_2{}^s$ grows to unity.[36] Therefore, in a steady state the discount rate δ_1 monotonously falls to $\delta_1 = \rho - (1-s)(\hat{x}_2 - \hat{x}_1)$ for $t \to \infty$. In general, it approaches the form $\delta_1(t) = \rho - (1-s)(\hat{x}_2(t) - \hat{x}_1(t))$. At the same time the value share of the environmental service stream $V_1{}^s$ falls to zero, implying that the social discount rate for the produced consumption stream falls to $\delta_2 = \rho$.

Case iii: *First*, I show that $V_2{}^s$ is strictly decreasing. From $\hat{x}_2(t) - \hat{x}_1(t) > 0\, \forall\, t$. As derived above, equation (20) holds. For $s < 0$ the expression $\frac{a_1 x_1(0)^s}{a_2 x_2(0)^s} e^{-s \int_0^t \hat{x}_2(t') - \hat{x}_1(t')\, dt'}$ is strictly increasing in time. Therefore, the value share of the produced consumption stream $V_2{}^s$ strictly decreases over time.

[36] Note that already the slightly weaker assumption in footnote 12 assures this limit.

Second, such a decreasing ${V_2}^s$ implies that the second term in the social discount rate for the environmental service stream $(1-s){V_2}^s\,(x_1(t), x_2(t))\,(\hat{x}_2 - \hat{x}_1)$ is decreasing in a steady state. As this term is subtracted from the constant rate of pure time preference, the social discount rate for the first commodity class $\delta_1(t)$ *grows* in a steady state.

Third, a strictly decreasing term ${V_2}^s$ implies a strictly increasing value share of the environmental service stream ${V_1}^s = 1 - {V_2}^s$. Such a strictly increasing term ${V_1}^s$ implies that the the expression $(1-s){V_2}^s\,(x_1(t), x_2(t))\,(\hat{x}_2 - \hat{x}_1)$ strictly increases in a steady state. This expression is added to the constant rate of pure time preference to yield the social discount rate for the produced consumption stream. Thus, the social discount rate $\delta_2(t)$ *grows* as well in a steady state.

Finally, by Assumption 1 there exists $\epsilon > 0$ such that $\hat{x}_1(t) < \hat{x}_2(t) - \epsilon\,\forall\, t$. In consequence, the expression $\frac{a_1 x_1(0)^s}{a_2 x_2(0)^s} e^{-s\int_0^t \hat{x}_2(t') - \hat{x}_1(t')\,dt'}$ grows without bounds and the value share ${V_2}^s$ falls to zero. Therefore, in a steady state, the discount rate δ_1 monotonously *grows* to $\delta_1 = \rho$ for $t \to \infty$. At the same time the value share of the environmental service stream ${V_1}^s$ grows to one, implying that the discount rate for the produced consumption stream *grows* to $\delta_2 = \rho + (1-s)(\hat{x}_2 - \hat{x}_1)$. Outside of a steady state, the same reasoning implies for $t \to \infty$ that $\delta_1 = \rho$ and δ_2 approaches the form $\delta_2(t) = \rho + (1-s)(\hat{x}_2(t) - \hat{x}_1(t))$. □

Proof of corollary 1: Consider the long run social discount rate for the environmental service stream. In the proof of Proposition 1 case *ii*) I have shown that the term ${V_2}^{0<s<1}$ monotonously grows to unity as $t \to \infty$. In particular, there has to exist

$t_1 \in [0, \infty)$ such that $V_2^{0<s<1} > \frac{2}{3} \,\forall\, t > t_1$, implying

$$\begin{aligned}
&(1-s)\, V_2^{0<s<1}\; (\hat{x}_2(t) - \hat{x}_1(t)) > (1-s)\, \frac{2}{3}\; (\hat{x}_2(t) - \hat{x}_1(t)) \\
\Rightarrow\ & \delta_1^{0<s<1}(t) = \rho - (1-s)\, V_2^{0<s<1}(t)\; (\hat{x}_2(t) - \hat{x}_1(t)) \\
&\qquad < \rho - (1-s)\, \frac{2}{3}\; (\hat{x}_2(t) - \hat{x}_1(t))
\end{aligned}$$

for all $t > t_1$.[37] Similarly, the fact that for $s < 0$ the proof of Proposition 1 case *iii)* has shown that $V_2^{s<0}$ monotonously falls to zero as $t \to \infty$, implies the existence of t_2 such that $V_2^{s<0} < \frac{1}{3}$. Then, for the social discount rate of the environmental service stream in the strong sustainability scenario it follows

$$\begin{aligned}
&(1-s)\, V_2^{s<0}\; (\hat{x}_2(t) - \hat{x}_1(t)) < (1-s)\, \frac{1}{3}\; (\hat{x}_2(t) - \hat{x}_1(t)) \\
\Rightarrow\ & \delta_1^{s<0}(t) = \rho - (1-s)\, V_2^{s<0}(t)\; (\hat{x}_2(t) - \hat{x}_1(t)) \\
&\qquad > \rho - (1-s)\, \frac{1}{3}\; (\hat{x}_2(t) - \hat{x}_1(t))
\end{aligned}$$

for all $t > t_2$. Setting $t_3 = \max\{t_1, t_2\}$ I find

$$\begin{aligned}
\delta_1^{s<0}(t) &> \rho - (1-s)\, \frac{1}{3}\; (\hat{x}_2(t) - \hat{x}_1(t)) \\
&> \rho - (1-s)\, \frac{2}{3}\; (\hat{x}_2(t) - \hat{x}_1(t)) \\
&> \delta_1^{0<s<1}(t)
\end{aligned}$$

for all $t > t_3$. Analogously, one derives for the social discount rate of the produced

[37] For the relaxation of Assumption 1 to the form pointed out in footnote 12 replace $t > t_1$ by $t > \max\{t_1, t^*\}$.

consumption stream the existence of $t'_1 \in [0, \infty)$ such that for $0 < s < 1$ it holds

$$\begin{aligned}\delta_2^{0<s<1}(t) &= \rho + (1-s)\, {V_1}^{0<s<1}(t)\ \ (\hat{x}_2(t) - \hat{x}_1(t)) \\ &< \rho + (1-s)\, \frac{1}{3}\ (\hat{x}_2(t) - \hat{x}_1(t))\end{aligned}$$

for all $t > t'_1$ (as ${V_1}^s$ goes to zero), and the existence of $t'_2 \in [0, \infty)$ such that for $s < 0$ it holds

$$\begin{aligned}\delta_2^{s<0}(t) &= \rho + (1-s)\, {V_1}^{s<0}(t)\ \ (\hat{x}_2(t) - \hat{x}_1(t)) \\ &> \rho + (1-s)\, \frac{2}{3}\ (\hat{x}_2(t) - \hat{x}_1(t))\end{aligned}$$

for all $t > t'_2$ (as ${V_1}^s$ grows to unity). Then setting $t'_3 = \max\{t'_1, t'_2\}$ delivers the relation

$$\begin{aligned}\delta_1^{s<0}(t) &> \rho + (1-s)\, \frac{2}{3}\ (\hat{x}_2(t) - \hat{x}_1(t)) \\ &> \rho + (1-s)\, \frac{1}{3}\ (\hat{x}_2(t) - \hat{x}_1(t)) \\ &> \delta_1^{0<s<1}(t)\end{aligned}$$

for all $t > t'_3$. Setting $\bar{t} = \max\{t_3, t'_3\}$ yields the statement of the proposition. $\square$

Proof of proposition 2: The proof of Corollary 1 brings about the existence of $\bar{t}$ and $\epsilon > 0$ such that

$$\begin{aligned} & \delta_i^{s<0}(t) - \delta_i^{0<s<1}(t) && > \epsilon && \text{for all } t > \bar{t} \\ \Leftrightarrow\ & \exp\left(\int_{\bar{t}}^{t} \delta_i^{s<0}(t) - \delta_i^{0<s<1}(t)\, dt'\right) && > \exp\left(\int_{\bar{t}}^{t} \epsilon\, dt'\right) && \text{for all } t > \bar{t} \\ \Leftrightarrow\ & \frac{D_i^{\chi^{0<s<1}}(t, \bar{t})}{D_i^{\chi^{s<0}}(t, \bar{t})} && > \exp\left(\int_{\bar{t}}^{t} \epsilon\, dt'\right) && \text{for all } t > \bar{t}\,. \end{aligned}$$

Define the strictly positive constant $C = \frac{D_i^{\chi^{0<s<1}}(\bar{t},t_0)}{D_i^{\chi^{s<0}}(\bar{t},t_0)} \in \mathbb{R}_{++}$.[38] Then, for any $t_0 \in [0,\infty)$ the following relation has to hold:

$$\begin{aligned}\frac{D_i^{\chi^{0<s<1}}(t,t_0)}{D_i^{\chi^{s<0}}(t,t_0)} &= \frac{D_i^{\chi^{0<s<1}}(\bar{t},t_0)}{D_i^{\chi^{s<0}}(\bar{t},t_0)} \, \frac{D_i^{\chi^{0<s<1}}(t,\bar{t})}{D_i^{\chi^{s<0}}(t,\bar{t})} \\ &> \quad C \quad \exp\left(\int_{\bar{t}}^{t} \epsilon \, dt'\right) . \end{aligned} \tag{21}$$

As the right hand side of equation (21) grows to infinity for $t \to \infty$ the left hand side in particular grows bigger than one. Hence it exists $\bar{\bar{t}}$ such that

$$D_i^{\chi^{0<s<1}}(t,t_0) > D_i^{\chi^{s<0}}(t,t_0) \quad \text{for all } t > \bar{\bar{t}} \, .$$

□

C Calculations and Proofs for Section 4

Calculation of the social discount rate for CES-CIES scenario: As pointed out in section 4 on page 24 the welfare representation (11) is a special case of (6) with $u_1(z) = u_2(z) = z^{1-\alpha}$, $A = \frac{1}{1-\alpha}$ and $s^* = (1-\alpha)s$. Moreover, for $\alpha \neq 1$, it can be transformed into the form of assumption 3 by an ordinal transformation (see footnote 24). Such an ordinal transformation does not change the corresponding social discount rates. For this special case, using $u_i'(x_i) = (1-\alpha)x_i^{-\alpha}$ and $u_i''(x_i) = -\alpha(1-\alpha)x_i^{-(1+\alpha)}$, equation (7) becomes

$$\begin{aligned}\delta_1(t) &= \rho + \alpha\hat{x}_1 - (1-s)\frac{a_2 x_2^{(1-\alpha)s}}{x_1^{(1-\alpha)s} + a_2 x_2^{(1-\alpha)s}}\Big((1-\alpha)\hat{x}_2 - (1-\alpha)\hat{x}_1\Big) \\ &= \rho + \alpha\hat{x}_1 - (1-s)(1-\alpha)V_2^{(1-\alpha)s}(\hat{x}_2 - \hat{x}_1) \end{aligned} \tag{22}$$

[38] C is strictly positive as it is the ratio of two values in the image of the exponential function.

$$\delta_1(t) = \rho + \alpha\hat{x}_1 - [(1-\alpha) - s(1-\alpha)]V_2^{(1-\alpha)s}(\hat{x}_2 - \hat{x}_1) \tag{23}$$

$$= \rho + \alpha\hat{x}_1 + \alpha V_2^{(1-\alpha)s}(\hat{x}_2 - \hat{x}_1) - [1 - s(1-\alpha)]V_2^{(1-\alpha)s}(\hat{x}_2 - \hat{x}_1)$$

$$= \rho + \alpha\left(1 - V_2^{(1-\alpha)s}\right)\hat{x}_1 + \alpha V_2^{(1-\alpha)s}\hat{x}_2 - [1 - s(1-\alpha)]V_2^{(1-\alpha)s}(\hat{x}_2 - \hat{x}_1)$$

$$= \rho + \alpha\left(V_1^{s^*}\hat{x}_1 + V_2^{s^*}\hat{x}_2\right) - (1 - s^*)V_2^{s^*}(\hat{x}_2 - \hat{x}_1). \tag{24}$$

It is easily verified that the same equation holds for $\alpha = 1$ where $\mathcal{U} = \int_0^\infty \frac{1}{s^*}\ln[a_1 x_1^{s^*} + a_2 x_2^{s^*}]e^{-\rho t}\,dt$. The derivatives needed for the calculation are

$$\begin{aligned}
\frac{\partial U}{\partial x_1} &= \frac{a_1 x_1^{s^*-1}}{[a_1 x_1^{s^*} + a_2 x_2^{s^*}]}e^{-\rho t} , \\
\frac{\partial^2 U}{\partial x_1 \partial t} &= -\rho\frac{a_1 x_1^{s^*-1}}{[a_1 x_1^{s^*} + a_2 x_2^{s^*}]}e^{-\rho t} , \\
\frac{\partial^2 U}{\partial x_1^2} &= \frac{-a_1^2 x_1^{2s^*-2} + a_1 a_2(s^*-1)x_1^{s^*-2}x_2^{s^*}}{[a_1 x_1^{s^*} + a_2 x_2^{s^*}]^2}e^{-\rho t} \qquad \text{and} \\
\frac{\partial^2 U}{\partial x_1 \partial x_2} &= \frac{-s^* a_1 a_2 x_1^{s^*-1}x_2^{s^*-1}}{[a_1 x_1^{s^*} + a_2 x_2^{s^*}]^2}e^{-\rho t} .
\end{aligned}$$

Plugging the derivatives into equation (2) yields equation (24) with $\alpha = 1$. For the special case $\alpha = 1$ and $s^* = 0$ find that $\mathcal{U} = \int_0^\infty [a_1 \ln x_1 + a_2 \ln x_2]e^{-\rho t}\,dt$ and $\frac{\partial U}{\partial x_1} = a_1 x_1^{-1}e^{-\rho t}$, $\frac{\partial^2 U}{\partial x_1 \partial t} = -\rho a_1 x_1^{-1}e^{-\rho t}$, $\frac{\partial^2 U}{\partial x_1^2} = -a_1 x_1^{-2}e^{-\rho t}$, $\frac{\partial^2 U}{\partial x_1 \partial x_2} = 0$ and $\delta_1 = \rho + \hat{x}_1$ in accordance with equation (24). □

Proof of proposition 3:

1) By equation (23) it is

$$\delta_1(t) = \rho + \alpha\hat{x}_1 - [(1-\alpha) - s^*]V_2^{s^*}(\hat{x}_2 - \hat{x}_1) \tag{25}$$

Thus, discounting for the environmental good is higher than pure time preference ρ

plus "Eigengrowth effect" $\alpha\hat{x}_1$ if and only if

$$-[(1-\alpha)-s^*] > 0 \;\Leftrightarrow\; \alpha + s^* - 1 > 0 \;\Leftrightarrow\; 1 - s^* < \alpha \;\Leftrightarrow\; \sigma^{\text{int}} < \sigma^*. \tag{26}$$

Discounting for the environmental good is lower if and only if the inequality sign in equation (26) is flipped around. By symmetry the discount rate for the produced consumption stream becomes

$$\delta_2(t) = \rho + \alpha\hat{x}_2 + [(1-\alpha)-s^*]{V_1}^{s^*}(\hat{x}_2 - \hat{x}_1). \tag{27}$$

and the analogous reasoning yields the stated result.
2) Trivial consequence of equations (14) and (16). □

Proof of proposition 4:
i) Because ${V_1}^{s^*=0} = a_1$ and ${V_2}^{s^*=0} = a_2$ all terms but the growth rates in equations (14) and (16) become constant.
ii) In a steady state the only non-constant term in equation (25) is the value share of the produced good. As shown in the proof of proposition 1, ${V_2}^{s^*}$ increases for $s^* \in (0,1)$. This time behavior carries over to the discount rate if and only if equation (26) holds. It turns around if the inequality sign in (26) is flipped. For $\sigma^* = \sigma^{\text{int}}$ it is $1-s^* = \alpha$ and the term containing the non-constant value share vanishes. The analogous reasoning holds for the produced good using equation (27) with ${V_1}^{s^*}$ decreasing for $s^* \in (0,1)$. In the long-run, ${V_1}^{s^*} \to 0$ and ${V_2}^{s^*} \to 1$ as shown in the proof of proposition 1. Then, equations (14) and (16) take the stated functional form.
iii) In a steady state the only non-constant term in equation (25) is the value share of the produced good. As shown in the proof of proposition 1, ${V_2}^{s^*}$ decreases for $s^* < 0$. This time behavior carries over to the discount rate if and only if equation (26) holds

and turns around if the inequality sign in (26) is flipped. The analogous reasoning holds for the produced good using equation (27) with $V_1^{s^*}$ increasing. For $\sigma^* = \sigma^{\text{int}}$ it is $1 - s^* = \alpha$ and the term containing the non-constant value share vanishes. In the long-run, $V_1^{s^*} \to 0$ and $V_2^{s^*} \to 1$ as shown in the proof of proposition 1. Then, equations (14) and (16) take the stated functional form. $\square$

Proof of corollary 2: In a steady state the only non-constant term in equation (22) is the value share of the produced good. As shown in the proof of proposition 1, ${V_2}^{s(1-\alpha)}$ increases for $s(1-\alpha) \in (0,1)$, decreases for $s(1-\alpha) < 0$ and is constant for $s(1-\alpha) = 0$.

First, assume $1 - \alpha > 0$. Then, *for* $s < 0$, ${V_2}^{s(1-\alpha)}$ is decreasing and the term enters with a minus sign into δ_1. Thus, δ_1 is increasing over time. For $0 < s < 1$, ${V_2}^{s(1-\alpha)}$ is increasing and the term still enters with a minus sign into δ_1. Thus, δ_1 is decreasing over time. *For* $s > 1$, ${V_2}^{s(1-\alpha)}$ is still increasing (note that by $s(1-\alpha) = s^* < 1$), but the term now enters with a plus sign into δ_1. Thus, δ_1 is increasing over time.

Second, observe that making $1 - \alpha < 0$ simultaneously turns around the growth behavior of ${V_2}^{s(1-\alpha)}$ and the sign of the factor in front of ${V_2}^{s(1-\alpha)}$. Therefore the growth behavior of δ_1 is unchanged with respect to the first case.

For $s = 0$ the value share V_2^0 is constant and so is the discount rate. *For* $s = 1$ the term containing the non-constant value share vanishes and the discount rate is constant.

The discount rate of the produced good replaces ${V_2}^{s(1-\alpha)}$ by ${V_1}^{s(1-\alpha)}$ exhibiting the opposite growth behavior. At the same time the sign in front of the growth term flips around so that overall the growth of δ_2 has the same qualitative dependence on s as δ_1. $\square$

Proof of corollary 3:

1) The analysis can be reduced to the $\{\cdot\}$-bracketed term in

$$U(x_1, x_2, t) = \frac{\left\{[a_1 x_1^{s^*} + a_2 x_2^{s^*}]^{\frac{1}{s^*}}\right\}^{1-\alpha} - 1}{1-\alpha} e^{-\rho t}$$

as the transformation parameterized by α is strictly monotonic and, thus, does not change the characteristics of U to be analyzed.

First, let $s^* > 0$. Then $\lim_{x_2 \to \infty}\{\cdot\} = \lim_{x_2 \to \infty}[a_1(\frac{x_1}{x_2})^{s^*} + a_2]^{\frac{1}{s^*}} x_2 = \infty$. As $\{\cdot\}$ does not go to infinity for any finite pair $(x_1^*, x_2^*) \in \mathbb{R}_+^2$ the good x_1 does not limit welfare.

Second, let $s^* < 0$. Then $\lim_{x_2 \to \infty}\{\cdot\} = \lim_{x_2 \to \infty}[a_1 + a_2(\frac{x_2}{x_1})^{s^*}]^{\frac{1}{s^*}} x_1 = a_1^{\frac{1}{s^*}} x_1$. Thus $x_1^* = x_2^* = a_1^{\frac{1}{s^*}} \bar{x}_1$ satisfy the defining property of x_1 limiting welfare strictly.

Third, let $s^* = 0$. Then $\lim_{x_2 \to \infty}\{\cdot\} = \lim_{x_2 \to \infty} x_1^{a_1} x_2^{a_2}$. That expression converges to infinity if $x_1 > 0$ and is zero for $x_1 = 0$. Thus for $s = 0$ the good x_1 limits welfare, but does not limit welfare strictly (i.e. there only exist $(x_1^*, x_2^*) \in \mathbb{R}_+^2$ as in the definition if $\bar{x}_1 = 0$).

Finally, it is well known that the upper contour sets of a CES function are strictly convex for $x_1, x_2 > 0$ if and only if $s^* < 1$.

2 & 3) It is immediate from proposition 3 that discount rates are non-increasing for $\sigma^* \leq \sigma^{\text{int}}$ in the weak sustaintability scenario and for $\sigma^{\text{int}} \leq \sigma^*$ in the strong sustainability scenario. $\square$

Proof of propostion 5: By equation (25) it is

$$\begin{aligned}\delta_1(t) &= \rho + \alpha\hat{x}_1 - [(1-\alpha) - s^*] V_2^{s^*}(\hat{x}_2 - \hat{x}_1) \\ &= \rho + \frac{1}{\sigma^{\text{int}}}\hat{x}_1 - \left(\frac{1}{\sigma^*} - \frac{1}{\sigma^{\text{int}}}\right) V_2^{s^*}(\hat{x}_2 - \hat{x}_1)\,.\end{aligned}$$

For strict weak sustainability preferences it holds $\left(\frac{1}{\sigma^*} - \frac{1}{\sigma^{\text{int}}}\right) > 0$. For strict strong sustainability preferences it holds $\left(\frac{1}{\sigma^*} - \frac{1}{\sigma^{\text{int}}}\right) < 0$. The rates, factors, and parameters corresponding to the strict *w*eak respectively the strict *s*trong sustainability preference will carry an upper index w respectively s. It holds

$$\begin{aligned} & \delta_1{}^s(t) < \quad \delta_1{}^w(t) \\ \Leftrightarrow\ & \frac{1}{\sigma^{\text{int}^s}}\hat{x}_1 - \left(\frac{1}{\sigma^{*s}} - \frac{1}{\sigma^{\text{int}^s}}\right) V_2^{s^{*s}}(\hat{x}_2 - \hat{x}_1) \\ & \qquad < \frac{1}{\sigma^{\text{int}^w}}\hat{x}_1 - \left(\frac{1}{\sigma^{*w}} - \frac{1}{\sigma^{\text{int}^w}}\right) V_2^{s^{*w}}(\hat{x}_2 - \hat{x}_1) \\ \Leftrightarrow\ & \underbrace{\left(\frac{1}{\sigma^{\text{int}^s}} - \frac{1}{\sigma^{\text{int}^w}}\right)}_{positive}\hat{x}_1 \\ & \qquad < \underbrace{\left(\underbrace{\left(\frac{1}{\sigma^{*s}} - \frac{1}{\sigma^{\text{int}^s}}\right)}_{negative} V_2^{s^{*s}} - \underbrace{\left(\frac{1}{\sigma^{*w}} - \frac{1}{\sigma^{\text{int}^w}}\right)}_{positive} V_2^{s^{*w}}\right)}_{negative}(\hat{x}_2 - \hat{x}_1) \qquad (28)\end{aligned}$$

and equivalently with $<$ changed into $>$ in all three inequalities. For case *i*) where $\hat{x}_1 = 0$ it is immediate that the inequality always holds with $>$ (the left hand side of equation 28 is zero and the right hand side is negative). As the relation holds for all times, it is immediate from equation (3) that $D_1^{\mathrm{x}s}(t, t_0) > D_1^{\mathrm{x}w}(t, t_0)$.

To analyze the long-run social discount rate for cases *ii*) and *iii*) recall that for the strong sustainability preferences $V_2^{s^{*s}} \to 0$ and for the weak sustainability preferences

${V_2^{s^*}}^{w} \to 1$ (see proof of proposition 1). Then inequality (28) becomes for $\hat{x}_1 < 0$

$$\Leftrightarrow \left(\frac{1}{\sigma^{\text{int}^s}} - \frac{1}{\sigma^{\text{int}^w}}\right)\hat{x}_1 < \left(\frac{1}{\sigma^{*s}} - \frac{1}{\sigma^{\text{int}^s}}\right)(\hat{x}_2 - \hat{x}_1)$$

$$\Leftrightarrow \underbrace{\left(\frac{1}{\sigma^{*s}} - \frac{1}{\sigma^{\text{int}^w}}\right)}_{positive}\hat{x}_1 < \underbrace{\left(\frac{1}{\sigma^{*s}} - \frac{1}{\sigma^{\text{int}^s}}\right)}_{negative}\hat{x}_2 \tag{29}$$

$$\Leftrightarrow \frac{\frac{1}{\sigma^{*s}} - \frac{1}{\sigma^{\text{int}^w}}}{\frac{1}{\sigma^{*s}} - \frac{1}{\sigma^{\text{int}^s}}} < \frac{\hat{x}_2}{\hat{x}_1}\,. \tag{30}$$

In case *ii*) where $\hat{x}_1 < 0$ and $\hat{x}_2 < 0$ equation (29) is always satisfied (left hand side negative and right hand side positive). In case *iii*) where $\hat{x}_1 < 0$ and $\hat{x}_2 > 0$ the truth of the inequality depends on the precise growth rates. For any two (strict weak and strict strong) preference specifications the left hand side of equation (30) is some negative constant. For given $\hat{x}_1$ the inequality can always be satisfied by choosing $\hat{x}_2$ small enough (as it converges to zero for $\hat{x}_2 \to 0$). Similarly, for given $\hat{x}_2$ the inequality can always be satisfied by choosing $\hat{x}_1$ small enough (as it the right hand side converges to zero for $\hat{x}_1 \to -\infty$). Analogously, one can always pick $\hat{x}_2$ large or $\hat{x}_1$ large (close to zero) such that inequality (30) is violated. The above relation for the social discount rates holds for times greater than some $\bar{t}$ (as ${V_2^{s^*}}^{s} \to 0$ and ${V_2^{s^*}}^{w} \to 1$). The same argument as in the proof of proposition 2 shows that from ${\delta_1}^s(t) < {\delta_1}^w(t)$ for all $t > \bar{t}$ it follows that there exists $\bar{\bar{t}}$ such that ${D_1^{\mathrm{x}}}^s(t, t_0) > {D_1^{\mathrm{x}}}^w(t, t_0)$ for all $t > \bar{\bar{t}}$. $\square$