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Coping with Gray Markets: The Impact of Market Conditions and Product Characteristics

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Gray markets, also known as parallel imports, are marketplaces for trading genuine products that are diverted from authorized distribution channels. They have created fierce competition for manufacturers in many industries and each year billions of dollars worth of products are traded in these markets. Using a game-theoretic model, we analyze the impact of parallel importation on a price-setting manufacturer that serves two markets with uncertain demand. We characterize the optimal joint price and quantity decisions of the manufacturer which determine whether the manufacturer should ignore, block, or allow parallel importation. We also show that parallel importation forces the manufacturer to reduce her price gap while demand uncertainty forces her to lower prices in both markets. Moreover, we observe that parallel importation may force the manufacturer to exit the low-profit market. Through extensive numerical experiments, we explore the impact of market conditions (size and price elasticity) and product characteristics (a fashion item or a commodity) on the manufacturer's reaction to parallel importation. In addition, we provide interesting insights about the value of strategic pricing for coping with gray markets versus the uniform pricing policy that has been adopted by some companies to eliminate gray markets.

Key words: Gray markets, parallel importation, parallel markets, demand uncertainty, strategic pricing, uniform pricing, commodity product, fashion product

History:

1. Introduction

Manufacturers around the world confront new pressures with the trade of their brand name products in unauthorized distribution channels known as gray markets. Gray markets primarily emerge when manufacturers offer their products in different markets at different prices. Price differentials

may motivate enterprises or individuals to buy products from authorized distributors in markets with a lower price and sell them in markets with a higher price. Gray market channels can be internal or external, with internal gray markets operating inside the same region (e.g., country, state) as the authorized distributors and external gray markets bringing parallel imports from another region.

When manufacturers attempt to combat gray markets with trademark infringement suits, the gray marketer usually wins. In the United States, the Copyright Act of 1976 contains the first-sale doctrine entitling the first purchaser of a legal copy of a product to transfer the product without permission from the copyright holder. In the European Economic Area, the court's interpretation of the Treaty on European Union produced the exhaustion doctrine legalizing parallel importation. Unlike black markets, products traded in gray markets are genuine.

Each year products worth billions of dollars are diverted to gray markets. In the IT industry alone, the approximate value of gray-market products was \$58 billion dollars and accounted for 5 to 30 percent of total IT sales, according to a 2008 study conducted jointly by KPMG and The Alliance for Gray Market and Counterfeit Abatement (AGMA). In the pharmaceutical industry, 20% of the products sold in the United Kingdom are parallel imports (Kanavos and Holmes 2005). In communications, nearly 1 million iPhones were unlocked in 2007 and used on unauthorized carriers in China (*New York Times*, 2008). International versions of college textbooks, drinks, cigarettes, automobile parts, luxury watches, jewelry, electronics, chocolates, and perfumes are among the numerous products that are traded in gray markets (Schonfeld 2010).

Growing numbers of efficient global logistics networks help gray markets reach more customers faster. Advancing web technology and a rapidly growing online retail sector also boost gray markets. To name only a few, Amazon, eBay, Alibaba, Kmart, and Costco are among the retailers known to have sold gray goods.

Although the price differential between markets is the primary driver of gray markets, demand uncertainty and misaligned incentives in the supply chain make fertile ground for proliferation. Consider a retailer who is left with unsold inventory because demand failed to meet expectations. That retailer can sell the remaining inventory to an unauthorized retailer at a discount, thus reducing holding costs and achieving sales targets. Contracts offering quantity discounts can subvert manufacturer incentives, resulting in retailers inflating order sizes with the intent to sell excess inventory to unauthorized dealers.

As to benefit and harm, opinions about gray markets are mixed, depending on one's perspective. Manufacturers generally consider gray markets harmful because:

1. Products diverted to gray markets end up competing with those sold by authorized distributors.

2. Brand value may erode as products become available to segments that the manufacturer deliberately avoided.
3. Unauthorized channels get a free ride from expensive advertising and other manufacturer efforts to increase sales.

Manufacturers can benefit, however, when a gray market generates a new stream of demand and increases sales. They also provide a means for manufacturers to increase their market share and deter competitors.

From the customer viewpoint, gray markets are beneficial because they create competition that results in lower prices and give customers more purchase options. During the year 2000, many senior citizens in the United States whose prescriptions were not covered by insurance were importing price-controlled drugs from Canada. The Canadian drugs cost as much as 85% less than the same drugs in the United States, which put pressure on American drug makers. Nevertheless, President Clinton signed a bill that year, legalizing prescription drugs from Canada to provide cheaper medicines for elderly citizens who were paying a substantial portion of their incomes for medication (*USA TODAY*, 2003).

One problem with purchasing from unauthorized channels, however, is that customers often lose manufacturer warranty and after-sales services. Also, if the product is imported from another region of the world, it may lose some functionality due to incompatible parts or variations in standards.

Companies adopt a variety of strategies to counteract gray markets. They may differentiate their products for different markets to make it harder for parallel importers to sell the products as perfect substitutes. They may offer a simple uniform pricing scheme across the different markets, eliminating price differentials and the resultant parallel importation. Two champions of this scheme are TAG Heuer and Christian Dior, which price their products the same worldwide (Antia et al. 2004). With such a policy, however, comes the cost of foregone profits associated with pricing each market to its own locality. Another strategy involves educating consumers to increase their awareness about the consequences of buying from gray markets (lost warranty and after-sales services).

In this paper, we analyze the price and quantity decisions of a strategic manufacturer serving two markets with uncertain demand under the threat of competition from a parallel importer. If the manufacturer were to charge different prices across the markets, the parallel importer could buy the product in the market with the lower price and transfer it to sell in the market with the higher price. Consumers base purchase decisions on perception and price, often comparing the offering of the manufacturer to that of the parallel importer. In our scenario, consumers perceive gray markets to be inferior to authorized channels, valuing instead the peace of mind they get when they buy a product from an authorized distributor. This lower perception can also be attributed

to characteristics of the product under consideration, with gray markets for fashion items being less attractive than those for commodities.

We address the following questions in this research:

1. How does the presence of the parallel importer and/or demand uncertainty change the manufacturer's price and quantity decisions?
2. What are the implications of ignoring, blocking, or allowing the parallel importer on the manufacturer's price and quantity decisions? Is there any policy that dominates another one?
3. What are the impacts of product characteristics (such as fashion or commodity) and market characteristics (such as consumer price elasticity or market size) on the manufacturer's policy against the parallel importer?
4. When, if at all, should the manufacturer leave one of the markets, parting with the associated profits, due to the damage caused by a high level of parallel importation initiated from that market?
5. When, if at all, should the manufacturer expand to a new market despite the associated risk of parallel importation from there to the current profitable markets?
6. How does the proposed strategic policy, in anticipation of parallel importation, compare to policies that are aimed primarily at elimination of parallel importation such as the uniform pricing policy?

The paper is organized as follows. Section 2 reviews the relevant literature. In Section 3, we introduce our analytical framework. First, we analyze the parallel importer's reaction when the manufacturer's prices are given and propose an approach for market segmentation in the presence of a parallel importer. Then, we formulate a Stackelberg game problem with the manufacturer as the leader and the parallel importer as the follower. We characterize the optimal solution of the game and show the impact of parallel importation and demand uncertainty on the manufacturer's optimal decision. Section 4 summarizes numerical experiments that demonstrate the merits of a model that considers the joint effects of parallel importation and demand uncertainty on the manufacturer's decisions. Further, we evaluate the effects of model parameters on optimal prices, quantities, and profit. In Section 5, we draw a number of managerial insights about the impact of market and product characteristics on the manufacturer's policy adoption towards the parallel importer. We also examine the benefits of the strategic price and quantity decisions compared to a simple uniform pricing strategy in both markets. Finally, we conclude the paper in Section 6 with a summary of our results and future research directions.

2. Literature Review

Despite the ubiquity of gray markets and their operational implications, this topic occupies a relatively small niche in operations management literature. There exists a body of literature on optimal

pricing and quantity decisions with stochastic demand (Petruzzi and Dada 1999, and Chan et al. 2004); however, the studies ignore gray markets. Existing operations management, marketing, and economics research into gray markets largely focus on differential pricing. Dutta et al. (1994) use an economic model to study retailers selling products in the territories of other retailers. Myers (1999) survey factors that lead to the emergence of gray markets. Maskus (2000) and Ganslandt and Maskus (2004) provide empirical evidence of gray market activities as well as an overview of the policy debate. Antia et al. (2004) discuss the impact of different policies on gray markets and methods trademark owners should employ to cope with them. Bucklin (1993) presents a modeling approach to examine the claims made by trademark owners and gray market dealers and draws public policy implications. Richardson (2000) analyzes an economic model of countries deciding whether to prohibit gray markets or not. Matsushima and Matsumura (2010) and Chen (2009) use economic models to explore the ramifications of parallel imports for intellectual property holders and manufacturers. Results from these studies indicate that manufacturers should tolerate some level of territorial restriction violation. Ahmadi and Yang (2000) investigate the interaction between a manufacturer and a parallel importer in a deterministic setting with endogenous prices. They showed that not only does parallel importation increase total sales, but also it can increase manufacturer profit by serving customers with a low willingness to pay. Xiao et al. (2011) build on the foregoing by analyzing four combinations of channel structures; in each market, the manufacturer could sell directly or through a retailer. They show that the structure of the channel is critical to determining the increase or reduction in manufacturer profit due to parallel importation.

The role of demand uncertainty in boosting gray markets is addressed in Ahmadi et al. (2011). They consider a decentralized supply chain with exogenous pricing in which a retailer could salvage leftover inventory or sell it to the gray market. They show that a gray market can have a negative impact on manufacturer profit and propose a multiple replenishment mechanism with a buyback contract to alleviate the impact.

Decentralized supply chains that face gray markets are also addressed by Altug and van Ryzin (2010). They consider a manufacturer selling a product through multiple retailers. The retailers face stochastic demand and sell their excess inventory to an internal gray market at the end of the season. The authors derive the market-clearing price while assuming a large number of authorized distributors. They also examine the robustness of supply chain contracts in the presence of gray markets. Although they assume stochastic demand, the price of authorized distributors is exogenous.

Hu et al. (2010) consider a supplier offering a reseller quantity discounts for batch orders. The reseller could order more to benefit from the quantity discount and divert a portion of the order quantity to an internal gray market. The authors assume that the reseller follows an EOQ inventory

policy and show that, when the reseller's batch inventory holding cost is high, the gray market improves channel performance.

Su and Mukhopadhyay (2011) consider a manufacturer who offers quantity discount (QD) to sell a product through one dominant retailer and N fringe retailers. The QD contract can coordinate the channel but may cause gray trading between the dominant retailer and fringe retailers. The paper proposes a dynamic QD contract and a revenue-sharing contract to prevent gray market activities.

Krishnan et al. (2010) study the impact of gray markets on a decentralized supply chain with one manufacturer and two retailers. The type of gray market they consider is an alternative channel to which the retailers could divert the product, not parallel importation. They examine the impact of the gray market on manufacturer and retailer profit when one retailer or both sell to the gray market.

Our work differs from the foregoing in that we analyze the impact of parallel importation on a vertically integrated manufacturer who must set prices and quantities before demand uncertainty is resolved. By deriving the solution to a game model, we show when it is in the manufacturer's interest to allow parallel importation. We also obtain insights about the value of strategic pricing in handling gray markets and the effect of market and product characteristics on the manufacturer's reaction to parallel importation.

3. Analysis Framework

Consider a manufacturer who sells a single product in two separate markets. The manufacturer chooses price p_1 and quantity q_1 in market 1, and chooses price p_2 and quantity q_2 in market 2. Table 1 summarizes the notation used throughout this paper. The demand in each market is stochastic and additive, and defined as

$$\mathcal{D}_1(p_1, \epsilon_1) = d_1(p_1) + \epsilon_1 = N_1 - b_1 p_1 + \epsilon_1, \quad (1)$$

$$\mathcal{D}_2(p_2, \epsilon_2) = d_2(p_2) + \epsilon_2 = N_2 - b_2 p_2 + \epsilon_2, \quad (2)$$

where $d_i(p_i)$'s denote the deterministic and ϵ_i 's denote the stochastic portions of demand in each market, for $i = 1, 2$. The deterministic part of the demand is linearly decreasing in the price of that market ($d_i(p_i) = N_i - b_i p_i$) where N_i denotes the market size, and b_i represents the consumer price elasticity. We assume that ϵ_i 's take their values in the interval $[L, U]$ with the probability density functions $f_i(x)$ and cumulative distributions $F_i(x)$. We denote the standard deviation of ϵ_i with σ_i and normalize its expected value to zero without loss of generality. We also assume that the hazard rate functions of ϵ_i denoted with $r_i(x) = \frac{f_i(x)}{1-F_i(x)}$ satisfies the Increasing Failure Rate (IFR)

Table 1 Notations

<u>Parameters</u>	
N_1, N_2	size of market 1 and market 2
b_1, b_2	price elasticity in market 1 and market 2
ϵ_1, ϵ_2	demand uncertainty in market 1 and market 2
$[L, U]$	domain of ϵ_1 and ϵ_2
σ_1, σ_2	standard deviation of ϵ_1 and ϵ_2
$F_1(x), F_2(x)$	probability distribution of ϵ_1 and ϵ_2
$r_1(x), r_2(x)$	hazard rate function of ϵ_1 and ϵ_2
c	manufacturer's unit production cost
c_G	parallel importer's unit transfer cost
ω	consumer's relative perception of parallel imports
 <u>Manufacturer's variables</u>	
p_1, p_2, q_1, q_2	price and quantity in markets 1 and 2
$\bar{\pi}$	profit when there are no parallel imports
π	profit in the presence of the parallel importer
π^d	profit in the presence of the parallel importer for deterministic demand
 <u>Manufacturer's optimal variables</u>	
$\tilde{p}_1, \tilde{p}_2, \tilde{q}_1, \tilde{q}_2$	when there are no parallel imports
$\tilde{p}_1^d, \tilde{p}_2^d, \tilde{q}_1^d, \tilde{q}_2^d$	when there are no parallel imports and demand is deterministic
$p_1^*, p_2^*, q_1^*, q_2^*$	in the presence of the parallel importer
$p_1^{*d}, p_2^{*d}, q_1^{*d}, q_2^{*d}$	in the presence of the parallel importer for deterministic demand
 <u>Parallel importer's variables</u>	
p_G	order quantity
q_G	price
π_G	profit

property, i.e., $r'_i(x) > 0, \forall x \in [L, U], i = 1, 2$. This property holds for many common distributions such as normal and uniform distribution.

The manufacturer has to determine her¹ prices and quantities before demand uncertainties are resolved. As depicted in Figure 1, after the manufacturer sets her prices, a parallel importer may decide to transfer the product from the low-price to the high-price market if the price gap makes the venture sufficiently profitable. The parallel importer must choose the quantity to buy from the manufacturer in the low-price market and then set the selling price in the high-price market.

We make two assumptions about the importer's ordering from the manufacturer. First we assume that he places his order early enough to avoid stockouts. Most gray marketers hire agents to swiftly purchase products, sometimes within a few hours of release. Also, in most situations it is very difficult for a manufacturer to distinguish between orders received from end customers and orders placed by gray market agents, especially when orders are placed on the internet. Though volume

¹ Throughout the paper, we refer to the manufacturer as a female and to the parallel importer as a male.

may provide a clue, consider the case of most iPhones sold in the Chinese gray market. They were brought to China in ones and twos by airline passengers or flight attendants (*New York Times*, 2008). The second assumption is that the parallel importer makes his decisions based on an estimate of average demand and does not have the capability to estimate the uncertainty he would face. We believe this assumption is reasonable because most manufacturers that operate in international markets are large companies that have the experience and resources to study markets extensively and collect data to estimate the parameters of demand distribution. In contrast, gray marketers have low capital and cannot invest in market research. This assumption also keeps the model tractable and allows us to better analyze the impact of parallel importation on the manufacturer.

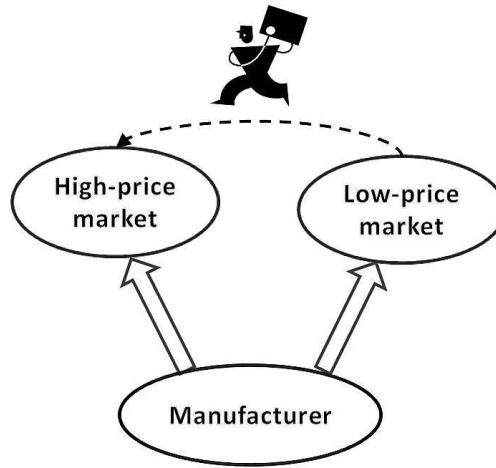


Figure 1 The manufacturer serves two markets and the parallel importer moves the product between markets

We model this problem in a Stackelberg game framework with the manufacturer as the leader and the parallel importer as the follower. To analyze the impact of the parallel importer, we first consider the case of no parallel imports. We assume there are no capacity constraints, and unsatisfied demand is lost. For ease of exposition, we assume holding costs, lost-sales costs, and salvage values are zero. With c denoting the per-unit production cost, the manufacturer's problem can be formulated as

$$\max_{p_1, p_2, q_1, q_2} \bar{\pi} = E \left\{ p_1 \min(q_1, \mathcal{D}_1(p_1, \epsilon_1)) + p_2 \min(q_2, \mathcal{D}_2(p_2, \epsilon_2)) - c(q_1 + q_2) \right\} \quad (3)$$

This is the classic price-setting newsvendor problem (Petruzzi and Dada, 1999) and $\bar{\pi}$ is strictly quasiconcave in p_1 and p_2 due to the IFR property (Xu et al., 2010). The optimal quantities and prices are

$$\begin{aligned} \tilde{q}_1 &= d_1(\tilde{p}_1) + z_1(\tilde{p}_1) \\ \tilde{q}_2 &= d_2(\tilde{p}_2) + z_2(\tilde{p}_2) \end{aligned}$$

where $z_1(\tilde{p}_1) = F_1^{-1}\left(1 - \frac{c}{\tilde{p}_1}\right)$, $z_2(\tilde{p}_2) = F_2^{-1}\left(1 - \frac{c}{\tilde{p}_2}\right)$ and \tilde{p}_1 and \tilde{p}_2 solve

$$N_1 - 2b_1p_1 + z_1(p_1) + cb_1 - \int_L^{z_1(p_1)} F_1(x) dx = 0 \quad (4)$$

$$N_2 - 2b_2p_2 + z_2(p_2) + cb_2 - \int_L^{z_2(p_2)} F_2(x) dx = 0 \quad (5)$$

To ensure the problem is not trivial, we assume $N_1 - b_1c + L > 0$ and $N_2 - b_2c + L > 0$ meaning that, when markets are priced at their lowest level, the market sizes are large enough to make the minimum possible demand positive.

3.1. Impact of Parallel Imports

If the manufacturer's price is larger in one market than in the other, the parallel importer may consider transferring the product to the high-price market for resale. Since we lack *a priori* information as to which monopoly optimal price is higher than the other, we can impose conditions on the model parameters without loss of generality to ensure that $\tilde{p}_2 > \tilde{p}_1$. The next proposition introduces the condition (All proofs are provided in Appendix 1.):

PROPOSITION 1. $\tilde{p}_2 > \tilde{p}_1$ if and only if

$$\frac{N_1 + L + \int_L^{z_1(\tilde{p}_2)} (1 - F_1(x)) dx}{b_1} < \frac{N_2 + L + \int_L^{z_2(\tilde{p}_2)} (1 - F_2(x)) dx}{b_2} \quad (6)$$

Inequality (6) describes the necessary and sufficient condition for charging a higher price in market 2. This proposition results in two sufficient conditions, which are provided in the following corollaries.

COROLLARY 1. Suppose $b_1 = b_2$. If ϵ_2 stochastically dominates ϵ_1 in the first order ($\epsilon_2 \succeq_{s.t.} \epsilon_1$), and $N_2 \geq N_1$, then $\tilde{p}_2 \geq \tilde{p}_1$.

COROLLARY 2. Suppose $N_1 = N_2 = N$. If $b_2 < b_1$ and $\epsilon_2 \succeq_{s.t.} \epsilon_1$, then $\tilde{p}_2 > \tilde{p}_1$.

Corollary 1 states that if price elasticity is identical in both markets, the manufacturer will charge a higher price in market 2 if it has the larger consumer base and demand stochasticity. On the other hand, Corollary 2 shows if the markets are the same size, the price in market 2 will be higher if its consumers are more price sensitive and demand is stochastically larger. Since the primary source of gray markets is price differential driven by different price elasticities, we will assume for the rest of the analysis that the demand parameters satisfy the conditions in Corollary 2 and the direction of import is from market 1 to market 2.

Now suppose the parallel importer decides to enter market 2. Define q_G to be the size of the order that the parallel importer places with the manufacturer in market 1. Also, define p_G to be the

price the parallel importer sets for his units. We assume that he incurs cost c_G to transfer one unit of the product to market 2. This cost represents the shipping cost and all other costs associated with distributing the product in market 2 (e.g., translating the user manual, repackaging, tariffs).

When there are no parallel imports and the manufacturer sets the price at p_2 , some customers buy the product and some do not. Once the parallel importer enters market 2 and offers the product at price p_G , the market divides into three segments as depicted in Figure 2. The first segment of the market is customers who continue to buy the product from the manufacturer. The second segment contains customers who buy the product from the parallel importer. Some of these customers initially bought from the manufacturer, but now switch to the parallel importer (the distance between the dashed lines) and some had not considered buying the product before due to higher price offered by the authorized channel. The third segment contains those who had not bought the product before and continue to refrain from doing so even after the parallel importer enters the market.

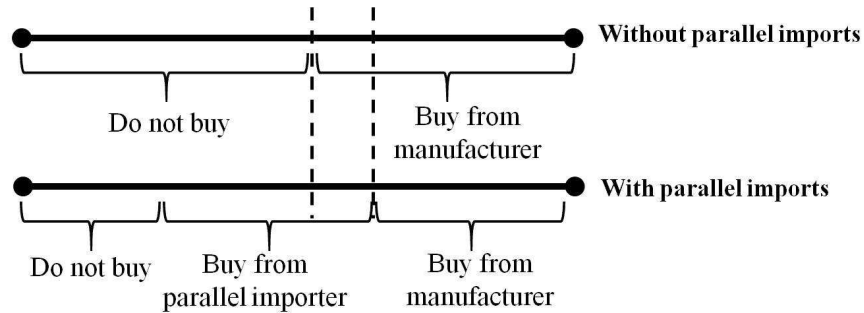


Figure 2 Segmentation of market 2 before and after parallel importation

The size of these segments is determined by the prices set by the manufacturer and the parallel importer. Size is also affected by the consumers' relative perception of gray-market products in market 2, whose valuation of parallel imports compared to their valuation of products provided by the manufacturer we denote with $0 < \omega < 1$. Higher values of ω correspond to high valuation of gray market goods.

To determine the market segments, we note that the manufacturer's linear demand model $N - b_2 p_2$ is equivalent to assuming that the customers' net utility of consuming the manufacturer's product is equal to $\theta - \frac{b_2 p_2}{N}$ where θ is uniformly distributed between 0 and 1.² Given that the reputation of the parallel importer is lower than that of the manufacturer, we assume that the net utility of consuming parallel imports is $\omega\theta - \frac{b_2 p_G}{N}$. Now, if θ_1 is the boundary between the segment

² Only consumers with a positive utility buy the product. Thus the manufacturer's total demand is equal to $N \int_{\frac{b_2 p_2}{N}}^1 d\theta = N - b_2 p_2$.

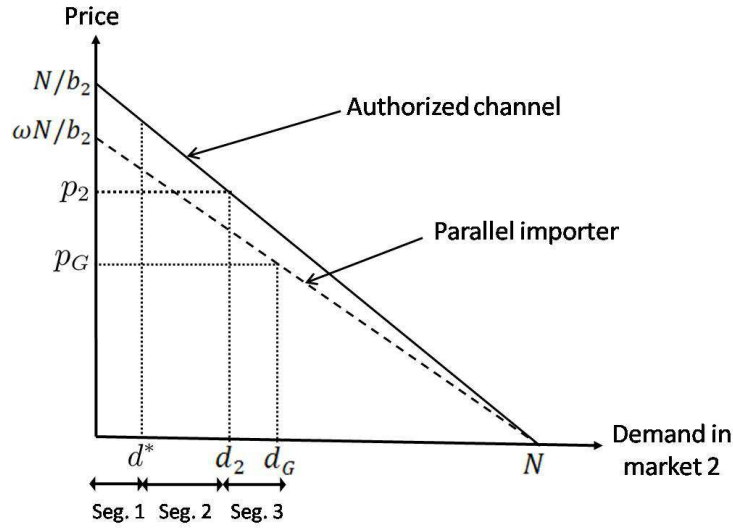


Figure 3 An alternative interpretation of segmentation of market 2.

that buys from the manufacturer and the segment that buys from the parallel importer, we obtain it by equating the consumption utilities:

$$\theta_1 - \frac{b_2 p_2}{N} = \omega \theta_1 - \frac{b_2 p_G}{N} \implies \theta_1 = \frac{p_2 - p_G}{N(1 - \omega)} b_2$$

Similarly, if θ_2 is the boundary between the segment that buys from the parallel importer and the segment that does not buy the product at all, we can write

$$\omega \theta_2 - \frac{b_2 p_G}{N} = 0 \implies \theta_2 = \frac{b_2 p_G}{\omega N}$$

Therefore, the net deterministic demand for the manufacturer is $N(1 - \theta_1) = N - \frac{p_2 - p_G}{1 - \omega} b_2$ and the net demand for the parallel importer is $N(\theta_1 - \theta_2) = \frac{\omega p_2 - p_G}{\omega(1 - \omega)} b_2$.

Figure 3 represents an alternative explanation of the approach for the segmentation of market 2 which was also used in Ahmadi and Yang (2000). The linear demand curves, $d_2(p_2)$ and $d_G(p_G) = N - \frac{b_2}{\omega} p_G$, can be thought of as a lineup of consumers with different tastes or willingness to pay. When both the manufacturer and the importer offer the product, one segment of consumers will stay with the manufacturer (segment 1), one segment switches to the importer (segment 2), and some consumers who did not buy the product before are now willing to buy from the importer (segment 3). The boundary between segment 1 and segment 2, denoted with d^* is the point at which the difference between the willingness to pay and price is the same for the authorized channel and the gray market:

$$\frac{N - d^*}{b_2} - p_2 = \frac{\omega(N - d^*)}{b_2} - p_G \implies d^* = N - \frac{p_2 - p_G}{1 - \omega} b_2$$

The parallel importer's demand will be the combined size of segments 2 and 3, which is $\frac{\omega p_2 - p_G}{\omega(1-\omega)} b_2$. However, because the parallel importer buys the product from the manufacturer in market 1, his order quantity is limited by q_1 . Therefore, the parallel importer's problem is

$$\max_{p_G} \pi_G = (p_G - p_1 - c_G) q_G$$

where $q_G = \min\left(\frac{\omega p_2 - p_G}{\omega(1-\omega)} b_2, q_1\right)$.

PROPOSITION 2. Define $\psi = \frac{\omega p_2 - p_1 - c_G}{2\omega(1-\omega)} b_2$. If $\psi < q_1$, the parallel importer's optimal price and quantity are

$$\begin{aligned} p_G &= \frac{\omega p_2 + p_1 + c_G}{2} \\ q_G &= \max(0, \psi) \end{aligned} \quad (7)$$

Otherwise,

$$\begin{aligned} p_G &= \omega p_2 - \frac{\omega(1-\omega)q_1}{b_2} \\ q_G &= q_1 \end{aligned} \quad (8)$$

If the manufacturer's quantity in market 1 is large enough, the parallel importer will order ψ if $\omega p_2 > p_1 + c_G$. The cost the importer incurs to transfer and sell a unit of the product in market 2 is $p_1 + c_G$. Clearly, if this cost is above p_2 , transferring the product will not be profitable. However, because consumers in market 2 have a lower perception of gray market products, $p_1 + c_G$ should be even lower (below ωp_2) to justify the importer's entry to the competition. If the manufacturer's quantity in market 1 is not enough and the importer only gets a portion of his order, he will choose the price that clears the market. However, as the next result shows, if a viable market exists for the parallel importer, the second scenario will not happen.

PROPOSITION 3. If the manufacturer's prices are such that $\omega p_2 - p_1 - c_G > 0$, then $q_1 \geq \psi$.

This proposition states that if the manufacturer chooses prices that leave a segment of market 2 to the parallel importer, she will not reduce her quantity in market 1 to restrict the importer's sales. In other words, when the parallel importer enters the competition, the volume of imports is only a function of the manufacturer's prices.

The next proposition describes the impact of parallel imports on the manufacturer's demand, which was also observed in Ahmadi and Yang (2000).

PROPOSITION 4. When the parallel importer enters the competition:

1. The manufacturer's demand in market 1 increases by q_G .
2. The manufacturer's demand in market 2 decreases by ωq_G .

From Figure 2, we see that the manufacturer's demand in market 2 is reduced because some consumers switch to the parallel importer. However, the segment of market 2 that buys the product from the parallel importer increases the manufacturer's demand in market 1. Overall, the manufacturer's demand goes up because parallel importation provides the product at a lower price and induces the consumers that have a lower willingness-to-pay to buy the product. The manufacturer could directly offer the product at a discounted price, but doing so through the authorized channel would lead to consumer confusion and severe demand cannibalization. Although the parallel importer also cannibalizes the demand of the authorized channel, this effect is alleviated because the importer is not affiliated with the manufacturer and has a lower reputation in the market.

The increase in total sales does not necessarily translate into higher profits because of the difference between the market prices. As a matter of fact, we will explain shortly that the manufacturer's expected profit is always lower in the presence of parallel importation. Before that, we formulate the Stochastic Stackelberg Game (SSG) as:

$$(SSG) \quad \max_{p_1, p_2, q_1, q_2} \pi = E \left\{ p_1 \min(q_1, \mathcal{D}_1(p_1, \epsilon_1) + q_G) + p_2 \min(q_2, \mathcal{D}_2(p_2, \epsilon_2) - \omega q_G) - c(q_1 + q_2) \right\}.$$

The manufacturer can respond to the gray market in one of three ways: (a) *Ignore* the importer and continue to use \tilde{p}_1 and \tilde{p}_2 from (4) and (5); (b) *Block* the importer by adjusting p_1 and p_2 so that $\omega p_2 - p_1 - c_G = 0$ and it is not profitable for the importer to enter market 2; (c) *Allow* the importer to enter and resell the product in market 2 by choosing prices such that $\omega p_2 - p_1 - c_G > 0$. Hereafter, we refer to the choice between ignore, block, and allow policies as the manufacturer's *strategy*, and we use words decision or solution to describe the price and quantity values. The next proposition describes the structure of the optimal solution and strategy to the SSG.

THEOREM 1. *Let \hat{p}_1 be the solution to the following equation*

$$\begin{aligned} \omega \left(N - 2b_2 \left(\frac{p_1 + c_G}{\omega} \right) + cb_2 + z_2 \left(\frac{p_1 + c_G}{\omega} \right) - \int_L^{z_2 \left(\frac{p_1 + c_G}{\omega} \right)} F_2(x) \right) \\ + \omega^2 \left(N - 2b_1 p_1 + cb_1 + z_1(p_1) - \int_L^{z_1(p_1)} F_1(x) \right) = 0. \end{aligned} \quad (9)$$

Then,

(a) \hat{p}_1 is unique.

(b) If $\omega \tilde{p}_2 - \tilde{p}_1 - c_G \leq 0$, then the manufacturer's optimal strategy is to ignore the parallel importer.

Thus, $p_1^* = \tilde{p}_1$ and $p_2^* = \tilde{p}_2$.

(c) If $\omega \tilde{p}_2 - \tilde{p}_1 - c_G > 0$ and $\mu > 0$, where

$$\mu = N - 2b_1 \hat{p}_1 + \frac{c_G}{2\omega(1-\omega)} b_2 + z_1(\hat{p}_1) + c \left(b_1 + \frac{b_2}{2\omega} \right) - \int_L^{z_1(\hat{p}_1)} F_1(x), \quad (10)$$

then the optimal strategy is to block the parallel importer by setting

$$p_1^* = \hat{p}_1, \quad p_2^* = \frac{\hat{p}_1 + c_G}{\omega}$$

(d) If $\omega\tilde{p}_2 - \tilde{p}_1 - c_G > 0$ and $\mu \leq 0$, then it is optimal to allow parallel importation. In this case p_1^* and p_2^* solve the following system of equations

$$\begin{aligned} N - 2b_1p_1 + \frac{2(\omega p_2 - p_1) - c_G}{2\omega(1-\omega)}b_2 + c \left(b_1 + \frac{b_2}{2\omega} \right) + z_1(p_1) - \int_{L_1}^{z_1(p_1)} F_1(x) dx &= 0 \\ N - 2b_2p_2 - \frac{2(\omega p_2 - p_1) - c_G}{2(1-\omega)}b_2 + c \frac{b_2}{2} + z_2(p_2) - \int_{L_2}^{z_2(p_2)} F_2(x) dx &= 0. \end{aligned}$$

(e) Let p_1^*, p_2^* be the optimal prices obtained above. Then the manufacturer's optimal quantities are

$$\begin{aligned} q_1^* &= d_1(p_1^*) + q_G(p_1^*, p_2^*) + z_1(p_1^*) \\ q_2^* &= d_2(p_2^*) - \omega q_G(p_1^*, p_2^*) + z_2(p_2^*). \end{aligned}$$

The manufacturer controls the parallel importer's order quantity through her prices. However, changing the prices also affects the demand of the authorized channels. Therefore, she should choose prices that balance these effects. For given prices p_1 and p_2 , if the parallel importer is allowed to transfer the product, the change in the manufacturer's total profit will be $(p_1 - \omega p_2 - c(1 - \omega))q_G$ which is negative because the importer would enter only if $\omega p_2 - p_1 > c_G$. This means that, while total sales increase according to Proposition 2, the manufacturer's profit would always be lower in the presence of a parallel importer. Thus, if \tilde{p}_1 and \tilde{p}_2 happen to block the importer (i.e., $\omega\tilde{p}_2 - \tilde{p}_1 - c_G \leq 0$), then the manufacturer should simply ignore the parallel importer. This situation can arise in several circumstances. First, if ω is very low, the manufacturer does not need to worry about the gray market because consumers significantly differentiate between the manufacturer and the importer and are not much inclined to buy the product from the gray market. Second, if the importer incurs a high cost (c_G) for transferring the product to market 2, the difference between \tilde{p}_1 and \tilde{p}_2 may not be large enough to cover the costs. Third, if the difference between price elasticities is small, then \tilde{p}_1 and \tilde{p}_2 will be naturally close and can prevent parallel importation, even if ω is moderately high or c_G is small.

When $\omega\tilde{p}_2 - \tilde{p}_1 - c_G > 0$, the manufacturer has to change her prices and deviate from \tilde{p}_2 and \tilde{p}_1 . In this scenario, the optimal strategy would be to either block the parallel importer (by setting $\omega p_2 - p_1 - c_G = 0$), or to allow him (by setting $\omega p_2 - p_1 - c_G > 0$). Thus, one can solve the SSG by imposing the constraint $\omega p_2 - p_1 - c_G \geq 0$. The parameter μ defined in (10) is simply the shadow price of this constraint for $(\hat{p}_1, \frac{\hat{p}_1 + c_G}{\omega})$. Theorem 1 shows that the optimal blocking price, \hat{p}_1 , and its corresponding shadow price, μ , are the factors that determine whether the optimal strategy is to

allow or block the parallel importer. If \hat{p}_1 makes the corresponding shadow price positive, then the constraint will be tight and the manufacturer will block the importer via \hat{p}_1 and $\frac{\hat{p}_1 + c_G}{\omega}$. However, if the shadow price is not positive, then allowing parallel importation is the optimal strategy.

Note that \hat{p}_1 is bounded from above as $\hat{p}_1 < \omega \tilde{p}_2 - c_G$ due to (4) and (5), and strict quasiconcavity of $\bar{\pi}$. Thus, μ will be positive when ω approaches one because $\frac{c_G}{2\omega(1-\omega)}b_2$ will be the dominating term in (10), making block the optimal strategy. In this case, the products in the gray-market become perfect substitutes for products in the authorized channel and the competition is highly intense. Therefore, if allowed, the parallel importer could gain a relatively significant size of market 2. Thus, the manufacturer's optimal strategy is to block the parallel importer when ω is high enough. Later in this paper, we define such blocking as *blocking from a position of weakness*.

Similarly when ω approaches zero, $\frac{c_G}{2\omega(1-\omega)}b_2 + c\frac{b_2}{2\omega}$ will be the dominating term in (10), making μ positive and hence blocking the parallel importer is the optimal strategy. In this case, the gray market could barely exist as products are highly differentiable. Thus, a simple and small closing of the price gap between the two markets could make parallel importation no longer profitable. Therefore, for low enough values of ω the manufacturer's optimal strategy is to block the importer by slightly altering her prices in both markets. We define such blocking as *blocking from a position of strength*.

Finally, for intermediate values of ω when $\frac{c_G}{2\omega(1-\omega)}b_2 + c\frac{b_2}{2\omega}$ is small enough, the value of μ may be negative and the manufacturer's optimal strategy would be to allow the parallel importer. This is because $\tilde{p}_1 < \hat{p}_1$ due to (4), (9), and strict quasi-concavity of $\bar{\pi}$. In this scenario, blocking the importer is simply too costly as it requires significant departure from optimal prices. This is because the importer is not too weak to be blocked easily and not extremely competitive to pose a significant threat to market 2. Therefore, for moderate values of ω the optimal strategy of the manufacturer is to let the importer enter market 2.

Next we look at how the presence of the parallel importer changes the manufacturer's prices.

PROPOSITION 5. *If $\omega \tilde{p}_2 - \tilde{p}_1 - c_G > 0$, the presence of the parallel importer forces the manufacturer to increase her price in market 1 and reduce her price in market 2; i.e., $p_1^* > \tilde{p}_1$ and $p_2^* < \tilde{p}_2$.*

The presence of the parallel importer forces the manufacturer to reduce her price gap, whether she allows or blocks the importer. When the manufacturer allows the importer to transfer the product, she increases her price in market 1 because the importer generates extra demand in that market. On other hand, because the movement of product to market 2 creates competition, the manufacturer needs to reduce her price in that market. If the manufacturer's decision is to block the importer, she has to choose prices so that $\omega p_2 - p_1 - c_G = 0$. Doing so by increasing p_1 or

reducing p_2 alone severely hurts the manufacturer's demand in the authorized channels. Therefore, she reduces the price gap by adjusting p_1 upward and p_2 downward.

Next we analyze the effect of demand uncertainty on the manufacturer when she faces parallel imports. For this purpose, we define the Deterministic Stackelberg Game (DSG) as the deterministic version of the SSG in which ϵ_1 and ϵ_2 are replaced by zero. The DSG is formulated as

$$(DSG) \quad \max_{p_1, p_2} \pi^d = (p_1 - c) (\mathcal{D}_1(p_1, 0) + q_G) + (p_2 - c) (\mathcal{D}_2(p_2, 0) - \omega q_G) \quad (11)$$

and $q_1^d = N - b_1 p_1^d$ and $q_2^d = N - b_2 p_2^d$. The next proposition compares the prices of the SSG with those of the DSG.

PROPOSITION 6. *The optimal prices in the SSG are always smaller than the optimal prices in the DSG; i.e., $p_1^* < p_1^{*d}$ and $p_2^* < p_2^{*d}$.*

In the literature, p_1^{*d} and p_2^{*d} are referred to as riskless prices. Dada and Petruzzi (1999) have shown that the optimal price for a manufacturer that serves a single market is *always* below the riskless price. The same result does not automatically follow in our problem because the manufacturer serves two markets that are linked to each other through the parallel importer. Nevertheless, with the IFR property the optimal prices in the stochastic environment are below their corresponding riskless prices.

4. Numerical Experiments

This section and the next present numerical experiments that respond to the motivating questions raised in the introduction. More specifically, we explore the effects of the parallel importer and demand uncertainty on the decisions made by a strategic manufacturer. In the next section, we demonstrate some managerial insights, which can be obtained from these numerical experiments that can help address some policy questions of interest to the manufacturer, such as market entry and exit decisions, and the effectiveness of myopic policies (e.g., uniform pricing) compared to the optimal strategic decisions, among others.

We implemented decisions and evaluated outcomes for more than 250 cases. In each experiment, we obtained the optimal price and quantity values using the MATLAB optimization toolbox, which produced result structures that are consistent across all cases. We based parameter values on the estimated production cost of the iPad and 2010 sales figures and average price in the United States (*Computer World* 2010, *eMarketer* 2010), assuming a linear demand-price curve. We set the manufacturer's per-unit cost to $c = \$250$, and the parallel importer's transfer cost to $c_G = \$10$. We also normalized $b_2 = 1$, varied b_1 from 1.25 to 4, and varied N from 1000 to 2500 in order to evaluate the model's sensitivity with respect to these parameters. To account for demand variability, we

assumed ϵ_1 and ϵ_2 have the same distribution. We focused on uniform and normal distributions as they are widely used in the literature (e.g., Yao et al. (2006), Schweitzer and Cachon (2000), Benzion et al. (2008)). Because the behaviors we observed for normal distribution were not significantly different from those for uniform distribution, we only present the figures for uniform distribution.

4.1. Joint Model Design

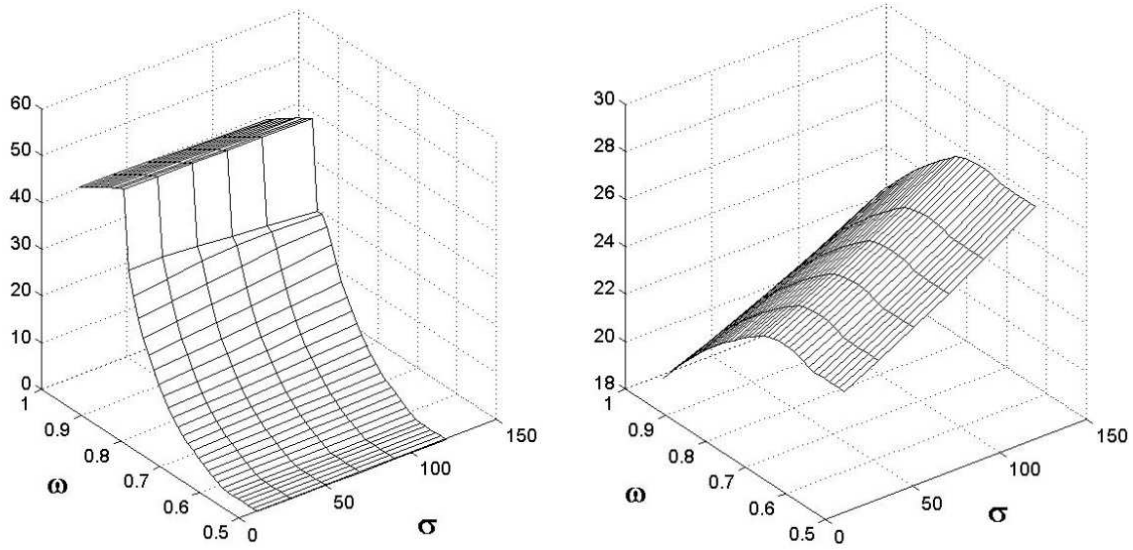
We have created a unified framework that depicts gray markets developing in a vertically integrated supply chain with uncertain demand. As pointed out in the literature review section, previous work in operations management largely dealt with price and quantity decisions in the presence of either a parallel importer or uncertain demand. The joint model presented here considers the effects of both parallel importation and demand uncertainty as they affect optimal price and quantity decisions.

4.1.1. Impact of ignoring parallel imports

In the first set of experiments, we study the impact of ignoring the presence of the parallel importer on the manufacturer's profit. More specifically, we determine by solving (3) how much profit the manufacturer would forfeit if she ignores the possibility of parallel importation emerging as a result of price differentials and treats each market independently. In general, we observe that the magnitude of profit losses can be between 1% and 89%, depending on the values of ω and σ . Figure 4(a) shows the percentage of the manufacturer's profit loss when she continues to use \tilde{p}_1 and \tilde{p}_2 for $b_1 = 2$, values of σ ranging from 10 to 120, and ω ranging from 0.5 to 0.95. We observe that when parallel imports have a very low reputation (small ω), the manufacturer loses a relatively small amount of 5% of her profit. This is intuitive as for small enough values of ω the parallel importer is not a major threat and the manufacturer's profit would not suffer greatly by ignoring parallel importation. As ω increases, however, the parallel importer emerges as a stronger competitor and the manufacturer's profit loss increases, as a result of ignoring parallel importation. We note that even for relatively moderate values of ω (e.g., between 0.7 and 0.8 for this case), the manufacturer can lose between 20% to 40% of her optimal profit by ignoring the parallel importer. For larger values of ω , the profit loss can even exceed 40%. Therefore, when making price and quantity decisions, it is crucial that manufacturers take gray markets into consideration for this range of ω . This issue can be of special importance when ω is close to 1. We study this case in more details in Section 5.1.

4.1.2. Impact of ignoring demand uncertainty

We now turn our attention to the case in which the manufacturer is aware of the presence of the parallel importer, but ignores demand uncertainty, and evaluate the corresponding profit loss to the manufacturer when variability in demand is not taken into account. For this purpose, we first solve the DSG in (11) and obtain its optimal solution, $(p_1^{*d}, p_2^{*d}, q_1^{*d}, q_2^{*d})$. We then evaluate



(a) Error percentage of using \tilde{p}_1 and \tilde{p}_2 as a heuristic solution for the SSG.

(b) Error percentage of using p_1^{*d} and p_2^{*d} as a heuristic solution for SSG.

Figure 4 Error percentage of ignoring parallel imports or demand uncertainty.

the profit of the SSG for the deterministic decision variables. In our test set, the percentage of reduction in manufacturer's profit varied between 17% to 36% for different values of ω and σ . Figure 4(b) illustrates one such example for $b_1 = 2$ and different values of ω and σ . One can observe that ignoring demand uncertainty can be detrimental to the manufacturer's profit. In this case, the manufacturer could lose between 18% (when $\sigma = 10$) and 26% (when $\sigma = 120$) of her profit, if she ignores variability in demand. Therefore, it is important for the manufacturer to account for both uncertainty in demand and parallel importation.

4.2. Impact of Parallel Importation on Manufacturer's Decisions

4.2.1. Quantities

Proposition 2 states that if there is a feasible market for the parallel importer, demand for the manufacturer in market 1 increases (due to orders placed by the parallel importer), while her demand in market 2 decreases (due to some customers switching to the gray market). Therefore, one would expect that a strategic manufacturer, who is aware of the presence of the parallel importer, would store more of the product in market 1 in order to maintain the same service level to her non-parallel importer customers; and, stock less in market 2 because she will lose the low-end of market 2 to the parallel importer. Interestingly in our numerical experiments, we observe that the opposite effect occurs; the manufacturer's quantity in market 1 will be below the quantity level in the absence of parallel imports, and her quantity in market 2 will be more than the quantity before

the presence of the parallel importer. Figures 5(a) and (b) depict the ratio of q_1^*/\tilde{q}_1 and q_2^*/\tilde{q}_2 when $b_1 = 2b_2$ for $\sigma = 30$ and 50. Notice that q_1^*/\tilde{q}_1 is always less than 1, whereas q_2^*/\tilde{q}_2 is always greater than 1.

This behavior can be explained as follows. Parallel importation influences the manufacturer's quantities in two ways. First it increases the demand in market 1 by q_G and decreases the demand in market 2 by ωq_G . Thus the manufacturer would like to increase q_1 and decrease q_2 accordingly. Second, as shown in Proposition 5, it forces the manufacturer to increase p_1 and reduce p_2 . Because demand is decreasing in price, the demand of the authorized channel in market 1 decreases while the demand of the authorized channel in market 2 increases. The second effect proves to be stronger, and ultimately the manufacturer keeps a lower stockpile in market 1 and a higher quantity in market 2. This tradeoff can be shown analytically when demand is deterministic. Proposition 7 formalizes this argument.

PROPOSITION 7. *In the DSG, the optimal quantity in market 1 (market 2) in the presence of parallel importation is smaller (larger) than the optimal quantity when there are not parallel imports, i.e., $q_1^{*d} < \tilde{q}_1^d$ and $q_2^{*d} > \tilde{q}_2^d$.*

While this proposition proves the result for the deterministic demand case, we observed the same behavior to that of Proposition 7 and Figure 5 across all the experiments when demand was random.

4.2.2. Price gap and profits

In this section, we extend the experiments to assess the effect of ω on the manufacturer's profit and price gap between the two markets. Figure 6(a) shows the manufacturer's price gap for values of ω and $b_1/b_2 = 1.5, 1.75, 2$ when $\sigma = 50$. We observe that the price gap is non-increasing in ω . This is hardly surprising because when consumers have high valuation for gray-market products, the competition intensifies and the manufacturer is forced to reduce her price gap in order to reduce the margin of the parallel importer.

The reduction in price gap leads to reduction in profit as we see in Figure 6(b). Although the total profit is non-increasing in ω , the profit in each market is not monotone. Figure 7(a) and (b) show the profit in market 1 and market 2, respectively. When ω exceeds $\frac{\tilde{p}_1 + c_G}{p_2}$, the profit in both markets goes down because the manufacturer increases p_1 and reduces p_2 to block the parallel importer. As ω increases further, the manufacturer is better off allowing parallel importation. Thus, the profit in market 1 increases by selling to the parallel importer. However, the profit in market 2 declines because the manufacturer is losing market share. When ω is very high, the revenue from selling to the parallel importer in market 1 no longer outweighs the loss of profit in market 2. At this point, it is better for the manufacturer to block the importer. Therefore, profit in market 2 goes up and profit in market 1 goes down.

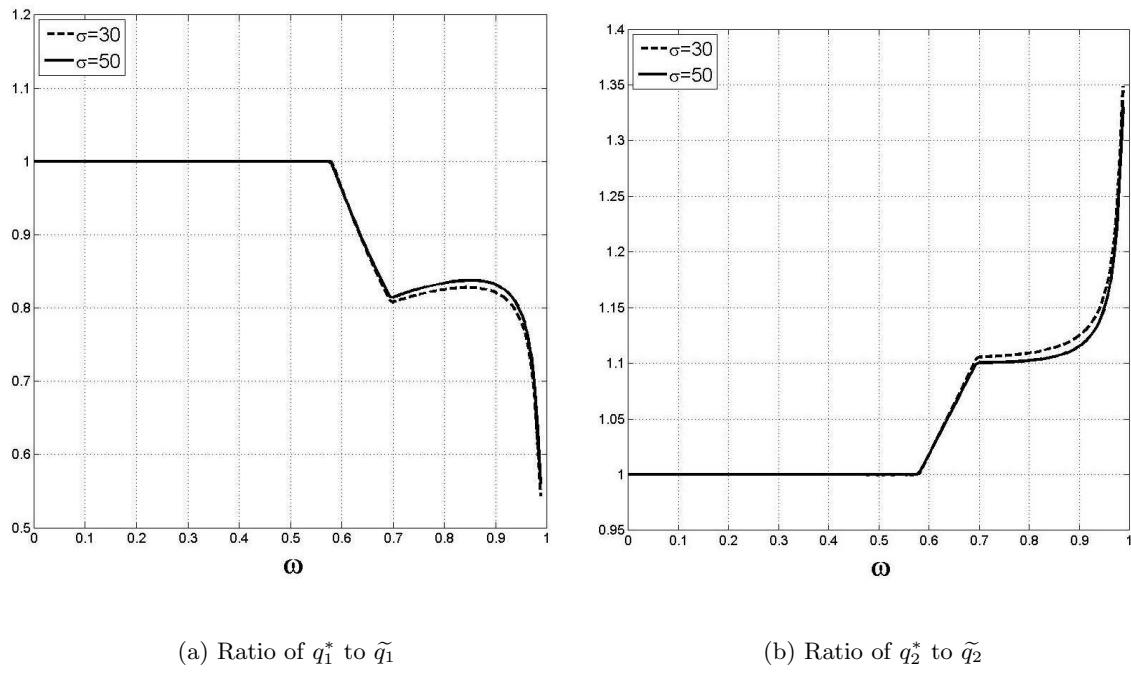


Figure 5 Ratio of optimal quantities with parallel imports to quantities without parallel imports.

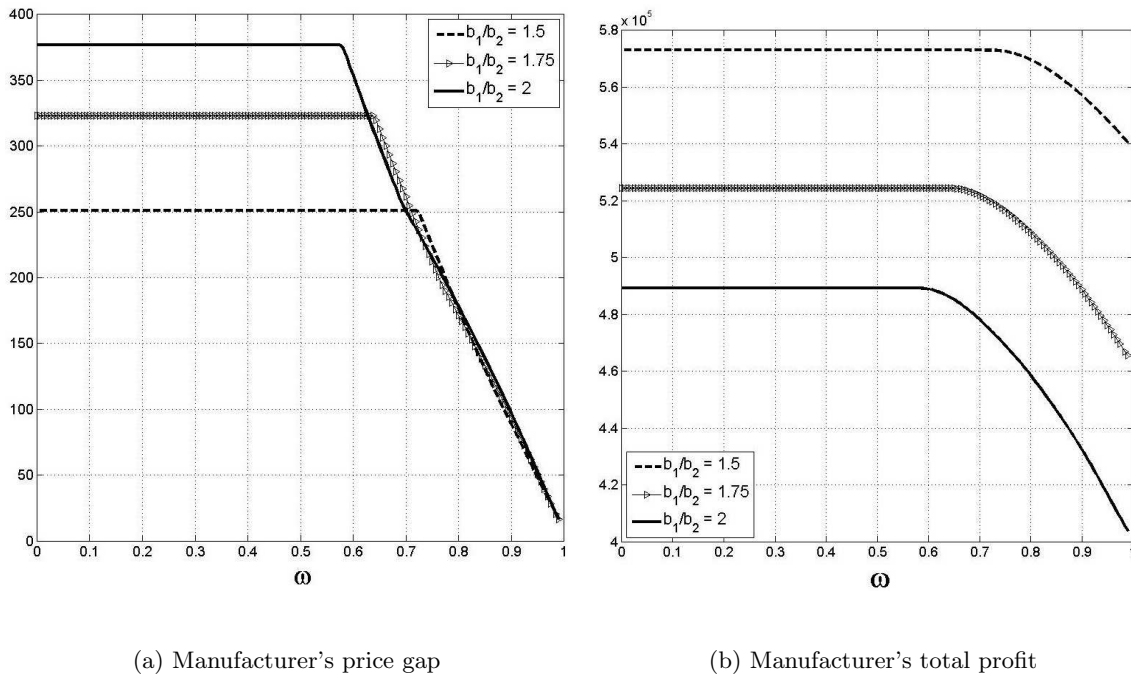
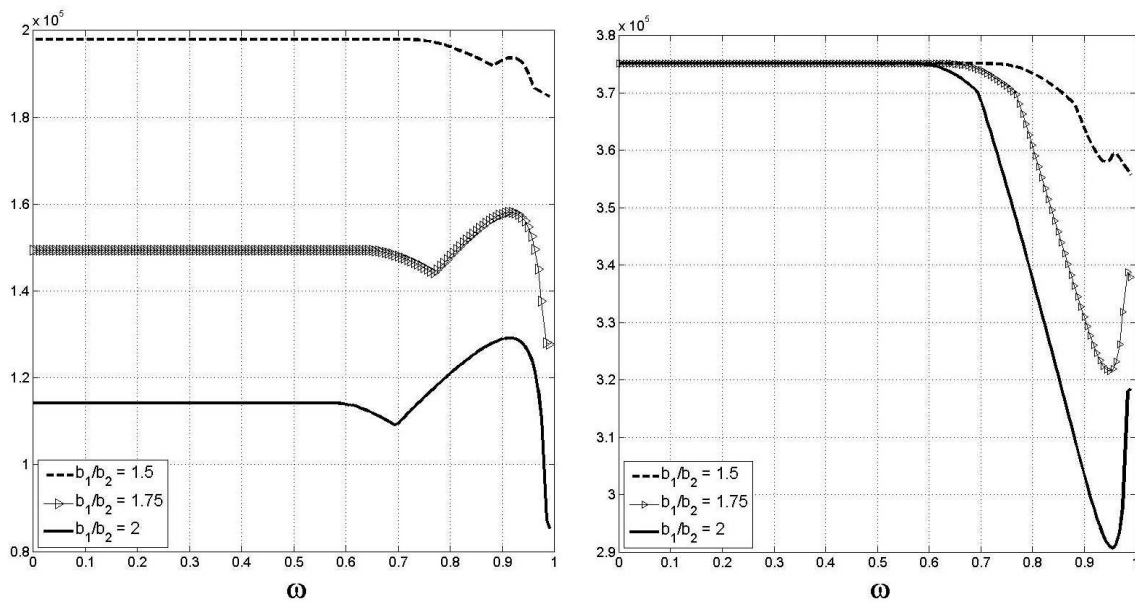


Figure 6 Manufacturer's price gap and total profit as a function of ω and b_1/b_2 .



(a) Manufacturer's profit in market 1

(b) Manufacturer's profit in market 2

Figure 7 Manufacturer's optimal profit in each market.

5. Managerial Insights

In this section, we generate managerial insights to inform the debate over policies and strategic decisions that a manufacturer facing the threat of parallel importation would consider. The analysis of the SSG in Section 3 was based on the assumption that the manufacturer had decided to enter both markets and establish her distribution channels before the gray market could potentially emerge. In this section, we expand our analysis and assume the manufacturer has the option to eliminate the parallel importer entirely by leaving market 1 and only serving market 2. Of course entering and exiting a market incurs costs, but we include this aggressive reaction and analyze the extreme behaviors to highlight the fact that the gray market not only affects the manufacturer's price and quantity decisions, it can also influence her decision whether to serve the low-profit market or not.

We refer to the strategy of leaving market 1 as the *single market strategy* to distinguish it from the block strategy. The reason the manufacturer may decide to abandon market 1 is that the block strategy calls for reducing the price of market 2 (hence losing the extra profit from higher prices) and increasing the price of market 1 (hence losing market share due to high price). Blocking may be a viable strategy as long as the price gap, $\tilde{p}_2 - \tilde{p}_1$, is not very large. However, if there is a very large gap between the prices, the manufacturer would have to sacrifice a lot of her profits in both markets if she wants to reduce the price gap and block the importer. Therefore, if the importer

becomes strongly competitive, it may be better for the manufacturer to give up the profit in market 1 for the sake of the much higher profit she would earn in market 2 when she serves market 2 without the parallel importer's presence.

Although we use the phrase leaving market 1, the single market strategy can be also interpreted as the market entry decision because quite often a product is released in different markets sequentially. Consider a manufacturer who serves only a highly profitable market (market 2) and is examining her entry to a less profitable market (market 1). One main barrier to entering the new market is the emergence of gray markets in her highly profitable market. Entering market 1 would be a viable and profitable strategy if changing the prices is not too costly and the risk of the gray market emerging is not too high.

Mathematically, the single market strategy corresponds to setting $q_1 = 0$ and offering the product at price \tilde{p}_2 in market 2. Because this is a boundary solution, it is difficult to derive a closed-form necessary and sufficient condition for the optimality of this strategy. If the optimal value of p_1 in Theorem 1 is such that $q_1 \leq 0$, then the single market strategy is optimal. However, this condition is not necessary. Therefore, we use numerical experiments to investigate the role of this strategy.

Most of the results presented in this section can be explained through either market-based or product-specific parameters. Market-based parameters describe the features of each market, such as size, price elasticity, and magnitude of demand uncertainty. Product-specific parameters describe the item under consideration. The main parameter that represents product characteristics in our model is ω , which measures the degree of differentiability between the products sold in authorized channels and those traded in the gray market. A low value of ω means that parallel imports and authorized-channel products are quite distinct in the eyes of consumers. The higher the ω , the more intense the competition between the manufacturer and the importer.

With that understanding, we define *market conditions* as the aggregate effect of relative market-based parameters, such as relative price elasticities (b_1/b_2) and relative market sizes (N_1/N_2). We say market conditions are *similar* if the parameters of the market are such that the price gap would naturally be small even if there are no parallel imports. On the other hand, we say market conditions are *different* if the price gap would naturally be large in the absence of parallel imports. We define *commodity* items as products for which consumers have a relatively high perception of parallel imports and do not distinguish between the authorized channel and the gray market (i.e., $\omega \approx 1$). At the other end of the spectrum, *fashion* items are ones for which consumers have a relatively low perception of parallel imports ($\omega \ll 1$).

We begin teasing out optimal strategies for the manufacturer by exploring the implications of various responses to parallel importation and the effects of product characteristic and market conditions on the optima strategy. Then, we compare the strategic pricing policy that our model

prescribes to a uniform-pricing policy in which the manufacturer eliminates parallel importation by charging the same price in both markets. Finally, we briefly describe the effect of product characteristic on the parallel importer's order size and profit.

5.1. How Do Market Conditions and Product Characteristic Determine Strategy?

Though determining the optimal price and quantity decisions for a strategic manufacturer has been the main focus of this paper, an important high-level question for any manufacturer is: *What is the best strategy to cope with the gray market?* In this section, we further build on the manufacturer's strategies discussed in Section 3 and examine the manufacturer's reactions to the gray market to better understand the effects of market conditions and product characteristics. Our numerical experiments indicate that neither of the responses completely dominates the others. In fact, each can emerge as the optimal strategy for a certain range of parameters. In Section 5.2, we consolidate the scenarios to propose the best strategy for a wide range of market conditions and product characteristics.

1. Ignore the parallel importer. Consider a scenario in which the product under consideration is highly fashionable and/or the parallel importer's transfer cost, c_G , is very high. Then, it would be too costly for the gray market to emerge, independent of the manufacturer's prices. Thus, the manufacturer can simply ignore the parallel importer. A similar outcome can occur if the market conditions are so similar that \tilde{p}_1 and \tilde{p}_2 are close to each other. In this situation these prices would render the gray market unprofitable unless ω is very high or c_G is very small.

To recap, the manufacturer can safely ignore the parallel importer if one or a combination of these factors is present: (1) the product is a fashion item; (2) market conditions are relatively similar; or (3) c_G is very large.

2. Block parallel imports. Suppose that product characteristics, market conditions, and transfer costs are such that the gray market could (barely) exist. However, a simple and small closing of the price gap between the two markets could make parallel importation no longer profitable. In this scenario, the manufacturer would slightly alter her prices in both markets and thus block the parallel importer from a position of strength.

Alternately, the manufacturer would block the gray market when the parallel importer could emerge as a strong competitor. This can happen when the product is a commodity and the market conditions are not too different. The gray market, if allowed, could undercut the manufacturer and gain a significant portion of the more profitable market. The manufacturer would make a more substantial reduction in her price gap and thus block the parallel importer from a position of weakness.

3. Allow parallel imports. Suppose the market conditions are moderately different and the product is in transition from fashion to commodity. Blocking the parallel importer in such a setting requires a significant deviation from otherwise optimal prices. Furthermore, because the product has not yet become a commodity, the parallel importer is not a grave threat to the manufacturer. In this situation, the manufacturer would allow the gray market to emerge simply because the cost of blocking the parallel importer (losing profits in both markets due to suboptimal prices) exceeds the cost of allowing parallel imports (lower sales in market 2).

4. Single market strategy: market exit (and entry) conditions. We mentioned that the manufacturer blocks the importer from a position of weakness when the product is a commodity. However, we observe that the manufacturer prefers the single market strategy over the block strategy when the product is a commodity and market conditions are sufficiently different. In this situation, blocking the parallel importer simply becomes too costly. The manufacturer would lose significant portions of her profit in the two markets if she insists to stay in both markets. Therefore, she foregoes the relatively small profit in market 1 entirely to eliminate the parallel importer.

5.2. Which Strategy to Choose?

By characterizing the four strategies as regions, which we then compare with one another, we can illustrate the transition from one optimal strategy to another as product characteristic and market conditions change. Figure 8 illustrates the transition between optimal strategies; in each case, the sequence of strategies remains the same.

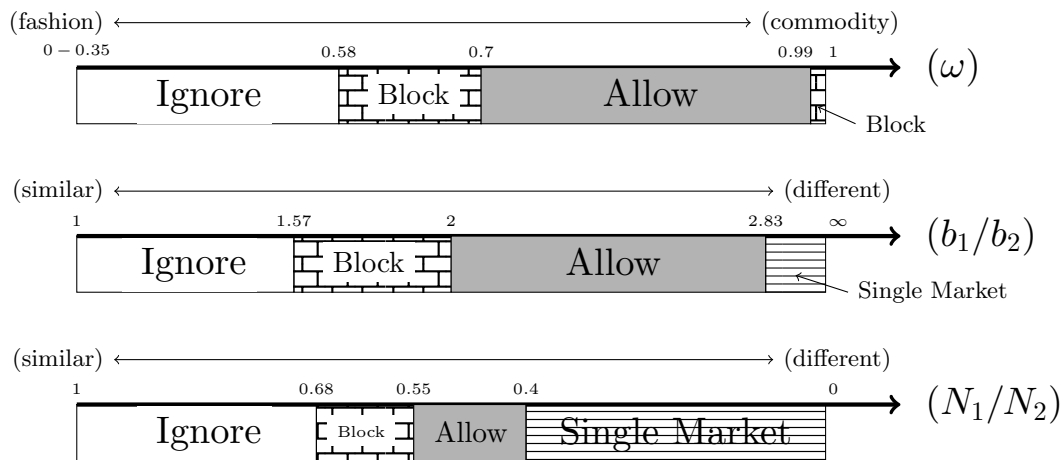


Figure 8 Optimal strategy as a function of product characteristic and market conditions.

Taking product characteristic as a key element, one can observe the following order for the optimal strategy: (1) When the product is highly fashionable (low consumer perception), the optimal strategy is to ignore the gray market. (2) As parallel imports gain acceptance from consumers and

the emergence of the gray market becomes feasible, the manufacturer's best strategy is to block the parallel importer (from a position of strength) by slightly reducing the price gap between the two markets (3) For a higher perception of parallel imports, it is no longer beneficial for the manufacturer to block the gray market as it requires large deviations from otherwise optimal prices for each market. In this region parallel imports are allowed into market 2. (4) Finally, when the product is a commodity, the manufacturer blocks the parallel importer (from a position of weakness) by closing the price gap between the two markets.

We can observe a similar order of regions when the effect of market conditions on the optimal strategy is taken into consideration. Figure 8 shows the effect of each element of b_1/b_2 and N_1/N_2 . The regions are similar except for region 4, which changes from the block strategy to the single market strategy. We note that in both realizations of region 4 (single market or block), the final outcome is the same: the manufacturer no longer tolerates the gray market as it imposes a significant threat to her profits in the high-price market 2. The elimination of the gray market can take one of the two forms (block or single market strategy), depending on the relative difference between markets 1 and 2 and the relative profit that the manufacturer can extract from each market.

Figure 9 provides a more complete analysis of the simultaneous effects of product characteristic and relative price elasticities as one measure of market conditions on the regions characterizing the optimal strategy. The three graphs also illustrate the effects of demand uncertainty on the optimal policy. In all three, the optimal policy is plotted against consumers' perception of parallel imports and the standard deviation of demand (assumed to be the same for both markets). Figure 9(a), (b), and (c) represent scenarios in which price elasticities are relatively similar to, somewhat similar to, and relatively different from each other respectively. Note that we observe the same order of regions in these plots as well. When the product is a highly fashionable item and price elasticities are fairly similar, the manufacturer should ignore the parallel importer. As these parameters grow, the optimal strategy changes to blocking the importer (from a position of strength) and then to allowing parallel imports. Finally when the product is a commodity and price elasticities are relatively similar, the manufacturer should stay in both markets, but block the parallel importer (from a position of weakness). When price elasticities are relatively different, however, the manufacturer of a commodity product should leave the low-price market and only serve the high-price market.

Moving from graph (a) to graph (b), one can observe that as price elasticities become more different, allowing the parallel importer is the optimal strategy for a wider range of parameters. This is not surprising because, when markets become different, blocking the parallel importer requires a more significant deviation from otherwise optimal prices. This holds true until the markets are sufficiently different so that the single market strategy becomes a better option, in which case the manufacturer should simply leave market 1 and serve market 2 only.

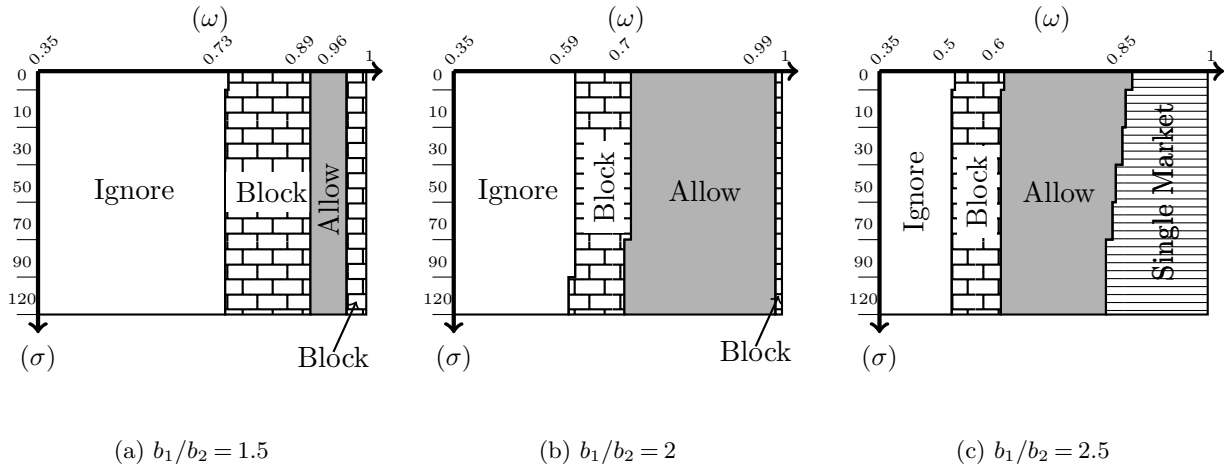


Figure 9 Optimal strategy regions for the manufacturer.

Another situation anticipated by these graphs is one in which the magnitude of standard deviation of demand forces the manufacturer to switch from one strategy to another. For example, the region in which the single market strategy becomes optimal is clearly growing with the size of the standard deviation of demand. This is because highly-uncertain demand makes market 1 even more unattractive to the manufacturer when the markets are sufficiently different. Proposition 6 states that demand uncertainty results in lower prices in both markets. However, this statement does not hold in the presence of the single market strategy because uncertain demand may force the manufacturer to leave market 1 sooner (lower values of ω) than when $\sigma = 0$. In this case, the manufacturer will offer the product at price \tilde{p}_2 , which can be higher than p_2^{*d} .

Figure 10 summarizes the effects of product characteristic and market conditions on the manufacturer's optimal strategy. We note that market condition is a relatively generic parameter representing the difference between the two markets and is generally dependent on relative market sizes and price elasticities. One can consider a more accurate value for relative market conditions based on the optimal price gap between the two markets when there are no parallel imports (i.e., $\tilde{p}_2 - \tilde{p}_1$). Also, for a more in-depth discussion one should consult Figure 9 for appropriate model parameters. Appendix 2 provides a closed-form solution for the profit of each strategy when demand is deterministic.

5.3. Strategic Versus Uniform Pricing

Implementing the optimal price and quantity decisions prescribed by the SSG requires estimating the value of parameters, such as the relative perception of parallel imports, ω , and the parallel importer's transfer cost, c_G , among others specific to the gray market. This requirement and all other issues related to gray markets urge some manufacturers to charge the same price for their

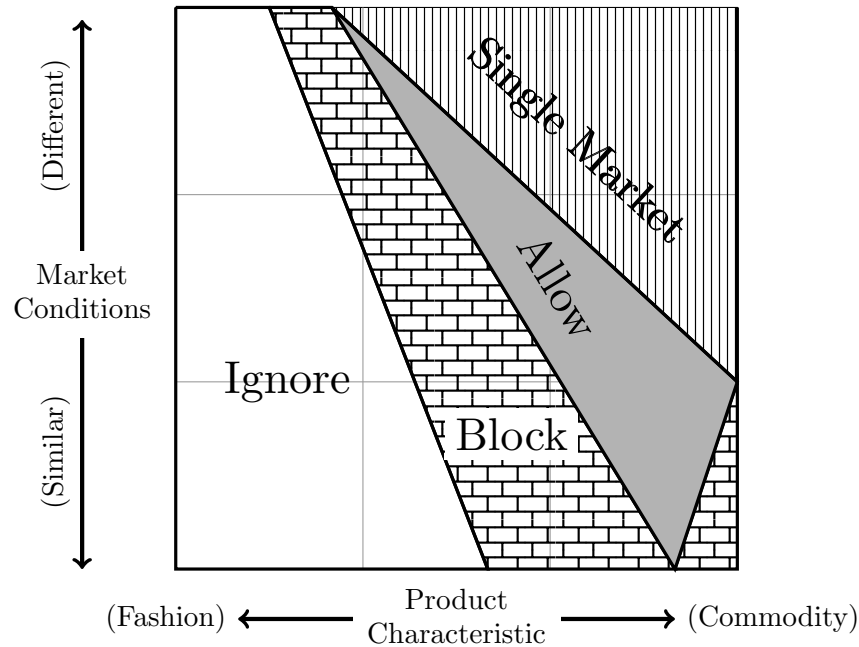


Figure 10 Optimal strategies as a function of product characteristic and market conditions.

products across all markets, parting with the added profit that can come from market-specific pricing (Antia et al. 2004).

Clearly, uniform pricing is a suboptimal pricing policy. We conducted a number of experiments to quantify the profit lost by adopting the uniform pricing policy instead of using the optimal prices of the SSG. Optimal uniform price can be obtained by solving (3) while enforcing $p_1 = p_2$. Figure 11(a) demonstrates a common behavior we observed in these experiments, illustrating the profit loss as a function of ω for various values of relative price elasticity, b_1/b_2 , when $\sigma = 50$ and $N_1 = N_2 = 1500$.

From this graph, we observe that despite the benefits of the uniform pricing policy such as easier implementation, this policy can result in a significant loss of profit as high as 25%. Therefore, choosing the prices strategically can indeed be very valuable.

Also for large b_1/b_2 , there is a value of ω beyond which the policies become identical (e.g., $\omega = 0.6$ for $b_1/b_2 = 3.25$) and the profit loss due to the uniform pricing strategy becomes zero. This situation arises when under both strategies it is optimal to leave market 1 and only serve market 2. The second market is simply attractive enough so that under uniform pricing, the optimal price is higher than the maximum price that market 1 would accept. Likewise, optimal strategic pricing determines that the manufacturer is better off leaving market 1. Furthermore, we observe that the value of ω beyond which the policies are identical decreases as relative price elasticity grows. This is expected because when b_1/b_2 grows, market 1 becomes less attractive.

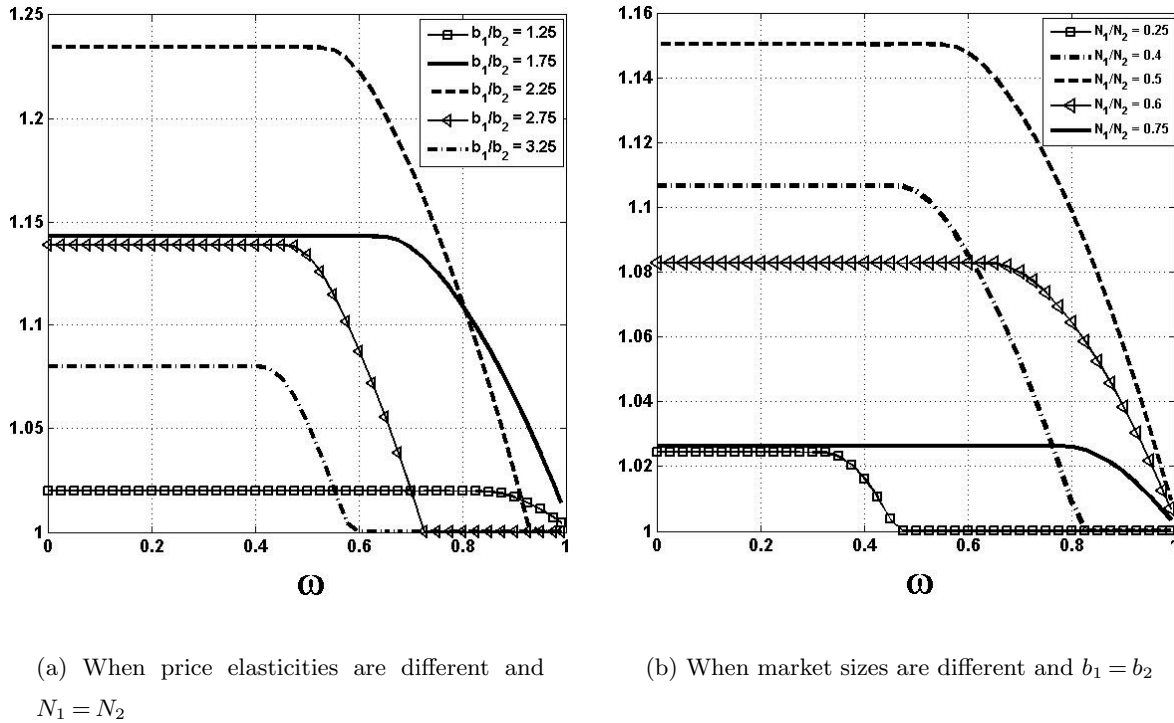


Figure 11 Ratio of optimal profit in the SSG to the uniform pricing profit.

The benefits of strategic pricing, moreover, are largest when price elasticities are not too close or too far apart, namely for moderate values of b_1/b_2 . For example when $b_1/b_2 = 3.25$ (high ratio), the additional benefit from strategic pricing is slightly above 5% and is even lower when $b_1/b_2 = 1.25$ (low ratio). This is because when price elasticities are close, both markets are similar and optimal strategic prices for both would be almost identical, thus making uniform pricing an appropriate policy. Alternatively, when price elasticities are disparate, as market 1 is very price sensitive, the manufacturer would not be able to charge a uniform price that attracts significant portions of both markets simultaneously. In this case, under the uniform pricing policy, the manufacturer abandons market 1 for the sake of the higher profit that she can earn from market 2. Although the manufacturer has the opportunity to boost her profit by charging markets differently under the strategic pricing policy, added profits from market 1 are relatively small compared to the more profitable market 2. The effect is exacerbated by the fact that the parallel importer also can exploit the high price differential and transfer a large amount of goods, reducing the manufacturer's share in the more profitable market 2. Put differently, the benefit from staying in both markets is highly restrained by the added cost of compromising on the price differential. Therefore as ω increases, a larger portion of market 2 is lost to the parallel importer to the extent that eventually it is no longer profitable for the manufacturer to continue serving both markets.

Finally, we observe that the added benefits of strategic pricing increase as the consumers' perception of parallel imports declines. This is intuitive because the manufacturer can exploit the difference between the two markets when the product under consideration is fashionable.

Figure 11(b) shows that when $b_1 = b_2$ and N_1/N_2 varies profit losses due to uniform pricing behave the same as when b_1/b_2 varies. The loss is highest for moderate values of N_1/N_2 and relatively lower when the market sizes are very close or very different. Thus, we determine that strategic pricing leads to a significant increase in profits for fashion products when the two markets are moderately different. In extreme cases, however, when markets are either too close or too different and when the product is a commodity, a simple uniform pricing policy can be considered a viable alternative to strategic pricing.

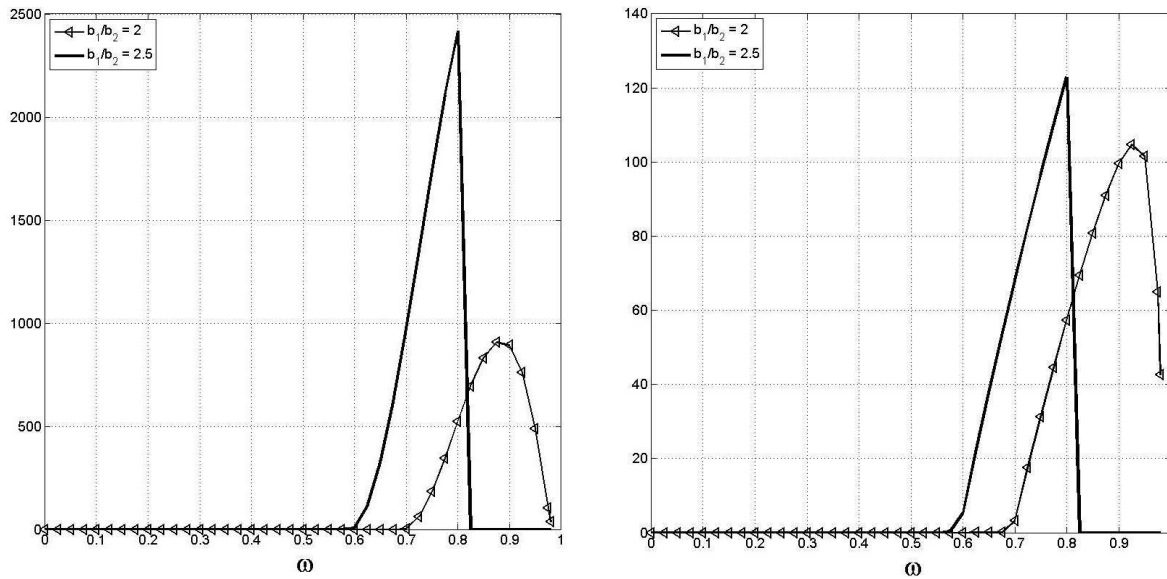
5.4. Parallel Importer's Problem

We close this section with Figure 12, which shows the parallel importer's profit and purchase quantities as a function of ω when $N_1 = N_2 = 1500$, $\sigma_1 = \sigma_2 = 70$, and $b_1/b_2 = 2, 2.5$. Note that the parallel importer's profit is positive for $\omega \in [0.7, 0.99]$ when $b_1/b_2 = 2$, and for $\omega \in [0.6, 0.83]$ when $b_1/b_2 = 2.5$. These ranges are consistent with the Allow regions of Figure 9. Interestingly, while in the Allow region, the parallel importer's profit is a non-monotone function of ω ; that is, as the product becomes more of a commodity, the parallel importer's profit decreases due to the manufacturer's aggressive pricing. What is noteworthy is that the parallel importer's profit is unimodal in ω and is maximized at intermediate values of ω . Even though higher ω strengthens the parallel importer's position, a degree of differentiability between the gray market and the authorized channel is actually something the importer needs to achieve maximum profit.

6. Conclusion

Gray markets pose a serious challenge to many companies in various industries. Appropriate reactions to the presence of a gray market is an important issue to manufacturers. In this paper, we analyze the impact of parallel importation on a manufacturer's price and quantity decisions in an uncertain environment.

We find that the manufacturer's reaction to the parallel importer depends heavily on market conditions and product characteristics. If the product is a fashion item, the manufacturer eliminates the importer. For similar market conditions, elimination may be possible without changing prices. However, prices need to be adjusted when market conditions are different. The manufacturer also eliminates the importer if the product is a commodity. This time she may be forced to leave the less profitable market and only serve the more profitable market. Finally, if the product is in transition from a fashion item to a commodity, the manufacturer allows the importer to operate if the market conditions are moderately different. We also find that strategic pricing is indeed more valuable than



(a) Parallel importer's profit

(b) Parallel importer's quantity

Figure 12 Parallel importer's optimal profit and order quantity for $\sigma_1 = \sigma_2 = 70$.

a uniform pricing policy especially when the product is closer to a fashion item and the market conditions are not too different or too similar.

Our work can be extended in several directions. First, one could consider a multi-period setting in which the manufacturer and the parallel importer interact repeatedly. It would be interesting to see the impact of the importer on the manufacturer's quantity and pricing decisions and also how the manufacturer would switch between the policies over time.

Second, we assume in our model that the importer only relies on an estimate of the average demand and does not have knowledge of the uncertainty in demand. One natural extension is to assume that the importer has the means to estimate the parameters of his demand distribution and analyze the ordering and pricing decisions of the importer.

Third, we consider an uncapacitated manufacturer. Most manufacturers, especially those that produce fashion products, have limited capacity. Limited capacity determines the manufacturer's allocation of quantities to each market, which then changes her prices. Also, because the importer acts as an agent who transfers the product between markets, he can influence the manufacturer's capacity investment decisions especially when the capacity costs are different across the markets.

Finally, we assume the manufacturer produces a single product. As mentioned earlier, product differentiation can give manufacturers more leverage to curb parallel importation. Offering a variety of products is a mechanism for reaching out to more segments of the market. We believe designing a line of differentiated products in the presence of a parallel importer merits future research.

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Supporting Document for “Coping with Gray Markets: The Impact of Market Conditions and Product Characteristics”

Appendix 1. Proofs

Proof of Proposition 1. Note that $z_1(p) - \int_L^{z_1(p)} F_1(x) = L + \int_L^{z_1(p)} (1 - F_1(x))$. Hence,

$$\begin{aligned} \left. \frac{\partial \pi}{\partial p_1} \right|_{p_1 = \tilde{p}_2} &= N_1 - 2b_1\tilde{p}_2 + cb_1 + L + \int_L^{z_1(\tilde{p}_2)} (1 - F_1(x)) \\ &= \frac{b_1}{b_2} (-2b_2\tilde{p}_2 + N_2 + cb_2) - \frac{b_1}{b_2} N_2 + N_1 + L + \int_L^{z_1(\tilde{p}_2)} (1 - F_1(x)) \\ &= \frac{b_1}{b_2} \left(-L - \int_L^{z_2(\tilde{p}_2)} (1 - F_2(x)) \right) - \frac{b_1}{b_2} N_2 + N_1 + L + \int_L^{z_1(\tilde{p}_2)} (1 - F_1(x)) \\ &= b_1 \left(\frac{N_1 + L + \int_L^{z_1(\tilde{p}_2)} (1 - F_1(x)) dx}{b_1} - \frac{N_2 + L + \int_L^{z_2(\tilde{p}_2)} (1 - F_2(x)) dx}{b_2} \right) \end{aligned}$$

The third equality is due to (5). Since the profit function is strictly quasiconcave in p_1 and p_2 , $\tilde{p}_2 > \tilde{p}_1$ if and only if the expression in the last line is negative. \square

Proof of Proposition 2. First consider the case of $\frac{\omega p_2 - p_G}{\omega(1-\omega)} b_2 < q_1$. Then $\pi_G = (p_G - p_1 - c_G) \frac{\omega p_2 - p_G}{\omega(1-\omega)} b_2$ is a concave function in p_G . The first order optimality condition and $q_G \geq 0$ give us (7), and the feasibility condition $\psi < q_1$. If $\frac{\omega p_2 - p_G}{\omega(1-\omega)} b_2 \geq q_1$, $\pi_G = (p_G - p_1 - c_G) q_1$ and it is optimal to increase p_G as much as possible. Thus $\frac{\omega p_2 - p_G}{\omega(1-\omega)} b_2 = q_1$, which gives us (8). \square

Proof of Proposition 4. The first part follows because the parallel importer buys q_G from the manufacturer in market 1. The change in the manufacturer’s demand in market 2 is $N - \frac{p_2 - p_G}{1-\omega} b_2 - (N - b_2 p_2) = \frac{p_G - \omega p_2}{(1-\omega)} b_2$. Because $\frac{\omega p_2 - p_G}{\omega(1-\omega)} b_2 \leq q_1$, the change of demand is equal to $-\omega q_G$. \square

Proof of Proposition 3. Suppose the manufacturer chooses her prices such that $\omega p_2 - p_1 - c_G > 0$, but she chooses $q_1 \leq \psi$. Then her profit will be

$$\max_{p_1, p_2, q_1, q_2} \pi = E \left\{ p_1 q_1 + p_2 \min(q_2, \mathcal{D}_2(p_2, \epsilon_2) - \omega q_1) - c(q_1 + q_2) \right\}.$$

Then $\frac{\partial \pi}{\partial q_1} = p_1 - \omega p_2 - c(1 - \omega)$. If the derivative is negative the manufacturer can increase her profit by setting $q_1 = 0$ and if it is positive she can increase the profit by increasing q_1 to ψ . Therefore, $q_1 \leq \psi$ is suboptimal. \square

Proof of Theorem 1. To prove that \hat{p}_1 is unique, define $h(p_1) = N - 2b_1 p_1 + z_1(p_1) - \int_L^{z_1(p_1)} F_1(x) + cb_1$. Then

$$\begin{aligned} h'(p_1) &= z_1'(p_1) \frac{c}{p_1} - 2b_1 = \frac{1}{c} \frac{[1 - F_1(z_1(p_1))]}{r_1(z_1(p_1))} - 2b_1 \\ h''(p_1) &= z_1''(p_1) \frac{c}{p_1} - \frac{c}{p_1^2} z_1'(p_1) \end{aligned}$$

where

$$z_1''(p_1) = \frac{-z_1'(p_1) f_1(z_1(p_1)) r_1(z_1(p_1)) - [1 - F_1(z_1(p_1))] z_1'(p_1) r_1'(z_1(p_1))}{c[r_1(z_1(p_1))]^2}$$

Thus,

$$\begin{aligned} h''(p_1) &= \frac{-z_1'(p_1)[1 - F_1(z_1(p_1))]}{c[r_1(z_1(p_1))]^2} \times \\ &\quad \left[f_1(z_1(p_1)) r_1(z_1(p_1)) + [1 - F_1(z_1(p_1))] \left[r_1'(z_1(p_1)) + [r_1(z_1(p_1))]^2 \right] \right] \\ &= \frac{-z_1'(p_1)[1 - F_1(z_1(p_1))]^2}{c[r_1(z_1(p_1))]^2} \times \left(2[r_1(z_1(p_1))]^2 + r_1'(z_1(p_1)) \right) < 0 \end{aligned}$$

Therefore $h(p_1)$ is concave, which means that $K(p_2) = N - 2b_2p_2 + z_2(p_2) - \int_L^{z_2(p_2)} F_2(x) + cb_2$ is also concave. Since $g(p_1, p_2) = \omega^2 h(p_1) + \omega K(p_2)$, $g(p_1, \frac{p_1 + c_G}{\omega})$ is concave. We find \hat{p}_1 when $\omega \tilde{p}_2 - \tilde{p}_1 - c_G > 0$. Because $\tilde{p}_1 > c$, we have $\frac{c + c_G}{\omega} < \tilde{p}_2$ and $K(\frac{c + c_G}{\omega}) > 0$. Therefore, $g(c, \frac{c + c_G}{\omega}) > 0$. Because $g(p_1, \frac{p_1 + c_G}{\omega}) < 0$ when p_1 is very large, we conclude that \hat{p}_1 is unique.

Now we prove parts (b) through (d). We consider two cases.

Case 1. $\omega p_2 - p_1 - c_G \leq 0$. In this case, $q_G = 0$ and the SSG can be written as

$$\begin{aligned} \max_{p_1, p_2, q_1, q_2} \pi &= E \left\{ p_1 \min(q_1, \mathcal{D}_1(p_1, \epsilon_1)) + p_2 \min(q_2, \mathcal{D}_2(p_2, \epsilon_2)) - c(q_1 + q_2) \right\} \\ &\quad s.t. \\ &\quad (\gamma) \omega p_2 - p_1 - c_G \leq 0 \end{aligned}$$

where $\gamma \geq 0$ is a nonnegative Lagrangian multiplier. For given prices p_1 and p_2 , π is concave in q_1 and q_2 . Thus

$$q_1 = N - b_1 p_1 + z_1(p_1)$$

$$q_2 = N - b_2 p_2 + z_2(p_2)$$

Replacing (14) in the profit function, we can write the KKT conditions:

$$\frac{\partial \pi}{\partial p_1} = N - 2b_1 p_1 + z_1(p_1) - \int_L^{z_1(p_1)} F_1(x) + cb_1 + \gamma = 0, \quad (12)$$

$$\frac{\partial \pi}{\partial p_2} = N - 2b_2 p_2 + z_2(p_2) - \int_L^{z_2(p_2)} F_2(x) + cb_2 - \omega \gamma = 0, \quad (13)$$

$$\gamma(\omega p_2 - p_1 - c_G) = 0, \omega p_2 - p_1 - c_G \leq 0, \gamma \geq 0.$$

If $\omega p_2 - p_1 - c_G < 0$, then $\gamma = 0$ and (12) and (13) reduce to (4) and (5). On the other hand, if $\omega p_2 - p_1 - c_G = 0$, then (12) and (13) reduce to $g(p_1, \frac{p_1 + c_G}{\omega}) = 0$ where

$$\begin{aligned} g(p_1, p_2) &= \omega^2 \left(N - 2b_1 p_1 + cb_1 + z_1(p_1) - \int_L^{z_1(p_1)} F_1(x) \right) \\ &\quad + \omega \left(N - 2b_2 p_2 + cb_2 + z_2(p_2) - \int_L^{z_2(p_2)} F_2(x) \right) \end{aligned}$$

Let \hat{p}_1 solve $g\left(p_1, \frac{p_1+c_G}{\omega}\right) = 0$. If $\gamma = -N + 2b_1\hat{p}_1 - z_1(\hat{p}_1) + \int_L^{z_1(\hat{p}_1)} F_1(x) - cb_1 \leq 0$, then the manufacturer ignores the importer. However, if $\gamma > 0$, then $\left(\hat{p}_1, \frac{\hat{p}_1+c_G}{\omega}, \gamma\right)$ is a solution to the KKT conditions. Because π is strictly quasiconcave, if $\gamma > 0$, then $\hat{p}_1 > \tilde{p}_1$ and $\frac{\hat{p}_1+c_G}{\omega} < \tilde{p}_2$, which means that $\omega\tilde{p}_2 - \tilde{p}_1 - c_G > 0$. Thus, if $\gamma > 0$, $\left(\hat{p}_1, \frac{\hat{p}_1+c_G}{\omega}, \gamma\right)$ is the only solution to the KKT conditions.

Case 2. $\omega p_2 - p_1 - c_G \geq 0$. In this case, $q_G = \psi$, and the SSG becomes

$$\begin{aligned} \max_{p_1, p_2, q_1, q_2} \pi &= E\left\{p_1 \min(q_1, \mathcal{D}_1(p_1, \epsilon_1) + \psi) + p_2 \min(q_2, \mathcal{D}_2(p_2, \epsilon_2) - \omega\psi) - c(q_1 + q_2)\right\} \\ & \text{s.t.} \\ (\mu) \quad & \omega p_2 - p_1 - c_G \geq 0 \end{aligned}$$

where $\mu \geq 0$. Similar to Case 1, for given p_1 and p_2 we have

$$\begin{aligned} q_1 &= N - b_1 p_1 + z_1(p_1) + \psi \\ q_2 &= N - b_2 p_2 + z_2(p_2) - \omega\psi \end{aligned} \tag{14}$$

The KKT conditions for this case are

$$\begin{aligned} \frac{\partial \pi}{\partial p_1} &= N - 2b_1 p_1 + \frac{2(\omega p_2 - p_1) - c_G}{2\omega(1-\omega)} b_2 + c\left(b_1 + \frac{b_2}{2\omega}\right) + z_1(p_1) - \int_L^{z_1(p_1)} F_1(x) - \mu = 0, \\ \frac{\partial \pi}{\partial p_2} &= N - 2b_2 p_2 - \frac{2(\omega p_2 - p_1) - c_G}{2(1-\omega)} b_2 + c\frac{b_2}{2} + z_2(p_2) - \int_L^{z_2(p_2)} F_2(x) + \omega\mu = 0, \\ \mu(\omega p_2 - p_1 - c_G) &= 0, \omega p_2 - p_1 - c_G \geq 0, \mu \geq 0. \end{aligned}$$

Case 2.1. If $\omega p_2 - p_1 - c_G > 0$, then $\mu = 0$ and

$$\begin{aligned} \frac{\partial \pi}{\partial p_1} &= N - 2b_1 p_1 + \frac{2(\omega p_2 - p_1) - c_G}{2\omega(1-\omega)} b_2 + c\left(b_1 + \frac{b_2}{2\omega}\right) + z_1(p_1) - \int_L^{z_1(p_1)} F_1(x) = 0, \\ \frac{\partial \pi}{\partial p_2} &= N - 2b_2 p_2 - \frac{2(\omega p_2 - p_1) - c_G}{2(1-\omega)} b_2 + c\frac{b_2}{2} + z_2(p_2) - \int_L^{z_2(p_2)} F_2(x) = 0. \end{aligned} \tag{15}$$

Case 2.2. If $\omega p_2 - p_1 - c_G = 0$, then

$$\frac{\partial \pi}{\partial p_1} = N - 2b_1 p_1 + \frac{c_G}{2\omega(1-\omega)} b_2 + z_1(p_1) + c\left(b_1 + \frac{b_2}{2\omega}\right) - \int_L^{z_1(p_1)} F_1(x) - \mu = 0, \tag{16}$$

$$\frac{\partial \pi}{\partial p_2} = N - 2b_2 \left(\frac{p_1 + c_G}{\omega}\right) - \frac{c_G}{2(1-\omega)} b_2 + z_2\left(\frac{p_1 + c_G}{\omega}\right) + c\frac{b_2}{2} - \int_L^{z_2\left(\frac{p_1+c_G}{\omega}\right)} F_2(x) + \omega\mu = 0. \tag{17}$$

One can see that solving (16) and (17) is equivalent to solving $g\left(p_1, \frac{p_1+c_G}{\omega}\right) = 0$ whose solution is \hat{p}_1 , similar to Case 1. Define

$$\mu = N - 2b_1\hat{p}_1 + \frac{c_G}{2\omega(1-\omega)} b_2 + z_1(\hat{p}_1) + c\left(b_1 + \frac{b_2}{2\omega}\right) - \int_L^{z_1(\hat{p}_1)} F_1(x).$$

If $\mu \leq 0$, the manufacturer should solve (15) and allow parallel importation. On the other hand if $\mu > 0$, then $\left(\hat{p}_1, \frac{\hat{p}_1+c_G}{\omega}, \mu\right)$ satisfies the KKT conditions. To show that it is indeed the only solution

to the KKT conditions, we show that the profit function for $\psi > 0$ is strictly quasiconcave in p_1 and p_2 (but not jointly). Suppose (15) has a feasible solution. Then

$$-b_1 - \frac{b_2}{\omega(1-\omega)} = \frac{1}{(p_1 - c)} \left[-N + b_1 p_1 - \frac{2\omega p_2 - c_G - c(1+\omega)}{2\omega(1-\omega)} b_2 + \int_L^{z_1(p_1)} F_1(x) - z_1(p_1) \right] \quad (18)$$

First because $p_1 > c$ we have

$$\frac{\partial^2 \pi}{\partial p_1^2} = z_1'(p_1) \frac{c}{p_1} - 2b_1 - \frac{1}{\omega(1-\omega)} b_2 < \frac{c}{p_1} \left[z_1'(p_1) - b_1 - \frac{1}{\omega(1-\omega)} b_2 \right]$$

where $z_1'(p_1) = \frac{c}{p_1^2 f_1(z_1)}$. Using (18), whenever $\frac{\partial \pi}{\partial p_1} = 0$ we have

$$\begin{aligned} \frac{\partial^2 \pi}{\partial p_1^2} &< \frac{c}{p_1} \left[\frac{\frac{c}{p_1^2}}{f_1(z_1(p_1))} + \frac{1}{(p_1 - c)} \left(-N + b_1 p_1 - \frac{2\omega p_2 - c_G - c(1+\omega)}{2\omega(1-\omega)} b_2 + \int_L^{z_1(p_1)} F_1(x) - z_1(p_1) \right) \right] \\ &= \frac{c}{p_1(p_1 - c)} \left[\frac{\frac{c}{p_1^2}(p_1 - c)}{f_1(z_1(p_1))} + \left(-N + b_1 p_1 - \frac{2\omega p_2 - c_G - c(1+\omega)}{2\omega(1-\omega)} b_2 + \int_L^{z_1(p_1)} F_1(x) - z_1(p_1) \right) \right] \\ &= \frac{c}{p_1(p_1 - c)} \left[\frac{F_1(z_1(p_1))}{r_1(z_1(p_1))} + \int_L^{z_1(p_1)} F_1(x) - z_1(p_1) - \left(N - b_1 p_1 + \frac{2\omega p_2 - c_G - c(1+\omega)}{2\omega(1-\omega)} b_2 \right) \right] \end{aligned}$$

If $K(z_1(p_1)) = \frac{F_1(z_1(p_1))}{r_1(z_1(p_1))} + \int_L^{z_1(p_1)} F_1(x) - z_1(p_1)$, then $K'(z_1(p_1)) = \frac{-z_1'(p_1)F_1(z_1(p_1))r_1'(z_1(p_1))}{r_1(z_1(p_1))^2} < 0$ because $r_1'(z_1(p_1)) > 0$ and $z_1'(p_1) > 0$. Thus $K(z_1(p_1))$ is decreasing in $z_1(p_1)$. Given that $z_1(p_1) > L$, we get $K(z_1) < k(L) = -L$. Note that for any p_1 and p_2 that allow parallel importation, the minimum demand in market 1 should be positive; i.e., $N - b_1 p_1 + \psi + L > 0$. Because $p_1, p_2 > c$, we have

$$N - b_1 p_1 + L + \frac{2\omega p_2 - c_G - c(1+\omega)}{2\omega(1-\omega)} b_2 > 0.$$

Therefore $\frac{\partial^2 \pi}{\partial p_1^2} \Big|_{\frac{\partial \pi}{\partial p_1} = 0} < 0$ and π is quasiconcave in p_1 for a given p_2 . Because the minimum demand in market 2, $N - b_2 p_2 - \omega \psi + L$ should be positive, we can show in a similar manner, that π is quasiconcave in p_2 for a given p_1 . Thus if (p_1, p_2) solve (15), because $\omega p_2 - p_1 - c_G > 0$, we can write

$$N - 2b_1 p_1 + \frac{c_G}{2\omega(1-\omega)} b_2 + c \left(b_1 + \frac{b_2}{2\omega} \right) + z_1(p_1) - \int_L^{z_1(p_1)} F_1(x) < 0 \quad (19)$$

$$N - 2b_2 p_2 - \frac{c_G}{2(1-\omega)} b_2 + c \frac{b_2}{2} + z_2(p_2) - \int_L^{z_2(p_2)} F_2(x) > 0 \quad (20)$$

Therefore, $\hat{p}_1 < p_1$. However, if this inequality holds, we must have

$$N - 2b_2 \left(\frac{p_1 + c_G}{\omega} \right) - \frac{c_G}{2(1-\omega)} b_2 + z_2 \left(\frac{p_1 + c_G}{\omega} \right) + c \frac{b_2}{2} - \int_L^{z_2 \left(\frac{p_1 + c_G}{\omega} \right)} F_2(x) < 0.$$

Again because of quasiconcavity, $p_2 < \frac{p_1 + c_G}{\omega}$, which is a contradiction. Therefore if $\mu > 0$, then (15) will not have a feasible solution.

To complete the proof, note that if $\mu \leq 0$, then $\gamma > 0$ and the solution of Case 1 is forced to the boundary (block). Also if $\gamma \leq 0$, then $\mu > 0$ and the solution of Case 2 is forced to the boundary (again block). \square

Proof of Proposition 5. Consider \tilde{p}_1 and \tilde{p}_2 such that $\omega\tilde{p}_2 - \tilde{p}_1 - c_G > 0$. Note that $g(\tilde{p}_1, \tilde{p}_2) = 0$. First suppose the solution to the SSG is to allow parallel imports. Then using (19) we see that

$$N - 2b_1p_1^* + c \left(b_1 + \frac{b_2}{2\omega} \right) + z_1(p_1^*) - \int_L^{z_1(p_1^*)} F_1(x) < 0,$$

which means $p_1^* > \tilde{p}_1$ because (3) is quasiconcave. Similarly, (20) gives us

$$N - 2b_2p_2^* + cb_2 + z_2(p_2^*) - \int_L^{z_2(p_2^*)} F_2(x) > 0,$$

so $p_2^* < \tilde{p}_2$. Now assume that the SSG suggests blocking the importer. From (4), (5), $\omega\tilde{p}_2 > \tilde{p}_1 - c_G$, and the quasiconcavity of the profit function we get $g\left(\tilde{p}_1, \frac{\tilde{p}_1 + c_G}{\omega}\right) > 0$. Therefore, $p_1^* = \hat{p}_1 > \tilde{p}_1$ must hold. Finally, because $g(\hat{p}_1, \tilde{p}_2) < 0$, $p_2^* = \frac{\hat{p}_1 + c_G}{\omega}$ must be smaller than \tilde{p}_2 . \square

Proof of Proposition 6. First suppose the DSG allows parallel importation. Then p_1^{*d} and p_2^{*d} solve

$$\frac{\partial \pi^d}{\partial p_1} = N - 2b_1p_1^{*d} + \frac{2(\omega p_2^{*d} - p_1^{*d}) - c_G}{2\omega(1-\omega)} b_2 + c \left(b_1 + \frac{b_2}{2\omega} \right) = 0 \quad (21)$$

$$\frac{\partial \pi^d}{\partial p_2} = N - 2b_2p_2^{*d} - \frac{2(\omega p_2^{*d} - p_1^{*d}) - c_G}{2(1-\omega)} b_2 + c \frac{b_2}{2} = 0 \quad (22)$$

and $\omega p_2^{*d} - p_1^{*d} - c_G > 0$. Because the expected value of ϵ_1 and ϵ_2 are normalized to zero, we have

$$z_1(p_1^{*d}) - \int_L^{z_1(p_1^{*d})} F_1(x) = - \int_{z_1(p_1^{*d})}^U (x - z_1(p_1^{*d})) f_1(x) \quad (23)$$

$$z_2(p_2^{*d}) - \int_L^{z_2(p_2^{*d})} F_2(x) = - \int_{z_2(p_2^{*d})}^U (x - z_2(p_2^{*d})) f_2(x) \quad (24)$$

If the SSG blocks parallel imports, then (21) and (23) imply that $\mu < 0$ when \hat{p}_1 is replaced with p_1^{*d} . Therefore $p_1^* = \hat{p}_1 < p_1^{*d}$ and $p_2^* = \frac{\hat{p}_1 + c_G}{\omega} < \frac{p_1^{*d} + c_G}{\omega} < p_2^{*d}$. If the SSG allows parallel imports, then

$$\begin{aligned} \frac{\partial \pi}{\partial p_1}(p_1^{*d}, p_2^{*d}) &= - \int_{z_1(p_1^{*d})}^U (x - z_1(p_1^{*d})) f_1(x) < 0 \\ \frac{\partial \pi}{\partial p_2}(p_1^{*d}, p_2^{*d}) &= - \int_{z_2(p_2^{*d})}^U (x - z_2(p_2^{*d})) f_2(x) < 0 \end{aligned}$$

which means $g(p_1^{*d}, p_2^{*d}) < 0$. Now using the quasiconcavity property we have

1. If $p_2^{*d} \leq p_2^*$, then $g(p_1^*, p_2^{*d}) \geq g(p_1^*, p_2^*) = 0$ and $\frac{\partial \pi}{\partial p_2}(p_1^*, p_2^{*d}) \geq 0$

(a) If $p_1^{*d} < p_1^*$, then $g(p_1^*, p_2^{*d}) < g(p_1^{*d}, p_2^{*d}) < 0$.

(b) If $p_1^{*d} > p_1^*$, then $\frac{\partial \pi}{\partial p_2}(p_1^{*d}, p_2^{*d}) > \frac{\partial \pi}{\partial p_2}(p_1^*, p_2^{*d}) \geq 0$.

2. If $p_2^{*d} \geq p_2^*$ and $p_1^{*d} \leq p_1^*$, then $\frac{\partial \pi}{\partial p_1}(p_1^{*d}, p_2^{*d}) \geq \frac{\partial \pi}{\partial p_1}(p_1^*, p_2^*) \geq \frac{\partial \pi}{\partial p_1}(p_1^*, p_2^*) = 0$.

All these cases result in a contradiction. Therefore, $p_1^* < p_1^{*d}$ and $p_2^* < p_2^{*d}$.

Now suppose the manufacturer blocks parallel imports in the DSG. Then $p_2^{*d} = \frac{p_1^{*d} + c_G}{\omega}$, and p_1^{*d} solves

$$N - 2b_1 p_1^{*d} + \frac{c_G}{2\omega(1-\omega)} b_2 + c \left(b_1 + \frac{b_2}{2\omega} \right) - \lambda = 0 \quad (25)$$

$$N - 2b_2 \left(\frac{p_1^{*d} + c_G}{\omega} \right) - \frac{c_G}{2(1-\omega)} b_2 + c \frac{b_2}{2} + \omega \lambda = 0 \quad (26)$$

where $\lambda \geq 0$ is the shadow price for $\omega p_2 - p_1 - c_G \geq 0$ in the DSG. First, consider the case when the SSG blocks parallel imports by $(\hat{p}_1, \frac{\hat{p}_1 + c_G}{\omega})$. If we replace \hat{p}_1 with p_1^{*d} and use equations (23) through (26), then

$$g \left(p_1^{*d}, \frac{p_1^{*d} + c_G}{\omega} \right) = -\omega \int_{z_2 \left(\frac{p_1^{*d} + c_G}{\omega} \right)}^U \left(x - z_2 \left(\frac{p_1^{*d} + c_G}{\omega} \right) \right) f_2(x) - \omega^2 \int_{z_1(p_1^{*d})}^U \left(x - z_1(p_1^{*d}) \right) f_1(x) < 0.$$

Therefore $\hat{p}_1 < p_1^{*d}$, which means that $\hat{p}_2 < p_2^{*d}$. Now if the SSG allows parallel imports, then

$$\begin{aligned} \frac{\partial \pi}{\partial p_1}(p_1^{*d}, p_2^{*d}) &= N - 2b_1 p_1^{*d} + \frac{c_G}{2\omega(1-\omega)} b_2 + z_1(p_1^{*d}) + c \left(b_1 + \frac{b_2}{2\omega} \right) - \int_L^{z_1(p_1^{*d})} F_1(x) < 0 \\ \frac{\partial \pi}{\partial p_2}(p_1^{*d}, p_2^{*d}) &= -\omega \lambda + z_2 \left(\frac{p_1^{*d} + c_G}{\omega} \right) - \int_L^{z_2 \left(\frac{p_1^{*d} + c_G}{\omega} \right)} F_2(x) < 0 \\ g(p_1^{*d}, p_2^{*d}) &< 0 \end{aligned}$$

The inequality in the first line comes from $\mu < 0$ and $\hat{p}_1 < p_1^{*d}$. This situation is similar to the first part of the proof (when both problems allow parallel imports). Therefore, we conclude that $p_1^* < p_1^{*d}$ and $p_2^* < p_2^{*d}$. \square

Proof of Proposition 7. When demand is deterministic and there are no parallel imports, the manufacturer sets $\tilde{p}_1^d = \frac{N+b_1c}{2b_1}$, $\tilde{p}_2^d = \frac{N+b_2c}{2b_2}$, $\tilde{q}_1^d = \frac{N-b_1c}{2}$, and $\tilde{q}_2^d = \frac{N-b_2c}{2}$. For the DSG, we have $q_1^{*d} = N - b_1 p_1^{*d} + q_G$ and $q_2^{*d} = N - b_2 p_2^{*d} - \omega q_G$ where

$$q_G = \max \left(0, \frac{2\omega(1-\omega)(\omega b_1 - b_2)N - b_2(b_2 + \omega(2-\omega)b_1)(1-\omega)c - (b_2 + \omega(4-3\omega)b_1)b_2c_G}{4\omega(1-\omega)(b_2 + \omega(2-\omega)b_1)} \right)$$

First suppose $q_G = 0$. Because $p_1^{*d} > \tilde{p}_1^d$ and $p_2^{*d} \leq \tilde{p}_2^d$, we have $q_1^{*d} < \tilde{q}_1^d$ and $q_2^{*d} \geq \tilde{q}_2^d$. If $q_G > 0$, then

$$\begin{aligned} p_1^{*d} &= \frac{(3\omega - \omega^2)N - b_2c_G}{2(b_2 + \omega(2-\omega)b_1)} + \frac{c}{2} \\ p_2^{*d} &= \frac{((1+\omega)b_2 + 2\omega(1-\omega)b_1)N + \omega b_1 b_2 c_G}{2b_2(b_2 + \omega(2-\omega)b_1)} + \frac{c}{2} \end{aligned}$$

which give us

$$\begin{aligned} q_1^{*d} &= \frac{N - b_1c}{2} - \frac{b_2c}{4\omega} - \frac{b_2c_G}{4\omega(1-\omega)} < \tilde{q}_1^d \\ q_2^{*d} &= \frac{N - b_2c}{2} + \frac{b_2c}{4} + \frac{b_2c_G}{4(1-\omega)} > \tilde{q}_2^d \quad \square \end{aligned}$$

Appendix 2. Regions for the Optimal Strategy for the DSG

Although the regions in Figure 9 are derived from numerical experiments, for the DSG they can be obtained analytically by comparing the profit for each strategy. The next proposition lays out the profit functions.

PROPOSITION 8. *The manufacturer ignores the parallel importer if $\frac{\omega(N+b_2c)}{2b_2} \leq \frac{N+b_1c+2b_1c_G}{2b_1}$. Otherwise, the manufacturer allows the importer, blocks the importer, or leaves market 1. The profit functions π_b (block), π_a (allow), and π_s (single market) are provided below*

$$\begin{aligned} \pi_b &= \frac{1}{4\alpha_1} \left[N^2(1+\omega)^2 + c^2(\omega b_1 + b_2)^2 - 4b_1b_2c_G^2 + Nc(2\omega b_1(1-3\omega) - 2b_2(3-\omega)) \right. \\ &\quad \left. + 4Nc_G(\omega b_1 - b_2) - 4b_1b_2c(1-\omega)c_G \right] \\ \pi_a &= \frac{1}{8\omega(1-\omega)\alpha_2} \left[N^2 \left(2\omega(1-\omega)(1+4\omega-\omega^2) + \frac{2\omega(1-\omega)b_1}{b_2} \right) + b_2(b_2 + \omega^2b_1)c_G^2 + 2\alpha_2b_2c(1-\omega)c_G \right. \\ &\quad \left. - 8\omega(1-\omega)\alpha_2Nc + (1-\omega)(2\omega b_1 + (1+\omega)b_2)\alpha_2c^2 - 4\omega(1-\omega)(b_2 - \omega b_1)Nc_G \right] \\ \pi_s &= \frac{(N - b_2c)^2}{4b_2} \end{aligned}$$

where $\alpha_1 = \omega^2b_1 + b_2$ and $\alpha_2 = b_2 + \omega(2-\omega)b_1$. The optimal prices in the order of the profits are

$$\begin{aligned} p_1^{*d} &= \frac{N(\omega + \omega^2) + c\alpha_1 - 2b_2c_G}{2\alpha_1} & p_2^{*d} &= \frac{N(1+\omega) + c\alpha_1 + 2\omega b_1c_G}{2\alpha_1}, \\ p_1^{*d} &= \frac{N(3\omega - \omega^2) + c\alpha_2 - b_2c_G}{2\alpha_2} & p_2^{*d} &= \frac{N \left(1 + \omega + \frac{2\omega(1-\omega)b_1}{b_2} \right) + c\alpha_2 + \omega b_1c_G}{2\alpha_2}. \\ p_1^{*d} &\geq \max \left\{ \frac{N}{b_1}, \omega p_2^{*d} - c_G \right\} & p_2^{*d} &= \frac{N + b_2c}{2b_2}, \end{aligned}$$

Proof. The manufacturer ignores the parallel importer if $\omega \tilde{p}_2^d \leq \tilde{p}_1^d + c_G$ or $\frac{\omega(N+b_2c)}{2b_2} \leq \frac{N+b_1c+2b_1c_G}{2b_1}$. Otherwise, she has to choose her strategy by comparing the profit functions for blocking, allowing, and abandoning market 1. We omit the details of obtaining the optimal profits and prices. Note that abandoning market 1 is equivalent to choosing a price in market 1 that is large enough to make the demand zero and block the importer. That is why for the single market strategy p_1^{*d} must be larger than $\frac{N}{b_1}$ and $\omega p_2^{*d} - c_G$. \square