Title
Dynamic bargaining in households (with application to Bangladesh)

Author
Ligon, Ethan

Publication Date
2002-05-09

DYNAMIC BARGAINING IN HOUSEHOLDS (WITH AN APPLICATION TO BANGLADESH)

Abstract. Much recent empirical work on intra-household allocation uses the axiomatic Nash Bargaining model to make predictions about how the distribution of consumption within the household will respond to individuals' income shocks. However, one of the basic axioms underlying this approach is that allocations will be Pareto optimal, so forward-looking, risk-averse household members ought to be expected to smooth away any such response to income shocks—Pareto optimality seems to be too strong in a dynamic setting. In this paper we use explicitly dynamic framework and replace the axiom of Pareto optimality with a weaker notion of efficiency. We give a simple algorithm for computing allocations, and construct an extended example, meant to model the effects of Grameen Bank lending on intra-household allocation in Bangladesh. The model resolves a puzzle in the literature, namely, it predicts that women borrowers will often voluntarily surrender control ("pipeline") their loans to their husbands.

Date: May 9, 2002.
Key words and phrases. Bargaining, Limited Commitment, Risk-sharing, Intra-household allocation, Grameen Bank.

1. Introduction

Economists have made much progress in thinking about modelling non-market institutions. Chief among the nonmarket arrangements which interest economists is the household. The standard 'unitary' model of the household, first enunciated in Becker (1974), is a model in which altruistic links among household members give rise to Pareto optimal intra-household allocations.

In recent years the unitary model has come under attack, principally on empirical grounds. The nature of this attack is very nicely surveyed in Lundberg and Pollak (1996), but for the reader's convenience we briefly review some of the salient evidence. One implication of the unitary household model is that changes in the allocation of resources across household members should lead neither to changes in the allocation of consumption nor to changes in investments made by the household. Yet a variety of authors have documented just these sorts of changes. Thomas (1990) shows that investments in child health (as measured by attained height) vary according to the share of income controlled by husband and wife. Lundberg, Pollak, and Wales (1997) show that a change in the recipient of a "child allowance" in the UK (from man to woman) led to increased expenditures on women's and children's clothing relative to men's clothing.

Economists have responded by developing new models of the household. Notable examples include the cooperative bargaining models of Manser and Brown (1980), McElroy and Horney (1981), Lundberg and Pollak (1993), as well as an axiomatic approach taken by Chiappori (1992). All of these models have Pareto optimal outcomes, and all are basically static.

Setting aside for a moment concerns regarding the mechanism which allocates goods within the household, it may be worth taking a moment to discuss the economic logic which lies behind the existence of households to begin with, as opposed to an understanding of how individuals behave once enmeshed within a household. Becker (1974) suggests that households exist because they allow household members to specialize in, say, market activities or child-rearing (and corresponding human capital investments in these activities receive increasing returns). More recently, authors who have used bargaining models to try and understand household allocations have tended to assume that households collectively produce some sort of public good as an alternative motivation (cf., Lundberg and Pollak).

In addition to returns to specialization and the household production of public goods, an additional possible motivation for household formation has to do with consumption-smoothing. If individuals are risk averse and income is uncertain then (so long as household members' incomes are not perfectly correlated) pooling household income *à la* Becker (1974) will be Pareto improving. However, adding uncertainty to existing bargaining models of intrahousehold allocation leads to problems, especially in a multi-period environment. The problem is that (as noted above) the approaches hitherto taken to intra-household allocation either assume (the axiomatic approaches) or predict (the unitary household models) that outcomes will be Pareto optimal. In an environment with risk averse agents and uncertainty, any Pareto optimal allocation will feature *full insurance* (Wilson 1968); that is, changes in individual marginal utilities will depend only on changes in household aggregates. More sharply, when utility from private consumption of one good is additively separable from utility from other goods, then an individual's consumption of that good will vary only with the aggregate household consumption of the good. Furthermore, in a multiperiod setting, individual *shares* of consumption will not depend on changes to individual shares of income; increases in one person's income won't produce any change to the household sharing rule.

It seems important for models of intra-household allocation to be able to accomodate forward-looking agents in a dynamic environment—after all, relationships between household members are among the longest term relationships most people are ever involved in. In this paper we develop a model in which household members can strike bargains over the intra-household allocation of resources. These bargains take the form of an agreement regarding a sharing rule to use after any history. The realized allocations are *ex post* Pareto optimal, in the sense that once the state is known and the consumption good allocated, there is no alternative Pareto-improving allocation of resources in that state.

Despite the *ex post* optimality of allocations, these allocations are *not* generally Pareto optimal *ex ante*, because of the lack of commitment available to household members. For example, should agent one realizes a very high endowment and agent two a very low endowment, then the first agent can, in effect, demand a renegotiation of the sharing rule in force from previous periods.

Thus, the model of this paper allows for the evolution of agents' positions within the household over time, and having made the model dynamic, it turns out to be very natural to also permit uncertainty. The addition of these elements permits us to contemplate risk-sharing within the household, which may be an important feature of households in many environments, perhaps particularly in developing countries. Further, the use an explicitly dynamic model allows us to characterize the evolution of the intra-household sharing rule over time. In particular, we provide a result (Corollary 1) which shows that if one household member is sufficiently more committed to the household than the other, then that person's eventual share of household resources will eventually depend only on the *worst possible* state for that person, in which their bargaining power is weakest. Accordingly, increasing the risk of the more committed person will tend to lead to worse outcomes for that person, even if that person's resources are must increased on average.

To gain some insight into the workings of the model, we compute some simple examples based on a stylized version of the sort of poor Bangladeshi household targetted by the well-known Grameen Bank. The computed example sheds some light on the phenomenon of loan "pipelining" noted by Goetz and Gupta (1996), whereby women who receive Grameen Bank loans turn over the proceeds of these loans to their husbands, rather than investing in productive activities of their own.

We proceed as follows. Section 2 begins by describing some of the shortcomings of the unitary household model and the cooperative bargaining framework when these models are used to describe dynamic problems. Section 3 modifies the axioms of the usual Nash bargaining model so as to address these shortcomings in a natural way, and provides an allocation rule which satisfies these modified axioms. Section 4 offers a sequence of computed examples based on a stylized Bangladeshi household. This example is meant to illustrate the sorts of outcomes one might expect from various policy interventions, such as the provision of credit to women famously undertaken by the Grameen Bank. Section 5 concludes.

2. An Illustration

We begin by describing a consumption-smoothing problem, which illustrates some of the issues involved in thinking about dynamic bargaining. Let $u_i(c)$ denote the utility from private consumption enjoyed by individual i at time t. We assume that every u_i is increasing and strictly concave. For the purposes of this example, we assume two individuals ($i = 1, 2$) and two time periods ($t = 1, 2$). Individuals 1 and 2 are 'married' at the beginning of the first period, and in each period each receive some endowment of the consumption good y_{it}. Should the two divorce, each receives some alienation utility $a_i(y_{it})$, which we also take to be increasing and strictly concave functions.

For the purposes of this illustration, we consider a particularly simple set of possible endowments. There are two possible states of the world, denoted by $s \in \{1, 2\}$. In state one, agent one receives an endowment of two, while agent two receives an endowment of zero. In state two, their positions are reversed; that is, agent one's endowment is zero, while agent two's endowment is two. Furthermore, these states simply alternate; in odd periods, $s = 1$, while in even periods $s = 2$.

Faced with these endowment processes, what might we expect a married couple to do? Both have concave utility functions, and *ex ante* would benefit from some sort of smoothing of consumptions over time. Thus, if both agents can commit themselves in advance (and it's natural to suppose that marriage is all about commitment), we might expect an egalitarian sharing arrangement to prevail. For example, consider a married couple, each of whom receives a bi-weekly paycheck, but on alternating weeks; here an egalitarian solution seems quite plausible. Modifying the example somewhat, if the length of a period is a decade rather than a fortnight, then the plausibility of an invariant, egalitarian split is rather lessened.

Taking this simple example as a sort of benchmark, we next compare each of several approaches to predicting the allocation of private consumption. We consider, in turn, a unitary household model, and a sort of repeated cooperative bargaining model, before turning our attention to a dynamic cooperative bargaining model.

2.1. **Unitary Household.** In the household model of Becker (1974), links between household members are sustained by altruism. For example, in addition to the utility that agent one derives from private consumption, suppose that he receives $\theta u_2(c_{2t})$ utils from the consumption of agent two. If we allow agent one to control the allocation of consumption within the household, then in each period he will solve

$$\max_{c_{1t}, c_{2t}} u_1(c_{1t}) + \theta u_2(c_{2t})$$

subject to the household resource constraint $c_{1t} + c_{2t} \leq y_{1t} + y_{2t}$ in each of periods one and two (note that, for simplicity, no savings are allowed).[1] The first order conditions from this problem imply that the ratio of marginal utilities of the two agents will be a constant, regardless of the income realization or the period, since

$$\frac{u_1'(c_{1t})}{u_2'(c_{2t})} = \theta. \tag{1}$$

[1]See Ligon, Thomas, and Worrall (2000) for a treatment of individual savings in a related model.

This equation determines a sharing rule. Because θ is taken to be a constant determined by one's feelings toward two, this ratio of marginal utilities must be a constant, and since the functions u_i are themselves time-invariant, then the sharing rule must itself be time invariant. Each agent's share of consumption depends on the preference parameter θ; the more one 'cares' about two (the higher the value of θ), the more consumption allocated to two. Because θ is a preference parameter which is presumably difficult to observe, the important empirical consequences of the unitary model are the Pareto optimality of allocations and the invariance of the sharing rule.

Notice that we've separated agent one's problem into two distinct subproblems, one for each period. For this particular problem, this separation of a dynamic allocation problem into a sequence of static problems doesn't affect the solution. This separability is due to the lack of an intertemporal technology such as savings, and due also to the invariance of the sharing rule implied by (1).

Also notice that any change in the distribution of income between agents one and two (or who receives the paycheck in a given period, per our example) will make no difference in the allocation of consumption so long as the total resources available to the household are unchanged, and so long as agent one remains in charge of the allocation decision.

2.2. **Cooperative Bargaining.** Following the analysis of, e.g. McElroy and Horney (1981), we consider the consequences of a change in the intra-household distribution of income in a Nash bargaining model. Nash's (1950) approach was to specify a set of axioms which any 'reasonable' solution ought to satisfy. Nash's set of axioms included a requirement of symmetry, and a requirement of Pareto optimality. Subsequent authors have investigated the consequences of modifying these axioms, in ways which we'll exploit. In particular, Harsanyi and Selten (1972) abandon Nash's symmetry axiom. With this modification, we're left with the following axioms, as applied to our simple two period example (note that feasibility is assumed).

- **INV:** Consumption allocations are *invariant* to affine transformations of agents' utilities.
- **IIA:** Consumption allocations are *independent of irrelevant alternatives*; that is, if the allocation $\{(c_{1t}, c_{2t})\}$ solves one bargaining problem when aggregate incomes are (y_1, y_2) and the same allocation is feasible in a second (otherwise identical) problem

with aggregate incomes $(\hat{y}_1, \hat{y}_2) < (y_1, y_2)$, then the allocation $\{(c_{1t}, c_{2t})\}$ also solves the second problem.

PO: The allocation is Pareto optimal.

Nash offers a simple algorithm to compute the allocations satisfying the axioms. McElroy-Horney applies this algorithm to the intra-household allocation problem by solving:

$$\max_{c_{1t}, c_{2t}} [u_1(c_{1t}) - a_1(y_{1t})]^{\alpha} [u_2(c_{2t}) - a_2(y_{2t})]^{1-\alpha}$$

with $0 < \alpha < 1$ for $i = 1, 2$, subject to the household budget constraint $c_{11} + c_{21} \leq y_{11} + y_{21}$. Then the first order conditions of this problem once again give us a set of restrictions on the ratio of marginal utilities of consumption,

$$\frac{u_1'(c_{1t})}{u_2'(c_{2t})} = \frac{1-\alpha}{\alpha} \frac{u_1(c_{1t}) - a_1(y_{1t})}{u_2(c_{2t}) - a_2(y_{2t})}. \tag{2}$$

Comparing this condition with (1), notice that although the left hand side of the two equations are identical, the right hand side of (2) is not time invariant; in particular, it will depend on the incomes realized. If, for example, there is an increase in y_{1t}, then this will result in a decrease in the marginal utility of agent one relative to agent two, and hence an increase in relative consumption. In our alternating paycheck example, each agent would take turns consuming a larger share of the total endowment.

This dependence of allocation on the *ex post* distribution of income is the feature of this model which has made it attractive to people seeking an interesting theory of intra-household allocation. However, as a consistent theory, this approach has a serious flaw; the solution to a sequence of static problems does not at all correspond to the solution to the dynamic problem if agents are forward-looking. To see this, suppose that agent i's time one utility if married is given by the time separable function $u_i(c_{i1}) + \beta u_i(c_{i2})$, while utility if divorced (in the first period) is $a_i(y_{i1}) + \beta a_i(y_{i2})$. Then the appropriate solution to the Nash bargaining problem is given by the solution to

$$\begin{aligned} \max_{c_{11}, c_{12}, c_{21}, c_{22}} & [u_1(c_{11}) + \beta u_1(c_{12}) - a_1(y_{11}) - \beta a_1(y_{12})]^{\alpha} \\ & \cdot [u_2(c_{21}) + \beta u_2(c_{22}) - a_2(y_{21}) - \beta a_2(y_{22})]^{1-\alpha} \end{aligned} \tag{3}$$

with $0 < \alpha < 1$, and subject to the constraints $c_{1t} + c_{2t} \leq y_{1t} + y_{2t}$ for $t = 1, 2$. The first order conditions from this problem once again give us an expression for the ratio of marginal utilities between agents one and two, given this time by

$$\frac{u_1'(c_{1t})}{u_2'(c_{2t})} = \frac{1-\alpha}{\alpha} \frac{u_1(c_{11}) + \beta u_1(c_{12}) - a_1(y_{11}) - \beta a_1(y_{12})}{u_2(c_{21}) + \beta u_2(c_{22}) - a_2(y_{11}) - \beta a_2(y_{22})}, \tag{4}$$

for $t = 1, 2$. This equation once more defines a sharing rule for consumption. The remarkable thing about this equation is that the implied sharing rule is, once again, time-invariant. The distribution of consumption in each period depends on the entire sequence of income realizations of both agents.

How should we interpret this invariance? It is as if the two agents, each knowing their income sequence, engaged in some borrowing-lending arrangement between them. In our alternating paycheck example, though agent two has no income in the first period, she will receive the entire endowment in the second. Knowing this, the first agent is perfectly happy to enter into an egalitarian sharing relationship in each period.

This example also makes it clear that the sharing rule implied by (2) is not in fact the solution to the Nash bargaining problem if agents are forward looking. The reason for this is that the sharing rule of (2) is *not* Pareto optimal; a move to the sharing rule implied by (4) makes each forward looking agent better off. Thus this sort of solution violates one of the key axioms underlying the Nash bargaining approach—the axiom of Pareto optimality.

Although the violation of Pareto optimality loses us the interpretation of solutions to (2) as Nash bargaining solutions, one might well argue that the abandonment of Pareto optimality represents a gain terms of realism (cf. Lundberg and Pollak (1996)). In the next section we offer an alternative axiomatic development of a dynamic cooperative bargaining model in which cooperative outcomes are not generally Pareto optimal.

3. Cooperative Dynamic Bargaining

The appeal of a sequence of static Nash bargaining problems as a description of intra-household allocation is due, I think, to the idea that binding legal contracts seldom constrain the members of a household with respect to their actions toward one another—within a household, the ability to *commit* to a sharing rule of the form suggested by (4) is limited. For example, we might think of a marriage in which the prospects of the wife unexpectedly turn out to be much better than those of her husband. After this state of affairs is realized, it seems likely that the wife might thenceforth claim a larger share of household resources.

This section develops a dynamic bargaining problem in which household members strike a bargain over the intra-household allocation of resources. A solution to the bargaining problem takes the form of an agreement regarding a sharing rule to use after any history. Realized allocations are *ex post* Pareto optimal, in the sense that once the state is known and the consumption good allocated, there is no alternative Pareto-improving allocation of resources in that state (this is similar to the "collective efficiency" of Bourguignon and Chiappori (1992)). However, these allocations are *not* generally Pareto optimal *ex ante*, because of the lack of commitment available to household members. In a state of the world in which agent one realizes a very high endowment and agent two a very low endowment, agent one can, in effect, demand a renegotiation of the sharing rule in force from previous periods.

The bargaining environment has two infinitely-lived agents, each with a time-separable von Neumann-Morgenstern utility function. Each agent discounts future utility at some positive rate $1/\beta - 1$. In any given period, one of a finite number of possible states $S = \{1, 2, \ldots, m\}$ occurs. If the current state is $s \in S$, then the probability that the state in the subsequent period is $r \in S$ is some number π_{sr}. For the sake of simplicity, we assume that $\pi_{sr} > 0$ for all s and r in S.

In this environment, bargaining involves deciding how to assign surpluses both now and in the future. Denote the set of possible (momentary) surpluses in state s by $W_s \in \mathbb{R}^2$, and let $\mathbf{W} = \{W_s\}_{s \in S}$. We assume that each W_s is convex, compact, and contains the origin. Let $\rho_s(w_1) = \max\{w_2 | (w_1, w_2) \in W_s\}$ trace out the Pareto frontier of the set W_s; we assume that each function ρ_s is monotone decreasing and strictly concave. Let $\mathcal{W}$ denote the set of $\mathbf{W}$ having surplus possibilities which satisfy these conditions.

A bargaining *problem* is completely described by the current state s and the set of surplus possibilities $\mathbf{W}$, so that a problem is simply some $(s, \mathbf{W})$. Thus, $\mathcal{B} = \{(s, \mathbf{W}) \in S \times \mathcal{W}\}$ denotes the set of bargaining problems we explore in this paper.

Any solution to the bargaining problem must assign surpluses in every possible sequence of states. Borrowing an idea from the literature on dynamic contracting (Green 1987; Spear and Srivastava 1987), we use surpluses in the previous period to summarize histories. Because of the Markovian nature of the environment, a solution to the bargaining problem $(s, \mathbf{W})$ is some initial assignment of surplus $w \in W_s$ and a collection of mappings $\Phi(\mathbf{W}) = \{\phi_{sr}\}_{(s,r) \in S \times S}$, with $\phi_{sr} : W_s \to W_r$. Thus, the collection $\Phi(\mathbf{W})$ defines a law of motion for surpluses, and thus a rule for assigning surpluses in every possible sequence of states. We write a solution to the problem $(s, \mathbf{W})$ as a pair (w, Φ); the pair w is the assignment of surpluses in the initial state s,[2] while the collection Φ gives rules for assigning all future surpluses, given the initial assignment.[3]

To make this notation clear, we define our earlier alternating paycheck example in this new notation. First, the possible states are simply $S = \{1, 2\}$, with probabilities $(\pi_{11}, \pi_{12}, \pi_{21}, \pi_{22})$ equal to $(0, 1, 1, 0)$, to capture the strict alternation of paychecks.[4] Momentary surpluses are given by $W_1 = \{(u_1(c) - a_1(2), u_2(2-c) - a_2(0)) | c \in [0, 2]\}$ and $W_2 = \{(u_1(c) - a_1(0), u_2(2-c) - a_2(2)) | c \in [0, 2]\}$.

Any solution (w, Φ) which assigns a sequence of surpluses also induces *values* $\{U_s(w)\}_{s \in S}$, which solve the equations

$$U_s(w) = w + \beta \sum_{r \in S} \pi_{sr} U_r(\phi_{sr}(w)),$$

for all $s \in S$. We interpret $U_s(w)$ as the values each agent derives from continuing the bargaining relationship, given that the current state is s and the current surplus assignment is w.

We next adapt Nash's axioms to the stationary dynamic environment described here. First is the axiom of invariance (INV). The only real difference from our earlier formulation is that we now work in a space of surpluses, rather than consumption allocations.

Let $\mathbf{W} = \{W_s\}_{s \in S} \in \mathcal{W}$, and define the operation of scalar multiplication on $\mathcal{W}$ by $\theta \mathbf{W} = \{\theta W_s\}_{s \in S}$ for any scalar θ.

INV: For all $\mathbf{W} \in \mathcal{W}$ and all $\theta > 0$, $\Phi(\theta \mathbf{W}) = \theta \Phi(\mathbf{W})$.

Since utilities are von Neumann-Morgenstern, surpluses are invariant only up to a positive affine transformation. Accordingly, any solution should be similarly invariant.

Next is independence of irrelevant alternatives. For the purposes of stating the axiom, let $\phi_{sr}(\cdot)$ denote part of the solution (w, Φ) to the problem $(s, \mathbf{W})$.

---

[2]Since the initial w may be chosen arbitrarily, the solutions given here are not generally unique. As in Nash, Jr. (1950), it would be a simple matter to also require that a solution to the bargaining problem satisfy some sort of *symmetry* which would guarantee uniqueness, though in the present case this requirement of symmetry would apply only to the *ex ante* discounted expected surpluses.

[3]We will often be working with pairs of real numbers. If a function $f(x)$ maps x into $\mathbb{R}^2$, then let $(f^1(x), f^2(x)) = f(x)$ denote the coordinates of the mapping. For any two real pairs (a, b) and (c, d), the strict inequality $(a, b) > (c, d)$ should be read as $a > c$ and $b > d$, while the weak inequality $(a, b) \geq (c, d)$ should be read as $a \geq c$ and $b \geq d$.

[4]This violates our assumption that all probabilities ought to be strictly positive, but the violation does no harm here.

IIA: For any $\mathbf{W}$ and $\hat{\mathbf{W}}$ such that $\phi_{sr}(w) \in \hat{W}_r \subseteq W_r$ for all $w \in \hat{W}_s$ and for all $(s, r) \in S \times S$, $\Phi(\hat{\mathbf{W}}) = \Phi(\mathbf{W})$.

The idea here is simply that if (w, Φ) is the solution to one problem, then if feasible it should also be the solution to a more constrained problem.

Both the INV and IIA conditions are essentially the same as conditions advanced by Nash. In particular, we can think of any of the values $U_s(w)$ as static bargaining surpluses. If we then work with the set of possible values, we can apply Nash's invariance and independence axioms directly to any of these sets. His axioms are then easily seen to be equivalent to the versions of INV and IIA given here.

As discussed above, the axiom of Nash's which is problematic from our point of view is Pareto optimality. This implies that agents can always commit *ex ante* to never renegotiate *ex post*, which seems too strong. We replace Nash's axiom with two alternative requirements. The first of these is a notion of *individual rationality*:

IR: For any problem $(s, \mathbf{W})$, $U_r(\phi_{sr}(w)) \geq 0$ for all $r \in S$; also, if there exists a state $q \in S$ and $\hat{w} > 0$ for some $\hat{w} \in W_q$, then $U_r^i(\phi_{sr}(w)) > 0$ for some $i \in \{1, 2\}$.

This requirement has two parts. First is the idea that neither agent can be made strictly worse off by continuing the bargaining relationship than by discontinuing it. Second, if there are positive surpluses for either agent in any state, then at least one agent must be made strictly better off by remaining in the relationship rather than terminating it.

Though IR is implied by Pareto optimality, Roth (1977) shows that in the static case $(\beta = 0)$ INV, IIA, and IR imply Pareto optimality.[5] This does not carry over to the dynamic problem; in particular, an *ex ante* Pareto optimal solution may not satisfy IR. We recover Nash's sentiment that agents ought to always be able to negotiate until they reach an efficient outcome by using a notion of *constrained Pareto optimality*:

CPO: Let $\{U_s\}$ be induced by the solution (w, Φ), and let $\{\hat{U}_s\}$ be induced by $(w, \hat{\Phi})$. If $\hat{U}_s(w) \geq U_s(w)$ with $\hat{U}_s^i(w) > U_s^i(w)$ for some $s \in S$ and some $i \in \{1, 2\}$, then $(w, \hat{\Phi})$ fails to satisfy at least one of INV, IIA, and IR.

The idea here is simple; we simply require that the solution be optimal within the class of surplus assignments satisfying the other three axioms.

[5] Our formulation of the IR requirement is slightly weaker than Roth's, who requires (in our notation) $U_r(\phi_{sr}(w)) > 0$. Thus, Roth's version of IR is not implied by Pareto optimality. Nonetheless, our weaker version of IR along with INV and IIA still implies Pareto optimality in the static case.

We next propose an algorithm for computing a solution satisfying INV, IIA, IR, and CPO. Given a problem $(s, \mathbf{W})$, we first define a set of functions for assigning surpluses in the current period. Let $C_s : [0,1] \to W_s$ for $s \in S$, with

$$C_s(\lambda) = \operatorname*{argmax}_{w \in W_s} \lambda w_1 + (1-\lambda) w_2. \tag{5}$$

Thus, in any state s, $C_s(\lambda)$ tells us how to maximize a sum of current period surpluses weighted by λ. Since W_s is compact, convex, and has a strictly concave frontier, $C_s(\lambda)$ is guaranteed to be single-valued for any $\lambda \in [0,1]$. $C_s(\lambda)$ can be regarded simply as a solution to a static bargaining problem. Note that when $W_s \subset \mathbb{R}^2_+$, this function also gives the solution to the asymmetric static bargaining problem (seen in the previous illustration):

$$N_s(\alpha) = \operatorname*{argmax}_{w \in W_s} w_1^{\alpha} w_2^{1-\alpha}. \tag{6}$$

Because we're interested in bargaining problems in which one agent may make a sacrifice in the current period in exchange for higher future surpluses, it's necessary to work with C_s, which is well defined for any W_s.

Next, we define two additional families of functions, designed to let us use surplus *weights* rather than surpluses as state variables.[6] The first, $\psi_s : [0,1] \to [0,1]$, maps the surplus weight λ in one state into a new weight given that the subsequent state is s. The second family, $V_s : [0,1] \to \mathbb{R}^2$, maps the current surplus weight into values. In particular, the $\{(\psi_s, V_s)\}_{s \in S}$ solve the set of functional equations

$$V_s(\lambda) = C_s(\lambda) + \beta \sum_{r \in S} \pi_{sr} V_r(\psi_r(\lambda)) \tag{7}$$

for $s \in S$, and

$$\psi_r(\lambda) = \begin{cases} \underline{\lambda}_r & \text{if } V_r^1(\lambda) < 0 \\ \overline{\lambda}_r & \text{if } V_r^2(\lambda) < 0 \\ \lambda & \text{otherwise} \end{cases} \tag{8}$$

[6]A number of earlier authors have used this same trick of using utility or surplus weights in models of efficient contracts, including Hayashi (1996) and Marcet and Marimon (1999).

where $\underline{\lambda}_r$ satisfies $V_r^1(\underline{\lambda}_r) = 0$, and $\overline{\lambda}_r$ satisfies $V_r^2(\overline{\lambda}_r) = 0$. The assumption that the Pareto frontier of each W_s is monotone decreasing along with the assumption that W_s includes the origin ensures that the $\{(\underline{\lambda}_r, \overline{\lambda}_r)\}$ exist, and so a solution to (7) and (8) exists.[7]

Very simply, the proposed class of solutions has the following character. Agents will efficiently divide any momentary surplus according to an invariant sharing rule until they reach a point such that continuing to use this rule would make one of the agents worse off than she would be on her own. At this point, the division of surplus will change so as to make this agent just indifferent between remaining in the relationship and leaving. Then this new sharing rule will remain invariant until it, in its turn, once more delivers too small a value to induce one of the agents to remain.

What can we say about this class of solutions? First, a solution to the bargaining problem satisfies the four axioms we've advanced if and only if it belongs to this class. More precisely, we have

Proposition 1. *There exists a solution (w, Φ) to the problem $(s, \mathbf{W}) \in S \times \mathcal{W}$ satisfying INV, IIA, IR, and CPO. This solution corresponds to some $\lambda \in [0, 1]$, with $w = C_s(\lambda)$ and $\phi_{rq}(w) = C_q(\psi_q(\lambda))$ for all $(r, q) \in S \times S$.*

Proof. Any solution to (7) and (8) corresponds to a family of proposed solutions to the bargaining problem $(s, \mathbf{W}) \in S \times \mathcal{W}$. Of these solutions, choose the one with the largest functions $\{V_s\}$ (such an ordering is unambiguous, since the mapping defined by (7) and (8) is isotone). We show that at least one proposed solution from this family satisfies the axioms, and then show that any solution not in the family would violate CPO. IR is satisfied by construction. (INV) Let $\hat{\mathbf{W}} = \{\theta W_s\}_{s \in S}$ for some scalar $\theta > 0$. Since the current surplus assignment $C_r^\theta(\lambda)$ on $(s, \hat{\mathbf{W}})$ is equal to $\theta C_r(\lambda)$, it follows that $V_s^\theta(\lambda) = \theta V_s(\lambda)$ for all $s \in S$ and $\lambda \in [0, 1]$. Hence $\psi_s^\theta(\lambda) = \psi_s(\lambda)$.

(IIA) Let $\hat{\mathbf{W}} = \{\hat{W}_s\}$ satisfy $\hat{W}_s \subseteq W_s$ for all $s \in S$, with $C_s(\lambda) \in \hat{W}_s$ for all $\lambda \in [0, 1]$ and all $s \in S$. Since current surplus assignments are the same on $\hat{\mathbf{W}}$ and $\mathbf{W}$, it follows that the values and the updating rules $\psi_s(\lambda)$ are also the same.

[7]More precisely, given the stated restrictions on the $\{W_s\}$, (8) and (7) together define an isotone mapping from a subset of the set of bounded functions on $\mathbb{R}^2$; existence of a fixed point to this mapping is then guaranteed by a theorem of Knaster-Tarkski (Moore 1985).

(CPO) Note that $\{\psi_r\}$ solves the planning problem

$$\max_{\{\lambda_r | V_r(\lambda_r) \geq 0\}_{r\in S}} \lambda[C_s^1(\lambda) + \beta \sum_{r\in S} \pi_{sr} V_r^1(\lambda_r)] + (1-\lambda)[C_s^2(\lambda) + \beta \sum_{r\in S} \pi_{sr} V_r^2(\lambda_r)]$$

Since, by construction there's a one-to-one mapping between solutions to the planning problem and the set of constrained Pareto optima, only this family satisfies CPO. □

The final observation of the proof, that there's a connection between the solution to the cooperative dynamic bargaining problem and a planning problem provides a connection to a literature which, for special cases of the environment explored here, characterizes the efficient subgame perfect equilibria of dynamic games (e.g., Binmore (1985), Kehoe and Levine (1993), Kletzer and Wright (2000)), and a closely related literature which computes efficient dynamic contracts in an environment with limited commitment (e.g., Kocherlakota (1996), Ligon, Thomas, and Worrall (2002)). Since the efficient subgame-perfect equilibria (or efficient contracts) of this literature deliver the same allocations as a solution satisfying our axioms, it's possible to view Proposition 1 as accomplishing the first part of the Nash program, with the noncooperative literature accomplishing the second (at least for some special cases).

An important implication is that it's possible to completely characterize the solution to the bargaining problem in terms of an initial surplus weight λ and a rule to update that surplus weights. We can use this fact to explore another important implication—that extreme but temporary shocks can have long-term consequences for the allocation of surpluses. Let the state $\tilde{s}$ be the worst possible state for agent two in the sense that $\underline{\lambda}_{\tilde{s}} = \min_{s\in S} \underline{\lambda}_s$. These weights are defined so that $\underline{\lambda}_s$ is the smallest weight that one can receive in state s, so that $V_{\tilde{s}}^2(\underline{\lambda}_{\tilde{s}})$ is an upper bound on the smallest surplus agent two can possible receive. Call $\tilde{s}$ agent two's *harsh* state.

In some settings (see the illustration of the next section), social attitudes may be such that women are likely to be treated very badly in the event of a divorce; thus, the surplus of married women may be quite large even in very bad states, relative to being divorced. Call $\min_{s\in S} V_s^2(\underline{\lambda}_{\tilde{s}})$ the *unconditional value* that marriage has for agent two; this provides an upper bound on the smallest possible value of marriage to agent two. With these additional definitions, we can state the following corollary to Proposition 1:

Corollary 1. *Suppose that period t is the first period in which agent two's harsh state $\tilde{s}$ is realized. If agent two's unconditional value is positive, then agent two's surplus in any state $s \in S$ in any period subsequent to t will be no greater than $C_s^2(\underline{\lambda}_{\tilde{s}})$.*

Proof. There are two cases to consider. In the first case $\lambda > \underline{\lambda}_{\tilde{s}}$. Then, using the definition of $\tilde{s}$, $\psi_r(\lambda)$ is bounded below by $\underline{\lambda}_{\tilde{s}}$ for all $r \in S$, so that $C_r^2(\psi_r(\lambda)) < C_s^2(\underline{\lambda}_{\tilde{s}})$.

In the second case, $\lambda \leq \underline{\lambda}_{\tilde{s}}$. Since $\min_{s \in S} V_s^2(\underline{\lambda}_{\tilde{s}}) > 0$, in this case the updating rule (8) can be simplified to become $\psi_r(\lambda) = \max\{\underline{\lambda}_r, \lambda\}$ for all $r \in S$, so that λ can only increase over time. Once state $\tilde{s}$ occurs at date t we have $\psi_{\tilde{s}}(\lambda) = \underline{\lambda}_{\tilde{s}}$ and for any subsequent state $\lambda_r = \psi_r(\underline{\lambda}_{\tilde{s}}) = \underline{\lambda}_{\tilde{s}}$, again by definition of the harsh state $\tilde{s}$. □

The point made by the corollary is that, if one agent has an unconditional surplus (as would be the case if that agent were quite committed to the relationship), then the surplus for that agent will eventually depend only on the bargaining power that agent can muster in her harsh state, the state in which her bargaining power is weakest; further, her position in the household in periods following the harsh state will never improve (we pursue an example of this in Section 4.3).

It may be worth noting that while Proposition 1 guarantees the existence of a solution to the bargaining problem, there may often be many possible solutions, each corresponding to a different initial surplus allocation w. At most, only one of these solutions treats the two agents symmetrically, in the sense that a renaming of agents shouldn't affect our predictions regarding outcomes. Just as in Nash, Jr. (1950), imposing this sort of symmetry delivers a unique solution to the bargaining problem. Note that the imposition of symmetry delivers equal expected surplus values for the two agents only *ex ante*; *ex post*, histories distinguish the two agents. Thus, if there are multiple solutions satisfying the conditions of Proposition 1, then imposing symmetry will typically influence the initial assignment of surplus w. However, symmetry may also influence the path of subsequent assignments. To see this, consider once more the alternating paycheck example introduced in Section 2, but repeated an infinite number of times, instead of only two. Suppose first of all that the two agents have very low rates of discount (coinciding, perhaps, with the case in which paychecks are handed out on a bi-weekly basis). For a sufficiently low rate of discount, a range of different initial assignments of w (corresponding to a range of λ weights) will solve the bargaining problem; further, the benefits from consumption smoothing will be great enough so that neither agent will ever prefer to terminate the relationship. The sharing rule in this case will be invariant to the distribution of income, and allocations will, in fact, be Pareto optimal. Now consider increasing the rate of discount dramatically (coinciding with the case in which paychecks are given out once per decade). If β is made small enough, then in the second period any rule which would give the first agent a share of half, say, will cause the second agent's value $V_2^2(1/2)$ to be negative, violating IR. To satisfy IR, the first agent's share must be made small enough that the immediate costs to agent two are matched by the benefits of receiving some consumption a decade hence.

4. An Application

In this section we work with an extended example motivated by some evidence regarding the impact of the Grameen Bank on intrahousehold allocation in Bangladesh. While necessarily extremely stylized, we'll argue that the bargaining arrangements we've described provide a qualitatively better description of the evolution of actual arrangements than do existing stylized models of intrahousehold allocation.

The account we'll give of intrahousehold allocations in Bangladesh has been heavily influenced by the accounts of Todd (1996) and Goetz and Gupta (1996), among others. These studies offer a much richer description of household arrangements than do other studies such as Hossain (1988) or Khandker et al. (1995). However, because they focus on very in-depth data collection from a small number of households, they are presumably less representative of rural Bangladesh than are these latter studies, a caution the reader should bear in mind.

4.1. **Seasonal Labor Markets.** We begin by describing an environment meant to capture some of the salient features of the problem facing a poor, two-person household in which the wife derives small quantities of income from a variety of activities (such as paddy-husking, raising vegetables, and caring for livestock), while the husband's income, while much larger on average, is earned in a highly seasonal agricultural labor market. This basic description matches the circumstances of several of the poorer households described in Todd (1996).

We begin by specifying the preferences of household members. Somewhat arbitrarily, we imagine that both husband and wife derive utility from consumption $u_i(c) = \log(c)$.[8] This implies that each household member's relative risk aversion is one, on the low end of estimated risk aversions in developing countries. Increasing risk aversion would have the

[8] Arrow (1965) enumerates several properties of logarithmic utility which make it particularly suitable for thinking about behavior in risky environments. For the purposes of the illustration we give here, our main motive is simply that with log utility the surplus weights λ can be interpreted as consumption shares.

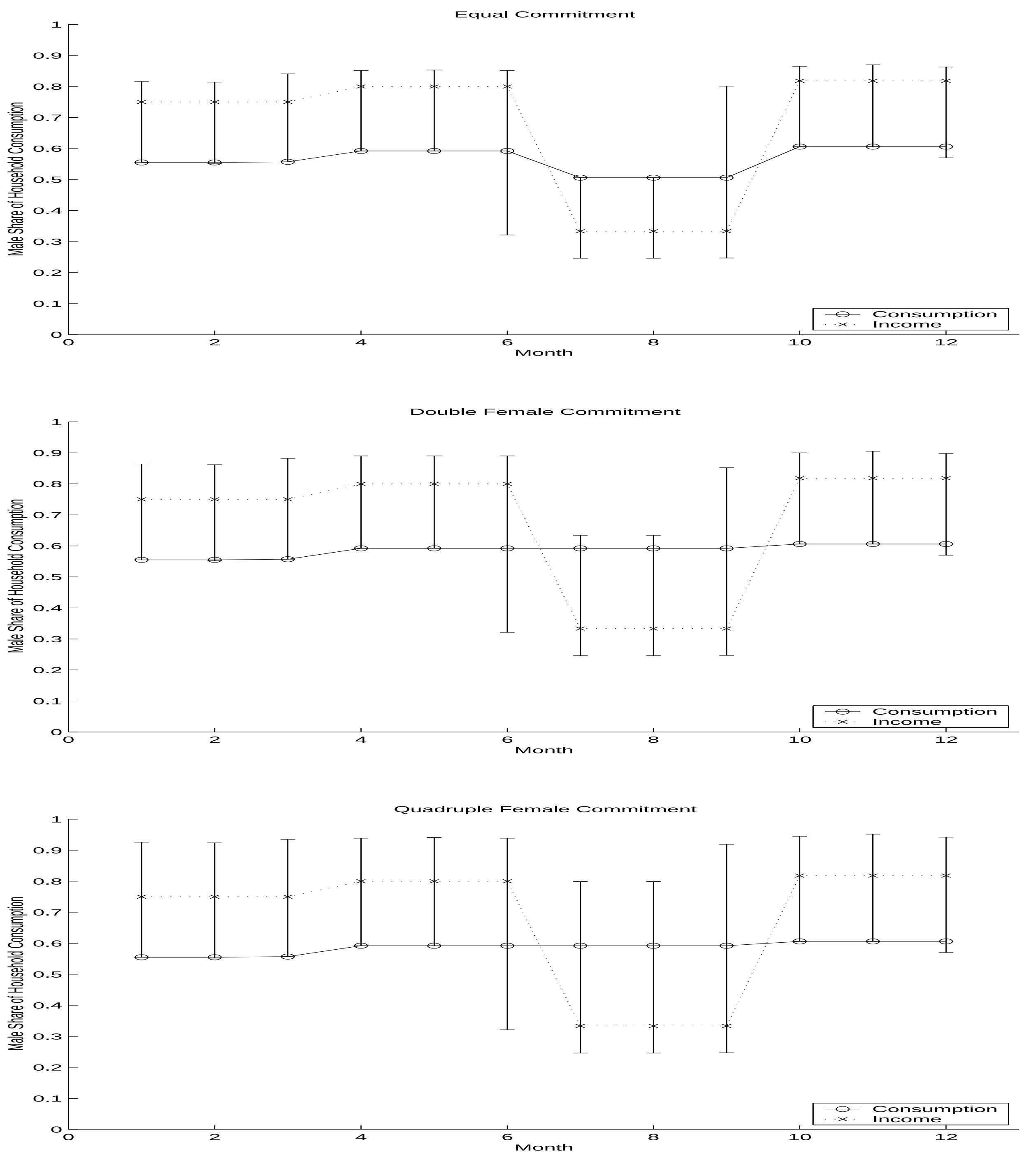


FIGURE 1. Consumption Shares with Seasonal Variation. The solid line traces out the husband's share of household consumption across time. At each date, an interval indicates consumption shares corresponding to a set $[\underline{\lambda}_s, \bar{\lambda}_s]$. Each panel differs only by the value of M, the value associated with marriage for the female.

effect of magnifying the differences between the dynamic bargaining framework and the static bargaining model; as we shall see, no such magnification is necessary to distinguish between these models. Agents are, of course, forward looking: each discounts the value of future utility using a discount factor $\beta = 0.7$.[9]

In addition to deriving utility from consumption, we suppose that both husband and wife derive utility from simply being involved in a family relationship with one another (relative to the utility each would derive in the event of a divorce or other separation).[10] The discrimination faced by divorced or widowed women in rural Bangladesh suggests that the costs involved in a separation will be much larger for women than for men. Accordingly, if in a relationship a man (agent 1) receives a utility bonus of $(1-\beta)$ in every period, while a woman (agent 2) receives a utility bonus of $(1-\beta)M$.

We abstract entirely from production. This is without loss of generality so long as production is efficiently arranged within the household and utility from leisure is additively separable from utility from consumption. Instead of engaging in productive activities, each household member i simply receives some endowment y_{it} at date t (here we take a period to be one month, since we're interested in modeling both variation within and across seasons). Then, motivated by the idea that the wife's income derives from a variety of small-scale activities, at least some of which can be carried on at any time of year, let $y_{2t} = 1$ for all $t = 1, 2, \ldots$. On the other hand the husband's income, derived from the agricultural labor market, varies across the season as follows:[11]

$$y_{1t} = \begin{cases} 3 & \text{if } t \bmod 4 \in \{1,2,3\} \\ 4 & \text{if } t \bmod 4 \in \{4,5,6\} \\ 0.5 & \text{if } t \bmod 4 \in \{7,8,9\} \\ 4.5 & \text{if } t \bmod 4 \in \{10,11,0\} \end{cases} . \tag{9}$$

[9]Choosing a relatively low discount factor such as this reduces the value of future intra-household insurance, and makes for easier-to-read graphs; using a larger discount factor wouldn't lead to a qualitative change in our results.

[10]Bergstrom (1996) is critical of the notion that divorce is necessarily the pertinent alienation point for a married couple, arguing that the threat of an outcome involving "burnt toast and harsh words" may often be a more credible threat than divorce. In our framework, this notion could be accomodated simply by defining the origin of the surplus sets $\{W_s\}$ to be the momentary utility associated with this non-cooperative outcome.

[11]These figures are chosen to approximately match the circumstances of a particular landless household, "Forman and Shireen," in the sample analyzed by Todd (p. 52); the units we use are thousands of Taka.

Since t indexes months, this specification of the husband's endowment process groups the months into four seasons. The seasons correspond roughly to the period from late February through mid May; late May through mid August, and late August to mid November (which includes *Khartik*, the "hungry season" in the Bengali calendar); and from late November through mid February, a period which includes both harvest and the planting of winter paddy crops. Note that on average, the woman's contribution to household income amounts to one quarter of the total, roughly matching the circumstances Todd reports for landless households in her sample.

Clearly there's considerable scope within this household for consumption smoothing over time, and it's clear from Todd's descriptive material that in fact the wife in this household (Shireen) undergoes considerable difficulty in order to feed her husband during the "hungry season." Of course, the unitary household model predicts *perfect* smoothing, consistent with Shireen's privation. However, other studies of intra-household allocation (e.g., Thomas 1990, Lundberg, Pollak, and Wales 1997) reject the hypothesis of perfect consumption smoothing while nonetheless yielding results which are consistent with a great deal of smoothing.

With logarithmic utility, λ can be interpreted as the male's share of household consumption. Figure 1 displays this predicted share under three different assumptions regarding the benefit to being married, M. In the top panel, the benefits to being married (consumption-smoothing aside) are assumed to be equal for the two partners ($M = 1$), with the consequence that the sets $[\underline{\lambda}_s, \bar{\lambda}_s]$(the vertical intervals in the figure) are somewhat smaller than they would be otherwise, and have a smaller intersection.

The consumption share in the first period is chosen to favor the woman as much as possible while still satisfying INV, IIA, IR, and CPO. The state in the first period is 1, so the initial value of λ is chosen such that $V_1^1(\lambda) = 0$, which necessarily lies at the lower endpoint of the first interval. Note that the woman's share of consumption in the first season (approximately 0.45) is considerably larger than her share of income (0.25).

In the second season ($t \in \{4, 5, 6\}$), male income increases from three to four, and his share of household income increases to 0.8. In the first two periods of the second season, this implies a larger male alienation point, with a corresponding increase in the male's share of consumption. However, in period 6 (immediately preceding the onset of *Khartik*) the male alienation point drops dramatically; if the household was dissolved just before *Khartik*, the wife wouldn't provide much needed support to her husband during the preharvest hungry season. However, the female alienation point changes little this period; she still benefits from higher consumption than her income. The onset of *Khartik* in period 7 changes this. The female alienation point increases dramatically; during this season the woman undergoes considerable privation to feed her husband. Since the upper bound on the male's share in period 7 lies below the lower bound of the male share of consumption in earlier periods, the model predicts that during the first two periods of *Khartik* the woman would *always* be indifferent to the dissolution of the household, regardless of her initial share consumption. Subsequent to period 7, the *only* possible path for consumption allocations is that detailed in the figure. The final period of *Khartik* gives rise to another large change in the female alienation point; in the subsequent period, both husband and wife anticipate a return to the wife eating more than she earns. The male alienation point, however, changes little until the beginning of the harvest season, at which time he demands a share of about 0.6. In all years following the first, consumption can follow only one path; the woman will have a share of roughly 0.4 in every season except *Khartik*, when her share increases to roughly 0.5.

The second panel of Figure 1 shows the same kinds of information, but for a household in which the benefits to marriage are less equal; in particular, $M = 2$, so that the state-independent costs to the wife of leaving the relationship are twice what they were in the first panel (and are twice that of her husband). The additional commitment this allows means that the intersection the intervals is non-empty; consequently, after the first year the consumption share of the woman is constant, between about 0.6 and 0.625, implementing perfect intra-household consumption smoothing. The additional commitment of the woman in this case means that (after the first year, at least) she is never tempted to leave during *Khartik*, and so never receives an increased share during the hungry season. Accordingly, her *average* share of consumption after the first year falls relative to the first panel. In the bottom panel, with even greater benefits to marriage, a woman will once again receive a constant share after the first year, but this share may be as small as 0.8, so that her share of consumption may never be larger than her share of income, at any date.

4.2. **Uncertainty.** The stylized seasonal environment just considered misses an obvious, important feature of the actual environment; namely, the real environment features not only variation in the resources available over time, but also features considerable uncertainty, or variation in resources across states. Floods, sickness, and occasional misfortune feature prominently in many detailed accounts of the lives of Bangladeshi households. Accordingly, in this example we add to the seasonality of the last account uncertainty regarding the endowments realized in any season. To make it easy to compare this account with the last, we assume that commitment is equal ($M = 1$), and suppose that the wife's endowments are unchanged (one unit of the consumption good at every date-state); however, the husband's endowments in the last example are the husband's *expected* endowments in this example. In each season, one of two different states can be realized: good for the husband or bad for the husband. In the good state, the husband's endowment realization is 150 per cent of his average endowment for that season; in the bad state it is only 50 per cent.

With the addition of uncertainty, the consumption smoothing described in the last section becomes even more important, and acquires the interpretation of *insurance*. Figure 2 illustrates the man's share of household consumption under one possible sequence of outcomes. As before, we choose the initial share to be as large as possible for the woman. For the particular history illustrated in Figure 2, this maximal share (about 0.53) is considerably larger than in the seasonal example, both because the male's endowment realization is bad in the first two periods, and because of the increased value of marriage to the male, with his risky income stream. A good shock for the male in period three leads to an increase in his consumption share to about 0.56; a subsequent pair of good shocks during the second season leads to a further small increase in the male's share. As in the seasonal case, the period (6) immediately preceding the hungry season of *Khartik* is associated with a dramatic increase in the male's appreciation of his partner. By the middle of *Khartik*, in period 8, the wife will receive a share of just over one half, despite the good shocks her husband receives in the first two periods of this season. A bad shock in period 9 does nothing to change the wife's share; she's anticipating the consumption her husband will share with her during the harvest season, during which his share increases to about 0.59.

This example illustrates an important point. The addition of uncertainty to the male's endowment improves the woman's standing in the household because she is well suited to act as an insurer. The bleaker her opportunities outside marriage, the greater her commitment to the relationship. This along with the relatively safe nature of her own endowment process imply that she can greatly increase the male's welfare by helping to smooth consumption. However, *she* controls this 'insurance' resource, and so uncertainty in the husband's endowment has a positive effect on her standing in the household.

4.3. **The Grameen Bank.** As is well known, the present-day loan programs of the Grameen bank (see Khandker et al. (1995) for a survey) focus almost exclusively on women. However,

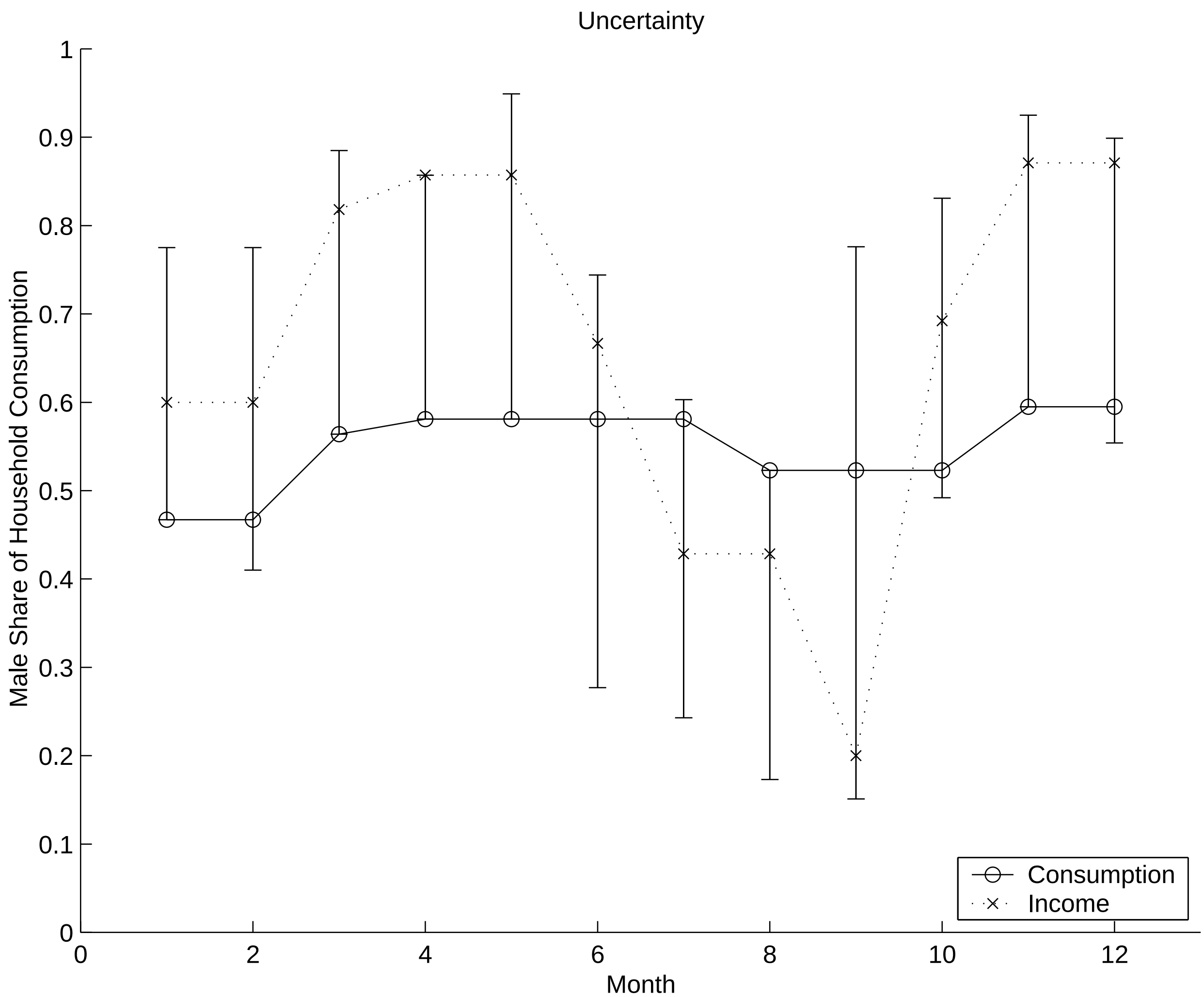


FIGURE 2. Consumption Shares with Seasonal Uncertainty. The solid line traces out the husband's share of household consumption across time for one possible history. At each date, an interval of consumption shares corresponds to a set $[\underline{\lambda}_s, \bar{\lambda}_s]$.

one of the rationales offered by the Grameen for lending to women is that such loans permit women to expand the range of productive activities in which they may engage. "Approved" uses of general loans in the Todd sample include loans to finance paddy-husking operations, the purchase or fattening of livestock, the purchase of mills for grinding mustard seed, and the operation of a grocery or purchase of inputs for making sweets. Any of these loan uses comports well with the aim of making women more self-sufficient. However, there seems to be a sharp disconnect between putative and actual loan use—in the Todd sample, only 13 per cent of loans were put to use for any approved purpose. Much the most frequent use of loans was to purchase or lease in land (49 per cent); the second most common was to repay existing loans (39 per cent); re-lending the borrowed amount to someone outside the household accounted for another 17 per cent of loan use![12] These these uses are generally forbidden. The purchase of land is frowned upon because land is ordinarily operated by men, and so apparently does little to make women less dependent, while borrowing to service existing debt raises obvious concerns for a bank.

The way in which we model the Grameen bank is meant to contrast two different understandings of the way in which Grameen Bank loans are used. First, we'll consider the possibility that the proceeds of a loan are used to finance some small scale productive activity which will increase expected endowments for the woman, while at the same time increasing the uncertainty she faces. Under this model the male's endowment process is unaffected. Second, we'll consider an arrangement in which the woman, upon receiving a loan, uses it to invest in an improved endowment process for her husband. This idea coincides with the hypothesis advanced in Todd that many loans are "pipelined" to their husbands. This idea receives some additional empirical support from Goetz and Gupta (1996), who find that women exercise full control over only 17.8 per cent of the loans they receive, and exercise absolutely none in 21.7 per cent of cases.

The way in which the loans operate is this: in the first period of each year, the woman borrows 4799 taka, of which she immediately (and in each subsequent month) repays one twelfth.[13] Under our first hypothesis regarding loan use, this loan is used to permanently change the woman's endowment process (gross of loan proceeds or repayments) to

$$\hat{y}_{2t} = \begin{cases} 0.4 & \text{with probability } 1/2 \\ 2.5 & \text{with probability } 1/2. \end{cases}$$

The male's endowment is assumed to follow the seasonal pattern described in Section 4.1. The change in the woman's endowment process is chosen so that the expected endowment is 1.45, a 45 per cent improvement over the original endowment process in risk-neutral terms, and more than enough to justify taking a loan such as the Grameen Bank loan with an interest rate of about 10 per cent (when amortized over the year). Furthermore, in expectation her

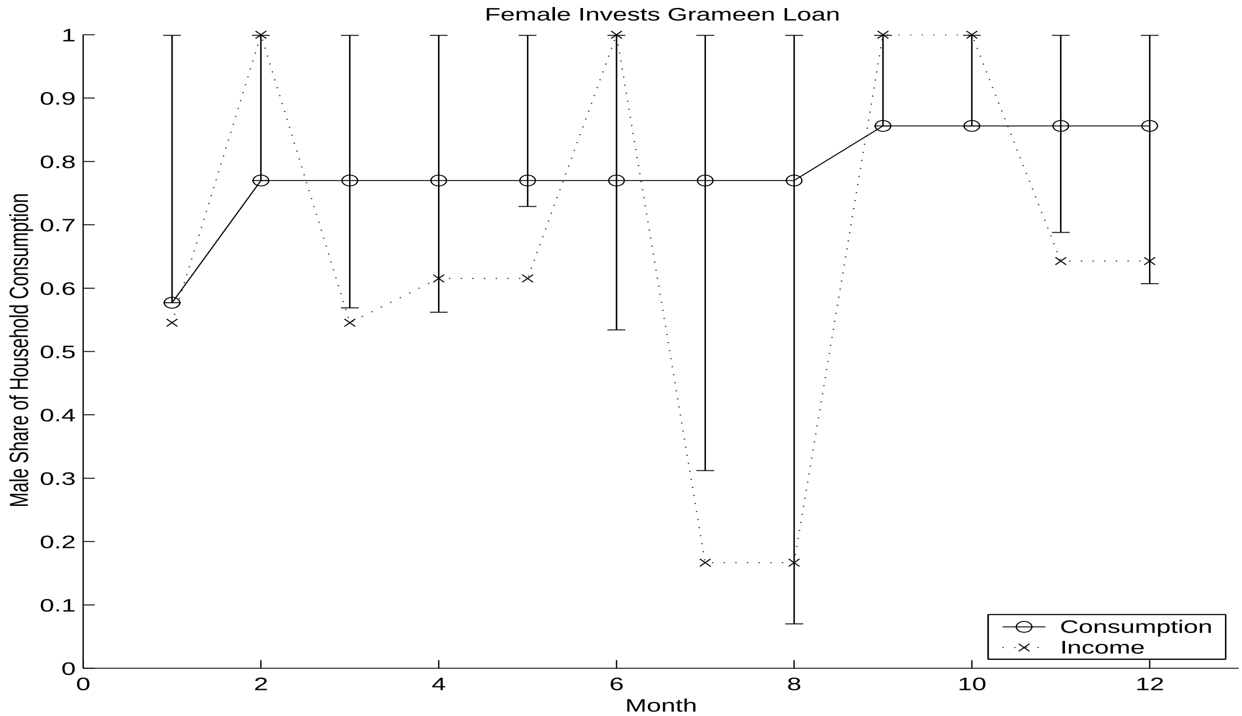
[12]These sum to more than 100 per cent because a single loan may be used for more than one purpose.

[13]This does a fair job of mimicking the actual loan process, except that women actually repay their annual general loans in 50 (almost) weekly installments, and make a terminal payment which includes an interest charge of 20%. In practice, the Grameen Bank induces women to make this final large repayment by immediately issuing a new, larger, loan.

alienation point is unchanged (at zero in every period); thus, with adequate risk-sharing household welfare must increase relative to what it would have been with the old technology.

Figure 3 displays some typical sample paths for each of these two scenarios. As before, we begin by giving the woman the largest possible share of consumption. In the top panel, the woman controls the loan in the first period, and happens to have a good endowment realization; as a consequence, she is able to claim more than 40 per cent of the household endowment in that period, just slightly less than her share of income. However, in the subsequent period, she has a bad endowment realization, and after making her mandatory loan payment, has almost no income to contribute to the household. Although this is a fairly bad shock for the household, it is not collectively disastrous: the household endowment is 1000 Taka, well below the average, but perfectly manageable. For the woman, however, it is a very serious crisis. She's forced into a very weak position with respect to her husband, and her share of the household endowment falls to only about 23 per cent. In subsequent periods she's lucky more often then not, but the damage has been done; because of the importance of her husband's help in states when her realization is poor, she's unable to subsequently negotiate a more equitable division of resources, even during *Khartik* when her endowment realization is high and her husband's income less than 20 per cent of the total. Still worse is to come: in the period immediately preceding harvest, she has another bad endowment realization. With the privation of *Khartik* a distant prospect, her share of consumption falls to about 16 per cent. This is the *harsh state* of Corollary 1, and as the intersection of all the λ-intervals is non-empty, the woman's unconditional surplus is positive. As a consequence, her share of household resources will henceforth be only 16 per cent, regardless of any future shocks, and despite the fact that her average share of household income is much greater than this.

The second model of loan use is illustrated in the bottom panel of Figure 3. The woman receives and is responsible for the loan, as before, but in this case the loan is employed to improve her husband's endowment process, changing his seasonal endowment realizations by a factor of either 0.4 or 2.5, each with probability one half. Now, although the woman continues to repay the Grameen bank, in the worst state of the world she still commands just over 500 Taka. Now it is the male who transfers resources (on average) to the woman in exchange for the consumption smoothing she can provide. Now the woman starts off with the lion's share of household consumption, nearly 70 per cent, but unlike the case illustrated in the first panel, in subsequent periods no crisis occurs which forces her into a terribly weak



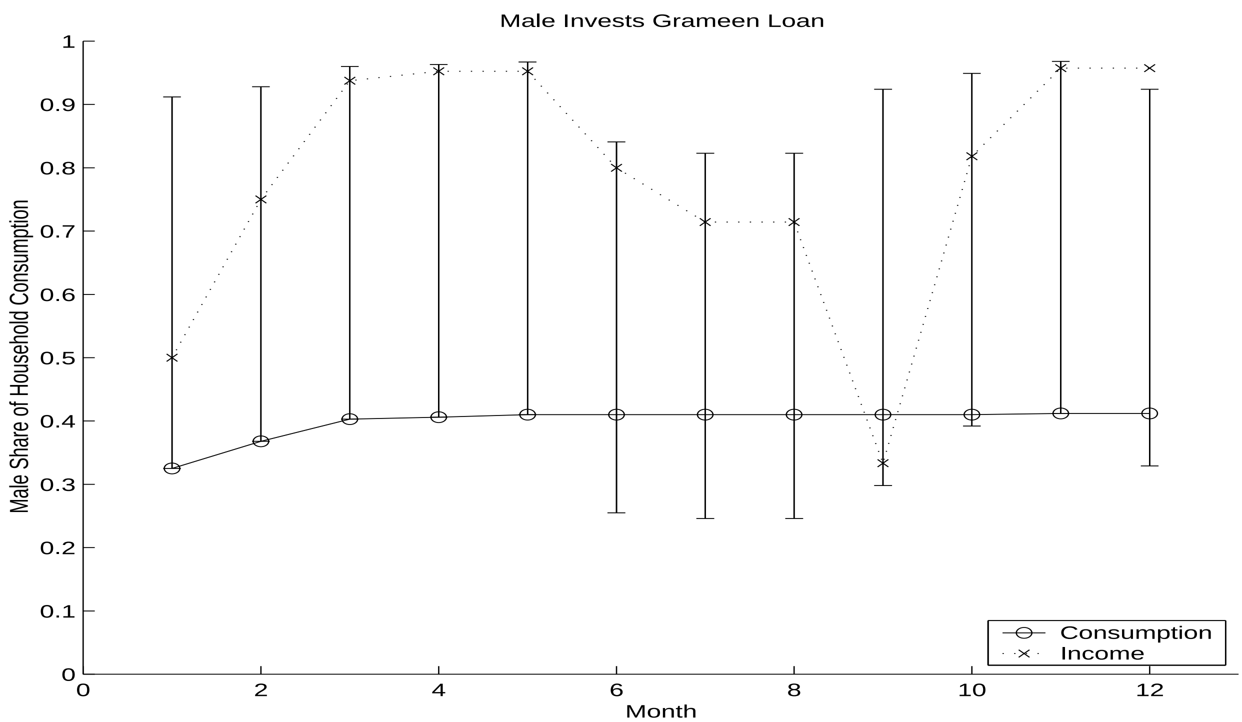


FIGURE 3. Consumption Shares With and Without "Pipelining." The solid line traces out the husband's share of household consumption across time. At each date, an interval of consumption shares corresponds to a set $[\underline{\lambda}_s, \bar{\lambda}_s]$. In the top panel, a loan is used to improve the woman's endowment process; in the bottom panel it is used to improve the man's.

position with respect to her husband. Her husband, though he receives bad endowment shocks, never has to personally undertake to repay the Grameen bank, and when bad shocks occur, he can rely to a great extent on his wife's forbearance.

Although the panels in Figure 3 simply show one of several different possible histories, they are typical. Both actually offer fairly good insurance within the household (after the first year, each offers full insurance to both members). The difference between the two is an illustration of Corollary 1, and is driven by the fact that the usual division of resources ultimately depends on the worst possible state (the "harsh" state of the corollary) that either person can fall into. Requiring the woman to make unconditional repayment of a large loan while at the same time making her income stream more risky is apt to lead to extreme, persistent, inequality within the household.

This simple example suggests a policy for the Grameen Bank or similar institutions—namely, that the bank should *not* discourage loan pipelining within the household. This policy recommendation is contrary to current bank policy, and at odds with the insights gained from a simple deterministic model with myopic agents, as in McElroy and Horney (1981), and much of the subsequent literature on intra-household allocation. This literature seems to suggest that increases in a woman's *expected* income are key to improving her expected bargaining position within the household. However, in the simple example here, with forward-looking agents and a stochastic environment, it's more nearly the case that the *worst possible* income realization determines the position of the woman in the household. Increasing the riskiness of women's incomes—even while greatly increasing the mean—may greatly exacerbate intra-household inequality.

5. Conclusion

In this paper we've constructed a novel model of dynamic intrahousehold bargaining, and related it to the often employed Nash bargaining model. While it's technically possible to apply the Nash bargaining framework in an intertemporal environment, the model seems somewhat ill-suited. Our model adds the requirement that no household member should ever want to terminate the bargaining relationship, and weakens the requirement of Pareto optimality with a weaker requirement that the solution be optimal within the class of solutions satisfying the other axioms. These modifications are meant to capture the idea that intra-household allocations are subject to ongoing renegotiation.

We construct a sequence of computed examples meant to illustrate some of the features of the solution, by applying the model to a stylized version of the problem facing a Bangladeshi household. We first show how the allocations of the model can deliver something like intertemporal exchange between husband and wife. We then show that the solution can provide a great deal of insurance, but that positions within the household may be renegotiated when there are dramatic changes in the control of resources.

Finally, we turn our attention to the impact of Grameen Bank membership on the woman's position in the household, and show that the model is capable of explaining a puzzle noted by Todd (1996). If loans from the Grameen Bank are used by women to engage in more productive but risky endeavors, far from improving the woman's standing in the household it is very likely to make it much worse. However, if women "pipeline" their loans to their husbands, so that the male undertakes most of the risk, both equity and efficiency can be dramatically improved.

References

Arrow, K. J. (1965). *Aspects of the Theory of Risk Aversion*. Helsinki: Yrjö Jahnssonin säätiö.

Becker, G. S. (1974). A theory of social interactions. *Journal of Political Economy 82*, 1063–1094.

Bergstrom, T. C. (1996). Economics in a family way. *Journal of Economic Literature 34*(4), 1903–1934.

Binmore, K. G. (1985). Bargaining and coalitions. In A. Roth (Ed.), *Game Theoretic Models of Bargaining*, pp. 269–304. Cambridge: Cambridge University Press.

Bourguignon, F. and P.-A. Chiappori (1992). Collective Models of Household Behavior: An Introduction. *European Economic Review 36*(2,3), 355–364. April.

Chiappori, P.-A. (1992). Collective labor supply and welfare. *Journal of Political Economy 100*(3), 437–67.

Goetz, A. M. and R. S. Gupta (1996). Who takes the credit? gender, power, and control over loan use in rural credit programs in Bangladesh. *World Development 24*(1), 45–63.

Green, E. J. (1987). Lending and the smoothing of uninsurable income. In E. C. Prescott and N. Wallace (Eds.), *Contractual Arrangements for Intertemporal Trade*. Minneapolis: University of Minnesota Press.

Harsanyi, J. C. and R. Selten (1972). A generalized nash solution for two-person bargaining games with incomplete information. *Management Science 18*, 80–106.

Hayashi, F. (1996). Analysis of household saving: Past, present, and future. *The Japanese Economic Review 47*(1), 21–33.

Hossain, M. (1988). Credit for alleviation of rural poverty: the Grameen bank in Bangladesh. Technical Report 65, International Food Policy Research Institute.

Kehoe, T. J. and D. K. Levine (1993). Debt-constrained asset markets. *Review of Economic Studies 60*(205), 865–888.

Khandker, S. R. et al. (1995). *Grameen Bank: Performance and Sustainability*. World Bank.

Kletzer, K. M. and B. D. Wright (2000). Sovereign debt as intertemporal barter. *American Economic Review 90*(3), 621–639.

Kocherlakota, N. R. (1996). Implications of efficient risk sharing without commitment. *The Review of Economic Studies 63*(4), 595–610.

Ligon, E., J. P. Thomas, and T. Worrall (2000). Mutual insurance, individual savings, and limited commitment. *Review of Economic Dynamics 3*(2), 216–246.

Ligon, E., J. P. Thomas, and T. Worrall (2002). Informal insurance arrangements with limited commitment: Theory and evidence from village economies. *Review of Economic Studies*. Forthcoming.

Lundberg, S. and R. A. Pollak (1993). Separate spheres bargaining and the marriage market. *Journal of Political Economy 10*(6), 988–1010.

Lundberg, S. and R. A. Pollak (1996). Bargaining and distribution in marriage. *Journal of Economic Perspectives 10*(4), 139–158.

Lundberg, S. J., R. A. Pollak, and T. J. Wales (1997). Do husbands and wives pool their resources? *The Journal of Human Resources XXXII*(3), 462–480.

Manser, M. and M. Brown (1980). Marriage and household decision-making: A bargaining analysis. *International Economic Review 21*(1), 31–44.

Marcet, A. and R. Marimon (1999). Recursive contracts. Manuscript.

McElroy, M. and M. J. Horney (1981). Nash-bargained household decisions: Toward a generalization of the theory of demand. *International Economic Review 22*, 333–350.

Moore, R. E. (1985). *Computational Functional Analysis*. Ellis Horwood Series in Mathematics and Its Applications. New York: Halsted Press.

Nash, Jr., J. F. (1950). The bargaining problem. *Econometrica 18*(2), 155–162.

Roth, A. E. (1977). Individual rationality and Nash's solution to the bargaining problem. *Mathematics of Operations Research 2*(1), 64–65.

Spear, S. E. and S. Srivastava (1987). On repeated moral hazard with discounting. *Review of Economic Studies 54*, 599–618.

Thomas, D. (1990). Intra-household resource allocation: An inferential approach. *Journal of Human Resources 25*, 635–64.

Todd, H. (1996). *Women at the Center: Grameen Bank Borrowers After One Decade*. Westview Press.

Wilson, R. (1968, January). The theory of syndicates. *Econometrica 36*, 119–132.