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Merger Simulation: A Simplified Approach with New Applications

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Abstract

Merger simulation is growing in importance as a tool to evaluate the unilateral competitive effects of mergers. This paper offers a relatively non-technical description of the principles of merger simulation. In addition, it introduces PCAIDS, a new and highly flexible "calibrated-demand" merger simulation methodology that is based on a simplified version of AIDS. PCAIDS can be implemented using market shares and two price elasticities; scanner or transaction-level data are not required. The paper offers some applications of merger simulation with PCAIDS that include comparisons with other simulation models. It also shows how PCAIDS can be applied to the analysis of efficiencies, divestiture, and product repositioning/entry. Finally, the paper offers an analysis of the Merger Guidelines safeharbors. A detailed mathematical appendix is included.

Keywords: antitrust; merger simulation; unilateral effects; empirical methods

JEL Classification: L40

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Merger Simulation:

A Simplified Approach with New Applications

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I. INTRODUCTION

In recent years there have been significant developments in the use of empirical economic methods to study the likely competitive effects of mergers.[1] These developments have been shaped by the increased use of unilateral effects analyses by the competition authorities, as is expressed in part in the 1997 *Horizontal Merger Guidelines*. Such analyses evaluate the ability of the post-transaction firm to raise the prices of some or all of its (often differentiated) products through unilateral decisions and without resort to overtly collusive activities.[2]

Unilateral effects analyses encompass a broad set of issues that arise when the differentiated brands produced by the merging firms constitute the first and second choices for some group of customers. Absent *de novo* entry or product repositioning,

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[1] For a recent survey see Jonathan B. Baker and Daniel L. Rubinfeld, "Empirical Methods Used in Antitrust Litigation: A Review and Critique," 1 *J. Am. L. & Econ. Ass'n* 383 (1999).

[2] The 1997 U.S. DOJ/FTC *Horizontal Merger Guidelines* and the 1995 USDOJ/FTC *Intellectual Property Guidelines* have also emphasized the potential effects of a transaction on innovation generally, and on the intensity of research and development efforts in particular. (For a general discussion and further references, see Daniel L. Rubinfeld and John Hoven, "Innovation and Antitrust Enforcement," in J. Ellig ed., *Dynamic Competition and Public Policy*, Cambridge University

a unilateral price increase may become profitable as the result of a merger if a substantial number of customers who previously would have been lost to competitors can now be retained because the merged firm also offers the customers' second choice. If, however, this "1–2" customer group is relatively small, then at best only a minimal price increase will be profitable.[3] In essence, the foregone profits from the lost sales to diverted customers would be comparable to the incremental profits from substantial price increases to customers that do not switch.

In recent years, the technique known as "merger simulation" has emerged as a promising framework for this analysis.[4] Simulation uses economic models grounded in the theory of industrial organization to predict the effect of mergers on prices in relevant markets. There is a common theoretical core to all simulation approaches in use today, although the details of a given simulation will depend on data availability and on the mathematical characterization of the market or markets at issue.

While merger simulation is not a panacea for all of the economic issues that arise in a difficult transaction, it nonetheless can offer assessments of competitive effects and remedies that are beyond the reach of other methods of inquiry. For example, simulation has been used to evaluate the likelihood that potential merger-

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Press, 2001, Chapter 3.) To our knowledge, merger simulation has yet to be applied to evaluate competitive issues that involve innovation markets explicitly.

[3] Unilateral effects simulation can predict price increases or decreases for a merger involving firms in the same market, depending on efficiencies and changes in market structure such as repositioning and divestitures.

[4] See, among others, Jerry Hausman, Gregory Leonard, and J. Douglas Zona, "Competitive Analysis with Differenciated Products," *Annales D'Économie et de Statistique* 34 (1994), Gregory J. Werden, "Simulating the Effects of Differentiated Products Mergers: A Practical Alternative to Structural Merger Policy," 5 *Geo. Mason L. Rev.* 363 (1997); Carl Shapiro, "Mergers with Differentiated Products," *Antitrust* Spring (1996) at 23; Gregory J. Werden, "Simulating Unilateral Competitive Effects from Differentiated Products Mergers," *Antitrust* Spring (1997) at 27; Jerry A. Hausman and Gregory K. Leonard, "Economic Analysis of Differentiated Products Mergers Using Real World Data," 5 *Geo. Mason L. Rev.* 321 (1997); see also, Gregory J. Werden "The Effects of Differentiated Products Mergers: A Practitioners' Guide," in Julie A. Caswell and Ronald W. Cotterill, eds., *Strategy and Policy in the Food System: Emerging Issues* 95 (1997), and Gregory J. Werden, "Simulating the Effects of Differentiated Products Mergers: A

specific efficiencies (associated with reductions in the marginal cost of production) are sufficiently great to offset predicted price increases. Simulation can also be used to analyze the competitive effects of product repositioning and *de novo* entry. Finally, simulation can help one to evaluate the adequacy of proposed divestitures.[5] With time, we believe that simulation techniques will be better understood and more widely used by antitrust lawyers and economists.[6]

A variety of different economic models can be utilized as the basis for a simulation analysis.[7] When sufficient data are available, demand models can be estimated econometrically. When these *estimated-demand simulation models* are not feasible, models requiring less data can be valuable if one is willing to make additional assumptions about the nature of demand. The logit demand model and "PCAIDS"—a new model to be introduced in this paper—both fit into this *calibrated-demand simulation model* category. We will suggest that PCAIDS offers advantages over a number of other calibrated-demand models.

We have undertaken this review and update of work on merger simulation with a number of goals in mind. First, we offer a relatively non-technical description of the principles of merger simulation—principles that are consistent with the methodologies currently in use by the competition authorities. Second, we describe PCAIDS, the new calibrated-demand merger simulation methodology. Third, we present examples that apply PCAIDS, including some applications that to our knowledge have not previously appeared in the literature on merger simulation.

Practical Alternative to Structural Merger Policy," 5 *Geo. Mason L. Rev.* 363 (1997).

[5] In recent years the agencies have begun to look critically at remedies involving restructuring. See, for example, "The Nature and Limits of Restructuring in Merger Review," Remarks by FTC Chairman Robert Pitofsky, Cutting Edge Antitrust Conference, Feb. 17, 2000 (available at www.ftc.gov), and Richard G. Parker and David A. Balto, "The Evolving Approach to Merger Remedies," *Antitrust Report*, May, 2000.

[6] For a lawyer's assessment of merger simulation, see James F. Rill, "Practicing What They Preach: One Lawyer's View of Econometric Models in Differentiated Products Mergers," 5 *Geo. Mason L. Rev.* 393 (1997).

[7] For an overview of publicly available merger simulation tools, see www.antitrust.org/economics/mergers/simulation.html.

Fourth, we suggest how simulation analyses might be used to evaluate the safeharbors of the *Merger Guidelines*.

Calibrated-demand models are relatively easy to implement and make detailed simulation feasible for nearly any transaction, since they require neither scanner nor transaction-level data. The PCAIDS model, in particular, requires only information on market shares and reasonable estimates of two elasticities. Estimates of these elasticities often can be obtained from marketing information or, when appropriate, through demand estimation. As with any calibrated-demand simulation model, one can test the sensitivity of the PCAIDS results to changes in the values of the estimated elasticities and to other simulation parameters.

We believe that calibrated-demand simulation models can offer valuable screening devices for "quick looks" by enforcement agencies and by merging firms. The models can be used to review the potential antitrust exposure resulting when unilateral effects issues are raised, but sufficient information is not available to estimate reliably a full set of cross-price elasticities. The models can also offer a useful means of working out the implications of the range of qualitative judgments an analyst might make based on documentary and interview evidence, and to test the sensitivity of competitive effects predictions to plausible variations in those assumptions. The analyses may be particularly useful for weighing opposing forces, as when comparing the potential anticompetitive loss of localized competition to the procompetitive gain relating to merger-specific efficiencies and product repositioning.

The balance of this article is organized as follows. Section II discusses the economic fundamentals of merger simulation. Because the pros and cons of merger simulation have been extensively debated elsewhere, we do not undertake such a treatment here. In Section III we introduce the PCAIDS approach to modeling demand. We explain how a key assumption about the relationship between market shares and the diversion of lost sales from price increases can be used to calibrate the PCAIDS model. Section IV offers some examples of merger simulation with PCAIDS that includes comparisons with other simulation models. In Section V we show how PCAIDS can be applied to the analysis of product repositioning and entry.

Section VI presents an analysis of the *Merger Guidelines* safeharbors using PCAIDS simulation, and section VII contains some brief concluding remarks. We have relegated the more technical mathematical details to the Appendix.

II. THE BASICS OF MERGER SIMULATION

Merger simulation models predict post-merger prices based on information about a set of pre-merger market conditions and certain assumptions about the behavior of the firms in the relevant market. Simulation models typically assume that firms' behavior is consistent with the Bertrand model of pricing, both pre- and post-merger. According to this theory, each firm sets the prices of its brands to maximize its profit, while accounting for possible strategic, non-collusive interactions with competitors. An equilibrium results when no firm can increase its profit by unilaterally changing the prices of its brands. This equilibrium can be interpreted as the outcome of the interactions between each firm's pricing decisions and its expectations of the price reactions of its competitors.[8]

Merger simulation requires a "demand model" that specifies the relationships between prices charged and quantities sold in the relevant market. A reasonable demand model must satisfy a number of conditions. The most basic is that the own-price elasticities (i.e., the percentage change in quantity for a given percentage change in its own price) should be negative. Increases in a product's own price should reduce the quantity demanded of that brand. Cross-price elasticities would normally be expected to be positive; a price increase for one brand normally leads to an increase in the quantity demanded of each of the remaining brands in the market (so long as the brands are economic substitutes for each other).[9] Implementation of the demand model requires particular values for these own and cross-price elasticities.

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[8] For a basic introduction to the "Nash-Bertrand" equilibrium, see, Robert S. Pindyck and Daniel L. Rubinfeld, *Microeconomics*, 5th Edition (2000), Chapter 12; a more advanced presentation appears in Jean Tirole, *The Theory of Industrial Organization* (1988).

[9] In a general demand model there is no requirement that own-price elasticities be equal for the different brands or that cross-price elasticities take on particular values.

In addition, simulation models require assumptions about supply or, more specifically, about how total cost responds to incremental changes in post-merger output. Most simulation analyses assume that incremental costs do not vary with output. The effects of any merger efficiencies are analyzed by changing the level of incremental costs (keeping the assumption that the level of incremental cost does not change as output changes).

A merger simulation analysis typically proceeds in two stages. First, one assumes that the market shares and own-price and cross-price elasticities for each brand in the pre-transaction market are known. The assumption of profit maximization then generates a set of mathematical "first-order conditions" (FOCs) that can be used to calculate pre-transaction gross profit margins for each brand.[10] Second, one takes into account the fact that the merged firm in general will set different prices than the pre-merger firms, to the extent that the merger removes some competition or there are potential efficiencies. The merged firm recognizes that, when it raises price on one of its brands, it keeps the profits from customers whose purchases are diverted to a brand of its merger partner. The demand model translates these price changes into corresponding changes in margins, elasticities, and shares. This second step in essence involves solving for the price changes that generate post-transaction margins, elasticities, and shares that are consistent with the merged firm maximizing the sum of its profits from all of the brands it now produces.[11]

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[10] See Appendix equation (A1). Using the first-order conditions to estimate margins avoids the distortions associated with the inclusion and allocation of fixed costs in accounting data, a particular problem for multi-brand firms. Moreover, relevant accounting data are likely only to be available for the brands sold by the merging parties. As a result, the FOC approach is particularly useful if one is to perform the simulation when there are more than two firms in the market and data sources are limited. We note, however, that the FOCs may yield negative margins, which are generally not consistent with the assumption that goods are substitutes. Because estimated margins depend on the price elasticities in the model, negative estimated margins could signal that the model is relying on inappropriate elasticities.

[11] See Appendix equations (A2) and (A3) for the solution to the relevant optimization problem.

III. THE PCAIDS MODEL

A. Background: Almost Ideal Demand Systems

Economists have explored a variety of demand models for merger simulation with a range of virtues: every model must strike a balance between theoretical rigor, tractability, and success in explaining the actual data. As might be expected, the simulated price effects of a merger will depend on the particular demand model chosen.[12] A demand model that we find particularly appealing is the Almost Ideal Demand System, or "AIDS."[13] AIDS is a widely accepted and intuitively reasonable model in economics that allows a flexible representation of own-price and cross-price elasticities. Moreover, its economic properties are arguably superior to alternatives that have often been used in merger simulation, including linear, constant-elasticity (log-linear), and logit demand models.

The major problem with AIDS is a practical one. AIDS typically requires econometric estimation of a large number of parameters, and it is not unusual for the estimated cross-price elasticities to have low precision and algebraic signs that are inconsistent with economic theory. We explain below how it is possible to implement a variant of the AIDS model in a manner that ensures the correct signs, without the use of complex econometric methods. This simplicity is not costless, however, since PCAIDS requires additional structural assumptions beyond the AIDS model.[14] We believe that these costs are often reasonable in comparison to the benefits associated with both the variety of applications that can be handled with PCAIDS or other calibrated-demand simulation models.

A simple example with three independent firms, each owning a single brand, will help explain the logic of AIDS (and PCAIDS). The AIDS model specifies that the share of each brand depends on the prices of all brands. More formally, the share

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[12] See, Philip Crooke, Luke Froeb, Steven Tschantz, and Gregory Werden, "The Effects of Assumed Demand Form on Simulated Post Merger Equilibria," *Rev. of Ind. Org.* 15(3) (Nov. 1999), pp. 205-217.

[13] For the original presentation of AIDS see Angus Deaton and J. Muellbauer, "An Almost Ideal Demand System," 70 *Am. Econ. Rev.* 312 (1980).

of the ith brand, s_i, as a percent of total market revenues is a function of the natural logarithms of the prices, p_i, of all of the brands in the relevant market:

$$s_1 = a_1 + b_{11} \ln(p_1) + b_{12} \ln(p_2) + b_{13} \ln(p_3)$$
$$s_2 = a_2 + b_{21} \ln(p_1) + b_{22} \ln(p_2) + b_{23} \ln(p_3)$$
$$s_3 = a_3 + b_{31} \ln(p_1) + b_{32} \ln(p_2) + b_{33} \ln(p_3)$$

The coefficients b_{ij} (for i, j = 1,2,3) must be determined to use this system to simulate the effects of a merger.[15] As shown in the Appendix (Section 3, Equations (A4) and (A5)), the *b*'s underlie the own-price and cross-price elasticities. The three "own-coefficients" b_{11}, b_{22}, and b_{33} specify the effect of each brand's own price on its share. These coefficients should have negative signs, since an increase in a brand's price should (all other prices held constant) reduce its share; indeed, these coefficients are closely related to and have the same signs as the own-price elasticities. The six other b_{ij}'s specify the effects of the prices of other brands on each brand's share. For example, b_{12} specifies the effect of an increase in the price of brand 2 on share 1, while b_{13} describes the effect of an increased price of brand 3 on brand 1's share. These "cross-effect" coefficients are expected to be positive (assuming the three brands are substitutes), since these terms are related to and have the same signs as the cross-price elasticities.[16]

When we use this AIDS (or PCAIDS) model to simulate a merger, we wish to predict changes in the share of each brand resulting from the transaction. These

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[14] Calibrated-demand models based on other types of demand systems also require comparably strong structural assumptions.

[15] In this presentation we have suppressed the aggregate expenditure terms from the original Deaton and Muellbauer specification. This "homotheticity" assumption is reasonable to the extent that changes in industry expenditure have no significant effects on share. Since we are concerned only with changes created by the merger, the a_i intercepts drop out in the analysis that follows.

[16] The market shares predicted by AIDS are required to sum to 100% -- the *adding-up* property. We also impose *homogeneity*, the assumption that equal proportional changes in all prices have no effect on market share (e.g., if all prices went up by 10 percent, the market shares for the various brands should not change). As explained in the Appendix, adding-up and homogeneity effectively reduce the number of brands to be analyzed in the AIDS model from *N* to *N*–1.

changes (obtained formally by differentiating each equation totally) are given by the following:

$$\begin{aligned} ds_1 &= b_{11}(dp_1/p_1) + b_{12}(dp_2/p_2) + b_{13}(dp_3/p_3) \\ ds_2 &= b_{21}(dp_1/p_1) + b_{22}(dp_2/p_2) + b_{23}(dp_3/p_3) \\ ds_3 &= b_{31}(dp_1/p_1) + b_{32}(dp_2/p_2) + b_{33}(dp_3/p_3) \end{aligned} \qquad (1)$$

We can see from (1) that there is a linear relationship between the change in each brand's market share (ds) and the percentage changes in the three prices (dp/p), where the b's provide the weights.[17] Note, for example, that an increase in p_1 leads to a decrease in s_1 (since dp_1/p_1 is positive and the weight b_{11} is negative), while an increase in p_2 leads to an increase in s_1 (since b_{12} is positive).

B. Econometric Estimation of Demand for Simulation Models

The simple 3-brand example also allows us to illustrate the difficulty in estimating elasticities. In the example, a model with 3 brands has 9 b parameters: 3 own coefficients and 6 cross-effect coefficients, which correspond to 3 own elasticities and 6 cross- elasticities. More generally, a market with "n" brands gives rise to a total of n^2 elasticities: n own-price elasticities and $n(n–1)$ cross-price elasticities. In the AIDS context, n^2 b_{ij} coefficients generate these elasticities.[18] While nine coefficients (n=3) may be easily tractable in this simple example, merger analysis can involve many more brands and parameters. In the ready-to-eat cereal industry, for example, there are approximately 200 brands. As a result, a complete cereal model could involve 40,000 elasticities! To estimate the parameters of a demand model with many brands, it is necessary either to have a large data set, or to impose assumptions that reduce the number of independent parameters to be estimated.[19]

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[17] The price changes will in general also affect the total size of the market (see the Appendix, section 1).

[18] Other demand models will also require a similar number of estimated coefficients.

[19] In addition to imposing adding-up and homogeneity, the number of parameters can also be reduced significantly by specifying a demand model that results from a multi-level decisionmaking process. For an evaluation of this approach, see Daniel L. Rubinfeld, "Market Definition with Differentiated Products: The Post/Nabisco Cereal Merger," 68 *Antitrust L. J.* 163 (2000) at 173-176.

Econometric estimation using supermarket scanner data is sometimes thought to be the only practical way to determine demand parameters for large simulation models (AIDS-based or otherwise). When available, these data can indeed be quite valuable. For example, they often track detailed price variations across many cities or market areas on a weekly or monthly basis, and provide important information concerning trade promotion, couponing, and other marketing practices. Nevertheless, there are important limitations that can handicap many applications.

First, scanner data are typically available only for brands sold in supermarkets and the largest drug stores and mass merchandisers. Unless supplemented by separate audits, retail sales data in smaller outlets are typically not available. Moreover, sales of many consumer goods, and nearly all intermediate goods, are not tracked by scanner data. Second, the scanner data describe the retail prices of consumer goods, whereas many mergers occur at the production or wholesale level. To use scanner data in such cases one must incorporate a set of assumptions about mark-ups and margins that link wholesale and retail prices. Third, scanner data generally must be analyzed with complex econometric procedures that can sometimes be open to criticism. For example, econometric issues involving model identification and estimation must be overcome before demand effects can be distinguished from supply effects. Finally, despite one's best efforts, econometric estimation may yield results at odds with common sense and intuition. With many parameters to be estimated, it is frequently the case that at least some of the empirically estimated cross-price elasticities suffer from low levels of statistical significance, implausible magnitudes, and/or wrong algebraic signs.

C. PCAIDS: Proportionality-Calibrated AIDS

Calibrated-demand simulation models offer an alternative to models that rely on econometric estimation of demand. Because they reduce the number of required demand parameters, these models are especially valuable when there are data limitations or estimation problems, or when a rapid and less costly analysis is

required.[20] We offer Proportionality-Calibrated AIDS, i.e., PCAIDS, as a calibrated-demand model that provides analytical flexibility while retaining many of the desirable properties of AIDS.

PCAIDS requires neither scanner data, nor data on pre-merger prices. It requires information only on market shares, the industry price elasticity, and the price elasticity for one brand in the market. The logic of PCAIDS is simple. The share lost as a result of a price increase is allocated to the other firms in the relevant market in proportion to their respective shares. In effect, the market shares define probabilities of making incremental sales for each of the competitors.[21]

We believe that the proportionality assumption is practical and often reasonable when data are limited.[22] With proportionality and PCAIDS, one can take a "quick look" at the likely price effects of a merger; these results are likely to be reliable when applied to markets with limited product differentiation, or when the merger brands are not unusually close (or distant) in terms of their attributes and substitutability. In this sense, proportionality reflects the analytical framework in the *Merger Guidelines*, which suggest that market share sometimes may be used to measure the relative appeal of the merging firms' products as first and second choices for consumers.[23] Moreover, as we discuss below, PCAIDS can be extended to situations where extensive product differentiation makes proportionality suspect.

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[20] See Baker and Rubinfeld, note 1 *supra*, for a survey of a variety of approaches to the calibration of demand systems, including auction models and conjoint survey methods.

[21] This approach has long been used in other settings involving economics and law when data are limited. For example, in *State Industries v. Mor-Flo*, one of the leading decisions in the patent damages area, the assumption is that the patent holder suffers lost sales equal to its market share applied to the infringer's sales (the remaining infringing sales would have been made by the other firms in the market in proportion to their respective shares). For a recent analysis of this decision see Roy J. Epstein, "State Industries and Economics: Rethinking Patent Infringement Damages," *Fed. Cir. B. J.* 9(3) (2000), pp. 367–381.

[22] Earlier discussions of proportionality in the context of merger analysis include Robert D. Willig, "Merger Analysis, Industrial Organization Theory, and Merger Guidelines," *Brookings Papers on Economic Activity: Microeconomics 1991*, ed. M. Baily and C. Winston, Washington, D.C.: Brookings Institution, pp. 299–305, and Shapiro, note 4 *supra*.

Indeed, PCAIDS can be used as an approximation of the AIDS model, with a structure that ensures proper signs and consistent magnitudes for the elasticities.[24] Another potential advantage compared to other simulation methods is that PCAIDS can be implemented on a conventional spreadsheet without additional specialized software. In summary, PCAIDS is a general method for calibrating AIDS demand with minimal data, and for which proportionality is a useful starting point.

The simplifications that flow from the proportionality assumption of PCAIDS can be illustrated in a simple example. The three equations in (1) above show that a change in the price of the first brand, p_1, affects the market shares of all three brands. Recall that the own-effect of the price of brand 1 on the share of brand 1 is b_{11}. The cross-effects of p_1 on the shares of brands 2 and 3 are given by b_{21} and b_{31}. With proportionality, sales are diverted to brands 2 and 3 in proportion to the market shares of the two brands. For example, if brand 2 has a share of 40% and brand 3 a share of 20%, an increase in the price of brand 1 will increase the share of brand 2 by twice as much as it increases the share of brand 3. Formally, the proportionality assumptions implies that the cross-effects associated with p_1 can be expressed in terms of b_{11} and the observed shares; b_{21} is equal to $-s_2/(s_2+s_3)b_{11}$ and b_{31} equals $-s_3/(s_2+s_3)b_{11}$.[25] The same relationships between own and cross effects hold for other prices; for example, b_{12} equals $-s_1/(s_1+s_3)b_{22}$.

The proportionality assumption reduces the number of unknown b's in (1) from 9 to 3. We only need to know the 3 own-effect coefficients (and market shares) to calculate the remaining 6 cross-effect coefficients. More generally, the proportionality assumption posits a direct relationship between all cross-effects

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[23] See *Horizontal Merger Guidelines* at ¶2.211.

[24] Our discussion of PCAIDS focuses on implementation with aggregate market share information. However, the method is also applicable as a set of restrictions that could be imposed when estimating standard AIDS with scanner data. We show in the Appendix that PCAIDS and its extensions to non-proportionality satisfy Slutsky symmetry, an important theoretical property for demand systems.

[25] The minus sign is necessary because b_{11} is negative (it is associated with the own-effect). It is easy to verify that the sum of the cross-effects in this case equals $-b_{11}$, which confirms that adding-up is satisfied.

associated with a particular price change and the corresponding own-effect.[26] The implication is that the only unknowns in the model are the n own-effect coefficients. The assumption that the predicted market shares sum to 100% eliminates one additional unknown, so the number of unknown parameters is then reduced from n^2 to n–1, or from 40,000 to 199 in our cereal example.

In fact, the proportionality assumption reduces the information requirement of PCAIDS even further. It is not necessary to know all n (or even n-1) own price effects or elasticities. The PCAIDS model can be calibrated with only two independent pieces of information (in addition to the shares): the elasticity of demand for a single brand and the elasticity for the industry as a whole. For example, only the industry elasticity and the own-price elasticity for brand 1 are needed as inputs in the calculation of the own-effect coefficient for brand 1, b_{11}:[27]

$$b_{11} = s_1(e_{11} + 1 - s_1(e+1)). \tag{2}$$

In Equation (2), e_{11} is the own-price elasticity for brand 1 and e is the industry elasticity. Then, as shown in Section 4.A. of the Appendix, proportionality implies that all remaining unknown own-effect coefficients can be determined as simple multiples of b_{11}, as Equation (3) illustrates:

$$b_{ii} = \frac{s_i}{1-s_1}\frac{1-s_i}{s_1} b_{11} . \tag{3}$$

We have already seen that once the b_{ii} own effects have been calculated, the cross-price effects can then be calculated from the own-price effects and market shares. This means that knowledge of the own-price elasticity of any one brand and the overall industry price elasticity is sufficient to obtain estimates of all relevant demand parameters of the PCAIDS model from the market share data. This is true whether there are 3 or 200 brands.

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[26] Note that elasticities derived using the assumption of proportionality may be sensitive to the market definition. If additional brands are thought to be in the market, and are therefore included in the model, the estimated price effects of the merger could change.

[27] More generally, the own-effect coefficient for any one brand can be determined from the industry elasticity and the own-price elasticity for that brand; the result is proven in the Appendix.

Elasticities can be calculated directly from the values for the *b* parameters, the market shares (s_i), and the industry elasticity (e), as follows (see Appendix equations (A4) and (A5) for details):

Own-price elasticity for the ith brand: $$e_{ii} = -1 + \frac{b_{ii}}{s_i} + s_i(e+1) \quad (4)$$

Cross-price elasticity of the ith brand with respect to the price of the jth brand:

$$e_{ij} = \frac{b_{ij}}{s_i} + s_j(e+1). \quad (5)$$

Under the assumption that the magnitude of the industry elasticity e is smaller than any brand own-price elasticity, PCAIDS implies that the cross elasticities will be positive. Moreover, it can be shown that all pre-transaction cross-elasticities corresponding to a given price change are equal, i.e., $e_{ij} = e_{kj}$ for all brands i, j, and k. This equality is a consequence of the assumption of proportionality.[28]

All the information required to calibrate PCAIDS should be available. Market shares typically are known with reasonable accuracy. It should be feasible to infer the own-price elasticity for at least one brand sold by the merging parties from marketing studies in the party's documents (including surveys and focus groups), from econometric analyses, or from accounting data.[29] The industry elasticity typically is considerably smaller than the price elasticity of any one brand, since brand substitution is easier than industry substitution.[30] Absent independent information about the magnitude of that elasticity, we suggest an industry elasticity of

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[28] The assumption of proportionality is equivalent to the assumption of "Irrelevance of Independent Alternatives" (IIA) that underlies the logit model. Unlike the logit model, however, the PCAIDS post-merger elasticities are not constrained by IIA.

[29] For an extensive discussion of the range of empirical methods that can be used to obtain estimates of demand elasticities, see Baker and Rubinfeld, note 1 *supra*, Section 3.

[30] Suppose the prices of all cereals rose by 10 percent. Since many consumers, particularly children, are likely to continue eating the similar quantities of cereal for breakfast (some, of course, will not and consumption of cereal for other purposes, such as snacks, may fall), ready-to-eat demand is not likely to be highly price sensitive. On the other hand, a 10 percent increase for a single brand, such as corn flakes, with no change in competitors' prices, will be more price sensitive, since it will likely result in substantial switching to other products within the cereal category.

–1 as a good starting point for a preliminary merger simulation. If the market under study is a relevant antitrust market, the industry elasticity will be equal to or greater than 1 in magnitude. As a result, this assumption will be conservative in its tendency to overpredict the price effects of mergers.[31]

To illustrate PCAIDS, reconsider the demand system in (1). Assume that the shares for the 3 brands (each sold by a different firm) are 20%, 30%, and 50%, respectively. Now, assume that there is a proposed merger between firms 1 and 2, the industry elasticity is –1, and the own-price elasticity for the first brand is –3. The formulas for PCAIDS given above and in the Appendix allow calculation of all parameters of the demand system (1) and all elasticities as shown in Table 1 below.

Table 1

PCAIDS Coefficients and Elasticities

PCAIDS Coefficient with Respect to:				Elasticity with Respect to:			
Brand	p_1	p_2	p_3	Brand	p_1	p_2	p_3
1	–0.400	0.150	0.250	1	–3.00	0.75	1.25
2	0.150	–0.525	0.375	2	0.50	–2.75	1.25
3	0.250	0.375	–0.625	3	0.50	0.75	–2.25

The calculated own elasticities—the negative values on the diagonal of the right panel of the table—can be either larger or smaller than the elasticity for the brand used to calibrate the system.[32] Reading down each column of elasticities, the cross elasticities corresponding to a given price are equal as expected given proportionality. PCAIDS simulation with these parameters predicts a unilateral post-merger price increase (absent efficiencies) of 13.8% for Brand 1 and 10.8% for Brand 2.

[31] This follows from the rule of thumb for pricing by a monopolist. See, for example, Daniel L Rubinfeld and Robert S. Pindyck, *Microeconomics*, 5th Edition (2000), Chapter 11.

[32] The PCAIDS coefficients satisfy adding-up and homogeneity and are symmetric, as required.

D. Deviations from Proportionality — PCAIDS with Nests

Proportionality will not always characterize the diversion of lost sales accurately when products are highly differentiated.[33] Fortunately, it is straightforward to modify PCAIDS to allow a more general analysis. Products that are closer substitutes for each other than proportionality suggests may be placed together in "nests." The approach is analogous to using nests in a logit context, but we believe it is easier and more flexible to calibrate PCAIDS with a nest structure.

To illustrate, return to the three-brand example discussed in the previous section. In that example, brand 2's market share of 30% and brand 3's share of 50% implied that 37.5% (30/80) of the share lost by brand 1 when its price increased would be diverted to brand 2 and 62.5% (50/80) would be diverted to brand 3. This effect can be characterized using an *odds ratio*. Here, the odds ratio between brand 2 and brand 3 is 0.6 (0.375/0.625). That is, under proportionality, brand 2 is only 60% as likely to be chosen by consumers leaving brand 1 as brand 3. Now suppose instead that brand 2 is relatively "farther" from brand 1 in the sense that that fewer consumers would choose brand 2 in response to an increase in p_1 than would be predicted by proportionality. For example, brand 2 may only be "half as desirable" a substitute as brand 3 and the appropriate odds ratio really only 0.3. It is straightforward to calculate in this case that the share diversion to brand 2 becomes 23.1% and the diversion to brand 3 increases to 76.9% (an odds ratio of 0.3=.231/.769). As expected, fewer consumers leaving brand 1 would choose brand 2.

We generalize PCAIDS to cover such situations by constructing separate "nests" of brands. Diversion among brands within each nest is characterized by proportionality. Share diverted to a brand in a different nest deviates from proportionality in the following sense: the odds ratio is equal to the odds ratio under

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[33] Cf. *Horizontal Merger Guidelines* at ¶2.211: "The market shares of the merging firms' products may understate the competitive effect of concern, when, for example, the products of the merging firms are relatively more similar in their various attributes to one another than to other products in the relevant market. On the other hand, the market shares alone may overstate the competitive effects of concern when, for example, the relevant products are less similar in their attributes to one another than to other products in the relevant market."

proportionality, multiplied by an appropriate scaling factor ranging from 0 to 1. The result is that brands within a nest are closer substitutes than brands outside the nest. PCAIDS with nests allows a more flexible pattern of cross elasticities, as the model is no longer fully constrained by the proportionality assumption.

Continuing with the example, we capture the effect of brand 2 being a less close substitute for brand 1 than indicated by market shares by placing brand 2 in a separate nest with a scaling or odds ratio factor of 0.5. We then use formulas in the Appendix to recalculate the *b* coefficients and resulting elasticities with this nesting assumption.[34] Table 2 reports the calculated elasticities for both the nested model and the original model.[35]

Table 2

PCAIDS Elasticities with Nests

Non-Nested Demand				Separate Brand 2 Nest, (Odds Ratio Factor = 0.5)			
	Elasticity with Respect to:				Elasticity with Respect to:		
Brand	p_1	p_2	p_3	Brand	p_1	p_2	p_3
1	–3.00	0.75	1.25	1	–3.00	0.46	1.54
2	0.50	–2.75	1.25	2	0.31	–2.08	0.77
3	0.50	0.75	–2.25	3	0.62	0.46	–2.08

The nest parameter rescales the cross elasticities in the right-hand panel; the cross elasticities measuring the responses of brands 2 and 3 to the price of brand 1, and those measuring the responses of brands 1 and 2 to the price of brand 3 are no longer equal. (The cross elasticities measuring the responses of brand 1 and brand 3 to the price of brand 2 remain equal, but at lower values, because brands 1 and 3 are in the same nest while brand 2 is outside.) With nesting, brand 2 becomes a poorer

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[34] It would be incorrect to scale the non-nested elasticities in the left-hand panel directly. Nests affect the impact of adding-up, homogeneity, and symmetry and the appropriate calculation takes account of these constraints to generate economically consistent elasticities.

[35] The calculations continue to assume an own-price elasticity of –3 for Brand 1 and an industry elasticity of –1.

substitute for brands 1 and 3 (as indicated by the smaller cross elasticities of brand 2 to the prices of brands 1 and 3 and of brands 1 and 3 to the price of brand 2), while brands 1 and 3 become better substitutes for each other (as indicated by the larger cross elasticities of brands 1 to the price of brand 3 and of brand 1 to changes in the price of brand 3).

Simulation of a merger of brand 1 and brand 2 using this nested PCAIDS model predicts a unilateral price increase (without efficiencies) of 10.1% for both brand 1 and brand 2, compared to the original increases of 13.8% and 10.8% without nests. The unilateral effects are smaller because the merging brands are less close substitutes for each other.

What remains is the difficult question of when proportionality is inappropriate, making nests necessary for accurate merger simulations. To our knowledge there has been very little empirical testing of this question.[36] We note, however, that if PCAIDS introduces the possibility of biased values for the *b* coefficients, it may still provide an economically useful approximation.[37] Fortunately, PCAIDS makes it easy to detect whether nesting is likely have economically meaningful effects through a sensitivity analysis of the odds ratio factors. We suspect that most simulations will justify very few nests, since simulation results appear to be robust to modest departures from proportionality. We also believe that a coarse grid (e.g., 0.75, 0.50, and 0.25) covering a range of odds ratio factors is adequate to assess sensitivity.

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[36] A statistical test procedure is described in Jerry A. Hausman and Daniel McFadden, "Specification Tests for the Multinomial Logit Model," *Econometrica* (1984). One recent AIDS analysis of a grocery item using scanner data indicates that proportionality is reasonable but it does not formally test the hypothesis. See David A. Weiskopf, "Assessment of the Relationship between Various Types of Estimation Bias and the Simulated Economic Impact of Certain Anti-Competitive Scenarios," Unpublished Ph.D. dissertation, Vanderbilt University, Department of Economics (1999) at 55 and Table B2.

[37] In econometric terms, coefficients estimated with the PCAIDS restrictions could have lower mean square error, i.e., the reduced variance of the estimates may more than balance any bias that is introduced. See, for example, Robert S. Pindyck and Daniel L. Rubinfeld, *Econometric Models and Economic Forecasts* (1998) at 29–32.

E. PCAIDS and Other Calibrated-Demand Simulation Models

The PCAIDS model shares some characteristics with models based on logit demand structures that have been used to simulate mergers. Both assume proportionality (the logit model makes a comparable assumption of "independence of irrelevant alternatives"), yield positive cross elasticities, and can be calibrated with only two parameters. We prefer PCAIDS to logit, however, for several reasons. First, PCAIDS does not require pre-merger price data. There will doubtless be occasions where prices are either not available for all firms in the market or are not measured accurately. Second, one can depart from proportionality in the PCAIDS framework using nested demands. Logit models can be generalized with nests as well, but we believe that logit is more difficult to calibrate econometrically and the additional nesting parameters are less intuitive.[38] Third, we prefer PCAIDS because it has mathematical "curvature" that approximates that of the standard AIDS model.[39] We suggest that the "curvature" of AIDS models is likely to fit data better than that of logit demand, although we recognize that this opinion invites further empirical research.[40] In essence, we view PCAIDS as a desirable mix of the best features of both logit (few parameters, correct signs) and AIDS (non-reliance on independence of irrelevant alternatives, ability to fit the data, curvature).[41]

Our approach is similar in spirit to one suggested by Carl Shapiro.[42] Shapiro offers a rule-of-thumb formula for calculating the predicted prices of the post-merger

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[38] For a discussion of estimation problems with nested logits, see Gregory J. Werden and Luke M. Froeb, "The Effects of Mergers in Differentiated Products Industries: Logit Demand and Merger Policy," *J. of Law, Econ., and Org.* 10(2) (1994) at 420.

[39] For an analysis of curvature of alternative demand models see Crooke, Froeb, Tschantz, and Werden, note 12 *supra*.

[40] We are aware of very few studies that directly compare AIDS and logit using real-world data. A recent article that uses grocery scanner data on white pan bread sales indicates AIDS fit the data significantly better than logit. See Atanu Saha and Peter Simon, "Predicting the Price Effect of Mergers with Polynomial Logit Demand," *Int. J. of the Econ. of Bus.* 7 (2)(2000) at 154.

[41] The informative discussion at www.antitrust.org/economics/simulation concludes that "much progress has been made using the linear and nested logit demand specifications….However, more progress can be made, by simulating the effects of mergers within the context of more flexible functional forms, like the AIDS model."

[42] Shapiro, note 4 *supra* and also www.usdoj/atr/public/speeches/shapiro.spc.

firm, assuming that the merger involves two firms and two symmetric merging brands. As inputs, he requires markups (or equivalently gross margins) and diversion ratios. Shapiro's diversion ratio-symmetry assumptions in his two-brand example are similar to our proportionality assumption. However, his approach differs from ours in a number of ways. First, in much of the paper Shapiro assumes that demand elasticities are constant, an assumption that can create simulation difficulties because (a) such models sometime fail to converge; (b) the price increases resulting from a merger tend to be overstated; (c) non-merging firms do not raise prices in response to unilateral increases by the merged entity. Second, his approach does not readily generalize to multi-brand firms. Finally, Shapiro does not discuss possible extensions when the proportionality assumption does not appear to be reasonable.

IV. USING PCAIDS

This section offers a number of examples of applications of PCAIDS that are intended to make some of the principles discussed above more concrete. Our goal is to demonstrate that PCAIDS can provide reasonable estimates of the simulated effects of mergers at relatively low cost and with some transparency. The examples demonstrate the calibration of the PCAIDS demand model using shares and elasticities, the incorporation of efficiencies, sensitivity analyses using nests, and divestiture. The examples utilize available data on toilet paper, baby food, and white pan bread.

A. The Kimberly-Clark/Scott Merger Revisited

We first use PCAIDS to re-examine the acquisition of Scott by Kimberly-Clark. A PCAIDS analysis of this 1992 merger may be compared to an earlier published simulation analysis by Hausman and Leonard that used supermarket scanner data to estimate econometrically a standard AIDS model.[43]

There were eight toilet paper brands pre-merger with national shares as shown in Table 3:

Table 3

Toilet Paper Market Shares

Brand	Share (%)
ScotTissue	30.9
Cottonelle	7.5
Kleenex	6.7
Charmin	12.4
Northern	8.8
Angel	16.7
Private Label	7.6
Other	9.4
Total	100.0

Scott produced both ScotTissue and Cottonelle. Kimberly-Clark produced only Kleenex. We calibrate PCAIDS using a price elasticity for Scott of –2.94 reported by Hausman and Leonard and an estimate of –1.17 for industry elasticity inferred from their article.

Table 4 compares PCAIDS price elasticities calculated using these parameters to the elasticities estimated econometrically by Hausman-Leonard.

[43] Jerry A. Hausman and Gregory K. Leonard, "Economic Analysis of Differentiated Products Mergers Using Real World Data," 5 *Geo. Mason L. Rev*. 321 (1997).

Table 4

PCAIDS and Hausman-Leonard Elasticities

	Own-Price Elasticity		Cross-Price Elasticity	
	PCAIDS	Hausman-Leonard	PCAIDS	Hausman-Leonard
ScotTissue	–2.9	–2.9	0.36	0.24
Cottonelle	–3.2	–4.5	0.14	0.22
Kleenex	–3.1	–3.4	0.16	0.13
Charmin	–2.6	–2.7	0.66	0.35
Northern	–3.0	–4.2	0.26	0.41
Angel	–3.1	–4.1	0.19	0.26
Private Label	–3.1	–2.0	0.16	0.09
Other	–3.1	–2.0	0.20	0.27
Average	–3.0	–3.2	0.27	0.24

The two methods yield similar results brand by brand, and on average there appears to be relatively little difference.[44] We take this as evidence that the proportionality assumption of PCAIDS is reasonably consistent with the toilet paper data. Moreover, differences between the elasticities yielded by the two methods may not be statistically significant. Hausman-Leonard report low precision for many of the estimated cross-price elasticities between the merging products in their model. For example, they report a Kleenex/Scott cross-price elasticity of 0.061 with a standard error of 0.066; this means that their estimated cross-elasticity is within two standard errors of our calibrated PCAIDS value of 0.16. Uncertainty about the true value of this cross-elasticity is particularly crucial to the merger simulation analysis since the magnitude of this cross-elasticity has a large effect on the price increases predicted from the merger.

[44] Each Hausman-Leonard cross-price elasticity in the table is calculated as the average of the cross-price elasticities with respect to the price of the brand given in the left-most column. The Hausman-Leonard study reported several negative cross elasticities (for non-merging goods) that we found difficult to interpret. The average values reported in the table exclude any negative cross-price elasticities.

The two simulation methods (taking into account the efficiencies assumed by Hausman-Leonard) yield predicted price changes for the merging firms as shown in Table 5:

Table 5

Simulated Unilateral Effects – Toilet Paper

	Price Change (%)	
	PCAIDS	Hausman -Leonard
ScotTissue	–0.3	–1.1
Cottonelle	0.7	0.5
Kleenex	4.3	0.2

The two models predict similar price changes for ScotTissue and Cottonelle. There is a greater difference between the price changes predicted by the two models for Kleenex, although even this difference may not be statistically significant. As a sensitivity test, we introduced a nest structure that lowered the PCAIDS Kleenex/Scott cross elasticity to 0.061 and left the other cross elasticities in the model essentially unchanged. The price increase for Kleenex predicted by this nested PCAIDS model fell to 1.7 percent. This experiment suggests that increasing the same cross-price elasticity by two standard errors in the Hausman-Leonard simulation would produce a Kleenex price change much closer to the PCAIDS result.

B. Efficiencies in a Baby Food Acquisition

The recently terminated effort by Heinz to acquire the Beech-Nut baby food assets raises many interesting questions about the role of efficiencies in merger analysis. We were not involved in that transaction, but it is our understanding that the litigation centered on coordinated effects. Indeed, we cannot ascertain from the published opinion whether either side presented testimony that relied on a merger simulation analysis of unilateral effects.[45] Nevertheless, we will use this proposed

[45] See *FTC v. H.J. Heinz Co. and Milnot Holding Corporation*, U.S. Court of Appeals for the D.C. Circuit, April 27, 2001.

merger as an example of how PCAIDS might be applied to evaluate unilateral effects issues.

According to the Court, there is a national relevant market for baby food in jars. The industry is concentrated, with three major firms and a small fringe (which we represent as a composite "private label" firm[46]). The market shares are given in Table 6:

Table 6

Baby-Food Market Shares

Brand	Share (%)
Heinz	17.4
Beech-Nut	15.4
Gerber	65.0
Private Label	2.2
Total	100.0

The pre-transaction HHI was 4,770, with a delta of 536, well above the safeharbor limits in the *Merger Guidelines*. Market shares and the HHI alone, however, do not provide sufficient information to analyze the potential magnitudes of a unilateral price increase or the mitigating effect of efficiencies.

We do not analyze individual brands, but instead treat each firm as if it produced a single aggregate. We also do not distinguish competition at the retail level (for customers) from competition at the wholesale level (for shelf space). Because the written opinion does not offer specific price elasticities, we have assumed an industry elasticity of –1.0 and we have estimated a price elasticity for Heinz of –2.60 from financial information.[47]

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[46] The use of composite goods or firms is common in merger simulation because, when appropriate, it greatly diminishes the number of parameters in the model and simplifies the analysis.

[47] The elasticity was calculated as negative of ratio of sales ($9,407,949) to gross profit ($3,619,424). At the profit-maximizing price for a firm, the negative of its markup of price over cost as a proportion of price equals the inverse of its elasticity. See H.J. Heinz Company Form 10-K for fiscal year ended May 3, 2000, Consolidated Statements of Income, available at www.edgar-online.com.

We consider three alternative simulations. First, we model the four firms as belonging in a single nest. Proportionality implies that most of the share lost by Heinz due to a price increase would be diverted to Gerber instead of Beech-Nut. The ratio of the Gerber to the Beech-Nut market share equals 65/15.4. This yields an odds ratio of 4.22, which indicates that consumers leaving Heinz would be more than four times as likely to shift to Gerber as to Beech-Nut. For the second simulation, we put Heinz and Beech-Nut in a separate nest from Gerber and private label, with an odds ratio factor of 0.5. This nest structure represents the hypothesis that one group of consumers strongly prefers Gerber to Heinz and Beech-Nut. In this scenario the Gerber Beech-Nut odds ratio falls by half to 2.11, indicating that Gerber becomes a poorer substitute (now only about twice as many consumers would choose Gerber). For the third simulation, we put Heinz and private label in a separate nest from Gerber and Beech-Nut, also with an odds ratio factor of 0.5. This scenario tests the implication of treating Gerber and Beech-Nut as closer substitutes because they are both premium priced brands. Since proportionality holds within a nest, the odds ratio would revert to 4.22 (the ratio of their market shares).

The simulated unilateral effects for each of these scenarios, in the absence of any efficiencies, are given in Table 7:

Table 7

Simulated Unilateral Effects – Baby Food

	Simulated Price Change		
Firm	No Nests	Heinz Beech-Nut Nest	Beech-Nut Gerber Nest
Heinz	6.2%	12.3%	3.9%
Beech-Nut	6.8%	13.3%	3.4%

These results illustrate the importance of the nesting assumption for the magnitude of the price increases. Predicted price increases are largest when the merging firms are in the same nest (which implies consumers view them as closer substitutes for each other than market shares alone suggest), and smallest when the merging firms are in separate nests (which implies consumers view then as less close substitutes for each other than market shares alone suggest).

PCAIDS can also be used to provide estimates of the efficiencies that would fully offset the predicted price effects. For the no-nest case, we calculate that reductions in marginal costs of approximately 8% for both Heinz and Beech-Nut would be required. If Heinz and Beech-Nut are closer substitutes and in the same nest, reductions in marginal costs of approximately 16% for each firm are necessary to offset the predicted price increase

The Court notes that the merging parties claimed expected efficiencies of 22.3% for Beech-Nut.[48] It is not clear to what extent the claimed cost-reductions for Beech-Nut would translate into merger-specific efficiencies for the merged entity.[49] However, our analysis in this hypothetical suggests that evidence on efficiencies would have been crucial to any argument that unilateral effects of the merger on price were not likely to be significant.

C. Merger with Divestiture

Some proposed transactions raise concerns about unilateral price effects that cannot be overcome by expected efficiencies or repositioning. Divestiture may be an option to "fix" such a deal, and simulation analysis can help evaluate whether and which divestitures would eliminate competitive concerns. We illustrate an analysis of divestiture using data from a recent study of a merger between two large white pan bread bakeries.[50] The pre-transaction market contained six firms with market shares as shown in Table 8:

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[48] *FTC v. H.J. Heinz Co. and Milnot Holding* at Section. II.C.2.

[49] We understand (from personal communication) that Jonathan Baker testified (on behalf of Beech-Nut and Heinz) to an expcted 15% reduction in marginal cost for the gains passed-through to the Beech-Nut brand. According to Baker, these gains would come from a price reduction; the gains to Heinz buyers would come from getting a brand that is 15% higher in quality (at the same price as their old brand according to the merging parties).

[50] See Saha and Simon, note 40 *supra*.

Table 8

Market Shares – White Pan Bread

Firm-Brand	Share (%)
A-1	14.2
A-2	8.05
A-3	7.6
B-1	8.8
C-1	7.0
D-1	7.6
Grocery	31.5
Other	15.2
Total	100.0

Firms A and B are the merging parties. "Grocery" and "Other" are composites of smaller suppliers. The pre-transaction HHI was 2,317 with a change of 524, values that could trigger detailed agency review.

According to the study, the industry elasticity was –1.0. We set the elasticity for B-1 to the study's estimate of –1.34 to complete the PCAIDS calibration of the demand model. Initially we assume proportionality. Table 9 shows the unilateral price increases for the merged firm predicted by PCAIDS in the absence of efficiencies.

Table 9

Simulated Unilateral Effects – White Pan Bread

Brand	Price Increase
A-1	10.0
A-2	10.0
A-3	10.0
B-1	28.7

The share-weighted average price increase for the brands in the merger is 14.3%. Further analysis shows that even if the merger yielded efficiencies that reduced the marginal costs of each brand by 10%, the PCAIDS simulation would predict a price increase of approximately 18% for B-1. The share-weighted average

price increase for the merged firm with these efficiencies is 4.4%, which may still raise concerns. We also experimented with nests, since A-3 was a premium priced brand and perhaps was less of a substitute for the lower priced B-4. However, we did not find that plausible nest structures significantly affected the results.[51] Without the prospect of timely entry or of efficiencies greater than 10%, the transaction would certainly raise anticompetitive concerns.

Divestiture by Firm A of one or more of its three brands is one possible strategy to restructure the deal. The effect of divestiture on unilateral pricing behavior will depend both on what brand or brands are divested and what firm acquires those brands. Simulation models can help analyze the effects on prices of specific divestitures. We first simulated the merger assuming a sale of A-3 to the smallest firm, C. For this merger and divestiture, assuming no efficiencies, the predicted share-weighed average price increase for the four brands originally sold by the merging firms is only 2.8%. Even modest merger-related efficiencies would eliminate this average price increase. Alternatively, we simulated the merger with divestiture of A-3 to a hypothetical new entrant and found a share-weighted average price increase of only 1.8% before efficiencies.

The evaluation of these simulated post-divestiture price effects also raises the issue of appropriate measurement of prices. In our example, the range of price changes for the various brands is quite wide. For example, if A-3 is divested to firm C its price is predicted to *decrease* by 11.0%, while A-1 and A-2 have predicted price increases of 1.3% and B-1 has a predicted price increase of 18.6%. Divestiture reduces considerably the predicted price increases for brands the merged firm retains and results in a predicted price decrease rather than increase for A-3. An important issue facing the merger authorities in this situation is whether a transaction should be judged by its effect on average prices in the relevant market, or by its separate effects on the prices for individual brands.

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[51] We even tried an extreme case of putting A-3 in a separate nest from all of the other brands in the market and setting the odds ratio factor to 0.01 to minimize the competitive overlap with B-1.

V. ANALYZING PRODUCT REPOSITIONING AND ENTRY WITH PCAIDS

The *Horizontal Merger Guidelines* acknowledge entry and product repositioning as competitive responses to a merger with unilateral price increases.[52] The *Guidelines* distinguish between "committed" entry, which requires significant sunk costs of entry and exit, and "uncommitted" entry, which does not.[53] Uncommitted entrants are capable of increasing output sufficiently quickly (e.g., by redeploying existing assets) that they are able to constrain the market pre-transaction. For this reason, the *Guidelines* focus on committed entry as truly new competition that may be generated by unilateral price increases. For committed entry to be an effective competitive check according to the *Merger Guidelines*, it must occur within two years (timeliness), must be profitable at *pre-transaction* prices (likelihood), and "must be responsive to the localized sales opportunities that include the output reduction associated with the competitive effect of concern" (sufficiency).

Merger simulation (which could be based on PCAIDS or other demand models) provides a prediction of the unilateral price increases that would occur absent entry or repositioning. Associated with any such price increase will be a reduction in output. The central question is whether repositioning or entry can increase output sufficiently to defeat the price increase.

A complete analysis of entry and repositioning raises difficult modeling issues that go beyond the scope of this article. It would require an assessment of sunk costs and minimum viable scale (the smallest scale at which its average cost is equal to the pre-transaction price) for committed entry, as well as a financial-accounting analysis to determine whether pre-transaction prices are adequate for long-run profitability. Nonetheless, we believe that PCAIDS can provide a useful framework in which to analyze under the conditions under which committed and uncommitted responses might be expected to constrain unilateral price increases.

We use the following procedure to identify the amount of entry that should be sufficient to eliminate unilateral price increases. For any brand sold by the merged

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[52] The *Horizontal Merger Guidelines* at footnote 23 indicates that the same analysis applies to both cases.

firm, the post-merger revenue can be defined in terms of the pre-merger revenue and the unilateral percent change in price (d^*) and percent change in quantity (denoted a) for the brand. Since the shares and industry elasticity are known, and the merger simulation yields the unilateral price changes, it is possible to solve for the percentage reduction in output α. Using the expression $p^{post}q^{post} = (1+d^*)p^{pre}q^{pre}(1-a)$, it can be shown that (see Section 4.D. of the Appendix for details)

$$a = 1 - \frac{s^{post}}{s^{pre}} \frac{(1+(e+1)\mathrm{d}P/P)}{(1+d^*)} . \qquad (7)$$

The predicted output reduction therefore depends on two price effects: the unilateral brand price increase and the average price change (dP/P) for the market as a whole.

The magnitude of the reduction in output in terms of the pre-transaction revenue market share for the brand is as^{pre}. If the entrant's sales were a close substitute for the restricted output, then we could expect sales at this share level for the new brand to be sufficient to constrain the merged firm at pre-transaction prices.[54] The rationale is that the sales opportunities of the entrant would effectively restore the restricted output to the market, implying a return to the pre-transaction prices.[55] This analysis can be applied to solve for the value of α for each brand sold by the merged firm for which unilateral price increases are a concern. The total required entry would then be the sum of the shares from the individual a factors.

The merger simulation may also indicate that other firms in the market would raise price and restrict output, generating additional sales opportunities. It may be appropriate to require additional entry to constrain these price increases as well, in order to make sure that the entrant is not diverted from pursuing the opportunities from the merged firm's output restrictions.

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[53] See *Horizontal Merger Guidelines* at ¶1.0 and ¶3.0.

[54] Normally, we would expect the entrant to offer a close substitute, since entry is intended to take advantage of the sales opportunities resulting from unilateral price increases.

[55] We implicitly assume that the combined sales of the entrant and the brand produced by the merged firm equal the pre-transaction level. That is, the entrant does not merely "cannibalize" sales from the incumbent.

This analysis can, in principle, be applied to both uncommitted and committed repositioning. In the uncommitted case, sunk repositioning costs are assumed to be zero. For committed repositioning, it is necessary to carry out additional analyses to determine required sunk costs and minimum viable scale. As the *Merger Guidelines* point out, entry is unlikely if the minimum viable scale is larger than the sales opportunities available to entrants. In addition, the profits on the sales opportunities at pre-transaction prices must be sufficient to justify the sunk costs.

To illustrate some of the issues involved in an analysis of entry, we consider a hypothetical transaction involving ready-to-eat ("RTE") cereals.[56] RTE cereal products are highly differentiated along several dimensions (e.g., sweetness, texture, grains, vitamin and fiber content, color and packaging). Because this example uses aggregated data and relies on other simplifying assumptions for purposes of illustration, we do not identify individual companies or their product lines. In our example there are six firms: firms A, B, C, and D are "majors," firm E is a private label composite, and firm F is another composite firm that represents an aggregation of other, smaller brands. Firms C and D each sell two brands. We use PCAIDS to analyze a hypothetical merger between firms A and B.

We account for the fact that the characteristics of firms' brands affect consumers' substitution patterns by placing the brands of the six firms in two nests, based on whether each firm's brands appeal primarily to adults or to children. (Each nest in the example could contain multiple brands.) The pre-merger shares and nests are given in Table 10.[57]

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[56] We wish to thank Kraft Foods for providing us with the breakfast cereal data.

[57] We use the notions of Kids and Adult nests for illustrative purposes only. We believe, nevertheless, that the relevant market for antitrust purposes is all ready-to-eat cereals. See *State of New York v. Kraft General Foods, Inc.*, 926 F. Supp. 321, 356 (S.D.N.Y. 1995).

Table 10

Pre-Merger Market Shares

Firm-Brand	Share (%)	Nest
A-1	13.0	Kids
B-1	4.2	Adult
C-1	26.5	Kids
C-2	8.8	Adult
D-1	21.8	Kids
D-2	5.4	Adult
Private Label	6.0	Kids
Other	14.2	Kids
Total	100.0	

Proportionality holds within each nest. We assume a scaling factor of 50% for share diversion across nests. That is, the share diverted from a Kids brand to an Adult brand (and vice versa) is only half as large as predicted by their market shares. This structure introduces a simple, but flexible alternative to strict proportionality (with a factor of 100%).

To complete the data requirements for the simulation, we assume an industry price elasticity of –1.0 and an own-price elasticity of –1.60 for A.[58] We also assume that a merger between A and B will generate efficiencies that lower incremental costs for each firm by 2%.

Taking into account the efficiencies (but not repositioning or entry), the PCAIDS simulation predicts that the merger will result in no change in A's prices. However, the predicted price increase for B is 4.9% and its share falls to 4.1%. This post-merger price increase could raise competitive concerns, but it might also induce other firms to enter de novo or to redesign and reposition their products to compete more directly with B.

We calculate the required entry to constrain B as follows. The value for α obtained from Equation (7) is 0.065. As a result, the value of the restricted output is

[58] The own-price elasticity for the example is calculated as the ratio of gross profits to sales from aggregate financial statements for A. A more refined estimate would require information on sales and costs by product line.

0.27 percentage points of market share (0.065 multiplied by the pre-transaction share of 4.2%). If an entrant could achieve this share with a new brand that is a close substitute for B then the unilateral price increase can be prevented.

The small amount of required entry in the example is not surprising, since B is a relatively small firm. (The amount of restricted output must be less than the size of B). This highlights the potential importance of the analysis of minimum viable scale, because entry on such a limited basis may not be economic. In the RTE cereal industry, one possibility for low-cost entry might be repositioning of existing brands (or capacity) from the Kids segment to the Adult segment.

Ultimately it is a matter of judgment as to whether an entrant would be capable of achieving the requisite share to make raising prices unprofitable for the merging firm. Additional analysis would also be necessary to determine whether the entrant would achieve minimum viable scale and be profitable at pre-merger prices. Nevertheless, we are optimistic that the approaches just described can provide a feasible and useful framework to evaluate the range of issues raised when entry and repositioning are discussed.

VI. PCAIDS AND THE MERGER GUIDELINES SAFEHARBORS

In this section we briefly discuss some applications of our simulation analysis to the evaluation of safeharbor rules for unilateral effects. A safeharbor offers a boundary below which transactions are not likely to be challenged, thereby reducing transactions costs and conserving enforcement resources. The *Merger Guidelines* suggest two alternative safeharbors with respect to unilateral effects. The first applies when the combined market share of the merging firms is less than 35%; the other is available when the change in the HHI is less than 50 (with a pre-transaction HHI over 1,800) or less than 100 (with a pre-transaction HHI between 1,000 and 1,800).[59]

[59] The *Horizontal Merger Guidelines* at ¶2.211 and ¶2.22 leave open the possibility of finding significant unilateral effects when the merging firms have combined market shares of less than 35%, indicating that this criterion is not equal in importance to the HHI safeharbor. For simplicity, however, we will refer to the 35% standard as a safeharbor and investigate its properties.

If taken literally, the 35% safeharbor would shelter transactions from review for unilateral effects when the merging firms have shares as large as 17.5% each, magnitudes that might not be uncommon. To evaluate this safeharbor, we used PCAIDS (and reasonable elasticity assumptions) to investigate potential unilateral effects when the merging firms have a combined share of 35%.[60] The results indicated price increases of 6% or more for at least one of the merging firms, irrespective of firm size. The simulations suggest that a 35% safeharbor runs too great a risk of sheltering anticompetitive transactions.

Moreover, we note that the 35% standard, if enforced, would make the HHI safeharbor virtually irrelevant for analyzing unilateral effects. The only mergers not already protected by the 35% rule that would be sheltered by the change in the HHI would be of minimal interest. Indeed, in these circumstances the smaller merging firm could have at most a 1.5% share (pre-transaction HHI between 1,000 and 1,800) or a 0.7% share (pre-transaction HHI over 1,800). These constraints are inherent in the mathematics associated with the existing safeharbors (see Section 5 of the Appendix for details), and are not dependent on our merger simulation analysis.

We have separate concerns about the HHI safeharbor in cases involving unilateral effects. The HHI safeharbor by itself shelters relatively few mergers because it is only satisfied when the smaller merging firm has at most a 7% share (pre-transaction HHI between 1,000 and 1,800) or a 5% share (pre-transaction HHI over 1,800). Again, as shown in the Appendix, these limits follow directly from the definition of the safeharbor in the *Merger Guidelines*. By "protecting" only mergers involving relatively low market shares, the HHI safeharbors pose a low risk of unilateral effects. This was confirmed by PCAIDS simulations that yielded maximum price increases under 5%.[61]

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[60] The simulations used an industry elasticity of –1, a brand elasticity of –3 for the first merger partner, and a third firm with a 65% share. There were no efficiencies or nests.

[61] The HHI simulations used an industry elasticity of –1, a brand elasticity of –3 for the first merger partner, and merging parties ranging from equal 5% shares to 24% and 1% shares, and a third firm with the residual share.

At the same time, it is natural to ask whether there is a basis for an alternative safeharbor (perhaps tied to the HHI or the sum of market shares) that could expedite a greater number of merger reviews while providing similar protection against anticompetitive transactions.[62] For example, our preliminary investigation suggests that a 25% safeharbor would typically generate unilateral effects below 5%, using similar assumptions as before. Moreover, the weighted average price increase for the merged firm will be even smaller when the merger partners are different sizes. We realize, of course, that the choice of an alternative safeharbor is a complex question that will involve substantial further study. However, the benefits in the form of reduced enforcement and transaction costs could make this a worthwhile effort.

VII. CONCLUDING REMARKS

Merger simulation can be used to evaluate many transactions that raise competitive concerns. It adds to the information provided by methods that rely on econometrically estimated demand systems, surveys of consumer preferences, and the analytical strategies described in the *Horizontal Merger Guidelines*. The PCAIDS simulation approach presented in this article represents a simplification over existing techniques that we believe offers advantages in many applications. It requires only aggregate market shares, the industry price elasticity, and the own-price elasticity for a single brand in the relevant market. We have also shown that this approach can be easily extended to accommodate additional information on substitution and diversion patterns by constructing product nests. It allows a range of sophisticated analyses at relatively low cost. We have provided examples that evaluate efficiencies, nesting, brand divestiture, and entry/repositioning.

Our work is also relevant to recent criticisms of the use of market shares, especially in the form of HHIs, for merger analysis. PCAIDS shows that market

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[62] Other researchers who advocate simulation have found little support for the 35% rule and have concluded that the existing HHI criterion "makes sense only if one believes either that mergers are likely to generate no efficiencies or that only consumer welfare should be considered in merger cases." See Gregory J. Werden and Luke M. Froeb, "Simulation as an Alternative to Structural Merger Policy in Differentiated Products Industries," in Malcolm B. Coate and Andrew N. Kleit, eds., *The Economics of the Antitrust Process* 77 (1996).

shares can be highly informative when combined with well-grounded economic principles. In our view, the PCAIDS model justifies renewed reliance on market shares as a pragmatic benchmark to assess competition. We note that the *Merger Guidelines* themselves spell out the option of using market shares in an analysis of unilateral effects when market shares are reliable indicators of the closeness of substitutes and demand (which are essentially the conditions under which the proportionality assumption is appropriate).

Merger simulation is evolving and its techniques are improving. We expect that PCAIDS can help establish simulation as a standard tool to analyze potential unilateral effects. We hope that the methods introduced in this article will provide a basis to evaluate options and possibilities that might otherwise be quite difficult to subject to quantitative analysis.

APPENDIX

Merger simulation builds on a demand-supply model that specifies a set of equations that relate three types of information for the brands in the relevant market: i) own and cross-price elasticities, ii) market shares, and iii) gross profit margins. The demand model implies a "first-order condition" ("FOC") for each brand, which specifies necessary mathematical relationships among these variables under the assumption that the firms in the market are maximizing profits without engaging in overt collusion. Each FOC involves the elasticities, shares, and margins both for that brand and for all of the other brands in the relevant market owned by the same firm. In this way the FOCs take into account possible trade-offs in pricing that are the primary source of unilateral effects.

1. Notation and Assumptions

A. There are n firms in the relevant market, each producing n_i brands. There are N brands in total.

B. The jth brand has the following characteristics:

   1. Average price p_j
   2. Quantity q_j
   3. Share s_j of revenues in the relevant market
   4. Own-price elasticity e_{jj} and cross-price elasticities e_{jk}
   5. Incremental cost c_j and profit margin $m_j = (p_j - c_j)/p_j$.

C. The average industry price is P, calculated as $\ln P = \sum s_i \ln p_i$, for i = 1 to N. Also, $\Delta P/P = \sum s_i(\Delta p_i/p_i)$.

D. The n firms face an aggregate industry demand curve with a (pre-merger) price elasticity of e. An estimate of the percentage change in industry revenue due to industry-wide price changes is $\Delta(\sum p_i q_i)/\sum p_i q_i = \Delta P/P(e+1)$.

E. There is at least one known own-price elasticity e_{jj}. Each known own-price elasticity is larger in magnitude than the industry elasticity e, $\text{abs}(e_{jj}) > \text{abs}(e)$, where abs(.) is the absolute value function.

F. Define the brand-specific vectors $s = (s_1, s_2, \ldots, s_N)'$ for market shares, $p = (p_1, p_2, \ldots, p_N)'$ for prices, $c = (c_1, c_2, \ldots, c_N)'$ for incremental costs, and $m = (m_1, m_2, \ldots, m_N)'$ for margins.

G. Define the brand-specific vector $d = (d_1, d_2, \ldots, d_N)'$ of exponential rates of price changes due to the transaction. Each $d_j = \ln(p_j^{post}) - \ln(p_j^{pre})$. Define the

brand-specific vector $g = (g_1, g_2, \ldots, g_N)'$ of percentage changes in incremental costs due to the transaction. Each $g_j = c_j^{post} / c_j^{pre} - 1$.

H. Define the matrices $S = \text{diag}(s)$, $G = \text{diag}(1+g)$, and $D = \text{diag}(\exp(d))$.

I. For the brands produced by the ith firm, define the n_i by n_i matrix E_i with element (k, j) equal to e_{jk}. That is, E_i is the transposed matrix of own-price and cross-price elasticities.

J. Define the solution vector d^* of price changes measured at compound rates as $\exp(d)-1$. The FOCs are solved using the d vector and the conversion to d^* expresses the solution in more convenient units.

2. General First-Order Conditions for Merger Simulation

There is a FOC equation for each brand in the market. A general expression for all of the FOCs is given by the matrix equation:

$$s + \text{diag}(E_1, E_2, \ldots, E_n)Sm = 0.$$

The first stage of a simulation is used to calculate the brand-specific margins m. Assuming the pre-transaction shares and elasticities are known, the margins are given by:

$$m^{pre} = -S^{-1}\text{diag}(E_1, E_2, \ldots, E_n)^{-1}s. \qquad \text{(A1)}$$

The second stage analyzes the FOCs to predict price changes due to the transaction. In general, the post-transaction shares, elasticities, and margins are functions of the price changes. To simplify the notation, assume that the merger involves firms $n-1$ and n. There are $n-1$ firms in the post-transaction market, but the number of brands remains N. The merged firm requires a new cross-elasticity matrix E^*_{n-1} for the n_{n-1} plus n_n brands it is now producing. The FOCs for the second stage are:

$$s + \text{diag}(E_1, E_2, \ldots, E^*_{n-1})Sm = 0, \qquad \text{(A2)}$$

where all variables are understood to be taken at their post-transaction values.

To understand the solution of (A2), consider the relation between m^{pre} and m^{post}. For the jth brand,

$$c_j^{pre} = (1 - m_j^{pre})\, p_j^{pre}$$

It follows from the definitions that $c_j^{post} = (1-g_j)c_j^{pre}$ and that $p_j^{post} = \exp(d_j)p_j^{pre}$. As a result,

$$m_j^{post} = 1 - c_j^{post}/p_j^{post}$$

$$= 1 - (1 - m_j^{pre})\,(1-g_j)/\exp(d_j)\,.$$

This relationship can be expressed in matrix notation for all brands as

$$m^{post} = \mathbf{1} - GD^{-1}(1 - m^{pre}),$$

where **1** is an N vector of ones.

The second stage FOC can now be written as a function of the percentage price changes:

$$s + \text{diag}(E_1, E_2, \ldots, E^*_{n-1})S\,[\mathbf{1} - GD^{-1}(1 - m^{pre})] = 0, \qquad \text{(A3)}$$

where the price changes also generate post-transaction shares and elasticities through the demand model. That is, the solution to (A3) is framed entirely in terms of finding the vector δ that solves the system of equations. Observe that the pre-transaction prices and costs p^{pre} and c^{pre} are not needed in the analysis.

Simulation of divestiture of a brand from the ith firm to the jth firm is accomplished by suitable definition of the price elasticity matrices. The rows and columns corresponding to the brands to be divested are deleted from E_i. When the jth firm is an incumbent in the market, E_j is augmented by a new row and a new column containing the own-price elasticity and the cross-price elasticities with the other brands for the firm. For divestiture to an entrant, the number of firms in the post-transaction market increases by one and an additional elasticity matrix is defined that consists of a single element equal to the own-price elasticity for the divested brand.

3. Properties of AIDS

A. Share Equations

Associated with the ith firm are n_i equations that model changes in brand-specific shares. They take the form $ds_{ik} = \sum b_{ij} dp_j/p_j$, where j = 1, …, N and k = 1, …, n_i. We omit the AIDS expenditure terms in our analysis as a convenient simplification. The system can be written in matrix notation as $ds = Bd$, where B is the N by N matrix of b's. The vector of pre-transaction shares s^{pre} is assumed known. The post-transaction shares are $s^{post} = s^{pre} + Bd$.

The "adding-up" property requires the shares of all the brands in the market to always sum to one. Since this identity holds for any set of price changes, it implies for any j that $\sum b_{ij} = 0$, i = 1, …, N. Adding-up makes one of the equations redundant

because its coefficients can be completely expressed in terms of the coefficients from the other equations.

The homogeneity property requires shares to be unaffected by a uniform percentage change in all prices in the model. It implies for any i that $\sum b_{ij} = 0$, j = 1, …, *N*. Homogeneity makes one of the prices in the model redundant because its coefficients can be completely expressed in terms of the coefficients for the other prices in the same equation.

B. AIDS Own-Price Elasticities

$$e_{ii} = \frac{\partial q_i}{\partial p_i}\frac{p_i}{q_i} = \left(\frac{-q_i}{p_i} + \frac{b_{ii}}{p_i}\frac{PQ}{p_i} + \frac{s_i}{p_i}\frac{\partial PQ}{\partial p_i} \right)\frac{p_i}{q_i}$$

$$= -1 + \frac{b_{ii}}{s_i} + \frac{p_i}{PQ}\frac{\partial PQ}{\partial p_i} = -1 + \frac{b_{ii}}{s_i} + \frac{p_i}{\partial p_i}\frac{\partial P}{P}(e+1)$$

$$= -1 + \frac{b_{ii}}{s_i} + s_i(e+1) \qquad \text{(A4)}$$

C. AIDS Cross-Price Elasticities

$$e_{ij} = \frac{\partial q_i}{\partial p_j}\frac{p_j}{q_i} = \left(\frac{b_{ij}}{p_j}\frac{PQ}{p_i} + \frac{s_i}{p_i}\frac{\partial PQ}{\partial p_j} \right)\frac{p_j}{q_i}$$

$$= \frac{b_{ij}}{s_i} + s_j(e+1) \qquad \text{(A5)}$$

4. Properties of PCAIDS

A. PCAIDS Calibration of the Demand System

We now show that PCAIDS can be fully calibrated regardless of the number of brands in the market, using only information on the own-price elasticity of demand for a single brand, the industry price elasticity of demand, and the market share data. The same result holds for the extension of the method using nests.

Each element of *B* can be written as $b_{ik} = q_{ik}b_{kk}$, where the *q*'s are known but the diagonal elements b_{kk} are unknown. The relative share diversion between brand i and brand j for a price change in brand k is given by the odds ratio q_{ik} / q_{jk}. For example, under strict proportionality $q_{ik} = -s_i/(1-s_k)$ and the odds ratio equals s_i/s_j.

Impose adding-up and homogeneity. The constraints imply a system of N–1 independent equations in the N unknown own-coefficients. Without loss of generality, assume that e_{11} is known. We normalize with respect to the first brand and define a vector b with N–1 elements equal to $b_{jj}/b_{11} = b_j$, j > 1. The equation system is then non-singular and can be written in matrix form as

$$\begin{pmatrix} ?_{12} & ?_{13} & \cdots & ?_{1N} \\ 1 & ?_{23} & \cdots & ?_{2N} \\ \vdots & & & \\ ?_{N-1,2} & \cdots & 1 & ?_{N-1,N} \end{pmatrix} \begin{pmatrix} ß_2 \\ \vdots \\ ß_N \end{pmatrix} = \begin{pmatrix} -1 \\ -?_{21} \\ \vdots \\ -?_{N-1,1} \end{pmatrix} \qquad \text{(A6)}$$

(A6) can be inverted to solve for the b vector, which will be a function of the market shares. It can be shown that each b_i equals $(1-s_i)/(1-s_1)(s_i/s_1)$.

Since ε_{11} and ε are known, we can invert the formula for own-price elasticity to find $b_{11} = s_1(e_{11} + 1 - s_1(e+1))$. The PCAIDS system can therefore be calibrated completely using market shares and the two elasticities.

We now prove that each PCAIDS own-price elasticity is larger in magnitude than the industry elasticity. By assumption, abs(e_{11}) > abs(e). Assume that abs(e_{ii}) < abs(e) for some i > 1. Substituting $b_{ii}=(1-s_i)/(1-s_j)(s_i/s_j)b_{11}$ in the expression for the own price elasticity for e_{ii} yields the contradiction that abs(e_{11}) < abs(e).

Finally, we prove that all PCAIDS cross-price elasticities are greater than zero. Suppose $e_{ik} < 0$ for some i, k. By substitution, this implies $-b_{kk}/(1-s_k)+s_k(e+1)<0$. Substitute for b_{kk} in terms of e_{kk}, and rearrange yielding the implication $((e_{kk}+1)-s_k(e+1))s_k > (1-s_k)s_k(e+1)$. However, since $e_{kk} < e$, this is a contradiction.

B. PCAIDS Nests

Assume that there are w nests, $w \le N$, with each brand assigned to a nest. Given a price increase for brand k in nest f_1, the diversion of share to brand i in nest f_2 deviates from proportionality by a multiplicative factor w(k, i) > 0. We assume that w(k, i) = w(i, k). Similarly, the diversion from brand k to brand j in nest f_3 deviates

from proportionality by $w(k, j)$. Proportionality is the special case where $w(k, i) = 1$. It can be shown in this general setting that:

$$?_{ik} = -s_i \frac{?(k,i)}{\sum_{m \neq k} s_m ?(k,m)},$$

The odds ratio under nesting is $q_{ik}/q_{jk} = (s_i/s_j)[w(k,i)/w(k,j)]$. In the case of proportionality for all nests this reduces to the familiar s_i/s_k.

C. Slutsky Symmetry of *B* with PCAIDS

We now show that the matrix B of PCAIDS coefficients is symmetric both under strict proportionality and with nests as we have defined them. The discussion in 4.A implies that, under adding up and homogeneity, $b_j = q_{j1} / q_{1j}$. It follows that

$$ß_j = \frac{s_j}{s_1} \frac{\sum_{m \neq j} s_m ?(j,m)}{\sum_{m \neq 1} s_m ?(1,m)}$$

and from before, $b_{jj} = b_j b_{11}$.

By the definition of B and substitution for b_i and b_j,

$$b_{ij} = -\frac{s_i s_j}{s_1} \frac{?(i,j)}{\sum_{k=2}^{N} s_k ?(k,1)} b_{11}.$$

for i≠j. Symmetry of B follows directly.

D. Required Market Share for Entry to Defeat Unilateral Effects

Let a represent the unilateral output reduction. For any brand produced by the merged firm, post-transaction revenue $p^{post}q^{post}$ is related to pre-transaction revenue $p^{pre}q^{pre}$ as follows:

$$p^{post}q^{post} = (1+d^*)p^{pre}q^{pre}(1-a),$$

where d^* is the unilateral percentage price increase. Total post-transaction market revenue equals pre-transaction market revenue PQ multiplied by $1+(e+1)dP/P$, where P is the average market price change ((see 1D). Dividing both sides of the equation by post-transaction market revenue yields

$$\frac{p^{post}q^{post}}{PQ(1+(e+1)dP/P)} = (1+d^*)\frac{p^{pre}q^{pre}}{PQ(1+(e+1)dP/P)}(1-a) \ .$$

Re-write in terms of shares as

$$s^{post} = (1 + d^*) \frac{s^{pre}}{(1 + (e+1)\mathrm{d}P/P)} (1 - a)$$

and solve for α as

$$\mathrm{a} = 1 - \frac{\mathrm{s}^{post}}{\mathrm{s}^{pre}} \frac{(1 + (\mathrm{e}+1)\mathrm{dP/P})}{(1 + \mathrm{d}^*)} \ .$$

5. Proof of Maximum Firm Sizes under *Merger Guidelines* Safeharbors

When the 35% safeharbor rule does not apply, then the HHI safeharbor has independent relevance only for transactions where one of the firms is very small. By the algebra of the HHI (see *Merger Guidelines* at footnote 18), the safeharbor for merging firms 1 and 2 can be expressed as:

$$2s_1s_2 < d,$$

where d, the maximum safeharbor change in the HHI, is either 100 (pre-HHI less than 1800) or 50 (pre-HHI greater than 1800). It follows that $s_2 < d/(2s_1)$.

By assumption, $s_1 + s_2 > 35\%$, so that $s_2 > 35\% - s_1$. Putting these two conditions together implies

$$35\% - s_1 < d/(2s_1),$$

or, equivalently,

$$s_1^2 - 35s_1 + d/2 > 0.$$

Apply the quadratic formula, assuming the expression is equal to zero, and solve for the two possible values for s_1. The inequality is then satisfied when s_1 is either smaller than the lower value (and $s_2 > 35\% - s_1$) or greater than the higher value (and $s_2 < d/(2s_1)$. By substituting for d, it can be seen that the HHI safeharbor limits the smaller merging firm to at most a 1.5% share (pre-transaction HHI between 1,000 and 1,800) and a 0.7% share (pre-transaction HHI over 1,800)

It also follows that when the maximum safeharbor change in the HHI is 50, then the smaller firm can be no larger than 5% (and must be below this level when the share of the larger firm is above 5%). When the maximum safeharbor change is 100, then the smaller firm can be no larger than 7.1% and must be below 5% when the share of the larger firm is above 10%.