Introduction

This article examines how smaller networks are formed and how they lay the basis for more complex systems. The intent is not to report new data nor to develop wholly new propositions. Rather, building on previous work, the observed characteristics of networks in small groups will be derived from a small set of simple assumptions. The synthesis is new; the propositions are not, though in the process of derivation I will put together ideas about small groups that may not heretofore have been seen as related one to another. It is important to understand the theoretical bases for network formation in small groups because they are the "primitives" of social network analysis. Typically, an analysis of networks in organizations focuses on the difference between the networks produced by formal systems of the organization and the networks created informally by office friendships and politics. Such an informal network is often called a "primary group," a term coined by Cooley (1909).

Primary Groups and Informal Systems: Propositions

A primary group is an interactive unit that is observed as such by an outsider, whether or not people identify themselves in these terms. Cooley (1909) said, "By primary groups I mean those characterized by intimate face-to-face association and cooperation." In Cooley's usage, primary groups can refer to intimate family groups and to informal associations. Let us focus here on the latter.

It is useful to consider what is likely to happen in "pure" groups in which there is a minimum of formal positions. In family or kinship small groups, roles are in part defined by the formally prescribed kinship terms and culturally expected relationships. In formal organizations, there are also primary or face-to-face groups but most of the relationships are prescribed by the organization. This is the case with all the primary groups of this sort analyzed by George Homans in his path-breaking book, *The Human Group* (1950), except for one. The "Norton Street Gang," reported in Whyte (1943), more closely resembles other primary groups existing independently of formal organizations. Homans observed that some interactions in human groups were prescribed by the formal system or what he called the "external system," but that others were based on "sentiments" that people had for one another. This he called the "internal system." Interaction and participation in common activities, whether or not dictated by the external system, generally led to positive sentiments; these sentiments in turn lead to further interaction. Moreover, the activities and interactions called for by the external system were always elaborated upon. The ties generated by this elaboration produced cliques[1]

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[1] Clique is a well worn sociological term that means some cluster in which the nodes are more connected with one another than they are with other nodes in the graph. Identifying cliques in this sense is hard to achieve with a network algorithm that works under all circumstances (Kleinberg 2002). The restrictive technical definition for clique in network mathematics is a maximal complete subgraph, which fails to satisfy the more general concept, which the network field has

that were characterized by common sentiments. These cliques and the social solidarity that developed were Homans' "internal system," (what I refer to as the informal system) and this was embedded within the external system. In Homans' terms, persons who became informal leaders were more often chosen by others and interacted and initiated interaction more often with others. Informal groups, through interaction, develop or reinforce certain common ideas or styles of relating to one another. These become the "norms" of the group or clique reinforced by mutual interaction. Informal leaders adhere more to these norms than do non-leaders, in part because, through leadership, they enforce the norms, and in part because groups choose leaders who exemplify the norms of the group. These are the core ideas of informal systems that are pegged to formal ones but that influence the way formal systems operate. These ideas are all found in Homans (1950) who, along with Bavelas (1948), was one of the originators of social network analysis for small groups and organizations.

The following seem, therefore, to characterize all informal small group systems: Over time, all seem to develop clusters or cliques. Inevitably, and related to the clusters, a ranking system develops in which some persons are preferred over others. Related to the ranking system, and perhaps both a cause and consequence of it, people in informal systems tend to develop feelings or sentiments about one another. Those who rank higher in the system are leaders. People tend to defer to them; members choose them more often (the very definition of rank in this system). Paradoxically, leaders tend more often to initiate interaction with members either directly or through others. These characteristics will be found within work groups in any formal organization. Homans insisted that they are true of all human groups, including families.[2]

These statements can be formalized as propositions. They all refer to small primary or face-to-face groups, where "small" for the moment remains undefined. All are understood to include a statement "other conditions being equal." What these "other conditions" are remains at the moment unclear.

1. Interaction-and activities that require interaction lead to sentiments or attitudes that group members have for one another.
2. The sentiments can be positive or negative. Positive sentiments lead to further interaction and negative sentiments lead to less interaction.
3. Thus 1 and 2 are in a feedback loop the limits of which are unclear but generally determined by the requirements of the formal system (one needs to get work done), or by the inherent limits on individual action in the internal system (there is some limit on the number of interactions or activities any person can sustain within a given period of time).
4. Since (a) the formal system requires certain differentiated activities and (b) these activities lead to interaction and interaction leads to sentiments and (c) all of this is in a feedback loop, it follows that in any small group there will develop clusters or cliques of individuals who have more dense patterns of interaction and more dense sets of feelings about one another. These clusters will generally follow the

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long been working hard to realize (see, for example, Moody and White 2003, White and Harary 2001).

[2] Some research finds similar characteristics in non-human groups (Faust and Skvoretz 2002, and the references they cite).

interactions dictated by the formal system. In this sense, an informal network is "draped upon" the formal (*external*) system.

5. Similarly, and the source of this is not clear at the moment, individuals will be evaluated differentially by the *internal* system and these differences in evaluation will lead to differences in interaction and, through the feedback process, to clusters or cliques of individuals that have more dense patterns of interaction and feelings for one another. These clusters are dictated by processes in the internal system.
6. The two kinds of clusters or cliques – those created by the external system and those by the internal system – will be related to one another. The form and nature of this overlap is not clear at the moment.
7. "Leaders" are those who are chosen more often by others as a result of the activity, interaction and sentiment feedback loops. Since the differential evaluation of persons in the system is based on something, leaders tend to have more of that "something" than others. This something is said to be a better match with the tacit standards or "norms" of the group than others in the group possess.
8. Because they are at the fulcrum of choice by members of the group (or by members of subgroups or cliques), leaders tend to initiate interaction more than others, reinforcing their leadership. In some respects, initiating interaction is the essence of leadership.

I would like to unpack this set of ideas. They are based on observation and Homans did not reduce them to axioms as I have done above.[3] The propositions may be shown, however, to be the result of the playing out of a smaller set of ideas. It is also obvious that at least analytically, the two systems, the internal and the external, need to be differentiated. It will be useful therefore to try to find informal (internal) systems that are "pure" or relatively so. It may then be possible systematically to incorporate the influence of external systems on small groups without adding too many additional ideas.

Pure Informal Systems

A pure informal system is of course an abstraction, but the parameters that define its variants can be specified. On the one hand, among the informal systems, the simplest cases to specify through observations are the "networks in a box" [4] – that is, networks such as those within the four walls of a classroom or, more generally, in which the boundaries are very clear. These networks are characterized by total visibility; everyone within the network can "see" everyone else or be aware that they exist. Informal political influence networks, on the other hand, do not meet these criteria, though the interactions and sentiments that emerge are not prescribed by the formal political system. Visibility in political and organizational influence systems is limited.[5] Within organizations, there are informal systems that cross "chimneys," where a chimney is defined as a chain of command of a division, say a parts department, an engineering development department,

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[3] See the appendix for some early efforts to model some aspects of Homan's system of interaction, though not his propositions on leadership.

[4] A term suggested by H. Russell Bernard.

[5] Some might argue that the very definition of informal political processes within organizations and in society generally is that they have limited visibility. Lifting the veil of invisibility is in part what both "muckraking" and organizational analysis have in common.

and a production department. The informal system enables communication across hierarchical systems without the communication having to pass up to the top of a department, across to the other department and then down again. Within work groups, there may be visibility but much communication is formally prescribed in reaction to either the formal or external system. I follow Freeman's insight that a pure informal system "permits observation of interaction that is voluntary and informal and ranges over a broad spectrum of activities, [and] there is no reason to suspect that the individuals are interacting in terms of external constraints" (Freeman 1992:163). Systems in which interaction is constrained by virtue of the needs of production, as for example in the Bank Wiring Room discussed by Homans (1950),[6] or systems in an office where there is a "boss" and other prescribed roles are hybrid systems in which an informal system is embedded within and partly constrained by the formally named statuses or, in Homan's terms, by the external system. These are indeed the most frequently found and the ones therefore most important to study, but a theory of what interaction patterns look like informally needs to be developed under the relatively rare situation in which there are no or few formal system constraints. Once such a theory is available it is possible to understand the relations between the formal and informal systems that constitute the heart of network analyses of organizations.

There is a further and an important consideration in delineating pure informal internal systems from those that are influenced by the external system. A pure informal system is most easily observed and described when the activity is merely interacting or hanging around. If we observe interaction, we can say who is interacting with whom without specifying who initiated the interaction. We simply note that two members of a group were seen together. In formal terms, the relationship is symmetric: if I interact with you, then you also interact with me. If we are friends, then I am your friend and you are mine. In terms of process, one might examine how the interaction came to be. Often, one individual might have been the first to initiate the interaction, but that is not the present concern in defining pure informal systems. In contrast, external systems tend to impose qualities and criteria on the interaction, and then it is easy to explain how the interaction came about.[7] It might be possible in pure informal systems to observe who initiated the interaction, or for that matter, ask people whom they choose as partners or to be with (as is the original Moreno study of girls' cottages[8]). These choices may or may not be

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6 The Bank Wiring Room was an experimental room of 14 workmen who assembled sets of central office telephone switches or "banks." It was intensively observed for a period of six and one half months from November 1931 to May 1932 by a research team at the Hawthorne plant of Western Electric and reported in (1939). The report contained detailed sociograms as well as detailed qualitative observations that Homans re-analyzed in his Human Group (1950), Chapter 3, "The Bank Wiring Observation Room."

7 In the Bank Wiring Room, for example, wiremen attached wires and then handed over the bank to a solderman who applied the final touch of fusing the connections. So the wiremen initiated the interaction. It was not symmetrical.

8 In the early 30's, Moreno (1953) artificially constructed primary groups of adolescent delinquent girls who were incarcerated in an institution that housed them in separate cottages. Using the technique of sociometry (a word he invented) he placed girls who liked one another into the same cottage rather than having cottages populated by warring cliques. When grouped into cottages with greater group cohesion, the girls were less likely to be cantankerous.

reciprocated. By definition, the more popular persons will not choose the less popular, but the less popular are more likely to choose the more popular than each other. These choices by the less popular are equivalent to social climbing. But these asymmetric situations are more likely when the external system intrudes on the internal system, as in more real-life situations.

How to Find Informal Systems?

A pure informal system is one in which relationships are likely to be symmetrical and with a minimum of named statuses and roles derived from the larger culture. Though sociograms as a depiction of social networks are most easily constructed on the basis of dichotomous ties – either a pair is related or not – in real life people are closer to some people than to others. Even when relationships are symmetric and depicted as dichotomous, it is possible to discern a hierarchy with those that interact most often with one another dominating those who are less frequent partners.

Freeman (1992) evaluates two empirical tests of the sociological concept of group. One, proposed by Winship (1977), formalizes a definition of a group that is based only on observed closeness of ties, combiing Rapoport's (1954) hypothesis that "the likely contacts of two individuals who are closely acquainted tend to be more overlapping than those of two arbitrarily selected individuals" with Heider's triadic balance theory (Freeman 1992:155). In a triad, "a balanced state exists if all three … relations are positive in all respects, or if two are negative and one is positive" (Heider 1946). For positive relations, this situation is equivalent to that of transitivity for directed ties: that is, if A affiliates with B, and B affiliates with C, then C affiliates with A.[9] Winship's definition of a group requires that within the group only transitive ties are present, a condition that also describes a strict hierarchy in a group (Freeman 1992 154). For symmetric ties, groups defined in this way also have the mathematically neat, if not always empirically useful, characteristic of being non-overlapping. There is only one problem: whether ties are symmetric or non symmetric such groups are almost never found. Most observed networks are messy, have overlapping circles, and have many intransitive triples.

To decide, in a situation in which a symmetric relationship is observed, whether or not a relatively pure informal system is present, I propose to use the second of Freeman's (1992) empirical tests of the sociological concept of group. Here, he uses a different adaptation of Rapoport's idea – Granovetter's (1973) famous concept of weak and strong ties. Weak ties are those between people who are not very well connected. But what do we mean by "not well connected"? The literature suggests almost as many meanings as you could possibly think of. But Freeman proposes a formal definition built from the very interaction matrix itself. First, consider a matrix of relationships such that the relationship is a matter of degree – people meet more often, have known one another for a longer time, or any such reasonable candidate for strength of tie. Freeman suggests that we define the highest level of attachment λ, in such a network, that will divide between weak and strong ties so that the latter produce no intransitive triads, that is, where a path of strong ties would connect a to b to c but a and c have no tie whatever.

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[9] Insights about equivalences such as these were formalized into rules for constructing triads in a sociogram by Davis (1967).

Granovetter's transitivity would allow a triple to be termed transitive if it has two people who are strongly connected but a third one who is weakly connected to the other two. Freeman called these triples "G transitive," and triples that do not meet at least this condition, "G intransitive." But how does one know the right level below which relations are weak and above which they are strong? This ambiguity has continued to plague the otherwise neat concept of strong and weak ties. Freeman developed an algorithm in which, by starting with the most closely linked nodes in a network and moving down to the less strongly connected, he keeps dropping the level of λ that defines strong ties in the network until a significant number of G intransitive triples are encountered. The lowest level of intensity of tie that retains G transitivity with very few exceptions defines λ. Everything above that level is termed "strong," and everything below it is "weak." Freeman applied this algorithm to seven data sets that had observed (not self-reported) counts of interactions. Strength or degree of tie was defined simply as the number of interactions, or for persons interacting at events, the number of common events attended. Strictly transitive relationships were very rare in all of them but the number of G transitive triples in four of them was far greater than would have been observed by chance alone. These were all situations in which "no evident constraints on interaction" (Freeman 1992:164) were imposed by the external or formal system. This suggests a roundabout way of ascertaining whether or not a situation is one involving–a relatively pure informal system: a pure informal system is one in which there are more G intransitive triads than one would expect by chance. The inner core of a system, the "clique" of insiders, is often comprised of people who are connected by strong ties; they are surrounded by hangers-on who are linked only by weak ties. Freeman provides a useful illustration with a famous dataset collected in the 1930s' – "Old South" (1941) – and reintroduced to students of small groups by Homans (1950). Based on observation by an ethnographer, "it provides records of women's coattendance at a series of small, informal social events," as shown in Figure 1. (For a comparative assessment of differing analyses of this dataset see Freeman 2003.)

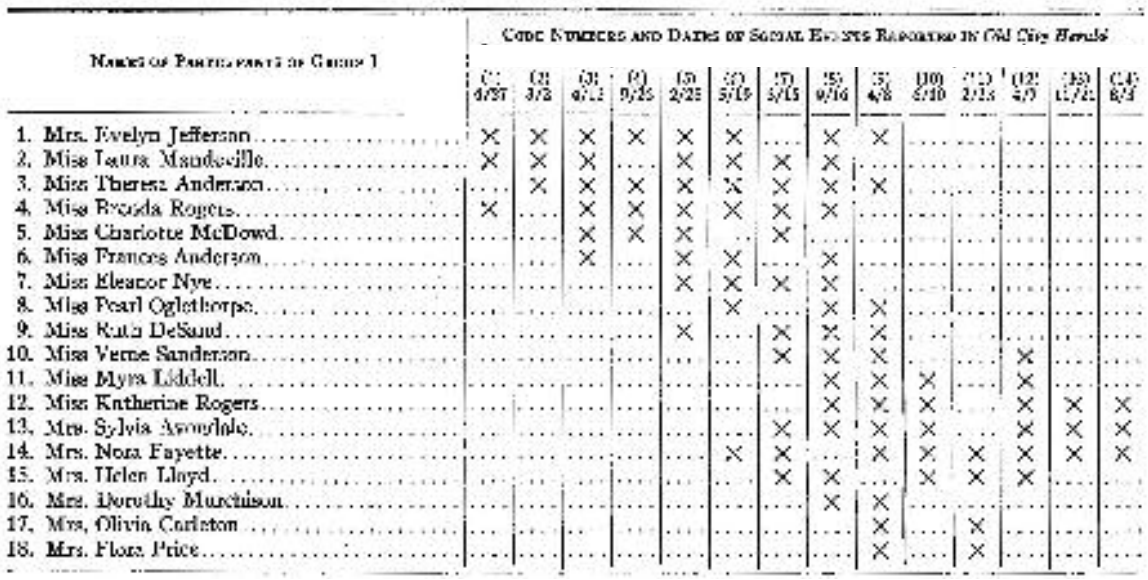

Fig. 1. Interparticipation of a Group of Women in Old City, 1936
18 Southern Women at 14 Events

The strength of the ties between the women is defined as the number of these social events they co-attended. While there are many G intransitive triples among those who co-attended 3 events, there are very few (Freeman mistakenly says none) among those who attended 4 events, and these differences are statistically significant. Hence, Freeman sets λ at level 4: Those who attended four or more events were defined as having strong ties, and the other ties were therefore said to be weak. Figure 2 shows the strong ties between the women according to λ = 4. [10] It identifies cliques that are very similar to those in the ethnographic report.[11]

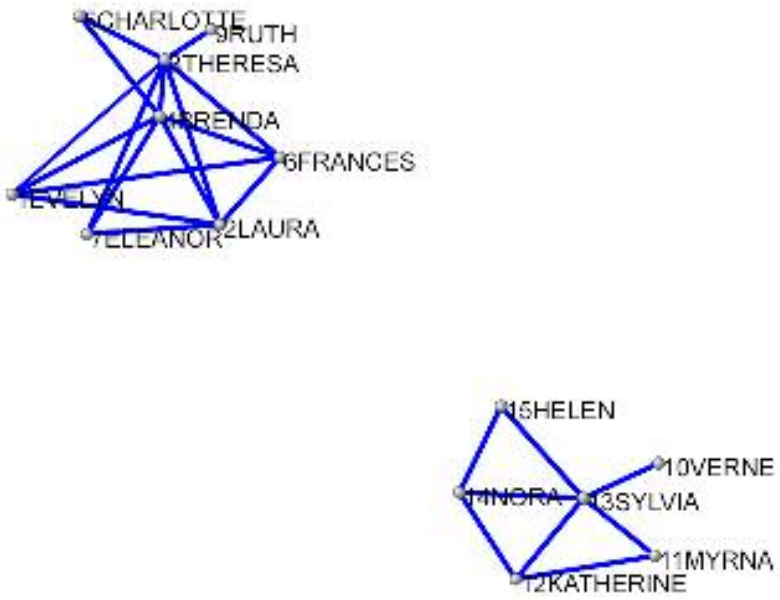

Fig. 2. Strong ties in the Old South data set

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[10] Editor Douglas White pointed out that Freeman's corresponding Fig. 3 needed correction: #16 does not attend 4 events with #11, #12, or #13 (she goes to only two events herself), and similarly for #12 with #15, who coattend 3 events. The present figures have been re-graphed by White.

[11] The classic graph-theoretic definition of a clique is too restrictive because it implies complete connectivity and is not the one used here. Rather, the intuitive sociological definition tends to correspond to the idea that members of a clique interact more with one another than they interact with others who are not in the clique, but while this seems simple enough, this idea is difficult to implement (Kleinberg 2002). Two different methods that produce different results have generally been employed. One is clustering based on internal cohesion ("safety"), and the other (as discussed in Kadushin 2002) groups together nodes whose patterns of linkage to other nodes, rather than to one another, is similar ("effectance"). Some statistical methods (e.g., Frank 1995) look at both patterns simultaneously but the assumption that cohesive groups must be defined as mutually exclusive rather than potentially overlapping or embedded is a common defect of measuring cohesion (White and Harary 2001) that is corrected by measures based on the graph-theoretic definition of multiconnectivity.

Figure 3 shows the weak ties between the women according to $\lambda = 4$, and includes another four women and the majority of the ties in the network. The weak ties shown as lighter lines are those between women who are connected because they mutually attended three, two, or one event(s). It is evident that there are many weak ties in the sense defined by Freeman and some serve as extensive bridges between the two cliques. Moreover, if one counts these weak ties, one can see that members of a clique may interact as much or more with outsiders, in number of ties through shared events, "than they do with some of their fellow group members" (Freeman 1992: 164). This is important to observe because we have asserted that pure informal systems can link people or groups in different organizational chimneys or even in different organizations. The bridge or link is likely to be some form of weak tie.

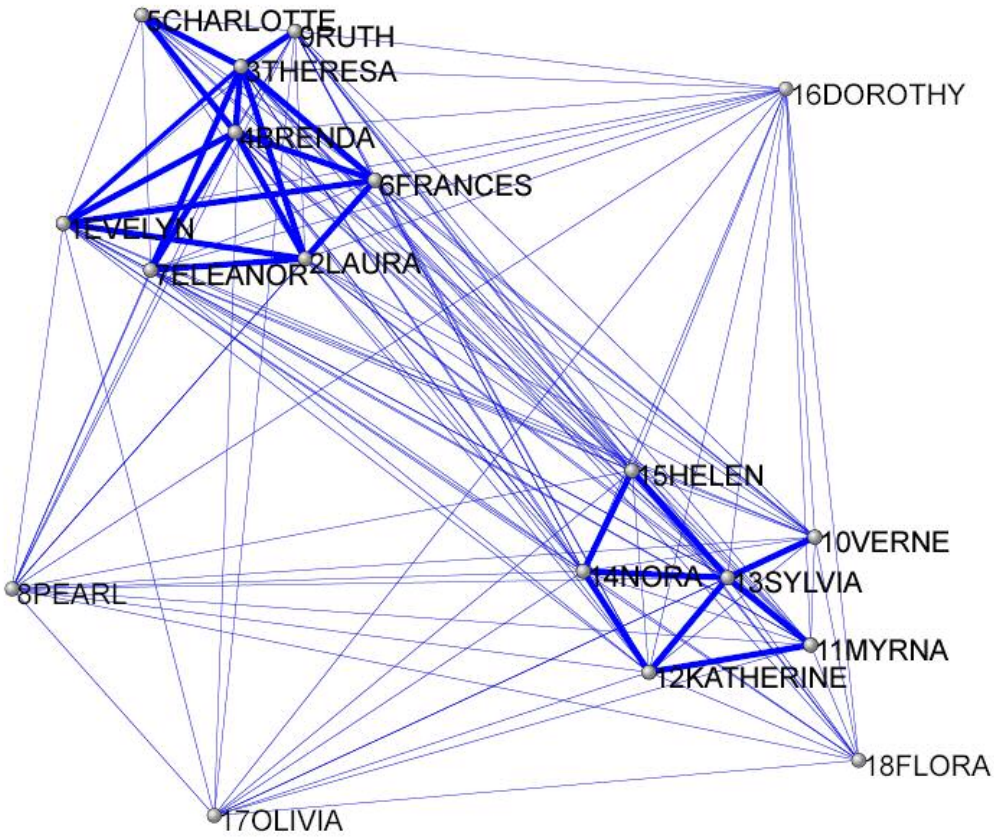

Fig. 3. Strong and weak ties in the Old South data set

It is still not clear why all of this happens: why do some persons in an informal system hang out together to such a degree that their ties with one another are in balance in terms of G-transitivity, while others are less systematically tied and remain on the periphery? Tying these phenomena to basic human styles of interaction formalized by Gould (2002) will be easier if we consider asymmetric ties.

Asymmetric Ties and the Influence of the External System

We now introduce relations that may not be symmetric. Some people are more popular than others. We noted that groups in which ties are not symmetric are closer to real life

groups, and that therefore there is generally some embedding of the informal system within the formal one. It was observed in the previous modeling of pure informal systems that, despite the symmetric nature of the ties as observed, a ranking system could nonetheless be discerned in which at the very least there were strong and weak ties. By considering asymmetric ties we can advance to an understanding of what might determine the interaction patterns in the first place. How is a ranking system developed even if interaction is not determined by the demands of an external system? And further, what happens when external system values are introduced into an informal system? To do this, we need to invoke some basic assumptions about motivations in networks.

Sociological theory tends to ignore motivations since they belong to a different "system reference" (Parsons and Shils 1951) or to introduce them in the most minimal fashion – often referring to outmoded psychological theories. But, when we work with the basic level of human interaction before even considering an external system, it is clear that some persons are more motivated to interact with others, that they do so in different ways and that these motivations are components of Homans' "sentiments." Here I draw on some previous work (Kadushin 2002). First there is the basic motivation of "safety" or affiliation. That is, people are motivated to interact with one another because they can meet each other's needs in this way. This goes all the way back to an infant's interaction with its mother as the source of its needs and is therefore a very primitive motivation. Safety or affiliation, a motive that leads to cohesive networks, is not all, however. Nothing would happen in life if that were there case. One can also observe in very early childhood the tendency to reach out, to take some risks and to move out of the zone of comfort. This tendency, present in all human beings, can be called the "effectance" (Kadushin 2002) motivation. This motivation has its counterpart in network structure as well and leads to the exploitation of "structural holes" (Burt 1992). Gould implicitly works with these motivations or basic human characteristics and, as will be seen, adds a third: the extent to which a person pays attention to others and is influenced by them. There may be a theory of how susceptibility to influence may be related to more primitive motivations and early child development patterns, but that is beyond the scope of the present paper. Interestingly, given the system reference of interaction in groups, these three motivations or human characteristics are treated in the theory as being present at random; they are outside the theory. A full practical theory that explains what happens in a concrete situation might have to drop the assumption of randomness since one might observe that certain kinds of external systems recruit persons with different mixes of these motivations. For example, some systems might prefer persons with stronger affiliative tendencies while others might tend to recruit persons with stronger motives for effectiveness. But for the present theory, the incidence of these motivations is random.

In Gould's theory, it is assumed that in any group there is some distribution of judgments about the attractiveness or quality of the members of the group. The bases for these judgments lie in the external system and in cultural values that are brought to the group and hence lie outside the theory. Gould assumes, therefore, that there is no inherently correct set of values or evaluations. Rather, "the uncertainties and subjectivity inherent in quality judgments gives rise to a self-reinforcing process in which collective adherence to socially provided assessments reproduces and thereby validates those very assessments" (Gould 2002). In other words, judgments in a group are a "self-fulfilling prophecy" in which the "Matthew effect" (Merton 1968) holds – "to he that hath shall be

given" – meaning that judgments of those who have greater rank in a group are given more weight than those with lesser rank and so the hierarchy is preserved. This matches the assumptions of a Nash equilibrium (made famous for non-game theory experts by the book and movie "A Beautiful Mind") that states, "If there is a set of strategies with the property that no player can benefit by changing her strategy while the other players keep their strategies unchanged, then that set of strategies and the corresponding payoffs constitute the Nash Equilibrium" (McCain 2002). In social terms, "[E]veryone's current choice of action is preferable to (or as good as) the alternatives as long as everyone else's choice of action remains constant" (Gould 2002 1148). The current distribution of social goods is the "right" one simply because it is socially confirmed. Note that this result is similar to Homans' feedback loops in which interaction leads to sentiment and in turn to interaction and in turn to a ranking system with leaders. The source of the evaluation of members is outside of this feedback loop and is located in either cultural norms or the requirements of the external system. Gould offers a model of this kind of system and also locates it within the context of large system Matthew effects. On the face of it, this is a rather uncomfortable assumption in that the source of the evaluation is outside the small group system. This is a conservative assumption in that it assumes, I believe correctly, that basic values upon which evaluations rest never emerge directly from a small group but rather are culturally or organizationally determined. There may be group norms, as for example, group "standard operating procedures" but these specify the way that values are implemented, not the values themselves.

Let us begin to build the model with safety because we first have to have people who want to interact. Recall that in all groups there is some evaluation or feelings by members about the other members – Homans' "sentiments." These evaluations lead to a differential ranking of each member. As noted, it is not necessary for the model that we know the basis of these sentiments, just that there is some distribution of them. We also add Homans' now familiar observation that sentiment leads to interaction and vice versa. Now introduce two corollaries of a basic motivation for interaction, safety and effectance.[12] First, in terms of sentiments towards the other, Gould observes that people have positive sentiments preferentially towards those who return those sentiments. This is a version of safety. (As he points out, this can also be stated in negative terms: people prefer to have negative views of those who have negative views of themselves; asymmetry can be painful."[I]ndividuals like to be tied to attractive others, but they don't like to be tied to people who do not reciprocate the attachment" (Gould 2002) 1152). In short, as Gould wryly observes, people like to have their phone calls returned. A second corollary takes account of the ranking in a group: it is sweeter to receive positive sentiments from those for whom you have positive sentiment yourself than from others you value less. "Consider how much more pleasant it is to be sought after by people you like than by people you do not." (Gould 2002:1153)

Now let us introduce striving, social climbing, or what I have called effectance. People enjoy the fruits of rank. "[I]ndividuals like to receive attachments that they do not repay, inasmuch as such attachments signify (and contribute to) their status." This is a result of the desire to direct positive sentiments to others of higher rank, mitigated of

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[12] Safety and effectance are my terms (Kadushin 2002), not Gould's.

course, by the pain of not having these sentiments returned. This is the risk factor in social climbing. One wants to do better, even though this may lead to less positive sentiments from some. But effectance is limited by safety. There is therefore a trade-off between people attaching themselves to the most desirable others and the most available ones. If asymmetry were totally painful, then people would "remain unattached, sort themselves into cliques in which everyone was tied equally to everyone else, or forge collections of symmetric dyads." (1150). On the other hand, if no one cared about the asymmetry, then everyone would attach him or herself to the most desirable person. Real life lies somewhere between these extremes, so although total fragmentation and winner take all do fit the requirements of a Nash equilibrium, this situation is rarely if ever encountered. In the more likely case, individuals who don't receive positive sentiments from the most desirable others tend to direct their sentiments to others who are less attractive than the "best" but who are at least likely to return the favor. This produces a ranking system like the ones described by Freeman above for symmetric ties and is likely to form a Nash equilibrium because no one has an incentive to change his or her pattern of sentiments given everyone else's pattern of sentiments. This definitely does not mean that the equilibria maximize social welfare but merely that they result in a state of affairs that everyone is willing to live with.

Note, too, that although the model finds the basis for evaluations to be outside the interaction system, the way effectance and safety play out is a function of the mix of persons in the interaction system, and this mix is also not part of the model. Since the motives of effectance and safety are basic personality traits developed at an early age, people will differ in the strength of these motivations. Thus the shape of the system is not entirely dependent either on the external system or the dynamics of interaction; in part, the basic character of those in the system also has an effect. Gould further takes these character traits into account by postulating that people differ in the extent to which they allow the opinions of others to influence them. However, this additional factor leads to an important consequence.

The equilibria operate in a closed system in which those producing the attributions of attractiveness are also those who are on the receiving end of these attributions.[13] There is, however, a further important aspect of these closed systems that Gould takes into account and that has already been noted. The perceived quality of another is not only a function of some exogenous, objective value of that other, but is also influenced by the evaluations that the individuals in the closed system have made of the other. This is Merton's "Matthew Effect." The opinions of some individuals in some social systems are more susceptible to the influence of others, and some are less susceptible. For those less susceptible to influence, the exogenous factors will be more important; for those more susceptible, the opinions of others in the group will be more important. Whichever the case, both external and internal system factors have to be taken into account. Thus, the influence of other persons on one's own evaluation must be taken as an axiom in addition to the safety and effectance motives.

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[13] This is obviously true in "networks in a box" but is also true of some open systems. Examples include the production of scientific knowledge in which people cite each others works; in the production system of scholarly books in which the readers, the writers and the editors are all part of the same social circle (Coser, Kadushin and Powell 1982); in systems of power elites (Higley et al. 1991); and in inter-organizational relations.

Formalizing the System

Gould offers a mathematical formulation that includes these axioms and ideas. The advantage of formalization is that one can draw a number of less than obvious conclusions from working out the algebra. The model predicts in general terms that the subjective judgment of the quality of another by any group member is influenced by three factors: (1) the intrinsic quality of the other – some of which judgments may come from the external system; (2) the interaction patterns of the internal system that develop from the safety and effectance motives; and (3) the weight of social influence in the internal system that affects this subjective evaluation of the quality of the other. Since this is summed over all the members of the group, the number of persons in the group also enters as a factor. Readers whose eyes glaze over when they encounter an equation can skip this and go directly to the next two paragraphs. For others, we may note that, for any individual, the model is stated in Gould's equation 5 and the summation in his equation 6 (Gould 2002:1157). [14]

$$a_{ij}^{*} = \frac{(1-\omega)[(2s+\omega)Q_j + sQ_i + \frac{s\omega(3s+2\omega)\sum_k Q_k}{(s+\omega)[3s-(3n-5)\omega]}]}{3s^2-(2n-7)s\omega-(n-2)\omega^2}. \qquad (5)$$

$$\sum_{i\neq j} a_{ij} = \frac{(1-\omega)[(2n-3)s+(n-1)\omega]}{3s^2-(2n-7)s\omega-(n-2)\omega^2}Q_j + \frac{s(1-\omega)(3s+2\omega)}{[3s-(3n-5)\omega](3s^2-(2n-7)s\omega-(n-2)\omega^2}\sum_i Q_i. \qquad (6)$$

Here a_{ij} is the received choice of an actor i by j; ω is the weight of social influence on i's judgment of j's quality Q_j; s stands for symmetry and is the extent to which people are hurt by non-reciprocity; and n is the size of the system or group. It is important that n, group size, is in the denominator because this means that the predicted effects decline in strength when group size is increased.

As Gould himself observes, equation 5 for the choices received for any individual actor is difficult to interpret, so he offers us a graphical representation assuming a modest group size (n) and a moderate degree of symmetry (s). In the figure below, Q, quality is centered on 0, as are the attachments received by an actor (otherwise known as social rank). Social influence, measured by ω, is allowed to vary from 0 to 1. A small group of size 10 is shown in the graph, and the extent to which reciprocity in evaluations is valued is also moderate, meaning that a person is willing to tolerate some imbalance or some negative consequences for choosing persons of higher rank than him or herself. Since size of group is a factor, working this equation through over time, will also show that low ranked actors can give high deference to higher ranked actors without losing much rank themselves in another round. Basically, this occurs because the larger numbers involved dilute the overall reduction in their individual rank that would occur because of their

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[14] I thank the University of Chicago Press for permission to reproduce these equations and figure 4. The equation numbers are those in the original text.

giving deference to others (recall that group size is in the denominator). This is less true of smaller systems, which means that smaller groups find a more egalitarian system easier to sustain than do larger systems, something that agrees with common intuition about groups.

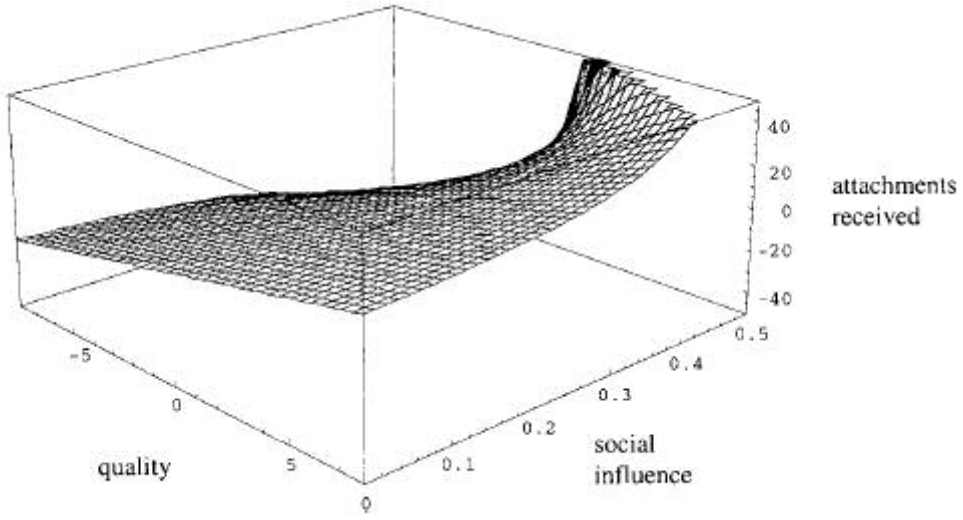

Fig. 4. Attachments received (choice status) as a function of actor quality and level of social influence (n = 10, s = 2.5; reproduced with permission from Gould 2002:1158)

Notice that inequality in rank increases sharply as social influence increases, showing that social influence or the Matthew Effect exacerbates whatever quality differences there are in the system. If there are no quality differences, then social influence has no effects except for the cascade of choices for a single actor. Gould quips, "Ironically, … the only status hierarchy that is stable when quality differences do not exist is the most extreme version of hierarchy – that of hereditary monarchs, for instance, or film celebrities" (Gould 2002 1158).

A number of interesting formal propositions follow from equations 5 and 6. Empirical tests of some of the propositions on real group data are congruent with Gould's mathematical predictions; proofs of the equations are found in Gould's appendix.). These propositions elaborate Homans' propositions. Only four basic assumptions or axioms are required as well as two corollaries: the safety motive and its two corollaries; the effectance motive as constrained by safety; the susceptibility to influence; and, finally, the assumption that interaction leads to sentiment and vice versa. A large set of non-obvious propositions is derived. Some of the key propositions are summarized here:

1. Asymmetry: The more popular actor receives a greater "vote" from the less popular actor than he or she receives in return. The asymmetry is in proportion to the differences in their ranks as determined by the choices or "votes" each receives.
2. But, similar to the point noted above, this asymmetry declines with group size.
3. Actors who are similar with respect to quality get similar choices from similar others and, consequently, direct similar choices to these others. This results in the

familiar observation of structural equivalence partitioning of groups.[15] All of this results from the logical consequences of asymmetry (effectance motive) coupled with the safety motive of preferring to choose those who are like oneself. Once again, however, the effects of asymmetry decline with group size.

4. Since actors direct choices to others in proportion to those received, actors with the highest number of choices received are also those with the highest degree of interaction directed towards others. But the distribution of popularity is more unequal than the distribution of choices of others. Interestingly, given any two actors, it can be shown from the equations that the one who is higher in choice rank directs more interaction or choices to any third actor even if that third actor is very low in rank. This is none other than the Homans' leadership paradox. Leaders direct more interaction to others even though the leaders have higher choice rank. No semi-mystical attribution of qualities or adherence to norms that Homans has attributed to leaders is necessary for this result. Leaders are those who enjoy and tolerate greater asymmetry in their choices than others, and, as a result of group process, regardless of differences in qualities they possess or whether these qualities are derived from the external system or not, their rank is magnified by social influence processes within the group. And, as we have generally seen, these processes are mitigated by group size. So that indeed we are talking about the "small group."[16]

We can say a bit more about leadership. In this theory, it is a mix of three factors. First, there is indeed a "personality" factor but it is quite different than the one usually attributed to leaders that involves some force of character or charisma. Rather, leaders are those who can tolerate both greater "unrequited love," as it were but who are also interested in the less popular members of the group; they can tolerate greater asymmetry of choice. Second, leaders do possess more of the attributes of value to the group: beauty, brains, brawn, or what have you. But, third, this differential distribution of attributes may be quite small; the differences are amplified by small group process so that the attribution of valued qualities by members of the group (see Feld 1991 who discusses how people's understanding of how many friends they should have is constructed by their friends) exceeds the objective differences.

Conclusion

We have seen that a limited set of simple assumptions can generate the major features of informal systems and networks in small groups. The first assumption is the safety motivation assumption, namely, that humans, from birth desire to interact with

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[15] There are several forms of structural equivalence. Here we mean that structurally equivalent individuals have the same patterns of relationships with the same others (because the others are in the same group). These structurally equivalent individuals may or may not themselves be interconnected. Equivalence can be a variable and the concept then is structural *similarity*. (See Burt 1987; 1992; Borgatti and Everett 1992).

[16] H. Russell Bernard, personal communication, points out that the leadership paradox relates to a problem in anthropology: how did pre-state chieftainship emerge? In chiefdoms, the chief can wind up being impoverished because he has to give so much to his constituents. This is illustrated by the phenomenon of "big men" in Malaysia (Sillitoe 1998).

others in order to feel secure and satisfy their needs. This interaction is first with a person's mother, then with the father and then with others. This assumption is now added in as the following corollary due to Homans: interaction with others leads to feelings or sentiments or evaluations about these others and these sentiments, positive or negative, in turn have consequences for future interaction. Obviously, if the sentiments are positive, a person engages in more activities (interactions) with the other; if the sentiments are negative, interactions are reduced. A second corollary is Gould's symmetry assumption. In interaction motivated by safety concerns, people prefer to extend choices or interaction with others who will return the favor. In symmetric interaction, for example, hanging out together, these assumptions lead to homophily of rank: pairs who hang out together or otherwise interact tend to have the same rank or share the same number of common activities. But networks are more than a collection of pairs of individuals. At least three are required to make the network non-trivial. A second assumption, based on Heider's balance theory, is now required. As we quoted above, "A balanced state exists if all three relations are positive in all respects, or if two are negative and one is positive" (Heider 1946). That is, if A is affiliated with B, and B is affiliated with C, then C is affiliated with A. We now have all the conditions that produce the interaction hierarchy modeled by Freeman. People are sorted into sets of mutual interest or regard. Above a certain number of common interactions, and this number differs according the group, the interactions are balanced and are said to be strong ties. Below that number not all the interactions are transitive, and these ranks are said to form weak ties. The motivation for safety, accompanied by a desire for symmetry in interaction plus the fact that interaction leads to sentiment, accounts for the development of strong and weak ties in a small group. The safety motivation and symmetry in interaction also lead to grouping people together according to the amount of interaction or activities that they have in common.

In potentially asymmetric situations in which choices or attractions to others are considered rather than merely whether people hang out together, two additional motivations comes into play. First, there is the effectance motive or the desire to control or make a difference in one's interaction environment. In this situation, some people are chosen more than others because rank homophily is not the only consideration. Social climbing is a factor. People like to choose others who are more attractive than they are; subject to the safety condition that too much unrequited love is painful. Attractiveness is of course not necessarily physical but the possession of any attribute valued by the group. A second motivational factor in asymmetric systems is the tendency to be influenced by the opinions of others that leads to the amplification of choices beyond the original differences in the possession of valued attributes. This leads to the ranking system Gould described and to the interesting consequences that ensue logically from this situation, namely, that leaders, i.e., those with more rank, have more of what is valued by the system, and they tend to direct interaction towards others of less rank, i.e., they lead. We also observe that the logic of rank dictates that the group becomes segmented by structural equivalence. That is, layers are formed in terms of people who have similar ties or relationships with others. Thus, an opportunity for further effectance or manipulation is possible because persons can bridge relationships between people whose ties are weak and reap the advantage of being brokers.

The main goals of this article have been achieved. We have been able to construct typical small group network situations by using a limited set of assumptions. What is

more, the sources cited have tested these propositions against some empirical data. More tests are necessary, but this is a good start. There is more work yet to be done. Conspicuously missing from the theory is a systematic account of the distribution of the three basic character traits: effectance, safety and influenceability. Gould's equations show that there will be different results depending on the relative distribution within the group of these traits, which in the theory are held to be random. Yet different social and cultural systems may recruit to a group persons with varying mixes of these traits. The outcomes then are likely to be different. For example, if there are more effectance minded persons and more of those susceptible to influence within the group, then the stratification within the group will be greater. Whether influenceability and safety are related to one another under any given circumstances is unknown.

Appendix: Simon and White on Homans

Attempts to formalize Homans are not new, though the approach used in my article is different. A number of years ago, Harrison White and Herbert Simon formalized Homans' ideas about sentiment, activity, and interaction. Simon, however, did not deal with leadership or stratification within the group:

> While the model described here was suggested by Homans' analysis of behavior in *The human Group,* we have attempted to present only part of his system: in particular we have omitted reference to phenomena of hostility, and to interpersonal differentiation (kinship and leadership). (Simon 1952: 210)

White (White 1970: 852-862) extended the formalization beyond averages and individuals become the unit of analysis. His interest remained in the relationship between friendship, interaction and activity and how these might change over time. As he wrote: "Simon extracts from Homans the idea of mutual interrelations among the levels of friendship, interaction, and activity in a concrete group of people subject to very strong pressure from the environment to carry out some level of purposeful activity."

By using stochastic models derived from Coleman, White suggested that the values for individuals, rather than the total group averages, might be measured. No direct applications, however, were given.

Gould and I are interested in how a few fundamental axioms can generate interaction and friendship and lead to group structure. Gould tests the propositions against data (I don't go into that). I believe that this represents a considerable advance over Simon and White. Gould cites neither White nor Simon.

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