A Bayesian account of two visual illusions involving lighthouse beams

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Abstract

Lighthouse beams are often perceived as bent rather than straight, and observers sometimes infer that a rotating lighthouse beam originates from a "phantom lighthouse" that lies in the opposite direction from the true source of the beam. We argue that both illusions arise as a result of Bayesian inference based on natural scene statistics and support our argument by implementing a formal computational model. In addition to capturing both illusions, our model makes the novel predictions that a beam viewed from the side is perceived to bend towards the observer, and that the phantom lighthouse illusion should only emerge at a critical point at which the observer is located around 75 metres in front of the true source of a rotating lighthouse beam. Our theory therefore motivates a future line of experimental work, and contributes to a broader body of research that explains perceptual phenomena (including visual illusions) in terms of Bayesian inference.

Keywords: visual perception; visual space; illusions; Bayesian inference; computational model

Introduction

Our visual systems mostly provide us with reliable information about the world, but occasionally they do not. Visual illusions are clearly important for understanding why our perceptual systems make errors, but are also valuable because they expose the mechanisms and principles that support accurate perceptual inferences under many circumstances (Gregory, 2006). Here we add to a body of work which argues that visual illusions are expected when a near-optimal perceptual system is applied to stimuli that depart from the statistical structure of natural scenes. We do so by presenting a simple Bayesian model and showing that it accounts for two illusions involving the perception of visual space.

The two illusions we consider both involve the perception of lighthouse beams. Lighthouse beams are normally parallel to the ground to allow them to travel as far as possible, but in many cases lighthouse beams are perceived to curve down towards the ground. A schematic illustration of this "bent beam" illusion is shown in Figure 1a. The second illusion involves an inferential error about the source of a moving lighthouse beam, and we refer to it as the "phantom lighthouse" illusion (Figure 1b). Under some circumstances, an observer standing with her back to a lighthouse will perceive the beams sweeping over her head not as originating from the actual lighthouse behind her, but as originating from a phantom lighthouse somewhere far away in front of her.

Although these illusions have received relatively little attention, we find them striking for two reasons. First, both illusions are readily experienced in everyday settings without specialized equipment. For example, the third author experienced both illusions on a seaside vacation without previously knowing about either one. Second, the phantom lighthouse illusion in particular is notable because of the magnitude of the error made by the visual system. In the experience of the third author, the distance between the locations of the true and phantom lighthouses can amount to hundreds of metres.

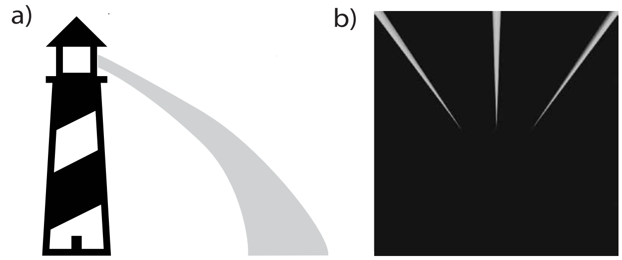

Figure 1: Two illusions involving lighthouse beams. (a) The bent beam illusion. Lighthouse beams are horizontal in reality but are often perceived as bending down towards the ground. b) The phantom lighthouse illusion. Shown here is a projection of three successive beams passing over the head of an observer who is standing with her back to a lighthouse located 75 metres behind her. In reality, the most distant points along the beams are far from each other, but the observer may infer that the beams meet at a distant source.

The earliest discussions of the phantom lighthouse illusion that we know date from the 1930s (Colange & Le Grand, 1937; Dunoyer, 1937; Rosenthal, 1938; Colange & Le Grand, 1938), and this illusion is also discussed by Minnaert (1993), who describes it as a "most impressive sight" (p 171) and by Floor (1982), who describes it as the "most impressive optical illusion associated with lighthouses" (p 233). Across this literature, three factors are proposed as contributing to the illusion. The first and most commonly-mentioned factor is an inadequate allowance for visual perspective (Colange & Le Grand, 1937; Rosenthal, 1938; Minnaert, 1993; Floor, 1982). In Figure 1b, for example, the three beams appear to converge in the image plane as a result of perspective projection, and an observer who takes convergence in the image as evidence for real-world convergence may incorrectly conclude that the beams are emitted from a source lying at the apparent point of

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convergence. Minnaert (1993) briefly mentions the "underestimation of faraway distances" as a second relevant factor, and suggests that the illusion depends in part on the perception that the beams do not extend infinitely but rather approach a point some finite distance away. A third factor is introduced by Colange and Le Grand (1937), who suggest that the photometric brightness of the beams is largest towards the horizon, which supports the inference that the source of the beams lies out towards the horizon.

Here we focus on the second factor — "the underestimation of faraway distances" — and argue that this underestimation occurs because the observer relies on Bayesian inference using a prior derived from natural scene statistics. Minnaert (1993) devotes only a single sentence to this factor, and we go beyond his verbal description by implementing a formal model and showing that it accounts for both the bent beam and the phantom lighthouse illusions. We also derive a key prediction from the model that distinguishes it from a *naive projection* account that attempts to explain the phantom lighthouse illusion based on perspective projection alone. The naive projection account predicts that the illusion will be experienced when the observer is standing relatively close to the lighthouse, but our account predicts that the illusion only begins to emerge when the separation between lighthouse and observer exceeds a critical distance of 75 m.

Our Bayesian approach builds on two related research directions. The first is a body of work that explores how perception is well adapted to the statistical structure of natural scenes (Purves & Lotto, 2003; Geisler, 2008). Our approach is related most closely to the work of Yang and Purves (2003), who used a laser range finder to construct an empirical prior on distances to the nearest surface at a range of elevation angles. Yang and Purves (2003) show that the resulting prior can account for a family of visual illusions, and we show that a similar prior accounts for the two lighthouse illusions considered here.

The second research thread is a broader body of work that models perception as Bayesian inference (Knill & Richards, 1996; Kersten & Yuille, 2003). Within this literature, prior distributions are often but not always based on natural scene statistics. Most relevant to us are previous approaches that show how perceptual illusions can arise from Bayesian inference (Weiss, Simoncelli, & Adelson, 2002; Geisler & Kersten, 2002; Yildiz, Sperandio, Kettle, & Chouinard, 2022; Khoei, Masson, & Perrinet, 2017). Gregory (2006), for example, presents a taxonomy of illusions and classifies each one according to whether it is consistent with Bayesian inference. Although comprehensive, Gregory's approach does not rely on computational models, which means that his claims about the Bayesian status of certain illusions are best treated as provisional. For example, Gregory considers the Ponzo illusion to be "counter-Bayesian," or contrary to what Bayesian inference would predict, but Yildiz et al. (2022) disagree and argue that the Ponzo illusion is indeed compatible with Bayesian inference. To avoid disputes of this kind it is useful to develop and test fully specified computational models (Weiss et al., 2002; Khoei et al., 2017), and we follow that approach here by implementing a Bayesian model and showing that is susceptible to the two lighthouse illusions.

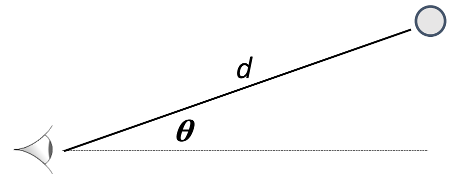

Figure 2: Distance estimation. An observer in a dark room sees a light at elevation angle θ and must infer the distance d to the light. A Bayesian observer computes a posterior distribution over d by combining a prior over distances with a noisy estimate of the distance of the light.

We begin in the next section by describing the Bayesian framework and the prior distribution that lie at the heart of our work. The following two sections demonstrate that this framework can account for both the bent beam and phantom lighthouse illusions. We show in addition that our theory makes two key predictions: first, that a beam viewed from the side is perceived to bend towards the observer, and second, that the phantom lighthouse illusion will be experienced only when the separation between the lighthouse and the observer exceeds a critical distance of 75 m. Testing these predictions is beyond the scope of the current work, but we hope that our theoretical analysis will lead to future field experiments conducted in the presence of a lighthouse at night.

Bayesian perception of visual space

Figure 2 shows an observer in a completely dark environment who is presented with a light at elevation angle θ. The observer is uncertain about the distance d at which the light lies, but we assume for simplicity that the elevation angle θ is perceived accurately and is therefore known. This assumption is consistent with prior empirical work suggesting that perception of θ is accurate in the absence of reliable distance cues (Ooi, Wu, & He, 2006).

Suppose that the observer's visual system provides her with an initial, noisy estimate d^* that will be combined with prior expectations to estimate a posterior distribution $P(d|d^*)$ over the distance d. A Bayesian observer computes this posterior by combining a prior $P(d)$ and a likelihood $P(d^*|d)$:

$$P(d|d^*) \propto P(d^*|d)P(d). \tag{1}$$

The likelihood $P(d^*|d)$ captures how probable the noisy estimate d^* would be if the true distance were d. This term could be formalized, for example, by assuming that d^* is noisy but unbiased and is drawn from a Gaussian distribution centered on the true distance d. The standard deviation of this Gaussian distribution could either be set to a constant, or set proportional to d, which captures the idea that the initial estimates for distant objects are noisier than those for nearby objects.

Here we adopt an extreme approach and assume that the likelihood function is constant, or so flat that it is effectively constant. This assumption is appropriate for perception under conditions so impoverished that the noisy estimate provides no real information about d. For example, an observer gazing out over a featureless ocean at midnight may find it impossible to distinguish between a small light that is close (small d) and a large light that is distant (large d). For simplicity, our primary model results in this paper are based on a constant likelihood function, but we have found that qualitatively similar results are obtained using a Gaussian likelihood function with distance-dependent noise (i.e. standard deviation proportional to d).

The prior $P(d)$ is assumed to be consistent with the observer's previous experience of natural environments. Yang and Purves (2003) constructed a prior of this kind by taking a laser range scanner into a series of natural environments and repeatedly measuring the distance to the nearest objects at a range of different elevation angles θ. The data collected by Yang and Purves (2003) are not publicly available, but they report that the empirical prior is roughly consistent with a prior derived by repeatedly generating synthetic scenes in which rectangular surfaces of different sizes were placed at different distances from the observer, and recording the distance to the nearest surface as a function of elevation angle. We used a related approach to compute the prior used for our analyses.

Our simulation assumes that each scene contains 5 surfaces, each of which is sitting on the ground with a depth uniformly distributed between 1 m and 150 m. The heights of the surfaces are uniformly distributed between 1 m and 25 m, and to simplify our computations we assume that all surfaces within a given scene have the same height. To further simplify the simulation we assume that the height of the observer is 0 m, which means that the observer is roughly equivalent to an eyeball sitting on the ground. We construct our prior by enumerating pairs that specify the depth of the nearest surface and the height of this surface, and for each pair storing the distance to the nearest surface as a function of elevation angle. With 5 surfaces uniformly distributed between 1 m and 150 m, the depth n of the nearest surface is distributed proportional to $(150-n)^4$, and we weight each of our pairs accordingly to produce a prior distribution over distance for each elevation angle. The resulting prior is summarized in Figure 3a, which shows the expected distance to the nearest surface as a function of elevation angle.

All of the numerical parameters used in our simulation, including the number of surfaces and the maximum depth of each surface, were chosen so that Figure 3a matched the analogous figure used by Yang and Purves (2003) to summarize their empirical distance data. For example, Figure 3a has a similar shape to the curve reported by Yang and Purves (2003), and spans a similar range of depths and heights. The

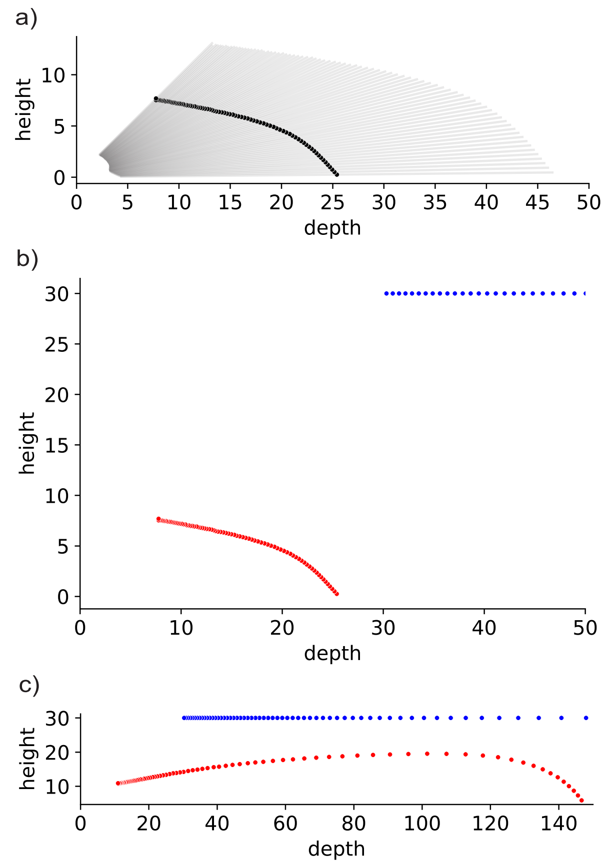

Figure 3: (a) A prior on distances at different elevation angles. The shape of the prior closely matches the empirical prior documented by Yang and Purves (2003). Error bars in light grey show the standard deviation associated with each elevation angle. (b) Inferred beam (red dots) perceived by an observer who is looking along a horizontal lighthouse beam (blue dots) and who combines the prior in (a) with a constant likelihood function to infer the positions of points along the beam. (c). Inferred beam (red dots) perceived by an observer who combines the prior in (a) with a Gaussian likelihood with constant standard deviation.

curve reported by Yang and Purves (2003) does not show results obtained for elevation angles that exceed 45 degrees or so, and we therefore consider only elevation angles between 0 degrees and 45 degrees to make sure that our prior has similar properties to their empirically-derived prior. An important limitation of the empirical prior reported by Yang and Purves (2003) is that the laser range scanner used in their work had a range of 2-300m, which means that the prior assigns zero probability to the possibility that the closest object is more than 300 m away. Our prior inherits this limitation, and we return to it in the Discussion.

In the next two sections, we show that a Bayesian observer who relies on the prior in Figure 3a is susceptible to both the bent beam and the phantom lighthouse illusions.

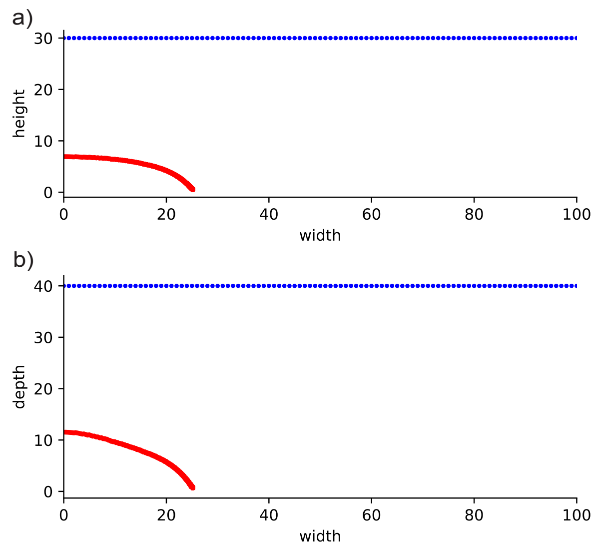

Figure 4: A Bayesian observer infers that a horizontal beam viewed from the side is bent in two respects. (a) The beam is inferred to bend towards the ground. (b) The beam is inferred to bend towards the observer.

The Bent Beam illusion

Suppose that an observer is standing with her back against a lighthouse and that a single, static lighthouse beam with a height of 30 m is running over her head. On a dark night no reliable depth cues will be available and the observer's inference about the distance of any given point along the light beam will be based entirely on her prior. The fact that the curve in Figure 3a bends down towards the ground therefore implies that the observer will perceive the lighthouse beam to have a similar shape.

To confirm this result we enumerated points along the lighthouse beam (blue dots in Figure 3b) and used Equation 1 and the prior in Figure 3a to compute the inferred distances of each of these points (red dots in Figure 3b). Our Bayesian approach specifies a posterior distribution over the depth of each point, and we take the mean of this posterior distribution as the final depth estimate for each point. Because we are using a constant likelihood function, the shape of the inferred lighthouse beam in Figure 3b (red dots) is identical to the shape of the prior in Figure 3a.

We considered two alternative likelihood functions and found in both cases that the perceived beam still bends towards the ground. A Gaussian likelihood that assumes distance-dependent noise (i.e. standard deviation proportional to the true distance) produces an inferred beam that looks virtually identical to the result in Figure 3b. A Gaussian likelihood with a constant standard deviation of 20 metres produces the inferred beam in Figure 3c, which has a different shape from the inferred beam in Figure 3b. Our theory therefore predicts that additional distance cues may alter the shape of the beam, and that there may be be conditions under which the inferred beam takes the shape of an inverted U. Although different likelihood functions produce inferred beams with different shapes, the key result that the inferred beam bends towards the ground appears to be very robust.

The results in Figures 3b and 3c assume that the light beam passes directly over the observer's head, but the model predicts that the bent beam illusion occurs regardless of the observer's position with respect to the lighthouse. Figure 4 is based on an observer 40 metres away from a lighthouse who is viewing a static beam perpendicular to her line of sight. Consistent with prior literature (Floor, 1982; Minnaert, 1993), Figure 4a shows that the beam is perceived to bend towards the ground as width (i.e. distance to the observer's right) increases. Figure 4b shows that the beam is also perceived to bend towards the observer: in other words, as width increases the depth of the beam decreases. To our knowledge, the perceived bending in Figure 4b is not discussed in the literature, and therefore constitutes a novel prediction that can be tested in future experiments.

The Phantom Lighthouse illusion

It is relatively obvious that the prior in Figure 3a accounts for the bent beam illusion, but not so obvious that this prior also accounts for the phantom lighthouse illusion. To see why distance estimates are relevant to this second illusion, consider the three projected beams in Figure 1b. Recall that the three beams diverge from a common source located behind the observer, but because of visual perspective the furthest points along the three beams lie nearby in the image. In reality, these points are projected to nearby image locations because each lies at a large distance from the observer. An observer, however, who believed that the distances of all three points were relatively small would conclude that the points lie close to each other in the image because they lie relatively close to each other in the real world. The example in Figure 1b therefore suggests that inferences about distance are critical for deciding whether or not the three lighthouse beams emerge from a single, distant source. We can therefore ask whether distance estimates computed using the prior in Figure 3a are compatible with a distant origin for a set of observed lighthouse beams.

Figure 5a provides a bird's eye view of the configuration we use to develop a formal analysis of the phantom lighthouse illusion. Consider an observer located at point O who is standing with her back to a lighthouse at L. The separation between the observer and the lighthouse is denoted by s. The lighthouse beam initially lies along the segment OB, but a moment later the beam has rotated and now lies along the segment OB'. Here we focus on the solid blue segments of the two lighthouse beams (AB and A'B'). The red segments (CD and C'D') show how these blue segments are perceived by the observer. Consistent with the prior in Figure 3a, the observer underestimates the distance of each point along the beam, and the discrepancy between inferred distance and true distance increases as the true distance increases. As a result,

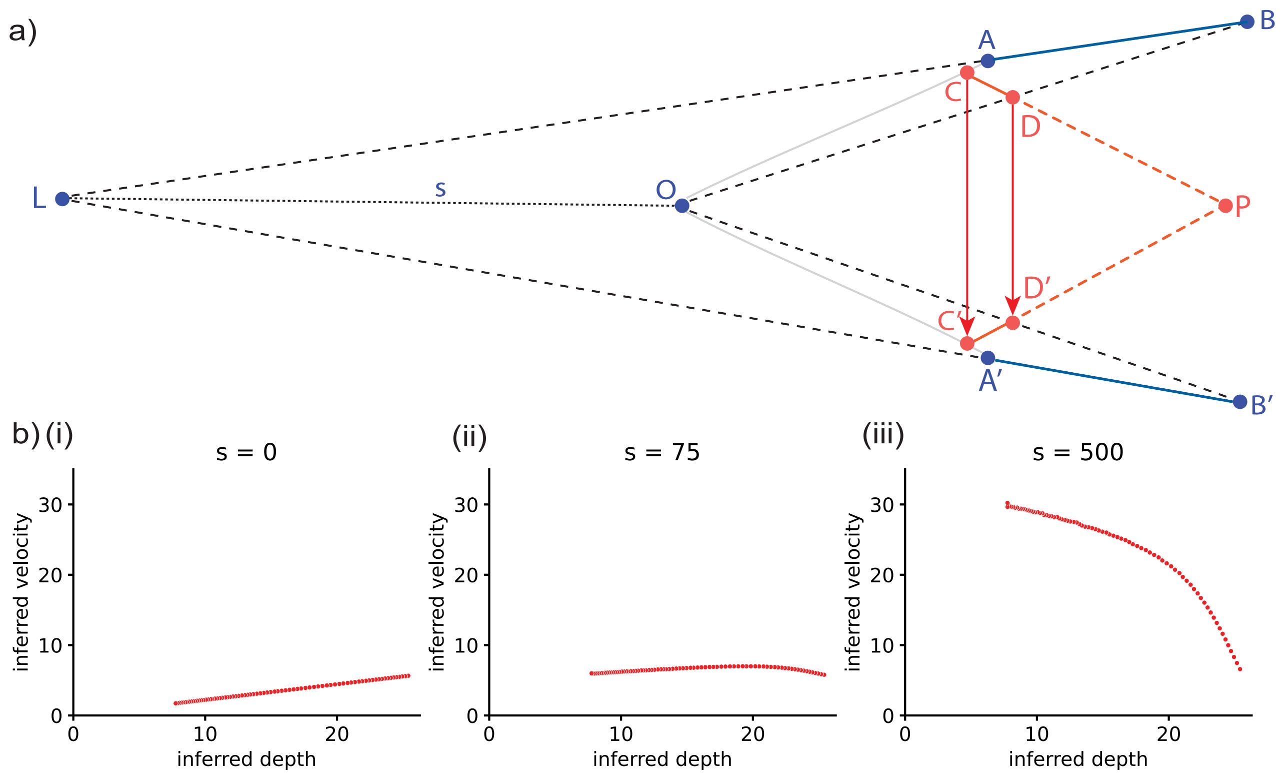

Figure 5: (a) Inaccurate estimates of distance can mean that two beams which in reality converge behind an observer are perceived to converge at a point in front of the observer. Here L and O are the locations of lighthouse and observer, and AB is a segment of the true lighthouse beam that rotates to become A'B' in the second snapshot of the beam. Bayesian depth estimates mean that segments AB and A'B' are perceived as segments CD and C'D', and these two inferred segments converge at point P, the location of a perceived phantom lighthouse. (b) Velocity estimates for different values of s, the separation between the observer and the lighthouse. When s is 0, more distant points along the beam are perceived as moving faster, consistent with the inference that the source of the beam is behind the observer (no phantom lighthouse). When s is 500, more distant points along the beam are perceived as moving slower, consistent with the inference that the beam originates at a phantom lighthouse in front of the observer. The transition between these two percepts occurs at a critical point near $s = 75$: at this location, the most distant point on the beam is perceived as having roughly the same velocity as the closest point.

the two inferred lighthouse beams shown in red converge in a way that suggests a common origin in a phantom lighthouse located at P.

We will say that an observer experiences the phantom lighthouse illusion if the most distant points along the inferred beam (e.g. point D in Figure 5a) are inferred to have lower velocities than the closest points along the beam (e.g. point C in Figure 5). The red arrows in Figure 5b show velocity estimates, and the reduced length of the arrow for D relative to the arrow for C indicates that the observer at O experiences the illusion. Based on the experience of the third author, the spurious source of the beams is sometimes perceived as moving rather than stationary, as if the phantom lighthouse were carried by a speeding boat far out at sea. Characterizing the illusion in terms of the relative velocities of close and distant points ensures that percepts involving a moving source still qualify as instances of the illusion.

Whether or not the illusion is experienced depends critically on the separation s between the observer and the lighthouse. When the separation is 0 m (ie the observer is standing at the base of the lighthouse), Figure 5b.i shows that velocity estimates increase monotonically with depth, which means that no illusion is predicted to occur. When the separation is 500 m, the inferred velocities for the most distant points are substantially lower than the inferred velocities for the nearest points along the beam, suggesting that a strong illusion is experienced. Based on the prior in Figure 3a, the critical point marking the transition between these two regimes is predicted to occur for values of s around 75 metres. At the critical point the inferred velocities of the most distant points match the inferred velocities of the closest points, which suggests that the observer may be uncertain whether the source lies in front of her or behind her.

The results in Figure 5b assume a constant likelihood function, and virtually identical results are obtained using a Gaussian likelihood with distance-dependent noise. If we assume a Gaussian likelihood with a constant standard deviation of 20 m, the critical point is predicted to occur at a separation of roughly 800 m. This result therefore suggests that the presence of additional depth cues may alter the separation at which the illusion begins to emerge. Importantly, however, our theory predicts the existence of a critical point for all likelihood functions that we considered.

The predicted existence of a critical point distinguishes our theory from the *naive projection* account mentioned in the introduction. This alternative account assumes that the observer takes retinal distance as a proxy for distance in the real world, and holds that the phantom illusion occurs because distant points along successive snapshots of the beam project to similar locations on the observer's retina. According to this theory, however, the phantom lighthouse illusion should be experienced for all values of $s > 0$, because in all of these cases visual perspective ensures that the most distant points along successive snapshots project to nearby locations on the retina. For example, the projection in Figure 1b was computed for $s = 75$, and when s is reduced the naive projection account still predicts that the illusion will occur because the three projected beams still converge in the image plane.

Future experimental work can test the prediction that a critical point exists, and the more fine-grained prediction that the point occurs at a value of s near 75 metres. If the critical point does indeed exist, experimental studies can also explore what people actually perceive near this point. It is possible that observers located at the critical point will simply be uncertain about the source of the light beams, but also possible that they will confidently perceive the lighthouse as situated in front of them on some trials and as situated behind them on other trials.

Discussion and Conclusion

We presented a formal theory which suggests that the bent beam and phantom lighthouse illusions both occur because observers use Bayesian inference to infer the distance of points along a lighthouse beam. Our theory suggests that both illusions depend critically on the observer's prior distribution over distances, and we used a prior that matched a prior derived by Yang and Purves (2003) from natural scene statistics. All numerical parameters of our prior were set to match empirical results reported by Yang and Purves (2003), and as a result our prior incorporates no free parameters. Our theory therefore makes parameter-free predictions that a beam viewed from the side is perceived to bend towards the observer, and that the phantom lighthouse illusion emerges at a critical point where the separation between the observer and the lighthouse is around 75 m.

We contrasted our theory with a naive projection account which suggests that the phantom lighthouse illusion occurs because observers fail to adjust adequately for perspective projection. To us, however, it seems likely that inadequate allowance for perspective (i.e. the focus of the naive projection model) and the underestimation of distance (the focus of our model) both contribute to the illusion. Future theoretical work can therefore attempt to develop and test a combined approach that includes both factors. Experimental support for a critical point would rule out the naive projection model, but a combined approach would be compatible with the existence of a critical point, and may account better for future behavioral experiments than either approach alone. Future work should also consider additional factors that may contribute to the illusion, including expectations about speed (Weiss et al., 2002) and other factors related to motion perception.

The two components of any Bayesian theory are a prior and a likelihood function. We have assumed that the prior is consistent with natural scene statistics, but our theory is agnostic about the origin of this prior. One possibility is that the prior is learned from previous experience, but it is also possible that this "learning" took place over the course of evolution and that the observer is innately provided with the prior. Our prior is based on the work of Yang and Purves (2003), but as mentioned earlier their prior does not allow for the possibility that the nearest object along a line of sight (e.g. a distant mountain) may be more than 300m away. Allowing for distances greater than 300 would not change any of the qualitative predictions of the model, but would increase the separation at which the critical point is predicted to emerge.

For simplicity, our primary analyses assumed that the likelihood function is constant which means that it makes no contribution to our results. This assumption seems appropriate for an observer viewing lighthouse beams in a dark and featureless environment, but will need to be adjusted for situations in which reliable cues to distance are available. Our analyses of alternative likelihood functions suggest that adding reliable depth cues may produce an inferred beam with an inverted U shape, and may change the location of the critical point associated with the phantom lighthouse illusion.

Although we have focused here on theoretical analysis, our approach suggests a number of directions for future empirical work. The most urgent priority is to empirically test our predictions that a beam viewed from the side is perceived to bend towards the observer, and that the phantom lighthouse illusion emerges at a critical point at which the observer is around 75 metres in front of the lighthouse. These predictions are best tested in the presence of an actual lighthouse, but could also be tested using displays presented on a virtual reality headset. Experimental work can also explore a wide range of other questions, including whether the shape of the beam in the bent beam illusion and the location of the critical point can be manipulated by presenting additional cues to distance (e.g. visual landmarks), and whether there is a transition for some value of s between perceiving a moving phantom lighthouse and a stationary phantom lighthouse. Although lighthouses have been around for more than two millennia, we contend that they remain underexplored as devices for studying perception.

Acknowledgments

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