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Near-Optimal Admission Control for Multiserver Loss Queues in Series with Light Traffic

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Abstract

This paper considers access control policies in multiserver loss queues in series, such as might arise in the context of computer and telecommunication networks. Each queue is presented with both served upstream customers and Poisson arrivals from outside the network, and it may route serviced customers out of the network or to the downstream queue. Revenue is gained by each station when it serves a customer, but the amount of revenue depends on whether the customer entered the network at this station or was routed from an upstream station. We propose a simple recursive method to solve the problem using dynamic programming on a set of reduced state spaces. This approach includes a rate estimation technique for upstream stations, and a revenue estimation technique for downstream stations, based on a light-traffic model. Numerical results demonstrate the performance of these near-optimal policies.

Keywords: Queueing; Dynamic programming; Connection admission control; Downsizing approximation; Optimisation

1. Introduction

This paper considers access control policies in multiserver loss queues in series. Each queue is presented with arrivals of customers of two types: a stream of "internal customers" from the previous queue, and a Poisson stream of "new customers" from outside the network. Each queue has an arbitrary number of servers, but no queue. Service times of each customer are i.i.d., but the revenue generated is differentiated, with internal customers typically paying more than new customers. Departures from the queue are Bernoulli routed either to the next queue or out of the network, independent of their type. We are interested in the access control policy, which will make a decision to accept or deny upon the arrival of any customer at each queue. The objective here is to maximize the total discounted revenue over an infinite horizon.

This problem arises in the context of computer and telecommunication networks, e.g. Internet and Asynchronous Transfer Mode (ATM), which transport real-time traffic [1], [3], [12], [21]. Computer networks were originally designed for "best-effort traffic", and typically serve traffic on a first-come first-served basis in a series of queues. Telecommunication networks, on the other hand, were originally designed for real-time traffic using a "circuit-switched architecture" -- each station has essentially no queue and reserves one server (defined as the transmission capacity required for one call) for each call passing through it.

Both types of networks are currently evolving toward multi-service architectures, in which transmission of traffic at each station is differentiated based on the service type. In particular, we are interested here in real-time traffic. The new network architectures strictly limit queueing time for real-time traffic, because of their inherent low-delay requirements. In addition, these architectures implement some type of reservation or priority mechanisms to give real-time traffic preference over best-effort traffic [13]-[16]. These reservation or priority mechanisms typically result in a connection admission control (CAC) policy on real-time calls, based on the capacity of each station.

In this paper, we model this type of network as a series of multi-server loss queues. Only a single series of queues is considered here; extensions to a general network topology will be considered at a later time. We assume that real-time traffic is given preference over best-effort traffic; hence only real-time traffic is modeled. Delay is minimized by not allowing real-time traffic to queue at any station. We only model the network traffic on an intermediate time scale. A "customer" corresponds to a burst of real-time traffic, i.e. a randomly sized batch of packets; we do not explicitly model long-term time scale phenomena, e.g. calls. The CAC policy is modeled by a server policy at each station that can reserve any number of servers for internal customers. In order to allow the policy to achieve the desired balance of blocking of internal and new customers, we assume that each customer pays revenue to each station it passes through. The amount of the revenue can depend on the station and on its type. In general, internal customers would be assumed to pay more, in order to bias the network in favor of completing transmission of traffic that has already received service at a previous station.

A good survey of research on access control policies in queueing networks can be found in [19], [20]. Most of the research on this topic, however, focuses on infinite queue models and uses holding costs as the performance metric. In contrast, here we assume a loss network, and use loss as the performance metric.

The typical approach to determine the optimal decision policy at each queue in a network, as a function of the number of customers at each station, is to use dynamic programming [2], [8], [9], [19]. However the size of the Markov chain for the entire network grows exponentially with number of stations, so this approach quickly becomes numerically infeasible.

Our goal in this paper is to reduce the amount of information contained in the state, and correspondingly the size of the state space. We borrow an idea often used in the stochastic modeling literature, where a common approach models only the bottleneck queue(s) in a network, by recursively estimating internal flow rates. We are unaware of such an approach being used in optimization of loss networks.

We propose determining a near-optimal policy for station i by creating a reduced model whose state contains only the number of customers at station i and the number of customers at its immediate upstream station $i-1$. This procedure ignores the states of stations upstream from $i-1$, so it requires estimation of the flow into that station. We also need to eliminate stations downstream from i. This reduction, however, contributes significant error to the decision as to whether to accept new customers. Therefore we propose a method to estimate the downstream revenue and to take this information into account at station i.

The control policy at one station affects upstream stations by altering their expected downstream revenue, and it affects downstream stations by altering their expected flow of internal customers. We therefore propose to apply these two techniques, rate estimation and revenue estimation, using simple recursions. Numerical results are presented to demonstrate the success of this approach.

The multi-server-loss-queues-in-series model and problem formulation is presented in section 2. In section 3, the state reduction method, along with rate and revenue estimation techniques, is proposed. In section 4, the numerical results are presented.

2. Model and problem formulation

Denote by n the number of stations in the tandem network. Station i is a non-preemptive loss queue with m_i servers. Each station is presented with a Poisson stream of new customers, with arrival rate λ_i. Service times are i.i.d. and exponential at rate μ_i. Departures from station i are routed out of the network with probability $\mathbf{P}_i$ or to station $i+1$ otherwise, independent where the customer entered the network, the customer's service time, and of other customers. This network is pictured in Fig. 1.

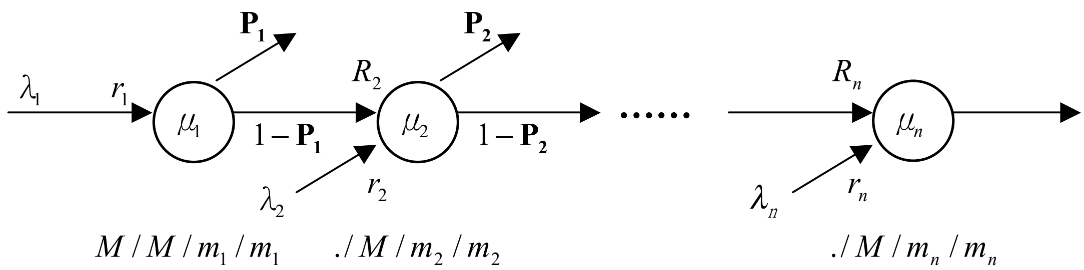

Fig. 1. A finite number of multi-server loss queues in series

Revenue is paid by each customer at the start of service. If the customer entered the network at station *i*, it pays an amount r_i at station *i*. If the customer entered at an upstream station, it pays an amount R_i at station *i*. We assume $R_i > r_i \ (2 \le i \le n)$, so that internal customers are preferable to new customers at station i, and that $1 > \mathbf{P}_i \ge 0 \ (1 \le i \le n-1)$.

We consider connection admission control policies that are capable of accepting or blocking arrivals of each customer type (internal or new) at each station. Blocked arrivals are lost. Our objective is to maximize the total discounted revenue over an infinite horizon. The optimal policy does not block customers at station 1 and does not block internal customers at any station [9], [10]. Admission policies therefore reduce to determining whether to block or accept new arrivals at station i, $2 \le i \le n$. For a two-station-in-tandem network, the optimal policy can be shown to consist of a switching curve in the two-dimensional state space [9], [10]. On one side of the curve, new arrivals are accepted; on the other side, new arrivals are blocked. Unfortunately, a similar characterization for the optimal policy in a series of more than 2 stations is unknown. Our objective here is to numerically determine a near-optimal policy.

3. State space reduction

Dynamic programming can be used to determine the optimal policy, which states whether to accept or block each new arrival at each station as a function of the number of current customers at each station in the network. However, the numerical complexity grows exponentially with the dimension of the state space, n. This renders computation of the optimal policy difficult for all but very small n. In this section, we propose a technique to reduce the dynamic programming problem to a collection of smaller problems, each of which consists of determining a switching curve for a single station.

Consider a particular station i. We propose determining a near-optimal policy for station i by reducing the state to a vector containing only the number of customers at station i and the number of customers at its immediate upstream station $i-1$. This procedure requires estimation of two quantities: (1) the flow into station $i-1$ and (2) the expected discounted reward paid at downstream stations. We will denote our estimate of the flow into station i by Γ_i, our estimate of the flow out of station i by d_i, and our estimate of the downstream discounted net revenue for a customer leaving station i by Θ_i.

The estimation of rates is discussed in the first subsection, and the estimation of rewards is discussed in the second subsection.

3.1 Estimation of upstream rates

Reduction of the state space will involve elimination of both upstream and downstream stations, reducing the state space for a single station to a vector containing the number of customers in that station and the number in its immediate upstream neighbor. This tandem queue can be easily analyzed. In this subsection, we consider elimination of upstream stations.

Focus on station *i*. Elimination of stations 1 through $i-2$ will reduce the information we have about short-term arrival rate variations of internal customers. The number of servers reserved for internal customers at station i will typically be increasing with the number of current customers present at each station upstream from *i*. However, we expect that the contribution of an upstream station to the reservation level at station i will be decreasing with the distance between the two stations. A reasonable approach, therefore, is to eliminate all but the immediate upstream neighbor, station $i-1$, as shown in Fig. 2. (A more accurate, but computationally more intensive, model is to keep additional upstream stations, e.g. from $i-2$ to i and estimate the arrival rate into station $i-2$.)

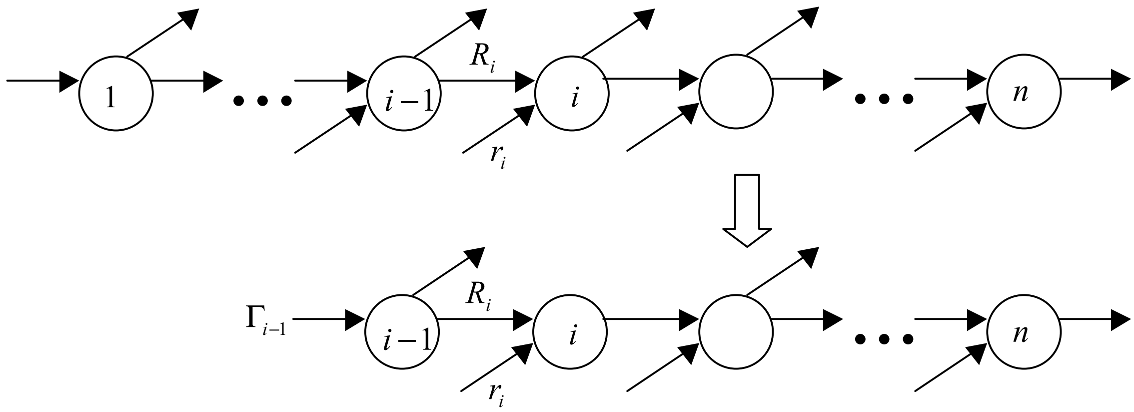

Fig. 2. Eliminating the upstream stations

We make two approximations to simplify the model. First, we assume that the arrival process into station $i-1$ is Poisson. The arrival process into station $i-1$ is the multiplexing of the departure process from station $i-2$, thinned by those departing the network, and the new arrivals into station $i-1$. Since the departure process from station $i-2$ is not Poisson, the arrival process into station $i-1$ is not Poisson. Nevertheless, we have found this approximation to yield good results, under light traffic.

Second, for purposes of estimating the arrival rate into station $i-1$, we model the upstream stations 1 through $i-1$ as uncontrolled loss queues. The rate of the departures from station $i-2$ depends on the control policy used in station $i-2$. However, under light traffic, ignoring this control policy is a reasonable approximation. Under moderate traffic, this second approximation can be avoided by explicitly using information about the control policies used in upstream stations. The resulting method then includes an iteration, which significantly increases the computational complexity [9], [10].

Our approach requires estimation of the arrival rate of customers into station $i-1$, Γ_{i-1}. We approximate the flow into station $i-1$ as a Poisson process, with a rate equal to the flow rate from station $i-2$ to station $i-1$ plus the rate of new arrivals into station $i-1$.

Every station in this system is a $M/M/m/m$ loss queue. Therefore, the loss probability is given by the Erlang B. If the arrival rate is λ and the service rate is μ, then:

$$\text{Blocking probability } p(\lambda,\mu,m)=\frac{\dfrac{(\lambda/\mu)^m}{m!}}{\displaystyle\sum_{k=0}^{m}\dfrac{(\lambda/\mu)^k}{k!}}.$$

For station j, $2\le j\le i-1$, we estimate rates using conservation of flow:

$$\Gamma_j \approx d_{j-1}\left(1-\mathbf{P}_{j-1}\right)+\lambda_j .$$

As noted above, this approximation method of the arrival rate depends on the departure rate of the previous station. Using the Erlang B formula, we can approximate the departure rate as:

$$d_{j-1}\approx \Gamma_{j-1}[1-p(\Gamma_{j-1},\mu_{j-1},m_{j-1})].$$

Denote $g_i(\lambda)\equiv \lambda\left[1-p(\lambda,\mu_i,m_i)\right]\left(1-\mathbf{P}_i\right)$. This approximation can then be implemented using a recursion, and the two preceding equations. For station 1, the estimated arrival rate is equal to the arrival rate of new customers:

$$\Gamma_1=\lambda_1 .$$

For station 2, the estimated arrival rate is based on the departure rate of station 1 customers and the arrival of new customers:

$$\Gamma_2\approx d_1\left(1-\mathbf{P}_1\right)+\lambda_2\approx \Gamma_1\left[1-p\left(\Gamma_1,\mu_1,m_1\right)\right]\left(1-\mathbf{P}_1\right)+\lambda_2=g_1\left(\Gamma_1\right)+\lambda_2=g_1\left(\lambda_1\right)+\lambda_2 .$$

The recursion continues to estimate the arrival rate into station $i-1$:

$$\begin{aligned}\Gamma_{i-1} &\approx d_{i-2}(1-\mathbf{P}_{i-2})+\lambda_{i-1} \approx \Gamma_{i-2}[1-p(\Gamma_{i-2},\mu_{i-2},m_{i-2})](1-\mathbf{P}_{i-2})+\lambda_{i-1} \\ &= g_{i-2}(\Gamma_{i-2})+\lambda_{i-1} \approx g_{i-2}(g_{i-3}(\cdots(g_2(g_1(\lambda_1)+\lambda_2)+\lambda_3)\cdots)+\lambda_{i-2})+\lambda_{i-1}\end{aligned}.$$

3.2 Estimation of downstream revenues

The second part of the reduction method of the state space involves elimination of the downstream stations. Again, we focus on station *i*. Customers departing from station *i*, with a positive probability will enter station $i+1$ (and possibly additional downstream stations). Therefore, these customers will generate further income to the network, which should be taken into account when the system managers are formulating the admission control policy at station *i*.

If we knew the number of customers at each downstream station, we could estimate (using dynamic programming) the future net revenue generated downstream. Indeed, the probability that a current customer at station i will be blocked at downstream stations increases with the number of current customers at these downstream stations; therefore the expected revenue this customer at station i will generate downstream is decreasing with the number of downstream customers.

Our estimation technique is based on the idea that the total expected discounted net revenue a customer will generate at downstream stations can replace the explicit effect of including all stations in the dynamic programming problem. Net revenue is defined as the revenue generated by this customer minus the lost revenue from blocking of new customers due to this customer.

Using this estimate, we base the control decision at station i on the reward a customer pays at station i plus the expected discounted net downstream revenue, denoted as Θ_i. The downstream revenue is independent of whether the customer at station i is internal or new. Our method therefore assumes a new customer at station i will contribute $r_i + \Theta_i$, and an internal customer coming from station $i-1$ will contribute $R_i + \Theta_i$. Accompanying this change in revenue, we eliminate all stations downstream from station *i*, leaving only stations $i-1$ and i as shown in Fig. 3.

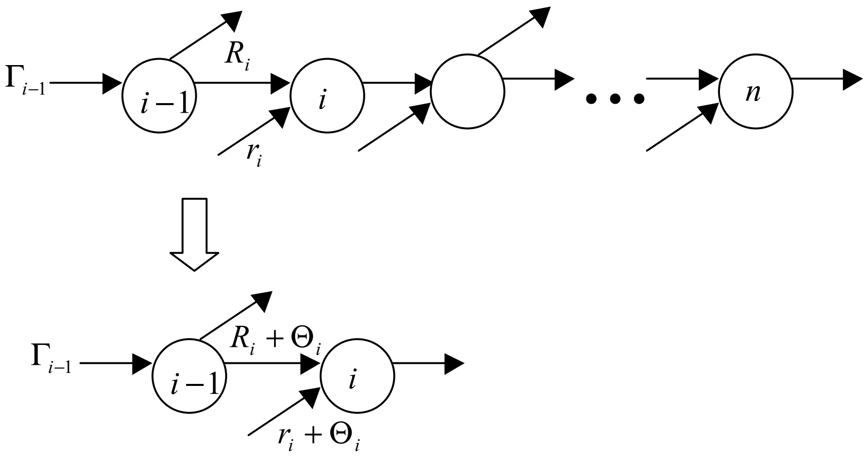

Fig. 3. Eliminating the downstream stations

The resulting model is a tandem queue, for which we can easily calculate the optimal admission policy. This optimal policy depends on the ratio of the revenues generated by internal versus new customers [9], [10]. If downstream stations were eliminated, but downstream revenue was not included in the decision at station *i*, then the admission policy would be based on $\frac{r_i}{R_i}$. By including an estimate of downstream revenue, the decision is based on $\frac{r_i + \Theta_i}{R_i + \Theta_i}$. Since this latter ratio is higher than the former one, the switching curve when downstream revenue is taken into account will be higher than (or equal to) the switching curve in the two-dimensional state space when the downstream revenue is not taken into account [9], [10]. Namely, consideration of future revenue will result in a lower reservation for internal customers at station i.

We now proceed to describe the method for estimating discounted net downstream revenue, Θ_i for $2 \le i \le n$. We start by observing that Θ_i depends on the expected net revenue earned at station $i+1$ plus Θ_{i+1}. Furthermore, the expected net revenue earned at station $i+1$ for an internal customer leaving station i is equal to the expected revenue paid at station $i+1$ minus the expected loss from any blocking of new customers due to the acceptance of this internal customer.

We first approximate the throughputs of internal and new customers at station $i+1$. We approximate the result of the admission policy at station $i+1$, in light traffic, as equivalent to internal customers receiving priority service. Consequently, we approximate the probability that an internal customer will obtain service at station $i+1$ as $1 - p(d_i(1-\mathbf{P}_i), \mu_{i+1}, m_{i+1})$, which we denote by p_{i+1}. Therefore, the throughput for internal customers at station $i+1$ is approximately $d_i(1-\mathbf{P}_i)[1 - p(d_i(1-\mathbf{P}_i), \mu_{i+1}, m_{i+1})]$. This gives us a basis for estimating the revenue gained at station $i+1$.

From our upstream flow estimation above, we approximate the total flow rate going through station $i+1$, which includes the internal customers and new customers at $\Gamma_{i+1}[1 - p(\Gamma_{i+1}, \mu_{i+1}, m_{i+1})]$. Hence, the throughput for new customers arriving from outside the network is approximately $\Gamma_{i+1}[1 - p(\Gamma_{i+1}, \mu_{i+1}, m_{i+1})] - d_i(1-\mathbf{P}_i)[1 - p(d_i(1-\mathbf{P}_i), \mu_{i+1}, m_{i+1})]$.

The difference between the arrival rate of new customers and the throughput of new customers is due to blocking. We attribute this blocking to the presence of internal customers.

Therefore, the reduction in new customer throughput at station $i+1$, per internal customer passing through station $i+1$, is approximately

$$\frac{\lambda_{i+1} - \Gamma_{i+1}\left[1 - p\left(\Gamma_{i+1}, \mu_{i+1}, m_{i+1}\right)\right] + d_i\left(1 - \mathbf{P}_i\right)\left[1 - p\left(d_i\left(1 - \mathbf{P}_i\right), \mu_{i+1}, m_{i+1}\right)\right]}{d_i\left(1 - \mathbf{P}_i\right)\left[1 - p\left(d_i\left(1 - \mathbf{P}_i\right), \mu_{i+1}, m_{i+1}\right)\right]}$$

which we denote by q_{i+1}. This gives us a basis to estimate for estimating the loss in revenue due to blocking of new customers at station $i+1$. To complete the estimates, we must also take into account the discounting of these revenues and losses. Denote the uniformization constant as Q, henceforth referred to as the reciprocal of one unit of time [9], [11]. Denote the discount factor per unit of time as α. We define $\beta(\mu) = \alpha^{Q/\mu}$ which represents the average discounting of future revenue for a customer receiving service at a rate μ.

We now use this discount factor, the rewards, and the losses obtained above to create an recursion to estimate the net discounted downstream revenue. We start with the last station n:

$$\Theta_n = 0.$$

A customer leaving station $n-1$ will gain service at station n with a probability of approximately $(1-\mathbf{P}_{n-1})\left[1 - p\left(d_{n-1}\left(1 - \mathbf{P}_{n-1}\right), \mu_n, m_n\right)\right] = (1-\mathbf{P}_{n-1})p_n$. Hence, our estimate is equal to this probability, times the discount factor, times the net expected revenue at station n:

$$\Theta_{n-1} \approx \left(1 - \mathbf{P}_{n-1}\right)p_n \beta\left(\mu_{n-1}\right)\left(R_n - q_n r_n + \Theta_n\right) = \left(1 - \mathbf{P}_{n-1}\right)p_n \beta\left(\mu_{n-1}\right)\left(R_n - q_n r_n\right).$$

The recursion continues with each further upstream station:

$$
\begin{aligned}
\Theta_i &\approx (1-\mathbf{P}_i)p_{i+1}\beta(\mu_i)(R_{i+1} - q_{i+1}r_{i+1} + \Theta_{i+1}) \\
&\approx (1-\mathbf{P}_i)p_{i+1}\beta(\mu_i)(R_{i+1} - q_{i+1}r_{i+1}) \\
&+ (1-\mathbf{P}_i)(1-\mathbf{P}_{i+1})p_{i+1}p_{i+2}\beta(\mu_i)\beta(\mu_{i+1})(R_{i+2} - q_{i+2}r_{i+2} + \Theta_{i+2}). \\
&= \sum_{j=i}^{n-1}\left[\left(\prod_{k=i}^{j}(1-\mathbf{P}_k)p_{k+1}\beta(\mu_k)\right)(R_{j+1} - q_{j+1}r_{j+1})\right]
\end{aligned}
$$

3.3 Calculation of near-optimal policies

In summary, the proposed procedure of reduction method operates as follows. First, we estimate the upstream rates for each station, resulting in Γ_i and d_i, as outlined in section 3.1. Then, we estimate the downstream revenues for each station, resulting in Θ_i, as outlined in section 3.2.

Finally, each tandem pair of stations ($i-1$ and i) is analyzed as a separate system, using Γ_{i-1} and Θ_i, as shown in the bottom part of Fig. 3. Then, the optimal admission policy is calculated using dynamic programming, and is guaranteed to be a switching curve [9], [10]. This policy is used, along with the policies for all other stations, as a policy for the network. Under light traffic, we believe that this simple recursive approach results in a near-optimal policy.

4. Numerical results

In this section, we will demonstrate the accuracy of the approach for a three-station-in-series network. Although our approach is computationally reasonable for much larger networks, only three stations are used here in order to compare the resulting near-optimal policy in two-dimensional state space to the optimal policy in three-dimensional one, for which the computation is geometrically increasing with the dimension. Numerical results were obtained by the method of successive approximation.

The system parameters are as follows: $m_1 = 8$, $m_2 = 7$, $m_3 = 6$, $\alpha = 0.99995$, $Q = 2000$, $\lambda_1 = 4.2$, $\lambda_2 = 4.18$, $\lambda_3 = 4.22$, $\mu_1 = 2.2$, $\mu_2 = 2.5$, $\mu_3 = 2$, $\mathbf{P}_1 = 0.1$, $\mathbf{P}_2 = 0.1$, $r_1 = 5$, $r_2 = 5$, $r_3 = 5$, $R_2 = 15$ and $R_3 = 15$. These parameters result in light traffic at stations 1 and 2, but moderate traffic at station 3.

4.1 Policy at station 3

The optimal admission policy imposed on new customers at station 3 is shown in Fig. 4. The policy has a threshold: a new customer will be accepted if and only if the state (i, j, k) of this system is below the switching surface.

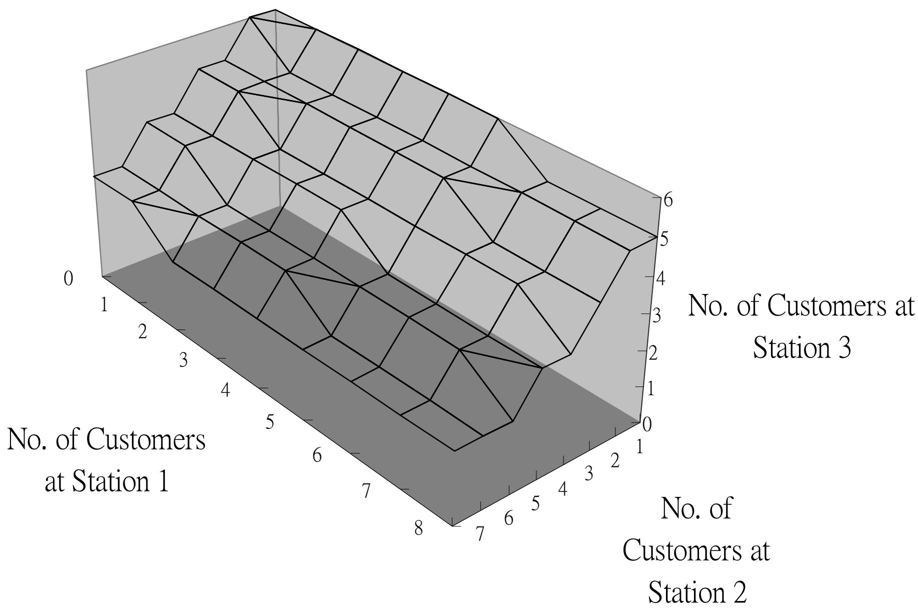

Fig. 4. The optimal admission policy on new customers at station 3

In order to estimate the near-optimal admission control policy for station 3, we eliminate station 1 and compute the expected arrival rate into station 2, i.e. Γ_2. Following our upstream rate estimation technique:

$$\Gamma_2 \approx \lambda_1[1 - p(\lambda_1, \mu_1, m_1)](1 - \mathbf{P}_1) + \lambda_2 = 4.2 \times (1 - 0.000649)(1 - 0.1) + 4.18 = 7.9575\,.$$

The optimal admission policy on new arrivals at station 3, for the resulting tandem queue, is shown in Fig. 5. As guaranteed, the policy is a switching curve. Acceptance decisions for new customers are now based solely on the occupancy of stations 2 and 3, ignoring station 1. This switching curve also coincides with the cross-section of the optimal switching surface when station 1 has 2-4 customers.

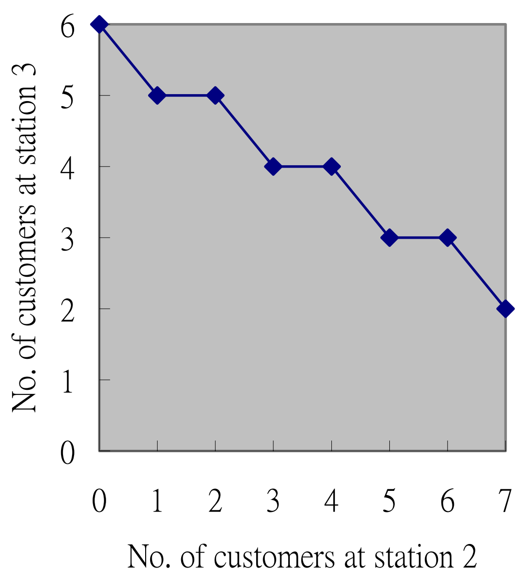

Fig. 5. The near-optimal admission policy on new customers at station 3

4.2 Policy at station 2

The optimal admission control policy imposed on new customers at station 2 is shown in Fig. 6. This policy also has a threshold. Since the load on station 2 is light, so the threshold is fairly high.

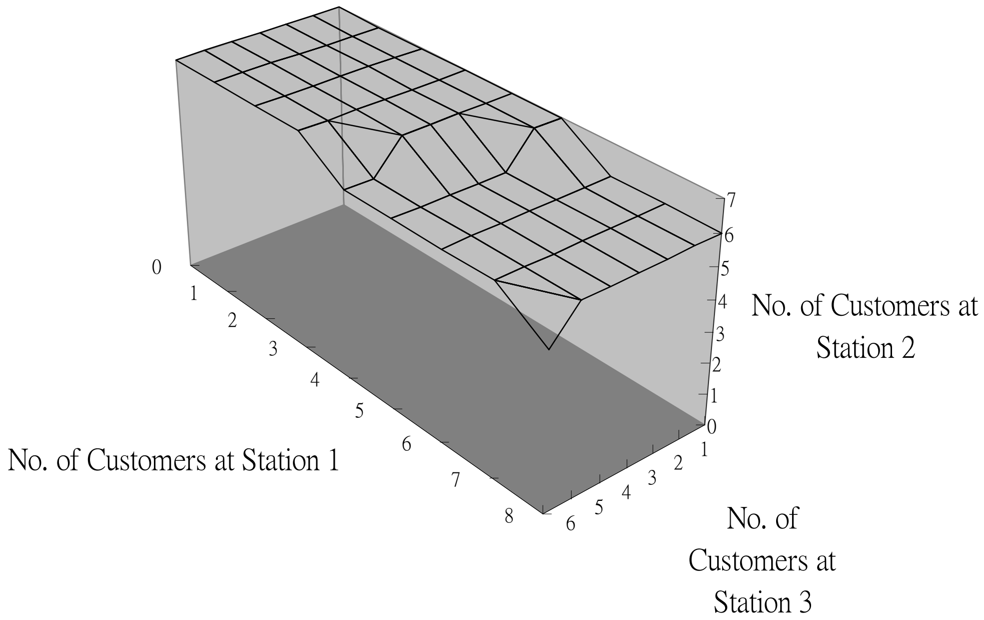

Fig. 6. The optimal admission policy on new customers at station 2

In order to estimate the near-optimal admission control policy for station 2, we eliminate station 3 and compute the discounted expected net downstream revenue for a customer leaving station 2, i.e. Θ_2. We first use the upstream rate estimation recursion to get:

$$d_2 \approx \Gamma_2[1 - p(\Gamma_2, \mu_2, m_2)] = 7.9575[1 - p(7.9575, 2.5, 7)] = 7.7372,$$

and:

$$\Gamma_3 \approx d_2(1 - \mathbf{P}_2) + \lambda_3 = 7.7372 \times 0.9 + 4.22 = 11.1835$$

We then apply our downstream revenue estimation technique:

$$p_3 = 1 - p(d_2(1 - \mathbf{P}_2), \mu_3, m_3) = 1 - p(0.9d_2, 2, 6) = 1 - p(6.9635, 2, 6) = 0.9187,$$

and:

$$q_3 = \frac{\lambda_3 - \Gamma_3[1 - p(\Gamma_3, \mu_3, m_3)] + d_2(1 - \mathbf{P}_2)[1 - p(d_2(1 - \mathbf{P}_2), \mu_3, m_3)]}{d_2(1 - \mathbf{P}_2)[1 - p(d_2(1 - \mathbf{P}_2), \mu_3, m_3)]} \approx 0.3235\,.$$

and finally:

$$\Theta_2 \approx (1 - \mathbf{P}_2) p_3 \beta(\mu_2)(R_3 - q_3 r_3) = 0.9 \times 0.9187 \times 0.9608 \times (15 - 0.3235 \times 5) = 10.6313$$

The admission policy at station 2 is then found by solving the dynamic programming problem associated with the tandem queues of stations 1 and 2 using $\frac{r_2 + \Theta_2}{R_2 + \Theta_2} = \frac{15.6313}{25.6313}$, and is shown in Fig. 7. As guaranteed, the policy is a switching curve. Acceptance decisions for new customers are now based solely on the occupancy of stations 1 and 2, ignoring station 3. This switching curve also coincides with the cross-section of the optimal switching surface when station 3 has 0 or 1 customer.

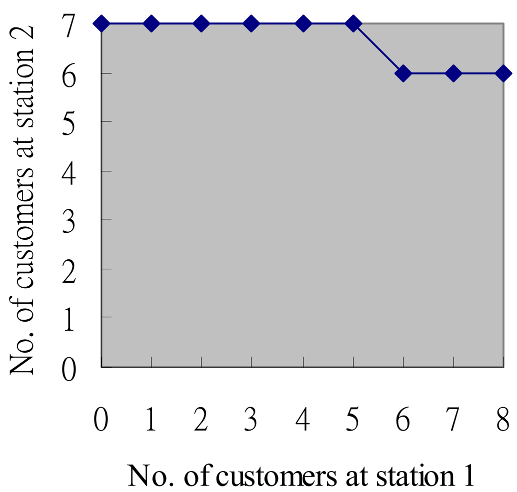

Fig. 7. The near-optimal admission policy on new customers at station 2

4.3 Accuracy

To evaluate the accuracy of the proposed techniques, the optimal and near-optimal admission control policies were numerically evaluated on the basis of the total discounted revenue they earn. For the purposes of comparison, an uncontrolled system was also evaluated. This discounted revenue depends on the initial state of the system. A range of initial states were evaluated. A sample result is shown in Fig. 8 and Table 1, wherein the initial occupancy of stations 2 and 3 are fixed, and the initial occupancy of station 1 is varied along its range.

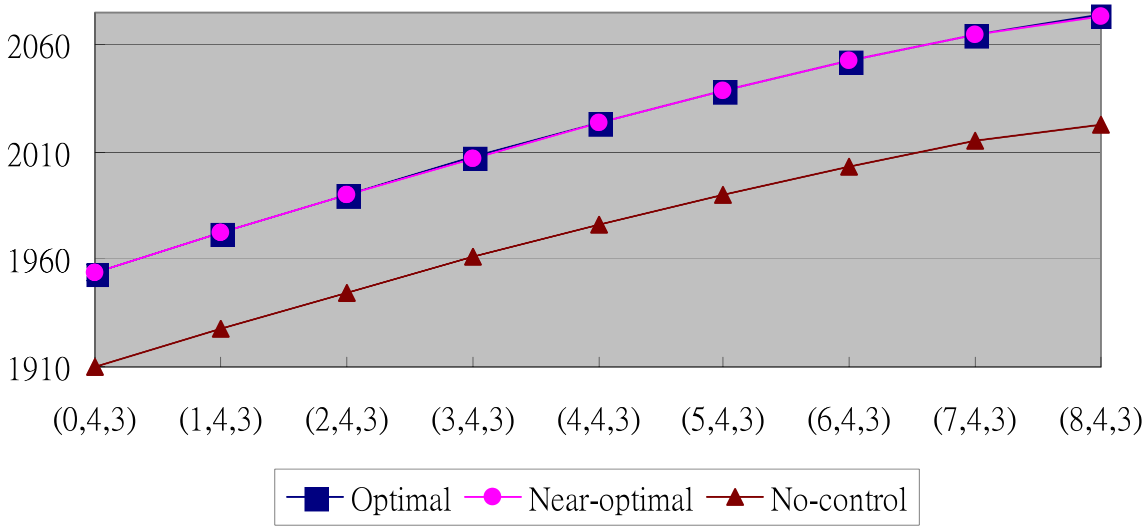

Fig. 8. The expected discounted revenues of optimal, near-optimal and no-control policies for three-queue-in-series system

State / Control	(0,4,3)	(1,4,3)	(2,4,3)	(3,4,3)	(4,4,3)	(5,4,3)	(6,4,3)	(7,4,3)	(8,4,3)
Optimal	1954.27	1972.74	1990.56	2007.60	2023.74	2038.90	2052.88	2065.19	2073.70
Near-optimal	1953.78	1972.29	1990.12	2007.17	2023.31	2038.42	2052.35	2064.59	2073.06
No-control	1910.27	1927.84	1944.73	1960.83	1976.07	1990.32	2003.42	2014.88	2022.69

Table 1. The expected discounted revenues of optimal, near-optimal and no-control policies for three-queue-in-series system

The revenue is increasing with the occupancy of station 1, due to the increased short-term (and thus lightly discounted) revenue that will be generated at stations 2 and 3 from these customers. Along the entire range, the near-optimal policy generates at least 99.9% of the revenue of the optimal policy. The difference between the revenue generated by the near-optimal policy and by no control remains at least 98.8% of the difference between the optimal policy and no policy.

4.4 Six-queue-in-series Example

When the network consists of more than 3 stations, the proposed approach remains accurate. The performance gap between the optimal policy and the proposed near-optimal policy increases slightly since more stations are eliminated. We present results for a six-queue-in-series system with the following parameters: $m_1 = 5$, $m_2 = 5$, $m_3 = 5$, $m_4 = 5$, $m_5 = 5$, $m_6 = 5$, $\alpha = 0.9999$, $Q = 2000$, $\lambda_1 = 5$, $\lambda_2 = 5$, $\lambda_3 = 5$, $\lambda_4 = 5$, $\lambda_5 = 5$, $\lambda_6 = 5$, $\mu_1 = 6$, $\mu_2 = 5.5$, $\mu_3 = 5$, $\mu_4 = 4.5$, $\mu_5 = 4$, $\mu_6 = 3.5$, $\mathbf{P}_1 = 0.2$, $\mathbf{P}_2 = 0.2$, $\mathbf{P}_3 = 0.2$, $\mathbf{P}_4 = 0.2$, $\mathbf{P}_5 = 0.2$, $r_1 = 1$, $r_2 = 1$, $r_3 = 1$, $r_4 = 1$, $r_5 = 1$, $r_6 = 1$, $R_2 = 15$, $R_3 = 15$,

$R_4 = 15$, $R_5 = 15$ and $R_6 = 15$. The optimal and near-optimal policies were numerically evaluated on the basis of the total discounted revenue they earn. For comparison purposes, an uncontrolled system was also evaluated. These revenues are shown in Table 2, for a range of initial states differing in the occupancy of the third station.

State / Control	(2,3,0,1,4,2)	(2,3,1,1,4,2)	(2,3,2,1,4,2)	(2,3,3,1,4,2)	(2,3,4,1,4,2)	(2,3,5,1,4,2)
Optimal	2869.767	2884.461	2897.215	2907.249	2913.160	2912.321
Near-optimal	2865.421	2880.006	2892.623	2902.483	2908.181	2907.230
No-control	2772.975	2786.745	2798.582	2807.728	2812.832	2811.499

Table 2. The expected discounted revenues of optimal, near-optimal and no-control policies for six-queue-in-series system

The optimal policy, found using dynamic programming on the six-dimensional state space, requires a couple of days of computation. In contrast, the near-optimal policy, found using the procedure proposed here, requires a few minutes of computation.

Along the entire range, the near-optimal policy generates at least 99.8% of the revenue of the optimal policy. The difference between the revenue generated by the near-optimal policy and by no control remains at least 95.5% of the difference between the optimal policy and no policy.

5. Conclusion

We have considered access control policies in multiserver loss queues in series. Such models arise in computer and telecommunication networks, in which continued downstream service to new customers is preferable to admission of new customers. Dynamic programming solution of the entire network results in a computational complexity that is geometric in the number of stations. We proposed a simple recursive method to solve the problem using dynamic programming on a set of reduced state spaces. This approach includes a rate estimation technique for upstream stations, and a revenue estimation technique for downstream stations, based on a light-traffic model. Numerical results demonstrate that these near-optimal policies are accurate.

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