Title
Cooperation with Rivals

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Publication Date
2006-04-11

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April 11, 2006

Abstract

The common characteristic of R&D joint ventures between oligopolistic competitors, arms reduction talks, and study groups in law school is cooperation with rivals. Players benefit from cooperation, but any gain by their partner weakens their own position when competing for profits, security, or a high class ranking. I construct a model in which players have different resource endowments and can increase them through bilateral cooperation. The final allocations enter a contest success function and determine each player's probability of winning a fixed prize. A refinement of Nash equilibrium, Pairwise Stable Nash Equilibrium (PSNE), is defined to deal with the need for mutual consent to establish cooperation. Results show that universal full cooperation is a PSNE in this zero-sum game without repeated play if no player predominates, and the only PSNE if players are free to negotiate side-payments. The model is then applied to trade between the US and China.

Cooperation with rivals is commonplace. Examples include the sharing of information between competing firms (Hippel, 1987), arms reduction talks, study groups in law school, trade between states engaged in military or economic competition (Liberman, 1996; Barbieri and Levy, 1999), and cooperation between co-workers in firms with a limited number of opportunities for promotion. Cooperation can be explicit, as when IBM and Microsoft jointly develop software (Economist, 2004), or implicit, as when two competing candidates who both lag the front runner refrain from using negative advertising targeted at each other.

In a situation of rivalry, a player's payoff increases with his own strength and decreases with the strength of other players, where strength refers to arms, patents, knowledge, or some other resource. There is thus a fundamental trade-off involved in cooperating with a rival: Gains from cooperation provide incentives to cooperate, but any gain by a partner weakens a player's position when competing head-to-head for a limited reward.

The problem is trivial when there are only two players: Any relative gain by one player is a relative loss for the other, who therefore refuses to cooperate.

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When there are more than two players, however, this simple reasoning no longer applies. If a player experiences a relative loss vis-à-vis his partner, but obtains an absolute gain, he improves his position relative to all other players, and cooperation becomes possible in equilibrium (Snidal, 1991a,b; Werner, 1997).

The literature discusses cooperation with rivals in the context of collusion in oligopoly (Cournot, 1971), R&D joint ventures (Veugelers, 1998; Caloghirou et al., 2003), relative gains (Waltz, 1979; Grieco, 1988a,b; Snidal, 1991a,b) and contests (Hirshleifer, 1991; Skaperdas, 1992; Powell, 1993; Morrow, 1997; Neary, 1997; Skaperdas and Syropoulos, 1997; Noh, 2002). In most contest models, a resource can be used either for production or appropriation. A player's payoff depends on the size of the prize and his probability of winning. The former increases with the total amount of resources used for production, while the latter is determined by a contest success function that increases in a player's resources dedicated to appropriation (arms) and decreases in his opponent's investment in arms.

This paper presents a model in which players have different resource endowments and can increase them through bilateral cooperation. The final allocations enter a contest success function and determine each player's probability of winning a fixed prize. Contrary to the convention in the literature, I disregard production and focus on the division of an exogenous prize. Any equilibrium with cooperation is thus a stronger result, since cooperation does not increase the size of the pie.

The main results are as follows: Full cooperation between all players is the only equilibrium if there are at least three players and side-payments are possible. If side-payments are not possible, universal full cooperation can still be an equilibrium if no player predominates.

The paper makes two contributions: it shows that cooperation between rivals can occur in the equilibrium of a one-shot, zero-sum game, i.e. even without repeated play or joint production in the widest sense. It also extends the contest literature by considering an unlimited number of players, without requiring them to be identical, and using a more general contest success function than most models.

The model and its main results are presented in sections 1 and 2, while section 3 develops two extensions. Section 4 discusses international trade between the US and China as an application, and section 5 concludes. The appendix contains the mathematical proofs.

1 A Model of Cooperation under Rivalry

The game under consideration is characterized by heterogeneity, gains from cooperation, and rivalry. Let $N = \{1, \ldots, n\}$ be a finite set of agents with $n \geq 2$. Players can be heterogeneous on one dimension, i.e. they might have different endowments of resources, but are identical otherwise. Player i's initial resource allocation is denoted r_i^0, with $r_i^0 \geq 1$ for all $i \in N$.

Secondly, cooperation with another player adds to the player's resources. Let c_{ij} denote the extent of player i's cooperation with player j, with $0 \leq c_{ij} \leq 1$.

Definition. Players i and j *cooperate fully* with each other whenever $c_{ij}c_{ji} = 1$. Full cooperation between all players is called *universal full cooperation*.

Let c^s_{ij} be player i's cooperation level vis-à-vis j under strategy profile s.

Definition. A *strategy* s_i for player i is a vector $(c^s_{i,1}, \ldots, c^s_{i,i-1}, c^s_{i,i+1}, \ldots, c^s_{i,n})$. A *strategy profile* s is the set of strategies $\{s_1, \ldots, s_n\}$.

Let r^+_{ij} denote the maximum gain in units of resources accruing to player i as a result of cooperation with player j, net of any costs. The value of r^+_{ij} is given exogenously for all i, j. The actual gain from cooperation is the product $c_{ij}c_{ji}r^+_{ij}$. The exact functional form of how the two cooperation levels are combined does not matter as long as each partner can unilaterally reduce the resulting gain to any level below the maximum that is determined by his partner's cooperation level.

Assumption 1 (Resource accumulation)**.** Player i's allocation of resources after playing s is

$$r_i(s) \equiv r^0_i + \sum_{j \neq i} \left(c^s_{ij} c^s_{ji} r^+_{ij} \right).$$

The assumption implies that there is no limit on the number of cooperation partners, that players strictly increase their resource allocations by further cooperation, and that they cannot end up with less resources than their original endowment. The next assumption specifies that both partners in any cooperative venture obtain equal and strictly positive gains from cooperation. It will be relaxed in extension 3.2.

Assumption 2 (Gains from cooperation)**.** $r^+_{ij} = r^+_{ji} > 0 \; \forall \; i, j \in N, i \neq j$.

The third element of the game is rivalry: Relative allocation of resources matters, since it determines a player's probability of winning an exogenous prize. Player i's probability of winning the prize, p_i, is a function of the strategy profile and is captured by the following contest success function (Skaperdas, 1996; Hirshleifer, 2000):

Assumption 3 (Contest success function)**.**

$$p_i(s) = \frac{r_i(s)^d}{\sum_{j=1}^n r_j(s)^d}.$$

Here, d is a decisiveness parameter, such that $d = 0$ corresponds to equal division of the prize irrespective of resources, $d = 1$ denotes strictly proportional division, and $d = \infty$ describes a situation where the winner takes all. Most of the contest models cited in the introduction assume $d = 1$.

The probabilities of winning the prize add up to one across all players. The term p_i can be interpreted as a player's share of the prize or his probability of winning the entire prize.

The cooperation game Γ is given by the (n+2)-tuple $(N, S_1, \ldots, S_n, u)$. Here, S_i is player i's strategy set, and the utility function u is the mapping $u : \prod_i S_i \longrightarrow \mathbb{R}^n$, with

$$u_i(s) = w \cdot r_i(s) + p_i(s).$$

The variable $w \geq 0$ is a weight determining the relative importance of resources and the probability of winning the prize in determining utility. The utility of winning the prize for certain, given $w = 0$, is normalized to one. The exact functional form of the utility function does not matter, as long as the weight on resources can switch between zero and positive.

A distinction needs to be made between games in which players care only about their probability of winning the prize, and games in which they also derive utility from their allocation of resources.

Definition. In a *pure contest*, $w = 0$; in a *mixed contest*, $w > 0$.

In a pure contest, players are only concerned with winning the prize, and any gains from cooperation are instrumental in the sense of increasing the player's chances of winning the prize. In a mixed contest, players care about gains from cooperation both for their direct impact on utility and their instrumental role. Whether resources are spent in the contest, and can thus be interpreted as a measure of effort, or remain in the possession of the players does obviously not matter in a pure contest.

Examples of pure contests include R&D races, performance evaluations at the workplace, the signaling theory of education for degree programs that publicize class rankings, and most elections. Examples of mixed contests include trade between countries engaged in economic rivalry, the formation of hunting parties in tribal societies, and study groups if students benefit directly from their education. See Table 1 for more details on these examples.

Table 1: Examples of Pure and Mixed Contests

Situation	Resource	Cooperation	Prize
Econ. competition*	GDP	Trade	Power
Education$^{(*)}$	GPA	Study group	High class ranking
Elections	Voter sympathy	No neg. ads	Win office
Managerial work	Goals met	Share inform.	Promotion
R&D race	Know-how	Joint venture	Profit
Tribal groups*	Meat provided	Hunting party	Status

Note: * denotes a mixed contest

The order of play is as follows: Players choose their strategies simultaneously. The players then implement their decisions and reap the gains from cooperation. Finally, the exogenous prize is allocated based on each player's final resources and the contest success function.

A player can unilaterally terminate a cooperative venture, but it takes the agreement of two players to establish a new one. Thus, a situation in which two players choose cooperation levels of zero with respect to each other can be a Nash equilibrium, even if cooperation would make both strictly better off without incurring any risk. An equilibrium refinement is required to rule out such situations.

The component of player i's strategy pertaining to player j is denoted by s_{ij}. Furthermore, let $\{s^*_{i-j}, s_{ij}\}$ denote a strategy for player i that is equal to s^*_i, except that the component s^*_{ij} is replaced by s_{ij}, and let s_{-ij} be the profile of strategies of all players except players i and j.

Definition. Strategy profile s^* is a *Pairwise Stable Nash Equilibrium (PSNE)* of the game Γ, if and only if it is a Nash equilibrium (NE) of Γ and, for all $i, j \in N$, $i \neq j$, $s_{ij} \in S_i$ and $s_{ji} \in S_j$:
If $\quad u_i(\{s^*_{i-j}, s_{ij}\}, \{s^*_{j-i}, s_{ji}\}, s^*_{-ij}) > u_i(s^*)$,
then $\quad u_j(\{s^*_{i-j}, s_{ij}\}, \{s^*_{j-i}, s_{ji}\}, s^*_{-ij}) < u_j(s^*)$.

In a PSNE, no two players can be made better off by changing those components of their strategies that refer to each other. The pairwise component of the solution concept is similar to pairwise stable networks as defined by Jackson and Wolinsky (1996), but allows for continuous choice variables. The concept differs from pareto dominance refinements of NE since every PSNE is pareto efficient in a pure contest: Given that the probabilities sum to one, the total payoff is always constant in a pure contest. In contrast to Coalition-Proof NE (Bernheim et al., 1987), PSNE considers only deviations from the equilibrium profile that are instigated by either one player acting alone, or two players changing only those components of their strategies that refer to each other.

2 Full Cooperation in Equilibrium

The following two subsections present the conditions for universal full cooperation in PSNE. Section 2.1 discusses the case without and section 2.2 the case with side-payments.

2.1 Cooperation without Side-Payments

Players can benefit from bilateral cooperation in a multi-player game even if their position relative to their cooperation partner deteriorates, since cooperation strengthens their position vis-à-vis all other players. The trade-off between benefiting oneself and a rival shows up in the marginal utility for increasing cooperation with another player, given some strategy profile s:

$$\frac{\partial u_i(s)}{\partial c_{ij}} = \frac{d r_i(s)^{d-1} \frac{\partial r_i(s)}{\partial c_{ij}} \sum_{k \neq i} r_k(s)^d}{\left(\sum_k r_k(s)^d\right)^2} - \frac{d r_i(s)^d r_j(s)^{d-1} \frac{\partial r_j(s)}{\partial c_{ij}}}{\left(\sum_k r_k(s)^d\right)^2}.$$

The first term on the RHS is the marginal benefit from increasing player i's allocation of resources through increased cooperation, while the second term is the marginal disutility from increasing the partner's allocation. Since all variables are positive, more cooperation is in i's interest only if the first terms exceeds the second. Further manipulation of the above equality leads to the following lemma. All proofs are given in the appendix.

Lemma 1. *Given assumptions 1 to 3, cooperation in a pure contest without side-payments is all-or-nothing in all NE for $d < 1$. For $d \geq 1$, intermediate cooperation levels are possible in equilibrium.*

The following proposition specifies the conditions under which full cooperation by all players is a PSNE. Let s^* be the strategy profile of universal full cooperation, and let r_i^* be player i's resource allocation under s^*: $r_i^* \equiv r_i(s^*) = r_i^0 + \sum_{j \neq i} r_{ij}^+$.

Proposition 1. *Given assumptions 1 to 3, universal full cooperation is a PSNE in a pure contest without side-payments if no player predominates, i.e. if, for all distinct i, j, $\sum_{k \neq i,j} {r_k^*}^d \geq (r_i^* - r_j^*)(r_j^* - r_{ji}^+)^{d-1}$ for $d < 1$, or iff $\sum_{k \neq i} {r_k^*}^d \geq r_i^* {r_j^*}^{d-1}$ for $d \geq 1$.*

The trade-off between benefiting oneself and a rival works in favor of cooperation if no player is too strong. The more resources player i has in equilibrium compared to some player j, the less likely that the conditions in the proposition will be satisfied. Intuitively, cooperation with equal sharing of the gains from cooperation hurts the larger partner in any pair, while benefiting the smaller partner. Nevertheless, cooperation is in the interest of the larger player if other rivals are sufficiently strong.

A numerical example helps illustrate the result. Consider a pure contest with three players, and assume that the maximum gains from bilateral cooperation, r_{ij}^+, are constant at one unit for all players. In the example in Table 2, full cooperation between players 2 and 3 only is the unique PSNE. No player can increase his payoff by adding or terminating a cooperative venture under his control, and no other constellation constitutes a PSNE. The payoff of the largest player, player 1, would decrease from .55 to .54 if he cooperated with one of the smaller players, and to .53 if he cooperated with both. If, on the other hand, player 1's initial resource allocation were smaller, say 4 instead of 6, or one of the other players had more initial resources, the condition for $d \geq 1$ in proposition 1 would be satisfied and universal full cooperation would be a PSNE.

Table 2: Example - Cooperation between Players 2 and 3 as PSNE

	Initial position		Full cooperation in pairs:									
			{2,3}		{1,2}, {2,3}		{1,3}, {2,3}		all		none	
i	r_i^0	p_i	r_i	p_i	r_i	p_i	r_i	p_i	r_i	p_i	r_i	p_i
1	6	.67	6	.55	7	.54	7	.54	8	.53	6	.67
2	2	.22	3	.27	4	.31	3	.23	4	.27	2	.22
3	1	.11	2	.18	2	.15	3	.23	3	.20	1	.11

Note: $d = 1$, $r_{ij}^+ = 1$ for all i, j; italics denote PSNE

The conditions stated in proposition 1 are sufficient for $d < 1$, and necessary and sufficient for $d \geq 1$. It is not, however, immediately obvious whether they are satisfied. The following corollary contains two simpler sufficient conditions.

Corollary 1. *Given assumptions 1 to 3, universal full cooperation is a PSNE in a pure contest without side-payments if, for all* i, $\sum_{k \neq i} {r_k^*}^d \geq r_i^*$ *for* $d < 1$ *or* $p_i(s^*) \leq 0.5$ *for* $d \geq 1$.

Finally, if no player benefits from reducing his cooperation levels in a pure contest, their utility would drop even more after limiting cooperation in a mixed contest, with $w > 0$. The proof of the following corollary is trivial and therefore omitted.

Corollary 2. *Given assumptions 1 to 3 and no side-payments, if universal full cooperation is a PSNE in a pure contest, it is also a PSNE in a mixed contest.*

Since many models with contest success functions assume $d = 1$, the following proposition determines the set of possible equilibria for this special case. Let l be the largest player given universal full cooperation, i.e. $l = argmax_i\{r_i^*\}$.

Proposition 2. *Given assumptions 1 to 3 and* $d = 1$, *there is always a unique PSNE in a pure contest with* $n > 2$: *universal full cooperation if* $p_l(s^*) < 0.5$, *arbitrary cooperation levels by* l *and full cooperation by all other players if* $p_l(s^*) = 0.5$, *and no cooperation by* l *and full cooperation by all other players otherwise.*

2.2 Cooperation with Side-Payments

In a game with side-payments, players can decide how to share the gains from cooperation.

Definition. In the game with side-payments, players i and j can agree on a *sharing arrangement* such that player i receives a fraction f_{ij} of the gains from cooperation, and player j receives a fraction $f_{ji} = (1 - f_{ij})$, with $0 \leq f_{ij} \leq 1$.

If players are able to negotiate side-payments, player i's gain can be expressed as a fraction of the total gains $r_{i+j}^+ \equiv r_{ij}^+ + r_{ji}^+$, such that player i's gain from the cooperation with j is $f_{ij} c_{ij} c_{ji} r_{i+j}^+$. The game without side-payments is thus a game with side-payments in which the fractions are given exogenously for every pair.

The definition implies that the fractions add up to one. If this is not true, some additional assumptions are necessary: If the sum of the fractions exceeds one, no cooperation occurs; if it falls short of one, the remaining gain is wasted. Given these assumptions, it is clear that the fractions must add up to one in any Nash equilibrium.

Definition. In the game with side-payments, a *strategy* s_i for player i is a pair of vectors $\big((c_{i,1}^s, \ldots, c_{i,i-1}^s, c_{i,i+1}^s, \ldots, c_{i,n}^s), (f_{i,1}^s, \ldots, f_{i,i-1}^s, f_{i,i+1}^s, \ldots, f_{i,n}^s)\big)$.

Given these definitions, we can specify resource accumulation in the game with side-payments.

Assumption 4 (Resource accumulation)**.** Player i's allocation of resources after playing s is

$$r_i(s) \equiv r_i^0 + \sum_{j \neq i} \left(f_{ij}^s c_{ij}^s c_{ji}^s r_{i+j}^+ \right).$$

The following proposition sums up the main result.

Proposition 3. *Given assumptions 2 to 4, full cooperation can be sustained as a NE in a pure contest with side-payments for $n = 2$. For $n > 2$, there is always a PSNE, and all PSNE are characterized by universal full cooperation.*

The proof in the appendix is based on ratio-preserving fractions. Let s^{-ij} be the strategy profile of universal full cooperation except for no cooperation between i and j. If two players split the gains from cooperation such that each player i obtains a fraction $r_i(s^{-ij})/(r_i(s^{-ij})+r_j(s^{-ij}))$, the ratio of their probabilities remains constant for any level of cooperation. This sharing arrangement is called ratio-preserving sharing.

A practical problem with ratio-preserving sharing is that the fractions must be determined iteratively, since each depends on all other cooperative ventures and their sharing arrangements. In applications of the model, $r_i(\cdot)$ can be approximated by r_i^0 if gains from cooperation are small relative to initial endowments, or it can be calculated numerically.[1]

In the example in Table 3, universal full cooperation is the unique PSNE. Terminating any cooperative venture reduces the payoff of the two players involved. Furthermore, though not shown in the table, any player terminating two ventures sees his payoff reduced even more. The maximum gain from cooperation is again assumed to be one unit for every pair of players. The fractions represent a set of ratio-preserving sharing arrangements.

Table 3: Example - Full Cooperation with Side-Payments as PSNE

	Initial position		Ratio-preserving fractions			Full cooperation in pairs:							
						all		{1,2},{2,3}		{1,3},{2,3}		{1,2},{1,3}	
i	r_i^0	p_i	$f_{i,1}$	$f_{i,2}$	$f_{i,3}$	r_i	p_i	r_i	p_i	r_i	p_i	r_i	p_i
1	6	.67	-	.72	.83	7.55	.63	6.72	.61	6.83	.62	7.55	.69
2	2	.22	.28	-	.66	2.94	.25	2.94	.27	2.66	.24	2.28	.21
3	1	.11	.17	.34	-	1.50	.13	1.34	.12	1.50	.14	1.17	.11

Note: $d = 1$, $r_{ij}^+ = 1$ for all i, j; italics denote PSNE

Ratio-preserving sharing, however, is not the only sharing arrangement supporting universal full cooperation, as stated by the following corollary.

Corollary 3. *For $n > 2$, sharing arrangements other than ratio-preserving sharing support universal full cooperation in equilibrium.*

[1] A sample Matlab code for a numerical solution is available from the author.

It should be obvious that if the result holds in a pure contest, it must also be true in a mixed contest, with $w > 0$. In that case, a range of fractions around ratio-preserving sharing allows for full cooperation in equilibrium, independent of the number of players. The proof is trivial and is omitted.

Corollary 4. *Given assumptions 2 to 4, every PSNE in a mixed contest with side-payments is characterized by universal full cooperation.*

3 Extensions

3.1 Generalized Rivalry with Side-Payments

In the game with side-payments, I can relax the assumption on the contest success function and still obtain an interesting result. The following assumption is weaker than assumption 3, and replaces it in this section.

Assumption 5 (Generalized CSF)**.**

$$p_i(s) = \frac{h(r_i(s))}{\sum_{j=1}^{n} h(r_j(s))},$$

$h(x) > 0 \; \forall \; x > 0$; $h(x)$ is continuous; $x < y \Rightarrow h(x) < h(y) \; \forall \; x, y > 0$.

To make the model more tractable, I restrict cooperation to be all or nothing, i.e. $c_{ij} \in \{0, 1\}$. Contrary to the result in section 2.2, the existence of a PSNE is no longer guaranteed in the game with more than two players. If there is a PSNE, however, it must still include universal full cooperation.

Proposition 4. *Given assumptions 4 and 5, cooperation can be sustained as a NE in a pure contest with all-or-nothing cooperation and side-payments for $n = 2$. For $n > 2$, every PSNE is characterized by universal full cooperation.*

As before, if the result holds in a pure contest, it must also be true in a mixed contest, with $w > 0$. The proof is trivial and is omitted.

Corollary 5. *Given assumptions 4 and 5, every PSNE in a mixed contest with all-or-nothing cooperation and side-payments is characterized by universal full cooperation.*

3.2 Unequal Gains without Side-Payments

This section allows for unequal gains from cooperation, thus relaxing assumption 2. Lemma 1 still holds and is restated here for completeness.

Lemma 2. *Given assumptions 1 and 3, cooperation in a pure contest without side-payments is all-or-nothing in all NE for $d < 1$. For $d \geq 1$, intermediate cooperation levels are possible in equilibrium.*

As before, let $r_i^* \equiv r_i(s^*) = r_i^0 + \sum_{k \neq i} r_{ik}^+$. Without loss of generality, assume that every player i orders the other players such that, whenever $j < k$,

$$r_j^{*d-1} r_{ji}^+ / r_{ij}^+ \leq r_k^{*d-1} r_{ki}^+ / r_{ik}^+. \tag{1}$$

Proposition 5. *Given assumptions 1, 3 and ordering* (1)*, universal full cooperation is a PSNE in a pure contest without side-payments if no player predominates, i.e. if* $\sum_{k \neq i,j} r_k^{*d} \geq (r_i^* r_{ji}^+ / r_{ij}^+ - r_j^*)(r_j^* - r_{ji}^+)^{d-1}$ *for* $d < 1$*, or if* $\sum_{k \neq i,j} r_k(s^{ij*})^d > \big(r_i(s^{ij*}) r_{ji}^+ / r_{ij}^+ - r_j^*\big) r_j^{*d-1}$ *for* $d \geq 1$*, for all distinct* i, j.

Here, s^{ij*} is the same strategy profile as s^*, except that, for all $k > j$, player i chooses cooperation levels c_{ik}^{ij*} such that

$$d\big(r_k^* + [c_{ik}^{ij*} - 1] r_{ki}^+\big)^{d-1} = r_j^{*d-1} \frac{r_{ji}^+}{r_{ij}^+} \frac{r_{ik}^+}{r_{ki}^+}, \tag{2a}$$

if this equality can be satisfied by choosing $0 < c_{ik}^{ij*} \leq 1$, and

$$c_{ik}^{ij*} = 0 \text{ otherwise.} \tag{2b}$$

Proposition 5 contains a rather complex condition for the case of $d \geq 1$, as it requires the determination of cooperation levels c_{ik}^{ij*} for all i, j, k. The following corollary states a simpler condition for universal full cooperation to be a PSNE, by placing restrictions on the maximum gains from cooperation. As before, let s^* be the strategy profile of universal full cooperation, and let $s^{i>j}$ be the same, except that player i chooses $c_{ik} = 0$ for all $k > j$.

Corollary 6. *Given assumptions 1 and 3, ordering* (1) *and* $d \geq 1$*, universal full cooperation is a PSNE in a pure contest without side-payments if, for all distinct* i, j*,* $\sum_{k \neq i,j} r_k(s^{i>j})^d \geq \big(r_i(s^{i>j}) r_{ji}^+ / r_{ij}^+ - r_j^*\big) r_j^{*d-1}$*, and* $r_{ij}^+ \leq \big(1 - d^{1/(1-d)}\big) r_i^*$.

The second condition in the corollary is easily satisfied for moderate levels of rivalry. For $d \leq 2$, it is satisfied whenever $r_{ij}^+ \leq 0.5\ r_i^*$, or in other words, whenever no player can more than double his entire allocation by cooperating with one more player.

As before, if no player benefits from reducing his cooperation levels in a pure contest, their utility would drop even more after limiting cooperation in a mixed contest, with $w > 0$. The proof of the following corollary is trivial and is omitted.

Corollary 7. *Given assumptions 1 and 3 and no side-payments, if universal full cooperation is a PSNE in a pure contest, it is also a PSNE in a mixed contest.*

4 Application to International Trade

Governments care not only about national welfare, but also about power (Cohen, 1990; Richardson, 1990). While economists have traditionally treated international trade as a purely economic activity, political scientists often view it as a situation of rivalry. Neorealists argue that international anarchy fosters competition and conflict among states since any disproportional economic gain by one state can be converted into military advantage which may then threaten the independence or even survival of other states (Waltz, 1979). Grieco (1988a, p. 498) affirms that "The fundamental goal of states in any relationship is to prevent others from achieving advances in their relative capabilities."

The importance of relative gains does not depend on the use of force. As external security threats diminish, states pursue relative gains more forcefully in economic relations so that even relations among potential allies are influenced by relative gains considerations (Mastanduno, 1991; Luttwak, 1993; Liberman, 1996). The main conflicts of interest between the major powers are likely to be over economic issues, since in the absence of military conflict, economic activity is the most important source of power (Huntington, 1993).

The current scapegoat for various economic ills is China. A poll taken in 2005 found that 54 percent of Americans viewed China's emergence as a superpower as a threat to world peace (Ipsos-Reid, 2005). American politicians often advocate a curtailment of trade with China (Economist, 2004). President Bush called China a "strategic competitor" and imposed quotas on a range of Chinese exports (Economist, 2001, 2003). Former President Clinton acknowledged, if not approved, the notion of China as a rival in saying "Some Americans believe we should try to isolate and contain China . . . in order to retard its capacity to become America's next great enemy" (Economist, 1998). Rousseau (2002, p. 667) neatly sums up the argument: "The net gains of trading with such a vast untapped market are huge . . . . But if agreeing to trade with China contributes to such a rapid rise in Chinese power that it becomes more threatening to US security, then the United States should still reject such opportunities of cooperation on grounds of relative gains concerns."

If the US government cared only about economic power, does it necessarily follow that free trade with China is not in its best interest? The model presented here can help answer that question. Economic power is a function of economic output, so that GDP can be treated as the resource in a pure contest in which p_i is country i's probability of being able to exert political or economic influence. The utility of producing the entire world's output is normalized to 1.

A reasonable assumption is that economic influence is roughly proportional to a country's share of global economic activity. In this model, the assumption translates to $d = 1$. I further assume that gains from cooperation are the same for both partners and that side-payments in units of GDP are not possible. I will relax the first two assumptions in a sensitivity analysis later.

Since gains from cooperation are relatively small for countries the size of the US and China, we can approximate each country's resources in the absence of a particular cooperative venture by its resources with universal full cooperation, i.e. its GDP. With equal gains from cooperation and $d = 1$, US marginal utility from cooperating more with China is non-negative whenever $\sum_{i \neq US} r_i^* \geq r_{US}^*$. In other words, the GDP of the rest of world, including China, must exceed American output for the US not to have an incentive to limit cooperation with China.

Purchasing power parity estimates of GDP are good for comparing living standards, but "for geo-political purposes or impact on the world economy, however, the ... figure based on market exchange rates is much more appropriate" (Cooper, 2000, p. 3). Using the IMF's IFS database from September 2004, and converting all national currency figures to US dollars at average annual exchange rates, in 2002 the US had a GDP of \$10.5 trillion, China \$1.4 trillion and the rest of world combined \$19.7 trillion. Since the condition was clearly satisfied in 2002, curtailing trade with China was not in the best interest of the US even if one assumes pure power maximization. Since China has grown more rapidly than the US, the condition has been satisfied hence. China also has an incentive to cooperate fully, due to its smaller GDP.

To see if the result holds for varying assumptions, I perform a sensitivity analysis for d and the ratio of gains from cooperation. As shown in expression (5) in the appendix, American marginal utility for more cooperation with China is non-negative whenever $\sum_{i \neq US,PRC} r_i^{*d} \geq \left(r_{US}^* \frac{r_{PRC}^+}{r_{US}^+} - r_{PRC}^* \right) r_{PRC} (s^{ij})^{d-1}$. Assuming $d = 1$, the maximum value for r_{PRC}^+ / r_{US}^+ to support full cooperation in equilibrium is

$$\frac{r_{PRC}^+}{r_{US}^+} \leq \frac{\sum_{i \neq US} r_i^*}{r_{US}^*} = \frac{19.7 + 1.4}{10.5} = 2.$$

The US would have an incentive to reduce cooperation only if China's benefit from full cooperation was more than double the US benefit. On the other hand, assuming again equal gains from cooperation, expression (5), evaluated at $s^{ij} = s^*$, holds for all values of d.

Therefore, a rather large discrepancy in benefits must occur for the US to have an incentive to limit cooperation. Furthermore, if we relax the extreme assumption of pure power maximization and consider mixed rather than pure contests, full cooperation between the US and China can occur in equilibrium for a wide range of relative benefits and values of d.

5 Conclusion

A player's rivals are all other players whose increased strength decreases the player's probability of winning an exogenous prize. The model developed in this paper analyzes situations of rivalry and determines the conditions under which cooperation between any two players can be sustained in equilibrium.

Players are allowed to differ on one dimension such that a player's allocation of resources, e.g. power, GDP or know-how, determines his probability of winning the prize. The main assumptions are that bilateral cooperation increases both players' resource allocations, and that the prize is allocated based on a contest success function of the power form.

A refinement of Nash equilibrium, Pairwise Stable Nash Equilibrium, is introduced to deal with the need for mutual consent to establish a cooperative venture. Results show that universal full cooperation is a PSNE in this zero-sum game without repeated play if third parties are sufficiently large for every pair of players, and the only PSNE if there are at least three players and side-payments in units of resources are possible.

The model is applied to trade between the US and China. Full cooperation between the US and China is an equilibrium strategy in a pure contest for power if China's benefit from full cooperation is not more than double the benefit to the US.

Appendix A. Proofs

Note that universal full cooperation must be a PSNE if it is a NE.

Proof of Lemma 1. Let s^{ij} be the strategy profile of universal full cooperation, except that player i chooses any c_{ij}. Using assumptions 1 and 2, noting that $r_k(s^{ij}) = r_k^*$ for all $k \neq i, j$, and $r_i(s^{ij}) - r_j(s^{ij}) = r_i^* - r_j^*$, shows that marginal utility $\partial u_i(s^{ij})/\partial c_{ij}$ is non-negative whenever

$$\sum_{k \neq i,j} {r_k^*}^d \geq (r_i^* - r_j^*) r_j(s^{ij})^{d-1}. \tag{3}$$

Since $r_j(s^{ij})$ increases strictly in c_{ij} by assumption 1, the RHS decreases in c_{ij} for $d < 1$ and increases for $d > 1$, while the LHS is independent of c_{ij}. The resulting plot of utility against c_{ij} is thus either monotonic over the range of c_{ij}, or it is U-shaped for $d < 1$ or inversely U-shaped for $d > 1$. For $d < 1$, only extreme cooperation levels, $c_{ij} \in \{0, 1\}$, can thus occur in equilibrium. □

Proof of Proposition 1. Satisfaction of expression (3) evaluated at $c_{ij} = 0$ ($c_{ij} = 1$) for $d < 1$ ($d \geq 1$) ensures non-negative marginal utility over the entire range of c_{ij}. Therefore, no player has an incentive to reduce cooperation with any one rival if expression (3) with $r_j(s^{ij}) = r_j^* - r_{ji}^+$ for $d < 1$, or $r_j(s^{ij}) = r_j^*$ for $d \geq 1$ holds for all distinct $i, j \in N$. The remainder of the proof, done separately for $d < 1$ and $d \geq 1$, demonstrates that no player has an incentive to reduce cooperation with more than one rival.

$\boldsymbol{d < 1}$ Since player i never has an incentive to choose an intermediate cooperation level for $d < 1$ (see lemma 1), I consider only deviations from universal full cooperation that reduce cooperation with some players to zero. Let s^D be such a deviation, where $D_i \subseteq N \backslash \{i\}$ is the set of players dropped by player i, i.e. $c_{ij} = 0$ for all $j \in D_i$, and C_i the complement of D_i, such that $c_{ij} = 1$ for all $j \in C_i$. Furthermore, let l be the largest player in the set D_i in the sense of $l = \text{argmax}_{j \in D_i} \left\{ (r_j^* - r_{ji}^+)^{d-1} \right\}$. Player i's marginal utility for cooperation with l, $\partial u_i(s^D)/\partial c_{il}$, is non-negative for all values of c_{il} whenever

$$\sum_{j \in C_i} {r_j^*}^d + \sum_{j \in D_i} (r_j^* - r_{ji}^+)^d \geq \Big(r_i^0 + \sum_{j \in C_i} r_{ij}^+ \Big) (r_l^* - r_{li}^+)^{d-1}. \tag{4}$$

The first equilibrium condition stated in proposition 1 can be written as $\sum_{j\neq i,l} r_j^{*d} + (r_l^* - r_{li}^+)^d \geq (r_i^0 + \sum_{j\neq i,l} r_{ij}^+)(r_l^* - r_{li}^+)^{d-1}$. Expression (4) is true if the LHS of (4) falls short of the LHS of the previous inequality by no more than the RHS: $\sum_{j\in(D_i\setminus\{l\})}(r_j^{*d} - (r_j^* - r_{ji}^+)^d) \leq (\sum_{j\in(D_i\setminus\{l\})} r_{ij}^+)(r_l^* - r_{li}^+)^{d-1}$. This inequality is clearly true if it is true for each summand, or if $r_j^{*d} - (r_j^* - r_{ji}^+)^d \leq r_{ij}^+(r_l^* - r_{li}^+)^{d-1}\ \forall\ j \in (D_i\setminus\{l\})$. Since l is the largest player dropped by i, we know that $(r_j^* - r_{ji}^+)^{d-1} \leq (r_l^* - r_{li}^+)^{d-1}$. Thus, (4) is true whenever $r_j^{*d-1} \leq (r_j^* - r_{ji}^+)^{d-1}$, which is always true for $d < 1$. Player i thus does not have an incentive to reduce cooperation with l. Since the same argument also holds for the next largest player in D_i after l, and all remaining players in D_i (in descending order), player i increases his utility by resuming full cooperation with all players. Universal full cooperation is thus a NE.

$\boldsymbol{d \geq 1}$ The test case for universal full cooperation to be a NE is the situation when the equilibrium condition holds with equality for all rivals of one player, say player i. In that case, marginal utility is zero initially; a given reduction in c_{ij}, for all j, lowers utility less than starting with strictly positive marginal utility. That is, for all $j \in N\setminus\{i\}$, $\sum_{k\neq i} r_k^{*d} = r_i^* r_j^{*d-1}$. For $d = 1$, marginal utility is always zero, and universal full cooperation thus a NE. For $d > 1$, r_j^* must be the same for all j. By implication, $r_i^* = (n-1)r_j^*$ in the initial state of universal full cooperation.

As i reduces some c_{ik} infinitesimally, the LHS of the equilibrium equality decreases by dr_k^{*d-1}, while the RHS falls by r_j^{*d-1}. Since $r_k^* = r_j^*$, the LHS decreases more than the RHS for $d > 1$. In that case, marginal utility turns negative and i has an incentive to reduce c_{ij} for all players $j \neq i, k$. To maximize utility, player i chooses some strategy s such that all conditions hold with equality again. Therefore, all of i's rivals must still be identical, and $r_i(s) = (n-1)r_j(s)$. Let q be the fraction of player j's resource allocation under s^* that is preserved under s. Player i's utility in a pure contest playing s is

$$u_i(s) = \frac{((n-1)r_j(s))^d}{((n-1)r_j(s))^d + (n-1)r_j(s)^d} = \frac{((n-1)qr_j^*)^d}{((n-1)qr_j^*)^d + (n-1)(qr_j^*)^d} = u_i(s^*).$$

With utility constant, player i does not have an incentive to reduce any cooperation level; and universal full cooperation is a NE. $\square$

Proof of Corollary 1. If $r_i^* < r_j^*$, then the inequalities in proposition 1 are necessarily satisfied, ensuring a NE. If $r_i^* \geq r_j^*$, $p_i(s^*) \leq 0.5$ implies $\sum_{k\neq i} r_k^{*d} \geq r_i^* r_j^{*d-1}$ for $d \geq 1$. For $d < 1$, $\sum_{k\neq i} r_k^{*d} \geq r_i^*$ implies $\sum_{k\neq i,j} r_k^{*d} \geq r_i^* - r_j^* \Rightarrow \sum_{k\neq i,j} r_k^{*d} \geq (r_i^* - r_j^*)(r_j^* - r_{ji}^+)^{d-1}$, thus satisfying the conditions in proposition 1, proving that universal full cooperation is a NE. $\square$

Proof of Proposition 2. If $p_l(s^*) < 0.5 \Leftrightarrow \sum_{k\neq l} r_k^* > r_l^*$ for $i = l$, it must also be true for all i. Therefore, the condition for $d \geq 1$ in proposition 1 is satisfied and universal full cooperation is a PSNE. If $\sum_{k\neq l} r_k^* = r_l^*$, $\partial u_l/\partial c_{lk} = 0$ for all k, so that l is indifferent between more and less cooperation. With equal gains from cooperation, the equality still holds even without any cooperation by l: $\sum_{k\neq l}(r_k^* - r_{kl}^+) = r_l^0$. By implication, $\sum_{k\neq i,l}(r_k^* - r_{kl}^+) + r_l^0 > r_i^* - r_{il}^+$ for all $i \neq l$. Therefore, all other players have an incentive to cooperate fully. Finally, if $\sum_{k\neq l} r_k^* < r_l^*$, l has an incentive not to cooperate with any player. As shown before, all other players still have an incentive to choose full cooperation.

For some strategy profile s to be another PSNE in either of the three cases, at least two players i and j must not have an incentive to cooperate fully with some other players, i.e. $\sum_{k\neq i} r_k(s) \leq r_i(s)$ and $\sum_{k\neq j} r_k(s) \leq r_j(s)$. Since both inequalities cannot be satisfied simultaneously for $n > 2$, universal full cooperation is the unique PSNE. $\square$

Proof of Proposition 3. Let s be a strategy profile in which player i chooses $c_{ij} < 1$, and let s^+ be the same profile except that i chooses some cooperation level $c_{ij}^+ > c_{ij}$. If the ratio of two players' probabilities of winning the prize is the same with more cooperation as without, both must be better off with more cooperation in the game with $n > 2$ players. Setting the ratios equal, $p_i(s^+)/p_j(s^+) = p_i(s)/p_j(s)$, and solving for the implied sharing arrangement reveals the following fraction: $f_{ij} = r_i(s)/(r_i(s) + r_j(s))$, which I call ratio-preserving sharing.

If two players divide the additional gain from full cooperation accordingly, their allocations increase while maintaining the same ratio of probabilities. Both will be better off. Therefore, less than full cooperation cannot occur in PSNE. If two players split the entire gain accordingly, neither has an incentive to limit cooperation with his partner in the game with $n > 2$ players.

In the game with two players, the two payoffs must add up to one. Ratio-preserving sharing is the only division that does not change either player's payoff as a result of cooperation. Any level of cooperation can thus be sustained as a NE with ratio-preserving sharing. This argument proves the first part of proposition 3.

I now show that in the game with $n > 2$ players, universal full cooperation with ratio-preserving sharing of all the gains from cooperation is a NE. Let s^* be the strategy profile of universal full cooperation, and let s^{ik} be the same except for i choosing a cooperation level $c_{ik} < 1$. Suppose player i reduces his cooperation with player k from full cooperation, thus switching from s^* to s^{ik}. Given that the ratio p_i/p_k stays constant due to ratio-preserving sharing, his utility must fall in response to the decrease in his resources.

The LHS of the following inequality gives the ratio-preserving fraction for player i when cooperating with j, under strategy profile s^*, while the RHS represents the sharing arrangement that keeps the ratio p_i/p_j constant given the limited cooperation levels between i and k as specified by s^{ik}:

$$\frac{r_i(s^*)}{r_i(s^*) + r_j(s^*)} > \frac{r_i(s^{ik})}{r_i(s^{ik}) + r_j(s^{ik})}.$$

Since $r_j(s^*) = r_j(s^{ik})$ and $r_i(s^*) > r_i(s^{ik})$, the LHS is greater than the RHS: Given the fractions specified by s^*, player i obtains a larger share of the gains from cooperation with j than is required to maintain a constant ratio under strategy profile s_{ik}. Decreasing c_{ij} thus reduces p_i/p_j, r_i, r_j, and consequently p_i. Therefore, reducing cooperation with any one player decreases utility, and reducing cooperation with any other player decreases utility even further, making universal full cooperation a NE. $\square$

Proof of Corollary 3. Replacing the exogenous gains from cooperation in expression (5) by $r^+_{ij} = f_{ij} r^+_{i+j}$ and $r^+_{ji} = (1-f_{ij}) r^+_{i+j}$ yields the following condition for player i: $\sum_{k\neq i,j} r_k^{*d} \geq \left(r_i^* \frac{1-f_{ij}}{f_{ij}} - r_j^*\right)\left(r_j^* + (c_{ji}c_{ij} - 1)(1-f_{ij}) r^+_{i+j}\right)^{d-1}$. For ratio-preserving sharing, $f_{ij} = \frac{r_i(s^{-ij})}{r_i(s^{-ij}) + r_j(s^{-ij})}$, the condition reduces to $\sum_{k\neq i,j} r_k^{*d} \geq 0$, and is thus always satisfied for $n \geq 2$ players. A sum greater than zero allows for deviations from ratio-preserving sharing. As a result, there is an interval around the ratio-preserving fractions such that full cooperation makes both players unambiguously better off. $\square$

Proof of Proposition 4. Let s^* be some strategy profile in which players i and j cooperate: $c_{ij}c_{ji} = 1$; let s^{-ij} be the same as s^*, except that players i and j do not cooperate, i.e. $c_{ij}c_{ji} = 0$. Cooperation between i and j can be sustained in equilibrium if no player has an incentive to terminate the cooperation, i.e. if $p_i(s^*) \geq p_i(s^{-ij})$ and $p_j(s^*) \geq p_j(s^{-ij})$. Solving for $\sum_{k\neq i,j} h(r_k(s^*))$ yields

$$\Leftrightarrow \sum_{k\neq i,j} h(r_k(s^*)) \geq \frac{h(r_i(s^{-ij}))h(r_j(s^*)) - h(r_j(s^{-ij}))h(r_i(s^*))}{h(r_i(s^*)) - h(r_i(s^{-ij}))} \text{ and}$$

$$\sum_{k\neq i,j} h(r_k(s^*)) \geq -\frac{h(r_i(s^{-ij}))h(r_j(s^*)) - h(r_j(s^{-ij}))h(r_i(s^*))}{h(r_j(s^*)) - h(r_j(s^{-ij}))}.$$

Since $p_i(s^*) < p_i(s^{-ij})$ for $f_{ij} = 0$ and $p_i(s^*) > p_i(s^{-ij})$ for $f_{ij} = 1$, there must be a unique $f^*_{ij} \in (0,1)$ such that $p_i(s^*) = p_i(s^{-ij})$, due to the strict monotonicity and continuity of $r_i(\cdot)$, $h(\cdot)$ and the contest success function (assumptions 4, 5).

In the game with $n = 2$, the LHS is zero. Therefore, $p_i(s^*) = p_i(s^{-ij})$ only if the numerator on the RHS also equals zero. Since both numerators are identical, both conditions must hold with equality. Therefore, there is a unique f^*_{ij} such that neither player is better off or worse off as a result of cooperation.

With $n > 2$ players, the LHS is strictly positive. Since the numerators are identical and both denominators are strictly positive, the RHS of one inequality must be non-positive, strictly satisfying the inequality. For any sharing arrangement, at least one partner must thus be strictly better off with cooperation.

As f_{ij} increases from 0 to 1, $p_i(s^*)$ rises above $p_i(s^{-ij})$ at some point, while $p_j(s^*)$ falls below $p_j(s^{-ij})$ at some other point. Since at least one player must always be strictly better off with cooperation, the equality of probabilities for

j must occur at a greater value of f_{ij} than the equality for i. Therefore, there is always a fraction, and in fact a range of sharing arrangements around this fraction, such that both players are strictly better off with cooperation. As a result, any strategy profile in which two players do not cooperate with each other cannot be a PSNE. Therefore, all PSNE include universal cooperation. □

Proof of Lemma 2. The proof is identical to the proof of lemma 1, except that marginal utility $\partial u_i(s^{ij})/\partial c_{ij}$ is non-negative whenever

$$\sum_{k\neq i,j} r_k^{*d} \geq \Big(r_i^* \frac{r_{ji}^+}{r_{ij}^+} - r_j^*\Big) r_j(s^{ij})^{d-1}. \tag{5}$$

□

Proof of Proposition 5. For $d < 1$, and l denoting the largest player dropped in the sense of $l = \operatorname{argmax}_{j\in D_i} \big\{(r_j^* - r_{ij}^+)^{d-1} r_{ji}^* / r_{ij}^*\big\}$, player i's marginal utility for cooperation with l is non-negative for all values of c_{il} whenever $\sum_{j\in C_i} r_j^{*d} + \sum_{j\in D_i}(r_j^* - r_{ji}^+)^d \geq \Big(r_i^0 + \sum_{j\in C_i} r_{ij}^+\Big)(r_{li}^+/r_{il}^+)(r_l^* - r_{li}^+)^{d-1}$. Following similar steps as in the proof of proposition 1, it can be shown that this inequality is satisfied whenever the expression for $d < 1$ in proposition 5 holds, thus making universal full cooperation a NE.

For $d \geq 1$, let $s^{i<j}$ be the strategy profile in which all players except i fully cooperate, and i cooperates fully only with the first $j - 1$ players, ordered according to (1), and chooses some arbitrary cooperation levels for all players $k \geq j$. Player j is thus the first player with whom i does not fully cooperate. Player i has an incentive to fully cooperate with j if and only if

$$\sum_{k\neq i,j} \Big(r_k^* + (c_{ik}^{i<j} - 1) r_{ki}^+\Big)^d + r_j^{*d} > \Big(r_i^0 + r_{ij}^+ + \sum_{k\neq i,j} c_{ik}^{i<j} r_{ik}^+\Big) \frac{r_{ji}^+}{r_{ij}^+} r_j^{*d-1}. \tag{6}$$

This condition is satisfied if the LHS has increased more or decreased less than the RHS, compared to the condition in proposition 5, rewritten as:

$$\sum_{k\neq i,j} \Big(r_k^* + (c_{ik}^{ij*} - 1) r_{ki}^+\Big)^d + r_j^{*d} > \Big(r_i^0 + r_{ij}^+ + \sum_{k\neq i,j} c_{ik}^{ij*} r_{ik}^+\Big) \frac{r_{ji}^+}{r_{ij}^+} r_j^{*d-1}. \tag{7}$$

If this relationship is true for each k, it must be true for the entire expression. The change in the LHS for a marginal change in c_{ik}^{ij*} is $d\big(r_k^* + (c_{ik}^{ij*} - 1) r_{ki}^+\big)^{d-1} r_{ki}^+ \, \mathrm{d}\, c_{ik}^{ij*}$, while the RHS changes by $r_j^{*d-1}(r_{ji}^+/r_{ij}^+) r_{ik}^+ \, \mathrm{d}\, c_{ik}^{ij*}$. The marginal changes are equal whenever

$$d\big(r_k^* + (c_{ik}^{ij*} - 1) r_{ki}^+\big)^{d-1} = r_j^{*d-1} \frac{r_{ji}^+}{r_{ij}^+} \frac{r_{ik}^+}{r_{ki}^+}. \tag{8}$$

Note that this condition is identical to expression (2a). The LHS clearly increases in c_{ik}^{ij*} for $d > 1$, while the RHS is independent of c_{ik}^{ij*}, and thus constant. For those players k for which c_{ik}^{ij*} falls into the interval $(0, 1]$, $c_{ik}^{i<j}$ can be smaller, equal or larger than c_{ik}^{ij*}. If $c_{ik}^{i<j}$ is smaller than c_{ik}^{ij*}, the LHS of (7) decreases by less than the RHS as player i reduces c_{ik}^{ij*} to $c_{ik}^{i<j}$, thus ensuring the truth of condition (6). If it is equal, there is no change. If $c_{ik}^{i<j}$ is larger than c_{ik}^{ij*}, the LHS of (7) increases more than the RHS as player i increases c_{ik}^{ij*} to $c_{ik}^{i<j}$, thus also ensuring the truth of condition (6).

Finally, for those players k for which $c_{ik}^{ij*} = 0$, the LHS of (7) must increase no less than the RHS, as i chooses $c_{ik}^{i<j} = 1$: $r_k^{*d} - (r_k^* - r_{ki}^+)^d \geq r_{ik}^+ (r_{ji}^+ / r_{ij}^+) r_j^{*d-1}$. Given the ordering of players, the preceding inequality is true if $r_k^{*d} - (r_k^* - r_{ki}^+)^d \geq r_{ik}^+ (r_{ki}^+ / r_{ik}^+) r_k^{*d-1}$, which simplifies to $r_k^{*d-1} \geq (r_k^* - r_{ki}^+)^{d-1}$. Since $d \geq 1$, this inequality is always true. Therefore, the LHS of (7) increases more than the RHS as i switches from $c_{ik}^{ij*} = 0$ to $c_{ik}^{i<j} = 1$. Since equality (2a) does not hold for any value of c_{ik}^{ij*} in case (2b), the LHS must always exceed the RHS for any value of $c_{ik}^{i<j}$. Condition (6) must therefore hold, concluding the proof that player i increases his utility by resuming full cooperation with player j. Since the same argument also holds for the next largest player and all remaining ones, player i does not have an incentive to reduce cooperation with any player in the first place. Universal full cooperation is thus a NE. $\square$

Proof of Corollary 6. The first condition in corollary 6 is the same as the condition in proposition 5, with all $c_{ik}^{ij*} = 0$ for $k > j$. Assume again that player i has reduced cooperation with one or more players $k > j$. Universal full cooperation is a PSNE if i has an incentive to fully cooperate with j, independent of his cooperation levels c_{ik}. Since the condition in the corollary ensures a positive marginal utility, $\partial u_i / \partial c_{ij}$, when evaluated at $c_{ij} = 1$ and $c_{ik} = 0$, it must still hold if the LHS increases no less than the RHS for any $c_{ik} > 0$. As we know from (8), this is the case whenever $d\big(r_k^* + (c_{ik}^{ij*} - 1) r_{ki}^+\big)^{d-1} \geq r_j^{*d-1} (r_{ji}^+ / r_{ij}^+)(r_{ik}^+ / r_{ki}^+)$. Since the LHS increases in c_{ik}^{ij*}, the condition is satisfied for all cooperation levels if it holds for $c_{ik}^{ij*} = 0$, i.e. if $d(r_k^* - r_{ki}^+)^{d-1} \geq r_j^{*d-1} (r_{ji}^+ / r_{ij}^+)(r_{ik}^+ / r_{ki}^+)$. Given ordering (1), this inequality is true if $d(r_k^* - r_{ki}^+)^{d-1} \geq r_k^{*d-1} (r_{ki}^+ / r_{ik}^+)(r_{ik}^+ / r_{ki}^+)$, which simplifies to $r_{ki}^+ \leq \big(1 - d^{1/(1-d)}\big) r_k^*$. This is the second condition contained in the corollary. Player i therefore has an incentive to resume full cooperation, first with j, and then all other players. $\square$

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