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September 2011

# Downtown Parking and Traffic Congestion

## A Diagrammatic Exposition\*

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### Abstract

Through an extended numerical example, this paper develops a diagrammatic analysis of steady-state parking and traffic congestion in an isotropic downtown. The model incorporates curbside parking, garage parking, and price-sensitive travel demand in a unified setting, and provides systematic policy analysis. In particular, we examine the deadweight loss associated with underpriced curbside parking, as well as first- and second-best curbside parking capacities. We also explore the transient dynamics and stability of various downtown traffic equilibria.

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## 1 Introduction

For many years, urban transport economists applied the economic and engineering tools developed in the 1960's and 1970's for the analysis of freeways to the study of downtown traffic congestion. Parking was treated crudely as a fixed cost incurred at the end of a trip, and the assumed form of the congestion function was that estimated for freeway traffic. Over the last decade, however, there has been increasing recognition that downtown traffic congestion differs in important ways from freeway traffic congestion. For one thing, parking is of central importance in downtown transportation. Parking is a major user of land downtown, curbside parking reduces street capacity, and cars cruising for parking slow downtown in-transit traffic (see Shoup (2005, 2006) and van Ommeren, Wentink, and Rietveld (forthcoming), van Ommeren, Wentink, and Dekkers (2011) for empirical evidence). For another, in heavily congested downtown areas, most of the congestion takes the form of queuing at intersections, while flow congestion is of dominant importance on freeways. As a result, the congestion function for downtown traffic may differ significantly from that for freeway traffic. Detailed analysis by transportation scientists of traffic sensor data indicate that, with heavy congestion, traffic *flow* on sections of freeways does not fall much as average density increases (**references from Coifman and Kim**) whereas in heavily congested downtown areas it does (**references by Geroliminis**). Put alternatively, at a macroscopic level (*e.g.*, a metropolitan network of freeways or large areas of downtown), hypercongestion (situations where traffic flow falls as traffic density increases) is more important in downtown traffic than in freeway traffic.

William Vickrey (1991) was the first urban transport economist to develop a model customized for the study of downtown traffic congestion. He conceived of downtown Manhattan

as a bathtub. Traffic density is analogous to the height of water in the bathtub; traffic flowing into Manhattan, as well as trips initiated within Manhattan, are analogous to water flowing into the bathtub; and traffic flowing out of Manhattan, as well as trips terminated within Manhattan, are analogous to water flowing out of the bathtub.<sup>1</sup> Traffic speed is negatively related to traffic density. Unfortunately, he did not fully develop the model before his death. In a series of papers (Arnott and Inci, 2006, 2010; Arnott and Rowse, 2009, 2011) we have been developing a sequence of models building on Vickrey’s conception. However, our models differ from Vickrey’s conception in that they put parking and the interaction between parking and traffic congestion at center stage. Arnott and Inci (2006, 2010) examines steady-state equilibria in a bathtub model of downtown traffic congestion with curbside parking (but no garage parking) and with price-sensitive demand. Arnott and Rowse (2009) extends that model to allow for garage as well as curbside parking but to keep it analytically tractable assumes demand to be *inelastic*.

In recent empirical work, Daganzo and Geroliminis (2007; xxxxxx) provides strong empirical support for Vickrey’s conception of downtown traffic congestion. In particular, they document a stable relationship between average velocity and average traffic density (which the transportation science literature terms the existence of a stable macroscopic fundamental diagram (MFD)) at the level of large urban neighborhoods. As a result, bathtub models are likely to attract more widespread attention than they have to date. Second, in the years to come, the focus of the literature will likely shift to the equilibrium intra-day dynamics of bathtub models, and it is important to have a good summary of their steady-state properties before this shift occurs. This paper is a step towards this goal. We develop an extended numerical example of a synthesized model that incorporates curbside parking, garage park-

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<sup>1</sup>The bathtub analogy is imperfect. At least for a frictionless tub, sink, and drain, the rate at which water flows out of a bathtub is positively related to the height of the water in a bathtub. In Vickrey’s bathtub model, the rate at which water flows out of the bathtub is positively related to the height of the water in the bathtub *up to a critical height* (which corresponds to capacity density), but above this critical height the rate at which water flows out of the bathtub is negatively related to the height of the water, until another critical height is reached (which corresponds to jam density) at which the drain becomes completely clogged.

ing, and price-sensitive demand. We provide a diagrammatic exposition of the results, which clarifies the basic insights. Working through an extended numerical example, with diagrams, circumvents the technical complexity of the earlier papers and puts the economic insights into sharper relief. We use the diagrammatic exposition to examine the deadweight loss associated with the underpricing of curbside parking, as well as first- and second-best (with the underpricing of curbside parking and traffic congestion being the distortions) curbside parking capacity, and to explore the possible multiplicity and stability of equilibria.

**A PARAGRAPH ABOUT WHAT WE EXACTLY FIND NEW IN THIS PAPER.**

The paper is organized as follows. Section 2 outlines the base model and adapts the fundamental diagram of traffic flow to downtown traffic. Section 3 adds curbside parking to the base model. Section 4 extends the analysis by incorporating both curbside and garage parking to the model. Section 5 investigates the stability of various equilibria of the previous sections. Section 6 provides concluding remarks for future research.

## **2 Traffic Congestion with No Parking**

To set the base for further analysis, we start by adapting the familiar diagrammatic analysis of congested traffic equilibrium with price-sensitive demand due to Walters (1961) to downtown traffic. For the moment, we ignore downtown parking, essentially assuming that parking is costless. We assume that downtown is isotropic; one can imagine a boundless Manhattan network of one-way streets. We also assume that the drivers are identical and that the demand for trips initiated per unit area-time is stationary and is a function of the full price of a trip,  $F$ :

$$D = D(F) \quad . \quad (1)$$

For simplicity we ignore the money costs of travel. Therefore, the user cost of a trip,  $UC$ , equals the travel time cost of a trip, which equals the distance traveled,  $m$ , times travel time per mile,  $t$ , times the value of time,  $\rho$ :

$$UC = \rho mt \quad . \quad (2)$$

Travel time per mile is an increasing, convex function of the density of traffic per unit area,  $V$ :  $t(V)$ , with  $t' > 0$ ,  $t'' > 0$ , and with  $t(0) > 0$  being free-flow travel time. In order to distinguish between the full trip price and user cost, we assume that a toll of size  $\tau$  is applied, so that the full price of a trip equals user cost plus the toll:

$$F = UC + \tau \quad . \quad (3)$$

In steady state, the number of trips initiated per unit area-time equals the number of trips terminated per unit area-time. We refer to this as *the steady-state condition*, and the steady-state number of trips per unit area time as *throughput*,<sup>2</sup> and denote it by  $r$ . The steady-state number of trips initiated per unit area-time is given by the demand function. The steady-state number of trips terminated per unit area-time equals traffic density divided by the length of time each car spends in traffic,  $mt$ . Thus, the steady-state condition is

$$D(\rho mt(V) + \tau) = \frac{V}{mt(V)} \quad . \quad (4)$$

This equilibrium can be derived geometrically using the four-quadrant diagram of Figure 1. Quadrant II plots the relationship between user cost and traffic density ( $UC = \rho mt(V)$ , which combines (2) and  $t = t(V)$ ). Quadrant III shows the 45-degree line. Quadrant IV

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<sup>2</sup>Throughput has units of cars per unit area-time. In steady state, throughput is the same as the entry flow and exit flow per unit area. We avoid the term *flow* to avoid confusion. The fundamental identity of traffic flow states that flow,  $f$ , equals density times velocity. Applying that identity in the current context gives  $f = V/t(V)$ . Flow, therefore, equals throughput times trip length. Then, throughput measures the exit rate (= entry rate) from the flow of traffic, which equals flow divided by trip length.

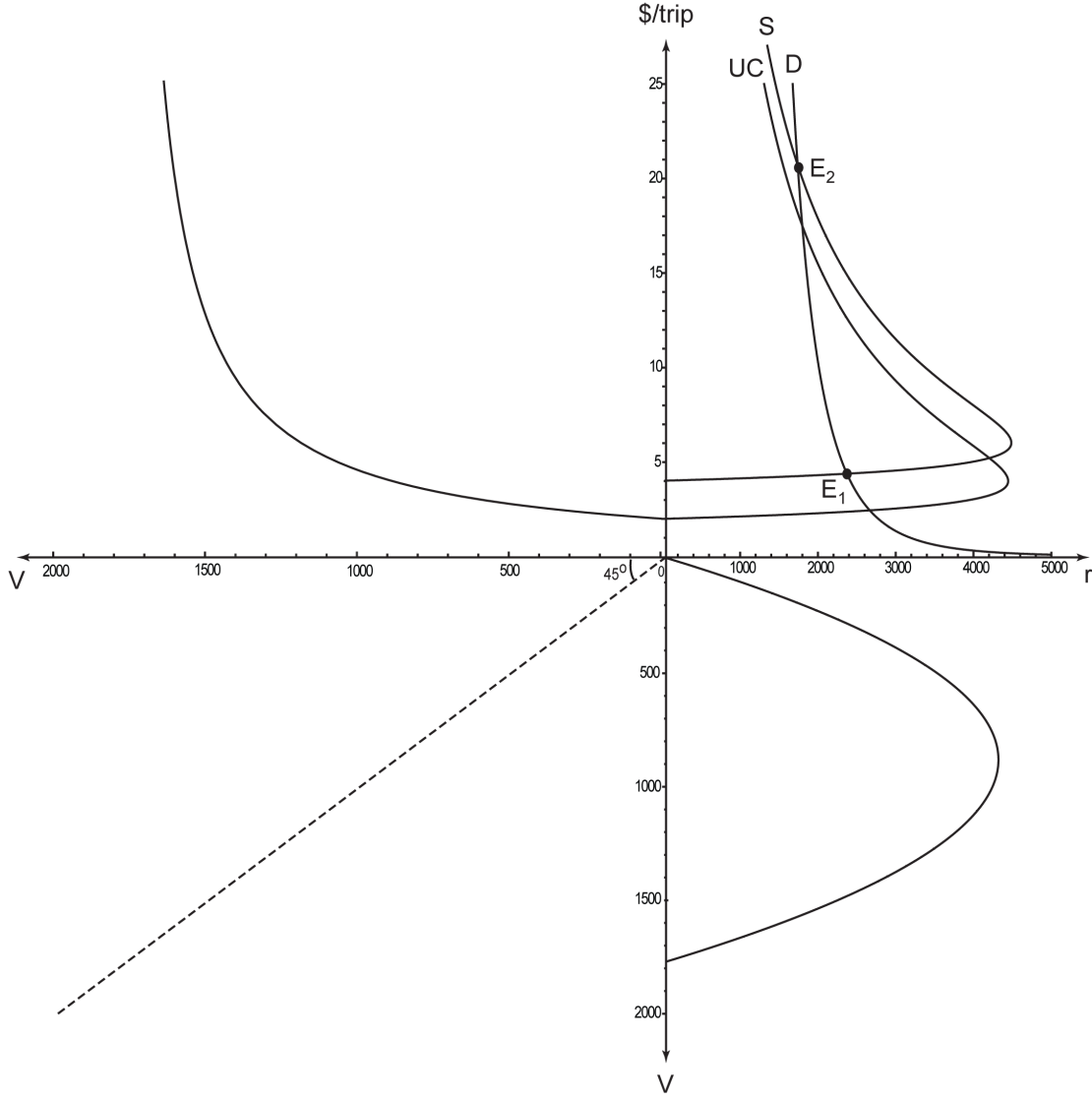


Figure 1: The Fundamental Traffic Diagram applied to downtown traffic

depicts the steady-state relationship between traffic throughput and density,

$$r = \frac{V}{mt(V)} \quad (5)$$

The user cost curve in Quadrant I, marked as  $UC$ , relates user cost to throughput.<sup>3</sup> The supply curve relates the full price of a trip to throughput, and is labeled  $S$  in the figure. It is obtained as a vertical shift of the user cost curve by  $\tau$ . The inverse demand function

<sup>3</sup>From (2) and  $t = t(V)$ ,  $V = t^{-1}(UC/(\rho m))$ . Substituting this into (5) gives  $r = t^{-1}(UC/(\rho m))/(UC/\rho)$ .



provides the demand relation between the full trip price and throughput, and equilibrium is given by the point of intersection of the demand and supply curves.

Figure 1 is plotted for specific functional forms and parameter values. The following are maintained throughout the paper:

$$D(F) = D_0 F^{-a} \tag{6}$$

$$t(T) = \frac{t_0}{1 - \frac{T}{V_j}} \tag{7}$$

with parameter values

$$a = 0.2, \quad t_0 = 0.05, \quad V_j = 1778.17, \quad m = 2.0, \quad \rho = 20.0 \quad . \tag{8}$$

The parameters chosen are the same as those assumed in Arnott and Inci (2006, 2010), and the basis for their choice is given in Arnott and Inci (2006). Demand is assumed to be iso-elastic, with demand elasticity equal to 0.2. The demand intensity parameter,  $D_0$ , is allowed to vary, in order to examine how equilibrium changes with demand. Travel congestion is described by Greenshields' Relation, which specifies a negative linear relationship between velocity and density, and hence the form of the relationship between travel time and density depicted in Quadrant II. Free-flow travel time per mile,  $t_0$ , is 0.05 hrs, which corresponds to 20 mph. Jam density,  $V_j$ , is 1778.17 cars/ml<sup>2</sup>. Trip distance is 2.0 mls and the value of time is \$20/hr. Figure 1 is drawn with the base case demand intensity of  $D_0 = 3190.94$ .

Following Vickrey, travel on the upward-sloping portion of the user cost curve is termed *congested* travel, and travel on the backward-bending portion is termed *hypercongested* travel. With congested travel, travel time and user cost increase with throughput. With hypercongested travel, travel time and user cost decrease with throughput. Congested travel corresponds to normal travel, and hypercongested travel to traffic jam situations.

Figure 1 shows two equilibria. At  $E_1$  traffic flow is congested, at  $E_2$  traffic flow is

hypercongested. There is also an equilibrium,  $E_3$ , that cannot be shown in the diagram, corresponding to gridlock – zero flow and an infinite full trip price. It is generally accepted that  $E_1$  is a stable equilibrium. The stability of equilibria on the backward-bending portion of the supply curve has been a matter of considerable dispute. Arnott and Inci (2010) examined the issue for a somewhat different model<sup>4</sup> that included curbside parking. If their model had excluded curbside parking, the stability analysis would have proceeded as follows. Stability is defined with reference to a particular adjustment dynamic. The natural adjustment dynamic in this context is that the change in the density of cars equals the demand inflow,  $D(F)$ , minus the outflow,  $T/(mt(T))$ . With demand based on either myopic foresight (when a driver is deciding whether to take a trip, he bases his expectation of the full price on current traffic conditions) or perfect foresight, the analog of  $E_1$  is indeed stable, while  $E_2$  is unstable<sup>5</sup> and  $E_3$  stable. Which of the two stable equilibria the traffic network attains depends on the density at the time when the demand function first became stationary.

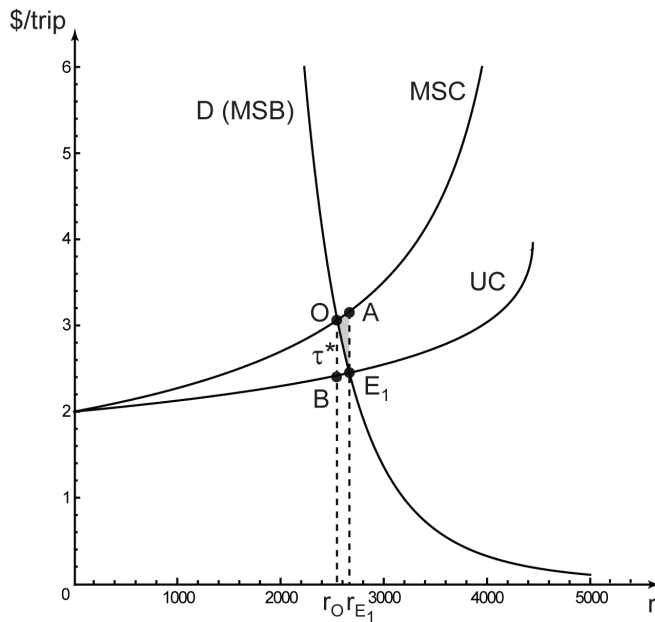


Figure 2: Equilibrium and social optimum without parking

<sup>4</sup>Visit length is assumed to be Poisson distributed with mean length  $m$ .

<sup>5</sup>A steady-state equilibrium is unstable if the measure of initial traffic conditions achieving this equilibrium is zero. One may call  $E_2$  to be *saddle-path stable* because it can be reached from initial traffic conditions on one of the arms of the steady state, which is a curve and thus measure zero.

Figure 2 focuses on the upward-sloping portion of the user cost curve. Aggregate user cost can be calculated as a function of throughput. On this portion of the user cost curve, marginal social cost is defined as the derivative of aggregate user cost with respect to throughput, and at a particular throughput equals the corresponding user cost plus the congestion externality cost (the cost to inframarginal users due to the increase in throughput slowing them down). Figure 2 displays the user cost curve and the marginal social cost curve, labeled  $MSC$ . In social surplus analysis, the demand curve is interpreted as the marginal social benefit curve, labeled  $MSB$ , so that optimal throughput occurs at the point of intersection of the marginal social cost and demand curves,  $O$ . The optimal throughput can be achieved by setting an optimal congestion toll equal to the congestion externality cost, evaluated at the social optimum,  $\tau^*$ . In the no-toll equilibrium,  $E_1$ , too many cars travel on the road since travel is underpriced due to drivers not paying for slowing other drivers down. The deadweight loss associated with having no toll is given by the area  $AE_1O$ , and equals the loss in social surplus from travel at throughput  $r_{E_1}$  compared to throughput  $r_O$ . These results are, of course, broadly familiar, but we have been careful to derive them precisely in the context of steady-state traffic congestion in an isotropic downtown area, since we shall build on them in the sections that follow, which add parking.<sup>6</sup>

### 3 Traffic Congestion with Only Curbside Parking

We now modify the model to take into account that drivers must park. In this section, we rule out garage parking and consider only curbside parking. Curbside parking affects the analysis in four ways. First, increasing the amount of curbside allocated to parking reduces the road space available for traffic flow, which has the effect of reducing jam density.<sup>7</sup> Second, the

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<sup>6</sup>We could extend the analysis to solve for optimal road capacity. But, here and throughout the paper we take road capacity as fixed.

<sup>7</sup>We assume that curbside allocated to parking reduces jam density by the same amount whatever the occupancy rate of the curbside parking. The rationale is that, under at least moderately congested conditions, even if only one curbside parking space is occupied on one side of the block, traffic flow is effectively excluded

amount of curbside parking constrains the throughput of the downtown traffic network to be no more than the curbside turnover rate, which we term *curbside parking capacity constraint* (CPC); if there are  $P$  curbside parking spaces per unit area and if the visit duration is  $l$ , then curbside parking capacity is  $P/l$ ; it is the maximum throughput that curbside parking can accommodate. Third, if there is insufficient curbside parking to ration the demand, given the curbside parking fee, cruising for parking occurs, with travel time costs, including cruising-for-parking time costs, adjusting to clear the market. And fourth, drivers pay a curbside parking fee per unit time (meter rate)<sup>8</sup>,  $f$ .

To simplify, we provide a crude treatment of parking search. We assume that each driver travels to his destination block. If a space is available, he takes it, and if it is not he drives around the destination block until a space opens up. Furthermore, we ignore the random variation that occurs due to the small number of parking spaces on each block, and assume that curbside parking is either *saturated* (fully occupied) everywhere, or *unsaturated* everywhere.<sup>9</sup> We shall consider optimal parking pricing and optimal curbside parking capacity, conditional on curbside parking being efficiently priced (first-best capacity) and inefficiently priced (second-best capacity). In all our analysis, we assume that no congestion tolling is employed.<sup>10</sup>

We have already distinguished between throughput and flow. Steady-state throughput is the rate at which trips are initiated and terminated per unit area-time. Steady-state flow is the number of car-miles traveled per unit area-time. When cruising for parking occurs, there is a further characteristic distinguishing throughput and flow – flow includes cars that are cruising for parking.

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from that lane for the entire block.

<sup>8</sup>To keep the analysis simple, we consider only linear curbside parking payment schedules.

<sup>9</sup>Realistically, at the level of the downtown area, there is a gradual transition between unsaturated and saturated parking. As the demand for curbside parking increases, curbside parking becomes saturated on an increasingly high proportion of blocks.

<sup>10</sup>Because the distance traveled and the visit duration are fixed, the first best can be achieved just by efficiently pricing curbside parking, even though congestion tolling is not employed; the efficient parking fee includes the optimal congestion toll. This is why we refer to the optimal capacity with efficient curbside parking pricing as first-best.

Two adjustments need to be made to the specification of the congestion technology to accommodate curbside parking. First, it is necessary to account for the reduction in road capacity due to curbside parking. We assume that *effective jam density* is related to the amount of street space allocated to traffic flow. In particular, where  $\Omega$  is the jam density with no curbside parking, effective jam density,  $V_j$ , equals jam density times the proportion of street space allocated to traffic flow,  $1 - P/P_{\max}$ , where  $P$  is the density of curbside parking spaces per unit area and  $P_{\max}$  its maximum value. Thus,

$$V_j = \Omega \left(1 - \frac{P}{P_{\max}}\right) \quad . \quad (9)$$

Second, the specification of the congestion technology needs to account for the congestion interaction between cars in transit and cars cruising for parking. We make the simple assumption that a car cruising for parking generates  $\theta$  times as much congestion as a car in transit. Thus, where  $C$  is the density of cars per unit area that are cruising for parking, the travel time function is

$$t(T, C, P) = \frac{t_0}{1 - \frac{T + \theta C}{V_j}} \quad . \quad (10)$$

We maintain the following parameters for the rest of the paper:<sup>11</sup>

$$\theta = 1.5, \quad \Omega = 2667.36, \quad P_{\max} = 11136 \quad . \quad (11)$$

We also assume that the curbside parking fee is \$1/hr, so that the parking fee for the trip is \$2, and that curbside parking is permitted on one side of the street everywhere, so that

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<sup>11</sup>The parameters are drawn from Arnott and Inci (2006) and were chosen to be broadly consistent with observation. A city block is assumed to be 1/8 ml long, the one-way streets to have three lanes, and roads to be 33 ft wide. Then each side of a block is 627 ft long. If parking is on one side of the street, so that two sides of every city block have curbside parking, the maximum length of curbside around each city block that could be devoted to parking is 1254 ft. But some of this curbside is used for crosswalks. On two sides of a city block, there are four crosswalks. We assume that each crosswalk is 9 ft wide, so that the amount of curbside around each city block allocated to parking is 1218 ft. With 21 ft devoted to each curbside parking space, the number of curbside parking spaces on each block is 58. And since there are 64 blocks per ml<sup>2</sup>, the number of curbside parking spaces per ml<sup>2</sup> is 3712.

$P = 3716$  and  $P/l = 1856$ . In the analysis that follows, we start with the short run, where the level of curbside parking is fixed, and then move to the long run, where the level of curbside parking is a policy choice variable.

### 3.1 The short run

We start with the first-best planning problem and its decentralization.

#### 3.1.1 First-best optimum in the short run

Consider a benevolent social planner who has direct control of the transportation system and its users. She would never choose cruising for parking because the same throughput can be achieved at lower cost without it. Since the amount of curbside parking is fixed, she chooses throughput to maximize social surplus. Resource cost per unit time is  $\rho T$ . Thus, where  $X(r)$  is the social benefit from throughput  $r$  (which equals the area under the inverse demand curve up to throughput level  $r$ ), she faces the maximization problem

$$\max_{r,T} X(r) - \rho T \tag{12}$$

*s.t.*

$$r = \frac{T}{mt(T, 0, P)} \tag{(i)}$$

$$r \leq \frac{P}{l} \tag{(ii)}$$

The first constraint is the steady-state condition and the second is the curbside parking capacity constraint, that throughput cannot exceed the curbside parking turnover rate. Figure 3 displays the solutions for two demand levels,  $D_1$  and  $D_2$ . The curbside parking capacity constraint is labeled *CPC*, and the marginal social cost of throughput, labeled,  $MSC(r)$ , is calculated as  $\rho[dT/dr]_{(i)}$  with  $[dT/dr]_{(i)}$  being calculated from the steady-state condition.

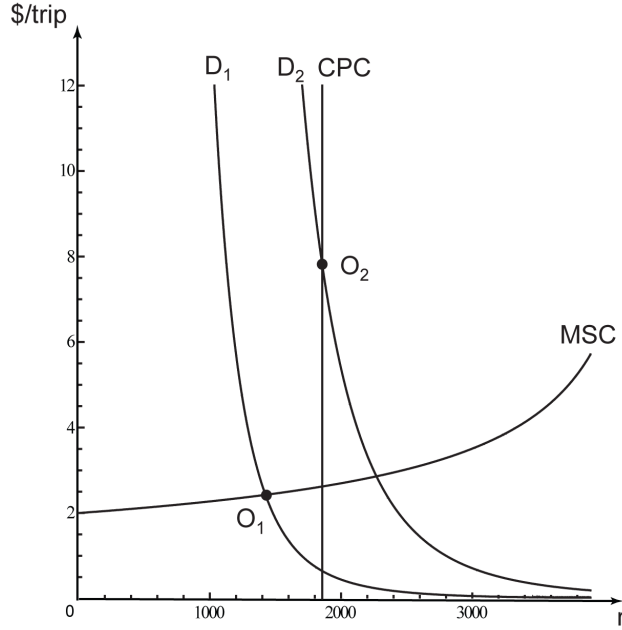


Figure 3: First-best optimum in the short run

With demand level  $D_1$ , the curbside parking capacity does not bind, and the first-best optimum, labeled  $O_1$ , is at the point of intersection of the demand (marginal social benefit) and marginal social cost curves. Since we have assumed that the reduction in road capacity caused by curbside parking depends on the amount of curbside that is *allocated* to curbside parking, independent of its occupancy rate, the marginal traveler generates no parking externality. Marginal social cost therefore equals user cost plus the congestion externality cost, so that the social optimum can be decentralized by setting the parking fee equal to the congestion externality costs.

With demand level  $D_2$ , the curbside parking capacity binds, and the first-best optimum, labeled  $O_2$ , is at the point of intersection of the demand curve and the curbside parking capacity constraint. The marginal social cost now equals the user cost plus the congestion externality cost plus a parking scarcity rent. The social optimum can then be decentralized by charging a congestion toll equal to the congestion externality cost evaluated at the social optimum plus a parking scarcity rent. Since in the model, each trip has a fixed length and visit duration, the social optimum can also be decentralized without an explicit congestion

toll by charging a parking fee equal to the congestion externality cost and the scarcity rent.

### 3.1.2 Second-best optimum in the short run

It is perhaps misleading to talk about a second-best optimum in the short run since the planner has no margins of choice. The second-best optimum is simply the equilibrium that generates the highest social surplus.

An equilibrium may entail unsaturated or saturated parking. Consider first equilibria with unsaturated parking. Since parking is unsaturated, there is no cruising for parking. The user cost is  $UC = \rho mt(T, 0, P)$  and the full price is  $F = UC + fl$ , where  $T$  satisfies the steady-state condition that  $T/(mt(T, 0, P)) = D(F)$ . From these results, the *unsaturated user cost curve* can be derived, which is completely analogous to the user cost curve derived in the previous section, except that curbside parking reduces road capacity. At levels of throughput where the curbside parking capacity constraint does not bind, the supply curve is derived as the unsaturated user cost curve shifted up by the curbside parking fee, and any point of intersection of the demand curve and this portion of the supply curve is an unsaturated equilibrium.

Now consider equilibrium with saturated parking. Parking is saturated because the parking constraint binds, and except in the situation where it just binds there is cruising for parking. Thus, equilibrium entails two density variables, the density of cars in transit and the density of cars cruising for parking. These are determined by two equilibrium conditions. The first is the familiar steady-state condition, but here modified to take into account curbside parking and cars cruising for parking:

$$D(F) = \frac{T}{mt(T, C, P)} \quad , \quad (13)$$

where the full price equals the cost of in-transit time, plus the expected cost of cruising-for-



parking time, plus the parking fee:

$$F = \rho mt(T, C, P) + \frac{\rho Cl}{P} + fl \quad . \quad (14)$$

Since  $C$  cars are cruising for parking and since the turnover rate of parking space is  $P/l$ , the probability that a car cruising for parking gets a space per unit time is  $P/(Cl)$ , so that expected cruising-for-parking time is  $Cl/P$ . The second equilibrium condition, *the cruising-for-parking equilibrium condition*, is that the rate at which cars exit the in-transit pool equal the parking turnover rate:

$$\frac{T}{mt(T, C, P)} - \frac{P}{l} = 0 \quad . \quad (15)$$

The steady-state condition and the cruising-for parking equilibrium condition provide two non-linear equations in the two unknowns,  $T$  and  $C$ . Their analysis is complex. Arnott and Inci (2006) derive the conditions under which the two curves intersect in  $T$ - $C$  space, and for which therefore there exists a saturated equilibrium. Furthermore, they prove that if a saturated equilibrium exists, it is unique. Here, we derive the properties we need for our diagrammatic analysis through heuristic argument. We ask: What are the minimum and maximum full prices consistent with the cruising-for-parking equilibrium condition being satisfied, and therefore for the existence of a saturated equilibrium. The equation  $T/(mt(T, 0, P)) = P/l$  has two roots, the smaller corresponding to congested travel, the larger to hypercongested travel. As intuition suggests, the minimum full price consistent with (15) being satisfied corresponds to the smaller root – a given level of throughput can be achieved with minimum congestion when there are no cars cruising for parking. Less obviously, the maximum full price consistent with (15) being satisfied corresponds to the larger root – the most jammed traffic consistent with a given level of throughput occurs when none of the cars is cruising for parking.

These two results greatly facilitate the analysis. Turn to Figure 4. First, imagine plotting

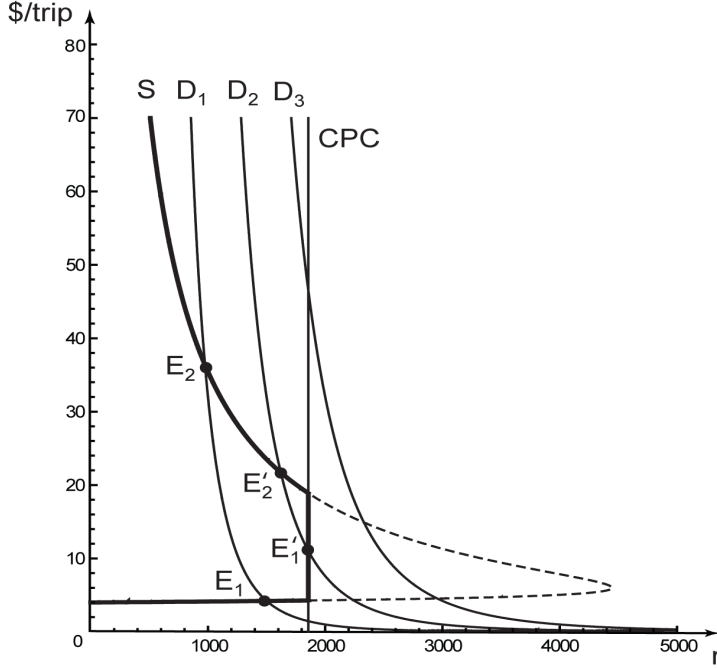


Figure 4: Equilibria with curbside parking

the unsaturated user cost curve, which relates user cost to throughput for a given level of  $P$  in the absence of cruising for parking, and which we denote by  $UC(r, P)$ . It is the same as the user cost curve in Quadrant I of Figure 1, except that some road space is allocated to curbside parking. Second, shifting this curve up by  $fl$  gives the *unsaturated full price curve*, denoted as  $F(r, P)$ , which indicates how the full price is related to throughput with curbside parking  $P$  when the curbside parking capacity constraint does not bind. Third, draw in the curbside parking capacity constraint, which constrains throughput to be no greater than  $P/l$ . The portions of the unsaturated full price curve to the left of the curbside parking capacity constraint – where it does not bind – form part of the supply curve. The portion to the right of the curbside parking capacity constraint is drawn as a dashed line since it is not relevant to the analysis. We argued above that there is a minimum and a maximum full price at which a saturated equilibrium can exist, and at each there is no cruising for parking. The minimum full price at which a saturated equilibrium can exist is therefore the lower point of intersection of the unsaturated full price curve and the parking capacity constraint, and

the maximum full price is the upper point of intersection. Thus, a saturated equilibrium must lie on the portion of the curbside parking capacity constraint above the upward-sloping portion of the unsaturated full price curve and below the backward-bending portion. This is the third piece of the supply curve. Equilibria then correspond to points of intersection of the supply curve so defined and the demand curve.

Figure 4 shows three demand curves, each corresponding to a different demand intensity. While not obvious from the diagram, for all three demand curves, gridlock, with zero throughput and an infinite full price, is an equilibrium. In the gridlock equilibrium, since there is zero throughput, parking is unsaturated, so that the parking capacity constraint does not bind, and the steady-state condition is satisfied since the entry flow and exit flow are both zero. Arnott and Inci (2010) show that the gridlock equilibrium is stable. With low demand intensity (in the figure,  $D_1$  with demand intensity  $D_0 = 2000$ ), there are three equilibria:  $E_1$ , which is unsaturated, congested, and stable;  $E_2$ , which is unsaturated, hypercongested, and unstable; and the gridlock equilibrium, which is unsaturated, hypercongested, and stable. The stability properties of the equilibria will be derived later. With medium demand intensity (in the figure,  $D_2$  with demand intensity  $D_0 = 3000$ ), there are again three equilibria:  $E'_1$ , which is saturated and stable;  $E'_2$ , which is unsaturated, hypercongested, and unstable, and the gridlock equilibrium. With high demand intensity (in the figure,  $D_3$  with demand intensity  $D_0 = 4000$ ), the equilibria corresponding to  $E_1$  and  $E_2$  disappear, with only the gridlock equilibrium remaining. Later we shall display the various equilibria, as a function of demand intensity, in a bifurcation diagram.

Let us consider the equilibrium  $E'_1$  in more detail. In this saturated equilibrium, the stock of cars cruising for parking and in-transit adjust to clear the market, such that the full price is at the point of intersection of the demand curve and the curbside parking capacity constraint.<sup>12</sup> The equilibrium values of  $T$  and  $C$  are 444.28 and 394.03, so that travel time

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<sup>12</sup>Without cruising for parking, the throughput demanded would exceed the throughput supplied (constrained by the curbside parking capacity constraint). Cruising for parking serves as a dissipative rationing mechanism. Here, cruising for parking is analogous to a bread line except that the service is random access

is 0.1197 per ml. This implies a velocity of 8.36 mph, which corresponds to hypercongested travel. The full trip price equals \$11.03, of which \$4.78 is in-transit travel time cost, \$4.24 is expected cruising-for-parking time costs, and \$2.00 is the parking fee.

We now consider the deadweight loss associated with inefficient pricing in the equilibrium  $E'_1$ , conditional on the level of parking capacity. The deadweight loss equals social surplus at the optimum minus social surplus in the equilibrium. In the example, the social optimum too is at the point  $E'_1$ . Since there is no cruising for parking in the social optimum, the socially optimal level of  $T$  is the smaller root solving  $T/(mt(T, 0, P) = P/l$ , which is  $T = 210.74$ , so that travel time is 0.0567 hrs, which corresponds to a velocity of 17.64 mph and an in-transit travel time cost of \$2.27. Thus, the deadweight loss due to inefficient pricing is \$6.75 per driver and \$12,528 per ml<sup>2</sup>-hr. The social optimum could be decentralized by charging each driver \$8.75.

Since in the model both trip length and visit duration are fixed, it makes no difference whether this charge takes the form of a congestion toll or a parking fee. But generally it does.<sup>13</sup> The congestion toll would be set equal to the congestion externality cost, and the parking fee would be set equal to the marginal social cost of a parking space for the duration of a visit, both evaluated at the social optimum. The congestion externality cost is familiar, and is computed as the increase in the total unsaturated user cost corresponding to a unit increase in throughput,  $r$ , holding fixed curbside parking capacity (and hence ignoring the curbside parking capacity constraint). Since total unsaturated user cost per unit time is  $\rho T$ , the marginal social cost of throughput, holding fixed parking capacity is  $\rho \partial T / \partial r$ , where

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rather than FIFO.

<sup>13</sup>Return to the first-best social optimization problem. Let  $\lambda$  be the Lagrange multiplier on the steady-state condition and  $\mu$  be the multiplier on the curbside parking capacity constraint. The first-order condition with respect to  $r$  is  $X'(r) - \lambda - \mu = 0$ . The first-order condition with respect to  $T$  is  $-\rho + \lambda(1/(mt) - Tt_T/(mt^2)) = 0$ , so that  $\lambda = \rho mt / (1 - Tt_T/t)$ .  $\rho mt$  is the user cost and  $\rho mt[1/(1 - Tt_T/t) - 1] = \rho mTt_T/(1 - Tt_T/t)$  is the congestion externality cost. Also,  $\mu = X'(r) - \lambda$ . If  $P$  were optimized (which it is in the long-run optimization problem), the corresponding first-order condition would be:  $-\lambda Tt_P/(mt^2) + \mu/l = 0 \implies -\lambda r t_P P/t + \mu r = 0 \implies \mu = \lambda t_P P/t \implies \mu = \rho m t_P P / (1 - Tt_T/t)$ . In the long-run optimum, a parking space has no scarcity rent and its rent equals the parking congestion externality cost. Thus, in the short run, we define the parking scarcity rent to be  $\mu - \rho m t_P P / (1 - Tt_T/t)$ .

$\partial T/\partial r$  is calculated along the condition that  $T/(mt(T, 0, P)) = r$ , which equals \$2.63. The first-best congestion toll would be set equal to the congestion externality cost, which equals the difference between this marginal social cost of throughput and the unsaturated user cost, and therefore equals \$0.36.

Even if there were no congestion, a parking fee would still be needed to ration the available curbside parking spaces. We term this the *parking scarcity rent*. Thus, the first-best parking fee would contain two components, the parking scarcity rent and the parking externality cost, which we define to be the increase in total congestion cost per unit time associated the additional parking needed to accommodate a unit increase in throughput, but *holding the level of throughput fixed*:  $\rho l \partial T / \partial P$ , where  $\partial T / \partial P$  is calculated along the condition that  $T/(mt(T, 0, P)) = r$ , holding throughput fixed, which is \$0.18. The parking scarcity rent is calculated as a residual. The social benefit from a marginal trip is \$11.03. To ensure that parking spaces go to those who value them the most requires that trip price equal this amount. In the decentralized first best, the trip price would have four components, the (unsaturated) user cost of a trip, the congestion externality cost, the parking externality cost, and the parking scarcity rent. The parking scarcity rent would therefore be  $\$11.03 - \$2.27 - \$0.36 - \$0.18 = \$8.22$ , and the first-best parking fee would be \$8.40.

Figure 5 is the same as Figure 4, except for plotting the user cost curve and focusing on the first-best and second-best optima with the demand function  $D_2$ . The social surplus at the optimum equals social benefit, the area below the demand curve and to the left of the curbside parking, minus aggregate user cost,  $0JHI$ . It therefore equals the area  $JAE'_1H$  plus the area above  $AE'_1$  and below the demand curve. The social surplus at the second-best optimum equals consumer surplus, the area above  $AE'_1$  and below the demand curve, plus parking fee revenue,  $JBKH$ . Thus, the deadweight loss due to the underpricing of curbside parking is given by  $BAE'_1K$ . Raising the parking fee does not alter consumer surplus but increases parking fee revenue, and therefore converts deadweight loss dollar for dollar into tax revenue. Thus, the extra revenue is raised not just with no excess burden but also with

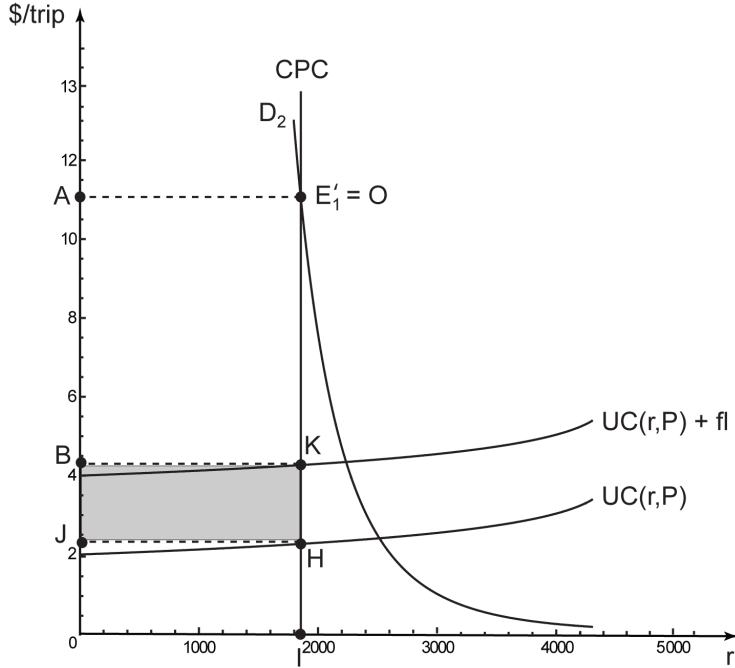


Figure 5: Equilibrium and social optimum with curbside parking

no burden. An obvious question is therefore why local governments choose to forgo such an efficient source of revenue.

## 3.2 The long run

We now turn to the determination of optimal first-best and second-best capacities.

### 3.2.1 First-best optimal capacity

Increasing curbside parking capacity constraint by a small amount has two effects, one positive and one negative. The positive effect is to raise throughput and hence the social benefit from travel, the area under the demand curve up to the curbside parking capacity constraint. The negative effect is to reduce the amount of road space available to traffic flow, which causes the unsaturated user cost curve to rise. These effects are displayed in Figure 6. Allocating more curbside to parking causes the curbside parking capacity constraint to

shift to the right, which generates the surplus to marginal travelers of  $HALJ$ , equal to the benefit they receive minus the user cost they incur. But it also causes the user cost curve to shift up, increasing the costs of inframarginal drivers by  $KBHG$ , and reducing their surplus by the same amount. At first-best optimal capacity the two areas are equal.

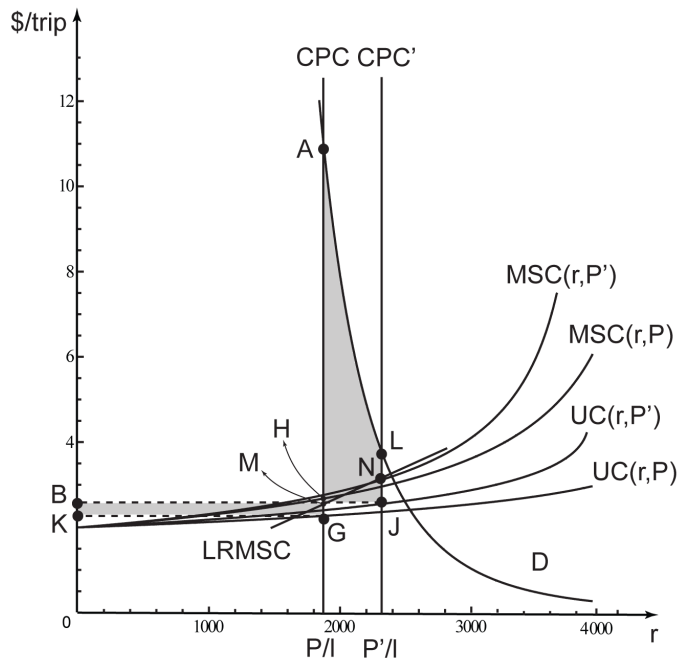


Figure 6: The first-best optimal curbside parking capacity

It will be instructive to determine first-best optimal capacity through an alternative geometric construct. The point  $M$  in Figure 5 gives the marginal social cost of a trip with parking capacity of 3712, which constrains throughput to be less than or equal to 1856. If parking capacity is reduced slightly, the throughput of 1856 cannot be achieved. If parking capacity is increased slightly, the throughput of 1856 can be achieved but at higher marginal social cost. Thus, with parking capacity endogenous,  $M$  gives the minimum marginal social cost associated with throughput of 1856. There is a point corresponding to  $M$  for every level of throughput, up to some maximum. Joining these points gives the *long-run marginal social cost curve*, labeled  $LRMSC$ . This long-run marginal social cost curve is defined up to the throughput at which the corresponding curbside parking capacity constraint is tangent to the corresponding unsaturated user cost curve,  $r_{\max}$ , at which point it is vertical.  $r_{\max}$

is the maximum level of throughput that can be accommodated on downtown streets with  $lr_{\max}$  units of curbside parking.

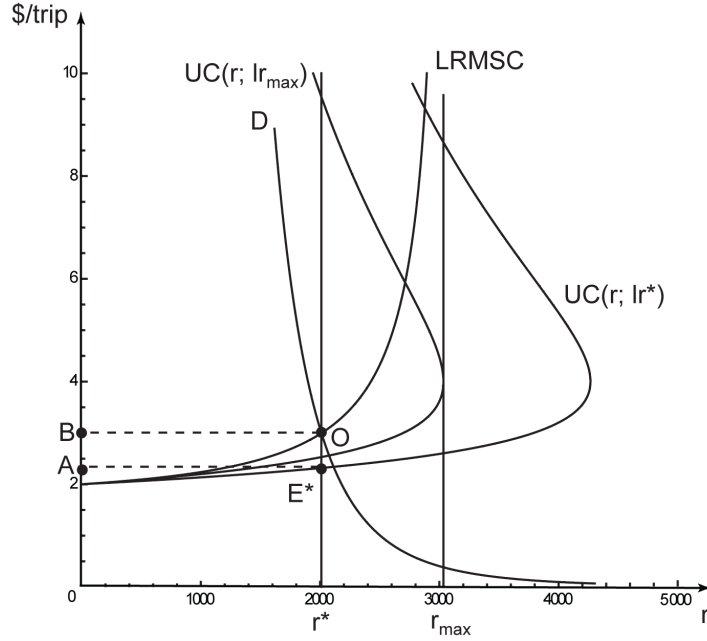


Figure 7: First-best optimal curbside parking capacity

If the demand curve lies everywhere below the long-run marginal social cost curve, the socially optimal level of throughput is zero. Otherwise, the first-best throughput,  $r^*$ , occurs at the point of intersection of the demand curve and the long-run marginal social cost curve, associated with which is first-best parking capacity,  $P^* = lr^*$ . Figure 6 shows how the *LRMSC* curve is constructed, and Figure 7 displays the full *LRMSC* curve, the parking capacity constraint  $P = lr_{\max}$  and the short-run user cost curve corresponding to  $r_{\max}$ ,  $UC(r; lr_{\max})$ . The socially optimal level of throughput occurs at  $O$ , the point of intersection of the demand curve and long-run marginal social cost curve, equals  $r^*$ , and corresponds to curbside parking capacity of  $lr^*$ .  $OA$  is the unsaturated user cost associated with throughput  $r^*$  and curbside parking capacity  $lr^*$ , and  $OB$  is the corresponding long-run marginal social cost. Decentralization of the social optimum entails charging a parking fee equal to  $BA$ . Since at the first-best optimum the curbside parking capacity constraint only “just binds”, the parking fee contains no scarcity rent, so that the parking fee equals the sum of the



congestion externality cost and parking externality cost.

First-best parking capacity has been derived on the assumptions that the transient dynamics will lead to the efficient equilibrium. Providing the first-best level of parking capacity and applying first-best pricing does not ensure this, however. Now consider the determination of second-best optimal capacity, where the distortion is underpriced curbside parking. Start at a saturated equilibrium. Increasing parking capacity a small amount unambiguously increases social surplus. With underpriced curbside parking, social surplus equals consumer surplus plus fee revenue, both of which are increased by an increase in throughput.

### 3.2.2 Second-best optimal capacity

Next start at an unsaturated, congested equilibrium. Reducing parking capacity a small amount unambiguously increases social surplus. Because parking is unsaturated, equilibrium lies at the point of intersection of the upward-sloping portion of the (short-run) supply curve and the demand curve. Reducing parking capacity lowers that portion of the supply curve, increasing throughput and hence social surplus. Thus, a second-best optimum that lies on the upward-sloping portion of the supply curve corresponding to the second-best level of curbside parking capacity entails the capacity constraint just binding (so that there is no cruising for parking). A more difficult argument establishes the same to be true for a second-best optimum that lies on the backward-bending portion of the supply curve.

This line of reasoning points to a method for determining second-best optimal throughput and capacity. Plot the  $UC(r;rl)$  curve, along which the parking capacity constraint just binds. Shifting the curve up by the amount of the parking fee generates the long-run supply curve, which we label  $LRS(r)$  in Figure 8. The second-best optimal throughput,  $r^{**}$ , corresponds to that point of intersection of the demand curve and the long-run supply curve with the highest level of throughput (and hence the highest level of surplus).<sup>14</sup> Similar to the

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<sup>14</sup>If the demand curve lies everywhere below the long-run supply curve, second-best capacity is zero.

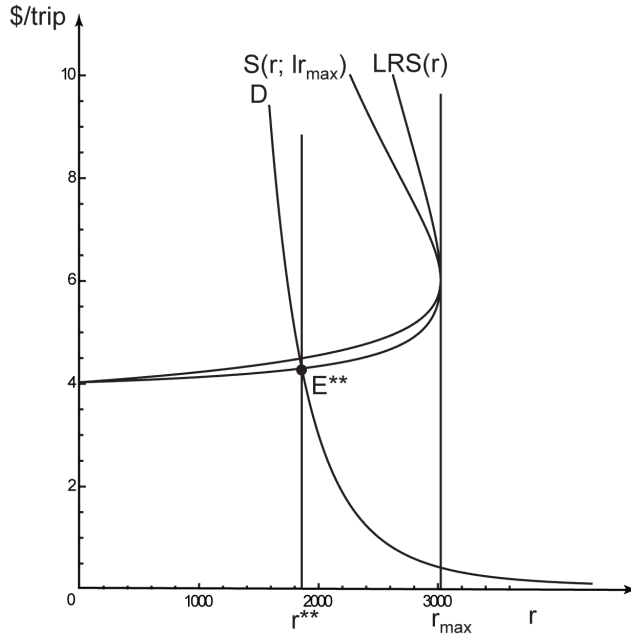


Figure 8: Second-best optimal curbside parking capacity

first-best optimum, simply setting curbside parking capacity at its second-best level,  $lr^{**}$ , does not ensure that the second-best optimum will be attained.

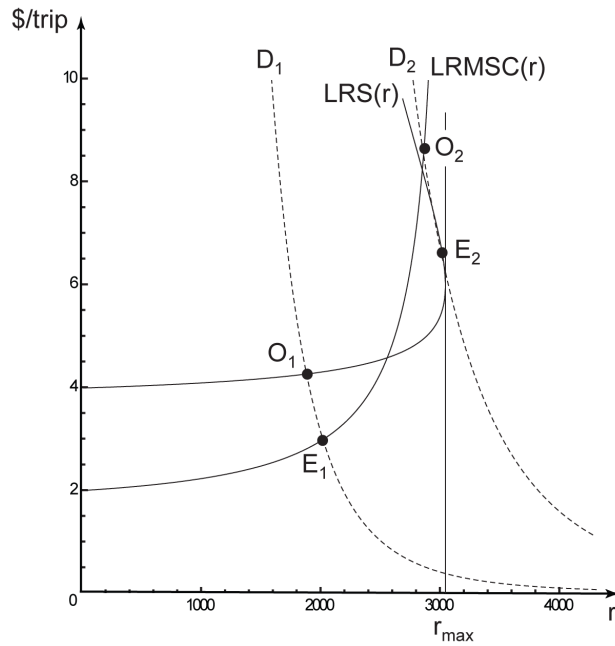


Figure 9: The relationship between first- and second-best optimal capacity

The relationship between first- and second-best optimal capacity is shown in Figure 9,

which plots the long-run marginal social cost curve, the long-run supply curve, and the demand curve. With both first- and second-best optimal capacity the curbside parking capacity constraint just binds, so that parking capacity equals throughput times visit length, and the analysis can be conducted in terms of throughput. The first-best optimum is at the point of intersection of the long-run marginal social cost curve and the demand curve. The second-best optimum corresponds to that point of intersection of the demand curve and the long-run supply curve with the highest level of throughput.

Consider first the case where demand is “moderate”, so that the demand curve intersects the long-run supply curve on its upward-sloping portion, as is illustrated by  $D_1$  in the figure. The long-run marginal social cost curve lies above the upward-sloping portion of the long-run user cost curve  $UC(r; rl)$ . Thus, if, with first-best capacity, curbside parking is underpriced (so that the long-run supply curve lies below the long-run marginal cost curve at  $r^*$ ), which we assume, second-best capacity and throughput exceed their first-best levels. In the figure, the first-best levels of throughput and capacity correspond to  $O_1$ , and the second-best levels of throughput and capacity correspond to  $E_1$ .

Consider next the case where demand is “high”, so that the demand curve does not intersect the long-run supply curve on its upward-sloping portion, as is illustrated by  $D_2$  in the figure. Again, if, with first-best capacity, curbside parking is underpriced, second-best capacity and throughput exceed their first-best levels. The intuition is that, with underpriced curbside parking and first-best capacity, throughput and hence social surplus can be increased by increasing capacity to the point where cruising for parking is just eliminated.

## 4 Traffic Congestion with Both Curbside and Garage Parking

In the downtowns of small towns, the suburbs of small cities, and the residential neighborhoods of medium-sized cities, there is typically enough parking space curbside to accommodate demand without severely impeding traffic flow. But in most locations where traffic congestion is a serious problem, curbside parking needs to be supplemented by off-street parking, whether in a parking lot or garage, or mandated by minimum parking requirements.

We shall treat off-street parking – which we shall refer to generically as garage parking – in the simplest possible way, by assuming that it is provided continuously over space at a constant cost of  $c = \$2.5$  per hour. In fact, in the downtowns of major metropolitan areas, because of economies of scale in garage construction, there is typically an irregular grid of parking garages, some public, some private, which engage in spatial competition with one another. Arnott and Rowse (2009) model this spatial competition, taking into account the technology of garage construction. But here we provide a simpler treatment in order to focus on the interaction between curbside and garage parking.

In what follows, we shall employ the terms “user cost” and “travel cost” to refer to the cost borne directly by users, in-transit travel time cost and cruising-for-parking time cost, and distinguish it from garage parking cost.

We start by considering first-best optimal curbside parking capacity when it is optimal to provide both curbside and garage parking, as it is in the example. Panel A of Figure 10 plots user cost for the first-best level of parking capacity. Aggregate resource costs equal aggregate user costs plus garage parking costs. Consider the effects of increasing curbside capacity by a small amount, holding throughput fixed. Doing so has two effects. The first is to decrease effective jam density, causing the user cost curve to shift up, from  $UC(r, P^*)$

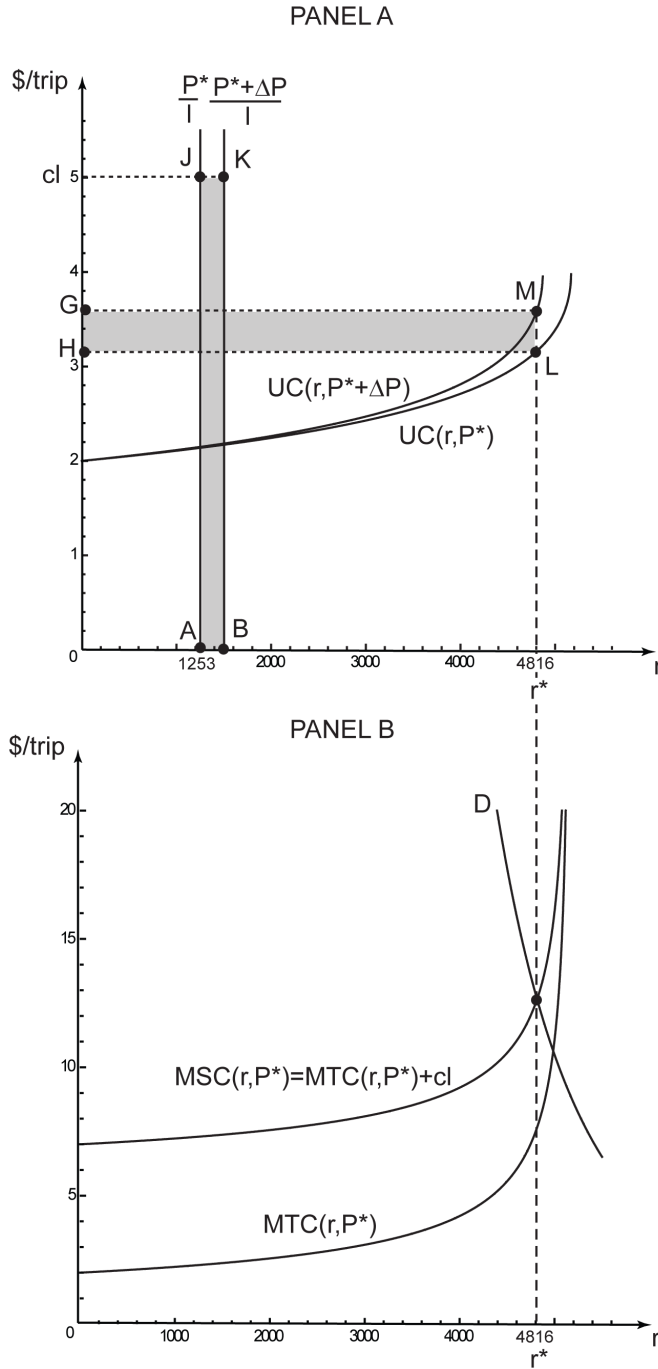


Figure 10: User cost for a given level of curbside parking capacity (Panel A) and the full first-best optimum (Panel B) with curbside and garage parking

to  $UC(r, P^* + \Delta P)$  in the figure, and aggregate user costs to increase. The second is to decrease the number of garage parking spaces that need to be provided. For the given number of travelers, first-best optimal curbside parking capacity is such that a unit increase

in capacity causes aggregate user costs to increase by  $cl$ , the saving in garage parking costs. We now need to determine optimal throughput. At the first-best optimum, the change in social surplus from an extra traveler is the same whether he is accommodated by increasing the amount of curbside parking or of garage parking. Assume the latter. The marginal social cost of the added traveler,  $MSC$ , is then the marginal travel cost,  $MTC$ , plus the garage cost,  $cl$ . And the optimum number of travelers is such that the marginal social cost of an added traveler equals the marginal social benefit. Panel B of Figure 10 displays the full first-best optimum. First-best optimal curbside parking capacity is  $P^* = 2504$ , and first-best optimal throughput is  $r^* = 4816$ .

We now turn to the situation where no congestion tolling is applied and where curbside parking is priced below the unit cost of garage parking, which is the case in most cities. When both curbside and garage parking are provided, drivers will choose whichever is cheaper. Thus, the stock of cars cruising for parking adjusts so that the full prices of curbside and garage parking are equalized:  $fl + (\rho Cl)/P = cl$ . Rearranging, we have that

$$C = \hat{C} \equiv (c - f) \frac{P}{\rho} \quad ; \quad (16)$$

thus, when both curbside and garage parking are provided in equilibrium, the stock of cars cruising for parking increases in proportion to the differential between the curbside and garage parking rates and to curbside parking capacity constraint. This yields the obvious but important result that cruising for parking can be eliminated by providing no curbside parking.

Figure 11 is like Figure 4 but adds garage parking. We start by defining two different short-run, quasi-supply curves for the same level of curbside parking,  $P$ . The first corresponds to the situation where a driver pays the curbside parking fee but experiences no cruising for parking since curbside parking is unsaturated, so that the full price of travel is  $F_1 = \rho mt(T, 0, P) + fl$ . The second corresponds to the situation where curbside parking is

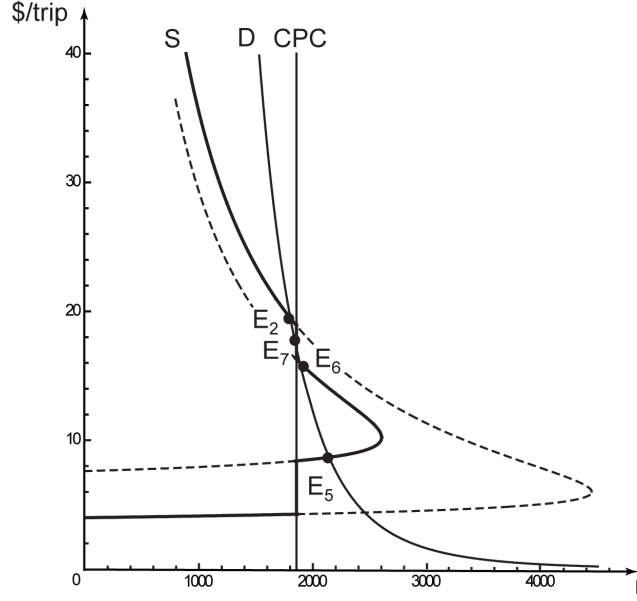


Figure 11: Equilibria with curbside and garage parking

saturated and garage parking occurs, so that there is cruising for parking with the stock of cars cruising for parking given by (16), so that the full price of travel is  $F_2 = \rho mt(T, \hat{C}, P) + cl$ , where  $\hat{C}$  is given by (16). Since the stock of cars cruising for parking reduces effective jam density, the second lies inside and to the left of the first. Now add the curbside parking capacity constraint of  $P = 3712$  (so that  $P/l = 1856$ ). To the left of the constraint, parking is unsaturated and the stock of cars cruising for parking is zero, so that the first quasi-supply curve applies. To the right of the constraint, garage parking is provided, so that (16) holds and the second quasi-supply curve applies. The supply curve, shown as the bold line  $S$  in the figure, contains five portions: (i) the portion of the second quasi-supply curve to the right of the parking constraint; (ii) the two portions of the first quasi-supply curve to the left of the parking constraint; and (iii) two segments of the parking constraint, each joining the first and second quasi-supply curves.<sup>15</sup> On (i), the scarcity rent on curbside parking is zero and there is no cruising for parking; on (ii) the scarcity rent on curbside parking is  $c$ ,

<sup>15</sup>The vertical segments of the supply curve on the curbside parking capacity constraint are an artifact of the model. In reality, curbside parking transitions smoothly rather than abruptly between unsaturated and saturated parking, and the supply curve of off-street parking is upward-sloping rather than flat, both of which would smooth the throughput supply curve.

and this rent is dissipated by cruising for parking, with the stock of cars cruising for parking given by (16); and on (iii) the scarcity rent on curbside parking is between 0 and  $c$ , and this rent is dissipated by cruising for parking, with the stock of cars cruising for parking varying between 0 and  $\hat{C}$ .

The demand curve is drawn for  $D_0 = 3300$ . With this demand curve and the assumed forms of the congestion and parking technologies, there are five equilibria, one of which, the gridlock equilibrium, cannot be displayed on the diagram. As expected, these equilibria alternate between stable and unstable equilibria. There are three stable equilibria. One is the gridlock equilibrium, which as before we label as  $E_3$ ; the second,  $E_6$ , is a hypercongested equilibrium with saturated curbside parking and no garage parking; and the third is a congested equilibrium with saturated parking and no garage parking. How the set of equilibria changes as demand intensity changes will be considered later.

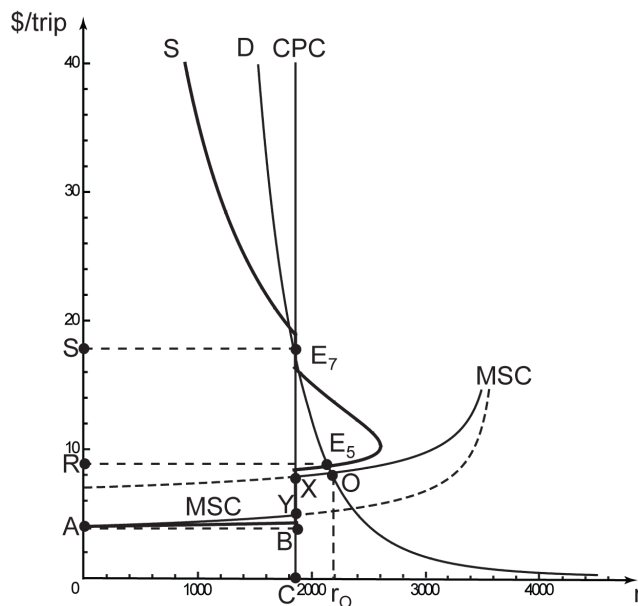


Figure 12: Deadweight losses associated with the two stable equilibria

Figure 12 displays the deadweight losses associated with the two stable equilibria when curbside parking may be supplemented with garage parking. It is analogous to Figure 5. The short-run marginal social cost curve corresponding to the curbside parking capacity



constraint  $P/l = 1858$ ,  $MSC(r; P)$ , is given by the locus  $AYXO$ , where  $O$  is the social optimum. Notice the discontinuous increase in the locus at curbside parking capacity, which indicates that the level of curbside parking is suboptimal. The social surplus at the optimum is given by the area between the demand and marginal social cost curves up to the first-best optimal level of throughput,  $r_O$ . The social surplus at the equilibrium  $E_5$  equals consumer surplus plus the curbside parking fee revenue,  $0ABC$ . Thus, the deadweight loss associated with equilibrium  $E_5$  is given by the area  $ARE_5OXY - 0ABC$ , which is **xxxx** per hour. The social surplus at the equilibrium  $E_7$  is determined analogously and is given by the area  $ASE_7OXY - 0ABC$ , which is **xxxx** per hour. The figure reinforces a point made in Figure 5, that the deadweight loss due to underpricing curbside parking can be substantial. It also illustrates another source of possible deadweight loss. If the transient dynamics are unfavorable, downtown traffic can end up at an inferior stable equilibria. In the example, ending up at equilibrium  $E_7$  rather than at equilibrium  $E_5$  generates deadweight loss of  $RSE_7E_5$ , which in the example equals **xxxx**.

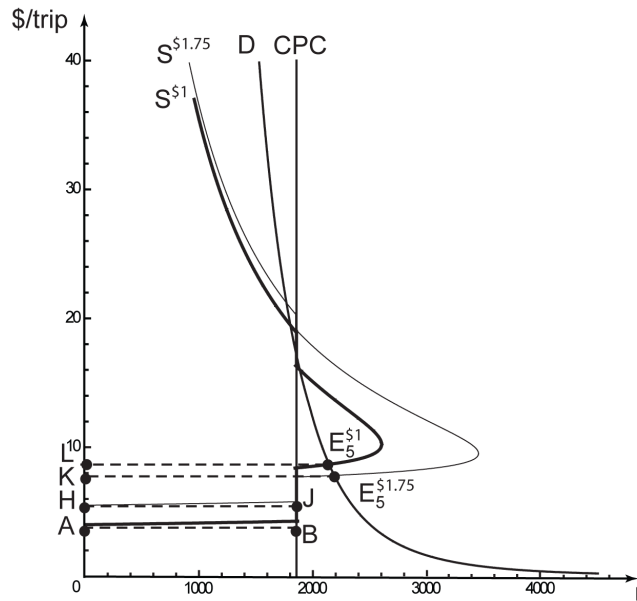


Figure 13: Social benefit from increasing the curbside meter rate

Figure 13 examines the social benefit from increasing the curbside meter rate such that the price differential between garage and curbside parking is reduced. Suppose that the curbside

parking rate is raised half the distance to its efficient level, from  $\$1.00/hr$  to  $\$1.75/hr$ . Doing so changes the value of  $\hat{C}$ . Denote the corresponding user cost curves by  $UC_{\hat{C}}^{\$1}$  and  $UC_{\hat{C}}^{\$1.75}$ , and the corresponding type-5 equilibria by  $E_5^{\$1}$  and  $E_5^{\$1.75}$ . The gain in parking meter revenue is given by the area  $ABJH$ , while the gain in social surplus equals the area  $LE_5^{\$1}E_5^{\$1.75}K$ . Thus, in contrast to the previous section where the gain in social surplus from increasing the meter rate exactly equals the increase in meter revenue, here the gain in social surplus may be several times the increase in meter revenue. The marginal burden of curbside parking fee revenue is then negative. Raising the meter rate, not only does the government obtain more revenue but also consumer surplus increases. In the previous section, the increase in the meter rate simply converted travel costs dollar for dollar into meter revenue. Here, the increase in the meter rate converts cruising-for-parking time costs dollar for dollar into meter revenue, with the added gain that the decrease in the stock of cars cruising for parking reduces traffic congestion, benefiting everyone.

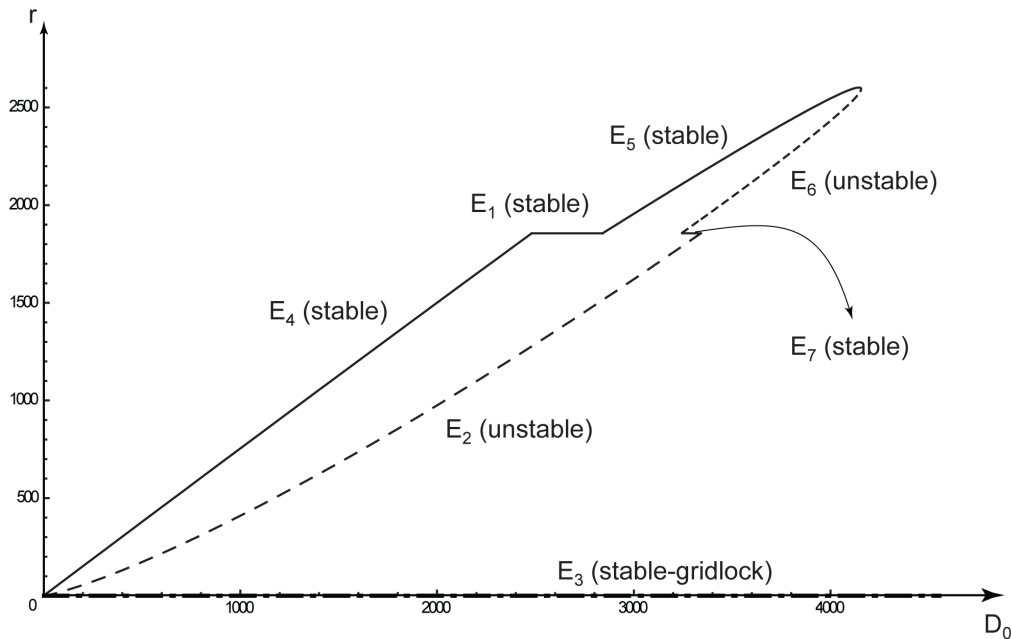


Figure 14: Social benefit from increasing the curbside meter rate

Figure 14 displays a bifurcation diagram, indicating the equilibrium throughputs at each level of demand intensity with the base-case parameter values. Equilibria of type 4 are

congested, stable, and unsaturated. Equilibria of type 1 correspond to equilibria of type 5 when there is no garage parking. The other equilibrium types are illustrated in Figure 11. Start at low levels of demand intensity. There is more than enough curbside parking to accommodate the demand. All three equilibria are unsaturated.  $E_4$  is the most efficient of the equilibria and is congested;  $E_2$  is hypercongested and unstable; and  $E_3$  is the dysfunctional, but stable, gridlock equilibrium. As demand intensity increases, a level is reached at which the efficient equilibrium becomes saturated, switching from a type 4 to a type 1 equilibrium. For an interval of higher demand intensities, the stable, saturated efficient type 1 equilibrium co-exists with the equilibria of types 2 and 3. The scarcity rent on curbside parking in the type 1 equilibrium is positive but not sufficiently high to make the construction of garage parking profitable, and traffic may be either congested or hypercongested. As demand intensity increases further, another level of demand intensity is reached at which the scarcity rent on curbside parking is sufficiently high to make garage parking profitable, and the type 1 equilibrium switches to a type 5 equilibrium. In the type 5 equilibrium, curbside parking is saturated, with the stock of cars cruising for parking being given by (16), traffic is congested, and garage parking occurs. As with the type 4 and type 1 equilibria, the type 5 equilibrium co-exists with equilibria of types 2 and 3. As the level of demand intensity increases further, another level of demand intensity is reached at which two new types of equilibrium emerge, types 6 and 7. A type 7 equilibrium has saturated curbside parking and no garage parking, and is hypercongested and stable. A type 6 equilibrium has both saturated curbside parking and garage parking, with the stock of cars cruising for parking being given by (16) and is hypercongested and unstable. As demand intensity increases yet further, the type 2 equilibrium becomes saturated, and the type 2, 6, and 7 equilibria collapse into a type 7 equilibrium. Finally, the demand intensity becomes so high that the downtown street use can be rationed only with gridlock.

Consider next second-best optimal capacity, when the distortion is again the underpricing of curbside parking. When demand intensity is sufficiently high that garage parking occurs in



consumer surplus of *BAML*. The increase in curbside parking revenue is given by *GHNR*. At an interior optimum of curbside parking capacity, the two areas are equal.

Three qualitative results are immediate. The first is that there should be no curbside parking when it is provided free; providing curbside parking reduces consumer surplus with no compensating increase in curbside parking revenue. The second is that, over the interval of throughputs for which an interior solution is optimal and with inelastic demand, second-best curbside parking capacity falls with demand intensity; as demand increases, the increase in curbside parking revenue from an increase in curbside parking capacity of  $\Delta P$  remains constant but the loss in consumer surplus increases due to the convexity of the congestion technology. The third is that below some level of demand intensity it is efficient to allocate all curbside to parking, while above another level of demand intensity it is efficient to allocate no curbside to parking.

Now consider the case where no garage parking occurs in the second-best optimum. We are then in the situation displayed in Figure 8 of the previous section. The curbside parking constraint just binds at the second-best optimum, which is given by the point of intersection of the demand curve and the long-run supply curve (conditional on only curbside parking).

**Reference to Figure 16 in the text.**

## 5 Stability Analysis

Arnott and Inci (2010) provided a thorough stability analysis of a variant of the model presented above with only curbside parking. Stability analysis of traffic congestion has proved difficult since it requires solving for the out-of-equilibrium dynamics of traffic flow over time and space. The treatment of downtown as isotropic simplifies the analysis considerably since at any point in time traffic flow is the same throughout the downtown area; the analysis then entails solving ordinary rather than partial differential equations. Arnott and Inci further

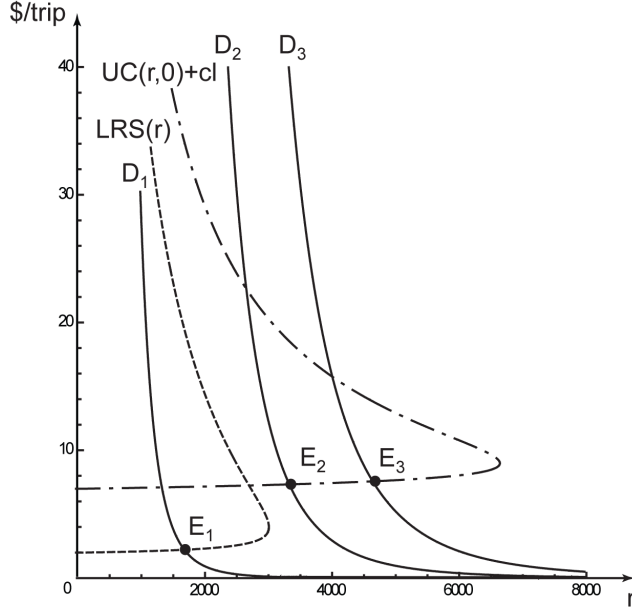


Figure 16: The possibility of no garage parking in the second-best optimum

simplified the problem by making some special assumptions<sup>16</sup> that render the differential equation system autonomous (time does not enter the analysis explicitly), which permits phase-plane/state-space analysis.

Figure 17 reproduces Figure 11 in their paper. The vertical plane corresponds to states of the downtown traffic system with saturated parking and therefore with cruising for parking. The state of the system is then characterized by  $T$  and  $C$ . The horizontal plane corresponds to states of the downtown traffic system with unsaturated parking and therefore no cruising for parking. The state of the system is then characterized by  $T$  and  $S$ , where, recall,  $S$  is the density or stock of *occupied* curbside parking spaces per unit area. In the vertical plane, the  $\dot{C} = T/(mt(T, C, P) - P/l = 0$  curve gives the locus of points for which the density of cars cruising for parking is unchanging over time, and the  $\dot{T} = D(F) - T/(mt(T, C, P) = 0$  (where  $F$  is given in (14)) lines are the corresponding locus of points for the density of cars in transit. One  $\dot{T} = 0$  line corresponds to jam density, for which both the inflow and outflow

<sup>16</sup>They assume that trip lengths and visit durations are negative exponentially distributed and that aggregate travel demand at a point in time is a function only of the density of cars in-transit and cruising for parking at that time.

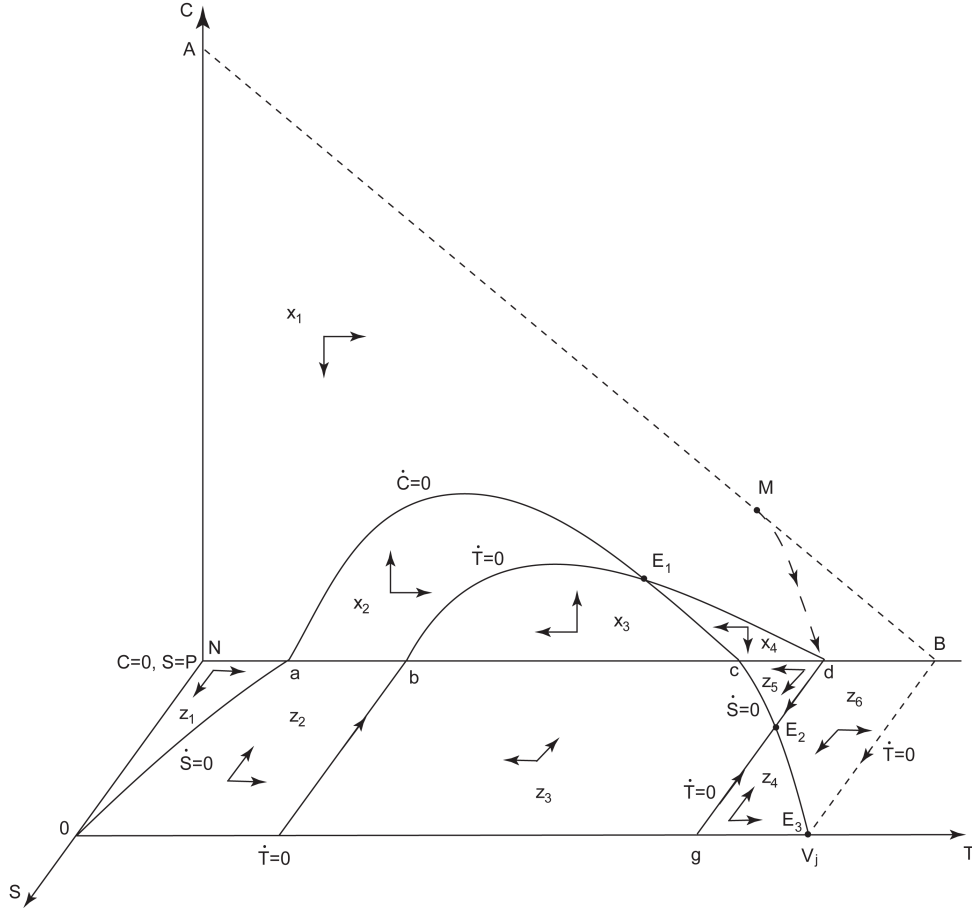


Figure 17: Transient dynamics of downtown traffic when there is only curbside parking

rates are zero. We shall refer to the other  $\dot{T} = 0$  line as the  $\dot{T} = 0$  locus. The arrows give the direction of motion. The vertical plane displays the  $\dot{T} = 0$  and  $\dot{S} = 0$  loci and the arrows indicate the direction of motion.

The details of the analysis and the results are quite complex. Suffice it for the moment to note that the results are consistent with the diagrammatic analysis in section 3. The  $E_1$ ,  $E_2$ , and  $E_3$  in Figure 17 correspond to the  $E_1$ ,  $E_2$ , and  $E_3$  in Figure 4. In section 3, we claimed that  $E_1$  and  $E_3$  are stable equilibria while  $E_2$  is unstable. Figure 17 presents these results and also makes precise the relevant notions of stability:  $E_1$  and  $E_3$  are locally stable, while  $E_2$  is unstable. We note too for future reference that an increase in demand intensity has no effect on the  $\dot{C} = 0$  and  $\dot{S} = 0$  loci but causes the  $\dot{T} = 0$  locus to shift downward.





to apply. When  $C \geq \hat{C}$ , however, the stability analysis of Figure 17 needs to be modified. If  $C > \hat{C}$ , garage parking is cheaper than curbside parking. We assume that when this occurs the number of cars cruising for parking falls instantaneously such that  $C = \hat{C}$  is satisfied. Thus, above  $C = \hat{C}$ , the direction of motion is vertically downward. The direction of motion along  $C = \hat{C}$  is as indicated. An intersection point of  $C = \hat{C}$  with the  $\dot{T} = 0$  locus is an equilibrium with garage parking if and only if the throughput associated with it exceeds  $P/l$ . Since the  $\dot{C} = 0$  line gives the locus of  $(T, C)$  such that throughput equals  $P/l$ , this condition is satisfied if and only if the point of intersection lies below the  $\dot{C} = 0$  line. Thus, an intersection point of  $C = \hat{C}$  with the  $\dot{T} = 0$  locus is an equilibrium with garage parking if and only if it lies below the  $\dot{C} = 0$  line. This implies that no equilibrium with garage parking exists when the parking equilibrium condition lies above the  $\dot{C} = 0$  line. Figure 18 displays the configuration of the  $T - C$  plane when the parking equilibrium condition is added, and the price differential between curbside and garage parking is sufficiently low that the  $C = \hat{C}$  line intersects the  $\dot{C} = 0$  locus.

There are three cases to consider. In case I, the parking equilibrium condition lies below  $E_1$ . In case II, the parking equilibrium condition lies above  $E_1$  and intersects the  $\dot{T} = 0$  locus. In case III, the parking equilibrium condition lies above  $E_1$  and does not intersect the  $\dot{T} = 0$  locus. We shall consider the three cases in turn.

Turn to Figure 18, which displays case I. With the introduction of the parking equilibrium condition, the equilibrium  $E_1$  disappears and is replaced by two candidate equilibria,  $E_4$  and  $E_5$ .  $E_4$  can be ruled out as an equilibrium with garage parking since the throughput associated with it is less than  $P/l$ .  $E_5$  meets the conditions for an equilibrium, and is stable. Now turn to Figure 19, which displays case II. With the introduction of the parking equilibrium condition,  $E_1$  remains an equilibrium, and  $E_4$  and  $E_5$  emerge as candidate equilibria. The throughput at both  $E_4$  and  $E_5$  exceeds  $P/l$ .  $E_4$  is unstable and  $E_5$  is stable. Now turn to Figure 20, which displays case III. With the introduction of the parking equilibrium condition,  $E_1$  remains an equilibrium, and there is no equilibrium with garage parking. In-

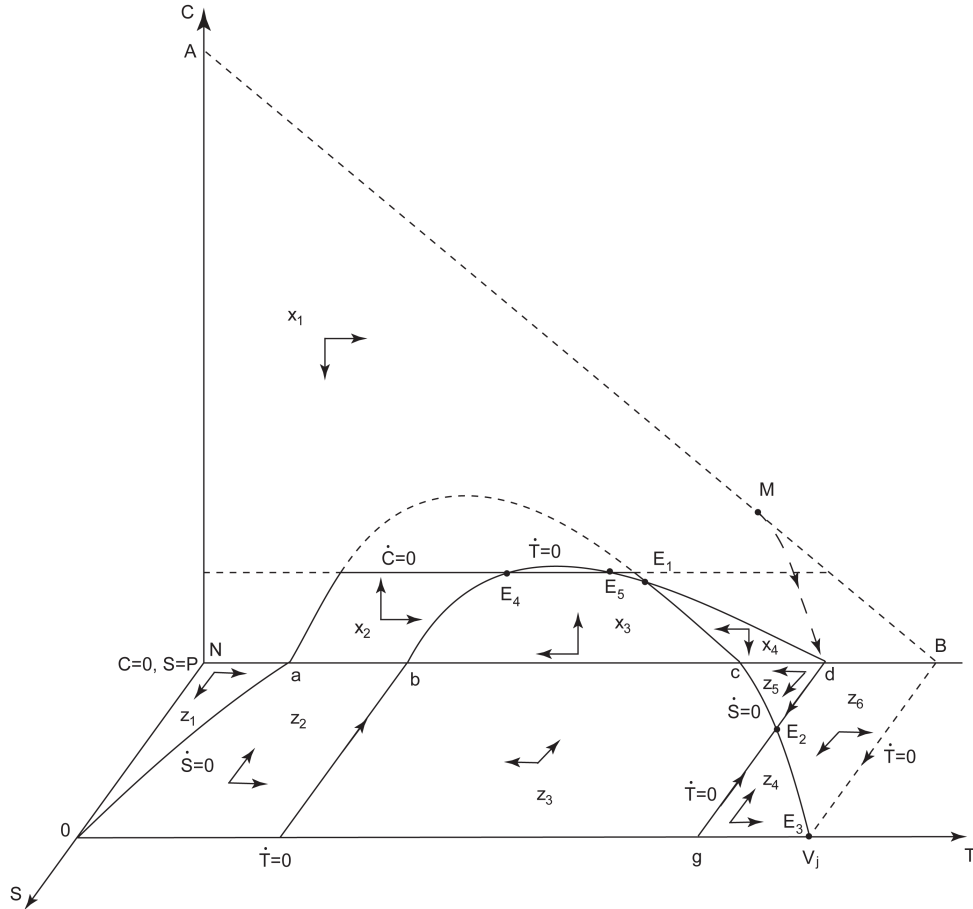


Figure 19: Transient dynamics of downtown traffic when there are curbside and garage parking, case II

creasing  $D_0$ , which causes the  $\dot{T} = 0$  locus to shift down, provides another way of generating the bifurcation diagram of Figure 14.

## 6 Directions for Future Research

Diagrammatic analysis is insightful since it draws on geometric intuition, but it can only go so far. The above analysis omits a number of important considerations, which cannot be easily handled via diagrammatic analysis. First, households are assumed to be identical, but of course driver heterogeneity is important. Arnott and Rowse (2008) explore how drivers who differ in their visit durations and values of the time sort themselves between curbside

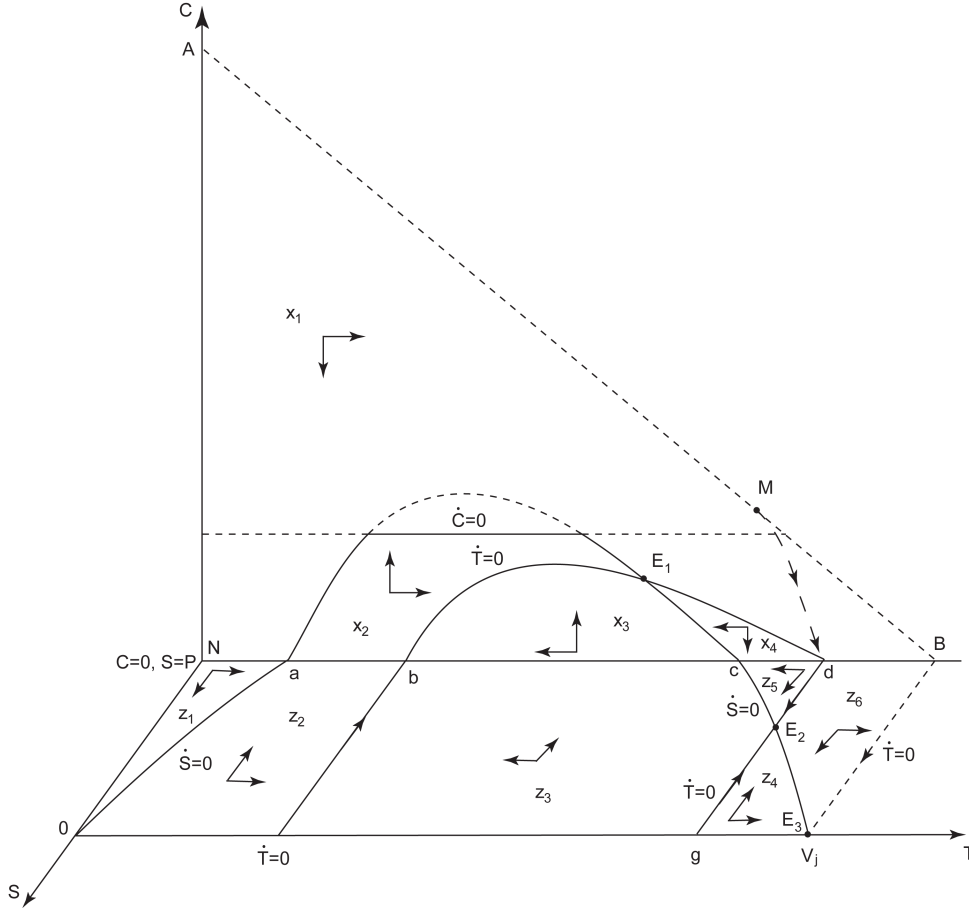


Figure 20: Transient dynamics of downtown traffic when there are curbside and garage parking, case III

and garage parking, and how, when drivers differ, curbside parking time limits can be used to reduce cruising for parking. Second, only steady-state equilibria are explored but the demand for parking spaces varies systematically over the course of the day. Ideally the meter rate would be adjusted over the day to clear the market for curbside parking. However, most cities apply single-step curbside parking fees, in which the curbside parking fee is fixed over the business day and free at other times, with the result that cruising for parking occurs during peak hours.

Third, mass transit is ignored. Via the Envelope Theorem, our analysis carries through if mass transit is organized efficiently, treated implicitly in the demand function. But if mass transit is not organized efficiently, welfare analysis should take into account how parking

policy affects the deadweight losses in the mass transit market. Fourth, our analysis assumes that garage parking is supplied and priced at constant cost. Extending the analysis to treat an upward-sloping supply curve for garage parking is straightforward, but extending it to treat spatial competition between garage parking operators is not. The spatial competition model presented in Arnott (2006) and Arnott and Rowse (2009) is coherent but its behavior is likely unrealistic since it ignores capacity constraints. The market power exercised by garage parking operators is likely sufficiently important that it should be explicitly treated in the analysis of downtown parking policy. Fifth, the analysis assumes downtown to be isotropic, but of course spatial variation in parking policy reflecting spatial variation in traffic is important; resident parking regulation in residential neighborhoods is one example.<sup>17</sup> Sixth, our analysis pays no attention to land use, except for the allocation of exogenous road space to parking. This may be a reasonable short-run assumption in the context of downtown traffic congestion, but over longer periods the allocation of downtown space to roads is an important aspect of downtown traffic policy, and the effects of downtown parking policy on land use both inside and outside the downtown area may be significant. Seventh, our models ignore two important aspects of downtown parking, heavily subsidized employer-provided parking<sup>18</sup> and minimum parking requirements. These may be treated as exogenous in a model of downtown parking, and better yet should be derived as properties of equilibrium. Seventh, our analysis assumes that downtown traffic congestion is appropriately modeled using classic traffic flow theory, which was developed from freeway traffic. One alternative is to model congestion using intersection queuing theory. Another is to employ a traffic and parking microsimulator. Eighth, much of our second-best analysis takes the underpricing of curbside parking to be an exogenous distortion. This can reasonably be challenged since typically the downtown parking authority determines both meter rates and the allocation of curbside to parking. Also, there seems to be widespread agreement that downtown curbside

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<sup>17</sup>For reasons explained earlier, treating spatial variation analytically is likely to prove intractable. In policy analysis it can be treated by employing a downtown traffic and parking microsimulator, such as VISSIM.

<sup>18</sup>Small and Verhoef (2007) make the informed guess that US urban commuters pay for at most 2.5% of their workplace parking costs. The percentage is higher in the downtowns of large metro areas.

parking is underpriced due to lobbying by downtown merchant associations, who argue that it is needed to compete with free suburban shopping center parking and to keep downtown vital. If this is correct, then parking policy should be evaluated either taking these objectives into account or taking them into account via political economy constraints.

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