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ABSTARCT

The value of the freezeout option is critical in many legal policy issues concerning corporate law. In this article, we present, for the first time, a method for determining the value of the minority stock and the freezeout option. We price the freezeout option with two different sets of assumptions regarding the controlling shareholder informational advantage, using both an exogenous and endogenous stock prices in our pricing. The result of our model indicates that the freezeout option has a low value and the minority stock is only slightly discounted. This result implies that the use of publicly known information, including market prices, in determining a fair value for minority stocks will not cause expropriation of minority shareholders and will not lead to inefficiency in corporate and controlling owners' decisions. Empirical studies support this view.

Introduction

Delaware's corporate law entitles a controlling shareholder to buy out - or "freezeout" - the minority shareholders.[1] Leaving aside the technical aspects of the freezeout, which is commonly performed through a merger, the most significant result is that the controlling shareholder is not just able to force the minority to sell their shares to her, but also determines the price of these shares. The right to freezeout, thus, carries the risk of minority shareholders expropriation. To counter that risk, the law offers protection to minority shareholders: a shareholder who is dissatisfied with the price offered for the shares in the merger is entitled to ask the court to determine the fair value of her shares. Without getting into the details, this result is accomplished either by using the "appraisal right",[2] or by claiming breach of fiduciary duties, thereby initiating the "entire fairness" test.[3]

The controlling shareholder's freezeout right is, in fact, a call option on the minority shares for an indefinite time whose exercise price is determined by the option holder. However, given the appraisal right and the duty to meet the entire fairness standard, the exercise price should not be lower than the expected fair price of the shares in the court's valuation process.[4]

Regardless of the particular valuation principles, courts rely on publicly known information about the corporation as a basis for the valuation process. One important piece of publicly known information is the market price of publicly traded corporations. The market

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[1] *See, e.g.,* Paramount Communications Inc., v. QVC Network Inc., 637 A.2d 34 (Del. 1994).

[2] *See,* Delaware General Corporations Law, Sec. 262(a) and (b).

[3] *See,* Weinberger v. UOP, 457 A.2d 701 (Del. 1983).

[4] *See,* Frank H. Easterbrook and Daniel R. Fischel, THE ECONOMIC STRUCTURE OF CORPORATE LAW, 148-150 (1991) (appraisal rights establish a floor under the freezeout price).

price enters the valuation process in variety of ways: as the main indication of the fair value;[5] as part of a "block" valuation that weighs and averages several different methods of valuation;[6] or as a component of a given valuation method.[7]

This reliance on publicly known information, however, might generate a risk of under-valuation of the minority shares even in efficient capital markets. This risk has two related possible sources. The first source is the controlling shareholder's informational advantage: since the controlling shareholder holds private information about the future value of the corporation, she can time the exercise of the option to her maximum benefit. Indeed, whenever the expected future price is higher than the current market price, the option can be exercised. Given that courts are unable to reflect the value of private information held by the controlling shareholder in the valuation process, the valuation process might result in under-valuation of minority shares. [8]

This initial under-valuation leads to a second source of under-valuation: investors expecting to receive fair value based on a discounted price -- due to the existence of the freezeout option -- discount the stock market price further to reflect the discounted expected "fair value". Given that the now discounted market price will in turn influence the valuation process, leading to an even greater under-valuation of the fair value of the minority shares, the

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[5] *See, e.g.,* Smith v. Shell Petroleum, Inc., 1990 WL 84218 (Del. Ch. 1990).

[6] Under the block method used by Delaware courts the appraiser computes separate values for market, earnings, and net assets, gives weight to each and adds them together.

[7] For instance, using the market price movements to calculates beta to be used in a capital assets pricing model.

[8] *See*, Lucian A. Bebchuk & Marcel Kahan, *The "Lemons Effect" in Corporate Freeze-Outs*, forthcoming, in Randall
Morck (ed.), Conrentrated Ownership (U. Chi. Press).

market price will be discounted further. This process, known as the "lemons effect", will repeat itself until the stock price drops to the lower end of the expected range of values.[9]

Indeed, if minority shares are under-valued in cases of freezeouts, this fact will be reflected, ex ante, in the stock price. That is, minority shareholders will discount the price that they are willing to pay for the shares, in response to the risk of under-valuation, and thereby avoid the risk of expropriation. However, even in this case, under-valuation of the fair value of minority shares will result in inefficiency. To the extent that the freezeout option provides the controlling owner with a private benefit of control, she will attempt, ex post, to capture this benefit. These attempts will result in inefficiency due to: inefficient investment in search for private information; inefficient business decisions designed to increase the value of the option; and unexpected (unpriced) instances of minority shareholders' expropriation.[10]

Against that view others have argued that in an efficient market minority shares are properly priced. Consequently, in capital markets where shares are traded frequently enough to have a market price, the pre-merger fair value is the pre-investment market price.[11] Thus, there is no need for complicated appraisal proceedings: every price above the pre-merger market price is a fair price. This view regards the risk of under-valuation due to the freezeout option as negligible.

How serious is the risk of under-valuation? In other words, how valuable is the freezeout option? If the freezeout option has only minor value, neither of the evils mentioned

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[9] Bebchuk and Kahan, *Id*. Easterbrook & Fischel, *supra note* 4, at pp.154-55, also voiced such a concern.

[10] Bebchuk and Kahan, *Id*.

[11] Benjamin Hermalin & Alan Schwartz, *Buyouts in Large Companies*, 25 J. Leg. Stud. 351 (1996).

above should be expected and the whole valuation process can indeed be simplified for firms having efficient pricing: every price above the pre-merger market price is a fair price. On the other hand, if the freezeout option is very valuable (i.e., the risk of under-valuation is substantial) inefficiency will result, and a proper policy for freezeout proceedings should be devised.

As the value of the freezeout option is critical to this debate, the importance of pricing the freezeout option becomes clear. Yet, pricing an option that is exercised based on private information regarding the future value of the stock is a complicated task. To date no such attempt was made. In this article, we present, for the first time, a method for determining the value of the minority stock and the freezeout option.

The result of our model indicates that the freezeout option has a low value and the minority stock is only slightly discounted. This result implies that the use of publicly known information, including market prices, in determining a fair value for minority stocks will not cause expropriation of minority shareholders and will not lead to inefficiency in corporate and controlling owners' decisions.

The intuition for this result is simple. Although the controlling shareholder holds the freezeout option for indefinite time, it can be exercised only once. Thus, the controlling shareholder will not exercise the option the first time that her private information indicates that the firm value is higher than the current market price. Rather, the controlling shareholder will attempt to capture the greatest expected divergence between the current price and the privately known future price. This strategy is aimed at the extreme cases of high expected values for the firm, leaving most of the expected range of values out of the reach of the freezeout option.

However, our basic model uses an exogenous stock price to price the freezeout option,

avoiding the possibility of a "lemons effect". To understand this point, assume that there are two countries: country A in which freezeout is allowed and country B in which freezeout is restricted. In country B, the stock will be traded for its full value, reflecting the whole range of expected probabilities of values. In country A, the stock will be discounted to reflect the value of the freezeout option. In our model, investors in country A expect the court determining the fair value to draw the market price from (hypothetical) country B.

On the other hand, when the stock price is endogenous the court determining the fair value of investors in country A draws the market price from country A, in which the stock is traded and a freezeout is allowed. Investors expecting a discounted "fair vale" further discount the stock price. It is this feature that gives rise to the "lemons effect".

We use an exogenous stock price because we believe that it better reflects courts' practices and investors' expectations. Courts rely on financial and accounting data that are *independent* of the firm's market price alongside the market price. The investors' expectation of the courts to use this data is similar to an expectation to use exogenous stock price in the valuation process. In other words, the value of the option is determined by the value of the private information held by the controlling owner, and not by her ability to capture the drop in the stock price due to the "lemons effect."

Nevertheless, to offer a complete analysis of the freezeout option, we present an extended general model that allows us to price the option based on endogenous stock price as well. The result of our model indicates that in this case the price of the minority shares will drop all the way to zero. This result contradicts the empirical reality of minority stocks trading with positive prices. The reason can either be that courts do not assign high weight to the market price in the valuation process, thereby leaving some positive value for the stock, or that

investors expectations are based on the assumption that courts will use exogenous stock price. Once investors hold such expectations, minority stock price is only slightly discounted, and can be used by the courts as the equivalent of an exogenous price. Empirical studies support this view.

The article is organized as follows: In the first part we price the freezeout option using a basic model with two different sets of assumptions regarding the controlling shareholder informational advantage. In the second part we price the freezeout option using an extended general model that allows us to use both an exogenous and endogenous stock prices in our pricing. This model is able to capture the “lemons effect”. In the third part we discuss the model’s implications for the debate surrounding the value of the freezeout option. In the fourth part we discuss some empirical studies that can supports our results. The fifth part is the conclusion.

I. A **Basic Model**

1. Pricing A One Day Option

Let’s assume that markets are efficient, and posit that the pre-merger market price is the fair price. The freezeout option is, then, a call option for an indefinite time to buy a share at today's market price. If the controlling shareholder has no information about the future price, and the demand curve for the shares is perfectly elastic, this option is valueless as the controlling shareholder can buy the shares on the market for the same price without it.

But if the controlling shareholder has private information about the future price of the share, the option will be valuable. She has a call option to buy a share at today's market price, while only she knows tomorrow's market price. This informational advantage provides the

option-holder the benefit of foresight; she knows two values -- today's market price and tomorrow's market price -- before she decides whether or not to exercise the option.

The option-holder, however, will not exercise the option the first time it gets into the money, i.e., as soon as tomorrow's market price is greater than today's market price. Rather, since the option is indefinite but can be used only once, the option-holder will wait for the time when the difference in expected value between the price today and the price tomorrow is large enough to maximize her profit. In other words, the option-holder will attempt to capture the highest range of the expected probabilities of values. This, of course, will influence the share's price, that will endure a drop equal to the value of the option. The share and the option, together, reflect the whole range of expected probabilities of values for the corporation. Thus, the value of the option is subtracted from the potential value of the share without the freezeout option.

To enable ourselves to price the option, we equalize the freezeout option to a perpetual call option to buy a share today for an exercise price equal to yesterday's market price. The equalization is proper given the continuous nature of time: today’s and tomorrow’s market prices will soon become yesterday’s and today’s market prices, respectively. Here, too, the option-holder has the benefit of hindsight. The option-holder knows two values -- yesterday's market price and today's market price -- before she decides whether or not to exercise the option. Here, too, the option-holder will attempt to capture the largest expected difference between today's price and yesterday's price.

Effecting this change in the description of the option will allow us to price the option using a simple model. We assume that the stock price follows a random process with changes in the price distributed normally over a short time horizon. In fact, this assumption is similar to an arithmetic Brownian motion, i.e. “random walk” (and not geometric as in Black-Scholes-

Merton), which is a reasonable assumption for a short time-horizon.

The perpetual option allows the owner to buy one share of stock at yesterday's price. First, it should be noted that since there is no explicit time dependence, the option's price does not depend on calendar time. Second, the option is homogeneous in price (there is no fixed strike), so its price is linear in the stock price. We can always normalize the stock price at $1. The price of the option is a function of the distribution of price changes, interest rates, time interval (one day in the current settings) and is proportional to the stock price.

Denote the value of the option by v, which is then equal to the expected payoff relative to the risk-neutral probability measure. The payoff is defined by the realized price change. The optimal exercise strategy is to exercise the option as soon as the price change is greater than the cash value of the option but to keep it alive otherwise.

Denote the risk neutral probability density distribution of the price change X by $\rho(x)$. Two events are possible: either the price jump is above v (then the option will be exercised) or the price jump is below v and the option is then worth more alive than dead:

$$ve^{rt} = v\int_{-\infty}^{v} r(x)dx + \int_{v}^{+\infty} xr(x)dx$$

Denote the yearly drift and volatility of the stock price by μ and σ. The corresponding drift and volatility of the price changes in time τ (τ =1 day = 1year /365) is then $\mu\tau$ and $\sigma\sqrt{\tau}$. Since we assume the price changes to be distributed normally, we can use the standard cumulative normal:

$$\int_{-\infty}^{v} r(x)dx = N\left(\frac{v - mt}{s\sqrt{t}}\right)$$

And correspondingly:

$$\int_{v}^{+\infty} x r(x)dx = mt\left(1 - N\left(\frac{v-mt}{s\sqrt{t}}\right)\right) + \frac{s\sqrt{t}}{\sqrt{2p}} e^{-\frac{1}{2}\left(\frac{v-mt}{s\sqrt{t}}\right)^2}$$

The option pricing equation becomes:

$$ve^{rt} = mt + (v-mt)N\left(\frac{v-mt}{s\sqrt{t}}\right) + \frac{s\sqrt{t}}{\sqrt{2p}} e^{-\frac{1}{2}\left(\frac{v-mt}{s\sqrt{t}}\right)^2}$$

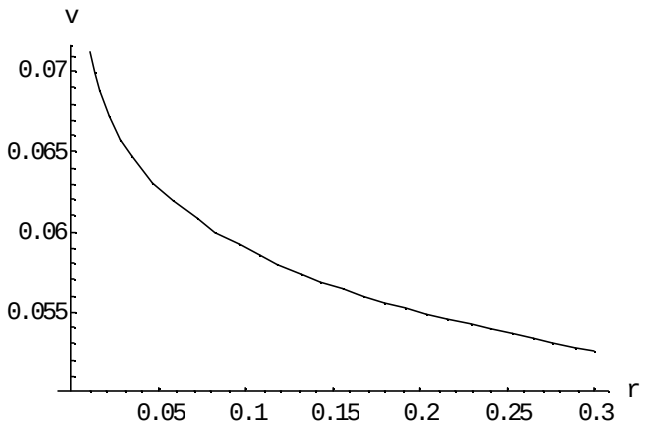
Its solution is not analytic, but can be easily found numerically.

The probability of the exercise is

$$\int_{v}^{+\infty} r(x)dx = 1 - N\left(\frac{v-mt}{s\sqrt{t}}\right)$$

The numerical results[12] are presented below. As a starting point we choose r = 10%, σ = 40%, μ = 15%. The price of this option is then \$0.059. The probability of an exercise tomorrow is 0.00268.

We plot below several graphs that show how the price of this option changes with changes in the parameters (interest rate, drift and volatility).



[12] Calculated with *Mathematica*.

Figure 1.
Value of a 1 day option when interest
rates change from 1% to 30%, μ=15%, σ=40%.

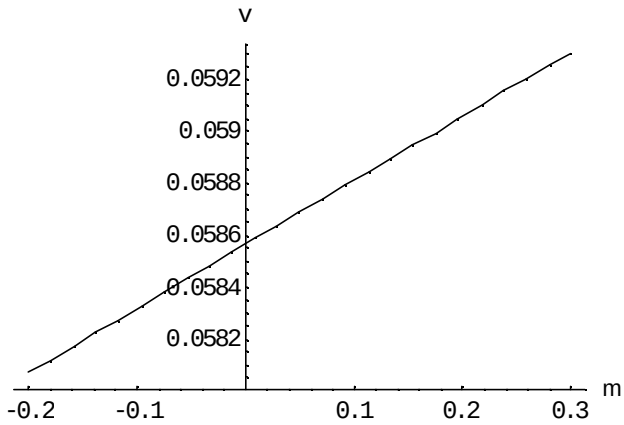


Figure 2.
Value of a 1 day option when drift μ
changes from -20% to 30%, r=10%, σ=40%.

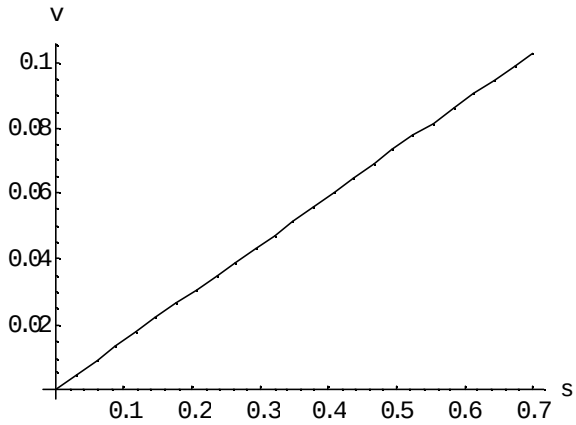


Figure 3.
Value of a 1 day option when volatility σ
changes from 0% to 70%, μ=15%, r=10%.

2. Pricing A Multi-day Option

The results suggest that the value of the option is not as high as might be expected. However, we priced an option for a single day. One may argue that the future event of which the

controlling shareholder has private information will be reflected in tomorrow's market price if is known to the market. In other words, it does not matter if the significant development in the corporation -- about which the controlling shareholder knows -- will take place next week, next month or next year, as once known, it will be reflected in tomorrow's market price.

However, this is not a correct valuation of the option. Whether the option holder has a wider range of values to choose from (information advantage) does make a difference. The option will have greater value if the option-holder can look to the future and see the share's prices for several days in advance before deciding whether or not to exercise. This foresight will increase the option-holder's ability to capture the largest difference between today's and tomorrow's prices. Thus, the longer the horizon, the greater the value of the option.

Using the same method, we price an option that allows the option holder to exercise the option, while looking backward for up to 100 days. Each day the option holder can exercise the option for a price equal to the stock price 100 days ago. This is equivalent to a freezeout option with a controlling shareholder having a horizon looking into the future for the same time.

Furthermore, we price the option assuming that the option holder knows, every day with certainty, the whole range of future prices within her horizon. In reality it seems more plausible that the controlling shareholder will have private information about the future price for only part of the time. Therefore, we add a probability factor q to our pricing, which reflects the probability with which the option holder knows the future prices within her horizon. It is clear that the higher the probability, the more valuable the option.

The freezeout option allows its owner to buy the stock at the price registered some time (τ) ago, but it is active with some probability (q) and inactive with probability (1-q). The idea is that the principal shareholder has important information about future price changes with some

probability (but not 100%) and can then buy shares back from other stockholders at the current price (or slightly higher). We assume that there is zero correlation between the information and the jump size. This is a useful assumption, even though in many cases the controlling shareholder will be informed of significant events first. However, as soon as q is a parameter, this can be incorporated in a risk-neutral version of q (in other words q is an adjusted probability).

In addition, we assume here a European type option. If the controlling shareholder has the information for a forthcoming period of length τ, but is prohibited from freezingout and then immediately reselling the stock, then the only information that matters is the information for the longest time horizon. In other words, if the private information predicts a price increase followed by a decline, the freezeout option should not be exercised.

Given the above assumptions, there are two possible events: either the price jump is above v and, with probability q, the option will be exercised, or the price change is not big enough to exercise the option. Since ρ is the risk-neutral probability measure, we can equate the future value of the current price with the expected payoff:

$$ve^{rt} = v\int_{-\infty}^{v} r(x)dx + q\int_{v}^{+\infty} xr(x)dx + (1-q)v\int_{v}^{+\infty} r(x)dx\,.$$

The option pricing equation becomes:

$$ve^{rt} = vN\left(\frac{v-mt}{s\sqrt{t}}\right) + \left((1-q)v + qmt\right)\left(1-N\left(\frac{v-mt}{s\sqrt{t}}\right)\right) + q\frac{s\sqrt{t}}{\sqrt{2p}}e^{-\frac{1}{2}\left(\frac{v-mt}{s\sqrt{t}}\right)^2}$$

Or:

$$ve^{rt} = vN\left(\frac{v-mt}{s\sqrt{t}}\right)+\left((1-q)v+qmt\right)N\left(\frac{-v+mt}{s\sqrt{t}}\right)+q\frac{s\sqrt{t}}{\sqrt{2p}}e^{-\frac{1}{2}\left(\frac{v-mt}{s\sqrt{t}}\right)^2}.$$

Its solution is not analytic, but can be easily found numerically. The probability of the exercise is: $q\int_{v}^{+\infty} r(x)dx = q\left(1-N\left(\frac{v-mt}{s\sqrt{t}}\right)\right)= qN\left(\frac{-v+mt}{s\sqrt{t}}\right).$

We plot below several graphs that show how the price of this option changes when the parameters (interest rate, drift and volatility) change.

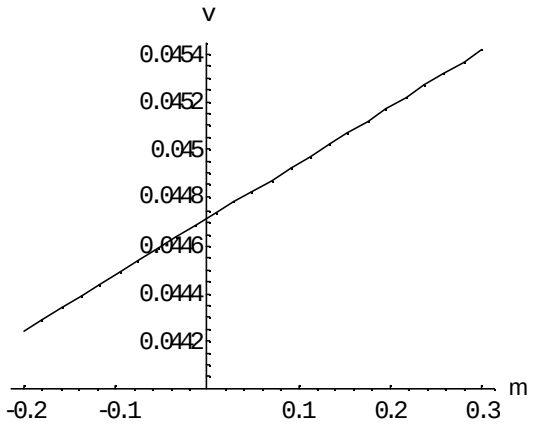


Figure 4.
Value of a 1 day option when drift μ changes from -20% to 30%, r=10%, σ=40%, q=0.1.

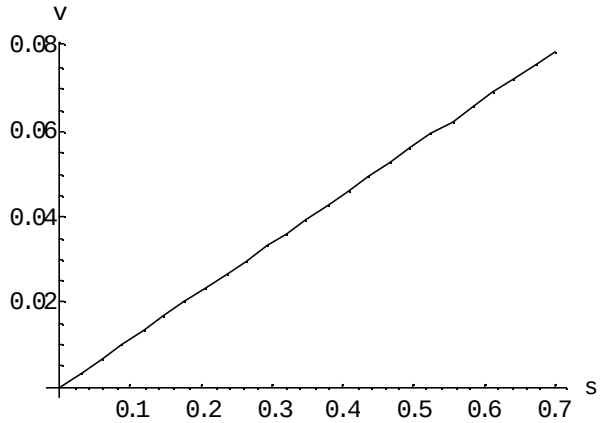


Figure 5.
Value of a 1 day option when volatility σ
changes from 0% to 70%, μ=15%, r=10%, q=0.1.

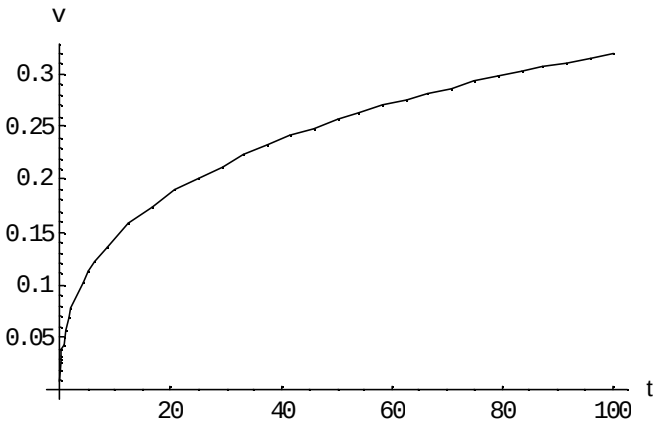


Figure 6.
Value of a multi-day option when the time horizon t
changes from 1 to 100 days, μ=15%, σ=40%, r=10%, q=1.

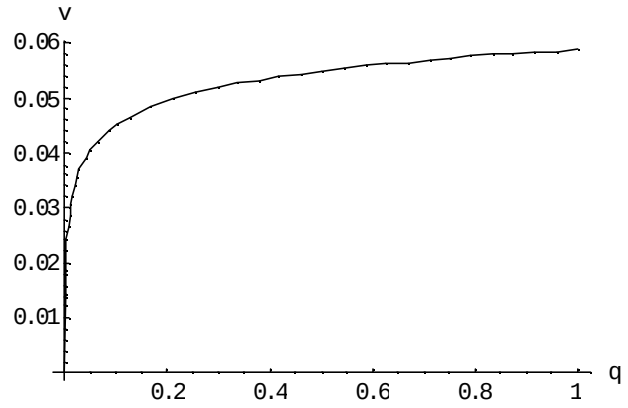


Figure 7.
Value of a 1 day option when the probability q
changes between 0 and 100%, μ=15%, σ=40%, r=10%.

According to this model the price of the freezeout option ranges from 4% to 6% of the stock price. This result is robust relative to all major parameters of the model as demonstrated in the figures above. As stated earlier, this basic model uses an exogenous stock price that avoids the under-valuation risk generated by the "lemons effect". Next, to complete the analysis, we

present a more general model that will allow us to price the freezeout option based on either exogenous or endogenous stock price.

II. An Extended General Model

We consider a company that distributes all profits and losses (P&L) to shareholders. The price of the whole company does not change and we set it equal to \$1. In addition we assume that the profits and losses over a short time horizon are distributed normally (i.e., follow the standard Arithmetical Brownian Motion). The longer the time horizon, the greater the mean of P&L and the greater the standard deviation of the distribution. For instance, a daily expected profit of 0.06% will translate into a yearly expected profit of 15%, and a daily volatility of 2.5% can be translated into a yearly volatility of 40% (drift is linear in time and volatility is proportional to the square root of time). Denote by m the infinitesimal drift mean and by s the infinitesimal standard deviation of the cash payoff. This means that over a time interval t the cash flow is distributed $N\left(mt, s\sqrt{t}\right)$, normally with a mean of tm, and a standard deviation of $s\sqrt{t}$. The downside of this assumption is that we ignore the limited liability principle, since an outcome of a normal variable can be a large negative number (exceeding \$1 by absolute value). In a realistic world this situation should lead to bankruptcy but we ignore this possibility, because under reasonable assumptions ($r=10\%$, $s=40\%$ and $t=2$ months), the probability of this event is less than $2.4 \cdot 10^{-10}$. For a shorter time interval the probability is even smaller.

Assume that there are two countries with two identical firms, but with different legal systems. The companies are of the same size, in the same industry, have the same clients and risk characteristics. If there were sole proprietorships there should not be any difference between them.

In the first country there are no freezeouts. There are two equivalent shares of the company each share is worth \$0.5. The future profits and losses are unknown at any time moment. They are random variables with some distributions. According to the results of Harrison & Kreps and Harrison & Pliska[13] if there is no arbitrage and no transactions costs, there exist other (not real) probabilities, so-called risk-neutral probabilities, which guarantee that the price is equal to the discounted expected payoff under these probabilities. We assume that the distribution of P&L of the company under these risk-neutral probabilities over a short horizon is $N\left(\mu t, \sigma\sqrt{t}\right)$.

The current value of the whole company -- \$1 -- must be equal to the discounted expected future payoff: $1 = (1 + \mu t)\, e^{-rt}$, this can be written also as $\mu t = e^{rt} - 1$ (continuous discounting used for simplicity). Since μ is the expected profit in the risk-neutral world, there is no risk premium and the discounting is by the risk free rate r.

In the second country there are also two shares but of different types, *A* and *B*. Denote their prices correspondingly by a and b. Together *A* and *B* constitute the whole company, thus $a+b=1$ at any time moment, since the company distributes all its profits (and losses) immediately. However, in this country the owner of the share *A* can force the owner of share *B* to sell him the share *B* for a price *K* that will be define later. Due to the no-arbitrage assumption the $\mu t = e^{rt} - 1$ relation is again satisfied.

Using the standard pricing technique we can write for the *A* share:

$$ae^{rt} = \int_{-\infty}^{z^*}\left(a + \frac{z}{2}\right)r(z)dz + \int_{z^*}^{+\infty}(a + b - K + z)r(z)dz \ ,$$

[13] Michael J. Harrison & David M. Kreps, *Martingales And Arbitrage in Multiperiod Securities Markets,* 20 J.Econ.Theory 381 (1979); Michael J. Harrison & S. R. Pliska, *Martingales and Stochastic Integrals in the Theory of Contingent Trading,* in STOCHASTIC PROCESSES AND THEIR APPLICATIONS, V.11, 261 (1981).

And for the *B* share:

$$be^{rt} = \int_{-\infty}^{z^*} \left(b + \frac{z}{2} \right) r(z)dz + \int_{z^*}^{+\infty} Kr(z)dz .$$

These two equations mean that the owner of each share calculates the expected payoff and compares it with the discounted current price.[14] For the owner of the *A* share the future payoff consists of the following scenarios. If the random variable z (company's profits) is small, she keeps her share *a* and receives exactly one half of the profits (losses). Otherwise, if the profits are high enough (above z^*) it is better to exercise the freezeout option and she will force the owner of the *B* share to sell her the *B* share, thereby receiving all the profits (z) and both shares ($a+b$), but paying the strike price *K*. Similarly the owner of the *B* share has the share and half of the profits in all cases when z is below z^*, otherwise he loses its *B* share and his portion of the profits, but is paid the amount *K*. The integration sign shows that each event is taken with its probability and then summed up.

The optimal exercise decision is determined by the value of z^*. We can view the share *A* as a combination of a simple share (like in the first country) and a call option on the *B* share. Since the freezeout option held by the owner of share *A* creates a possibility that part of the future profits will shift from *B* to *A*, the share *A* has price $a > 0.5$ (the price in the state without freezeout) and the price of share *B* is $b < 0.5$.

The owner of *A* makes her decision based on a comparison of an option alive and dead. The equation she arrives at is:

$$a + \frac{z^*}{2} = a + b - K + z^*$$

[14] The discount factor e^{rt} is moved to the left side of the equation.

This equation means that if she decides not to use the option she gets her share a and half of profits. If she exercises the option she receives both shares and all profits but must pay K. At the optimal exercise point she would be indifferent between these two possibilities. The optimal exercise price is:

$$z^* = 2(K - b) .$$

Using this result, we next consider the following two pricing cases: the first with an exogenous stock price and the second with an endogenous stock price.

1. Pricing with An Exogenous Stock Price

In the case that an exogenous stock price is used for pricing, the strike price K is set based on the share price in the first country (without freezout). For simplicity we set it as a percentage of the price in the no freezeout state $K=l/2$. The equations become:[15]

$$\begin{cases} ae^{rt} = \int_{-\infty}^{z^*} \left(a + \frac{z}{2} \right) r(z)dz + \int_{z^*}^{+\infty} \left(1 - \frac{l}{2} + z \right) r(z)dz \\ be^{rt} = \int_{-\infty}^{z^*} \left(b + \frac{z}{2} \right) r(z)dz + \frac{l}{2} \int_{z^*}^{+\infty} r(z)dz \end{cases}$$

A numerical solution of this system is straightforward. We provide some numerical results for λ=1 and different horizon τ (this is the time period – in yearly terms -- that the owner of share A can see into the future):

[15] The implicit solution is developed in the appendix.

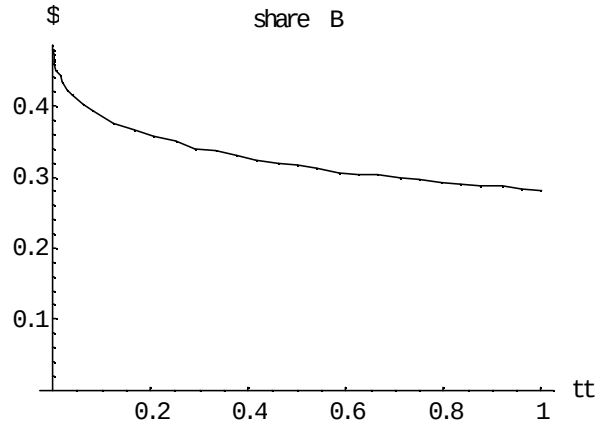


Figure 8.
Price of the *B* share for *K=0.5* as a function of t
when *r=1o%* and s*=40%.*

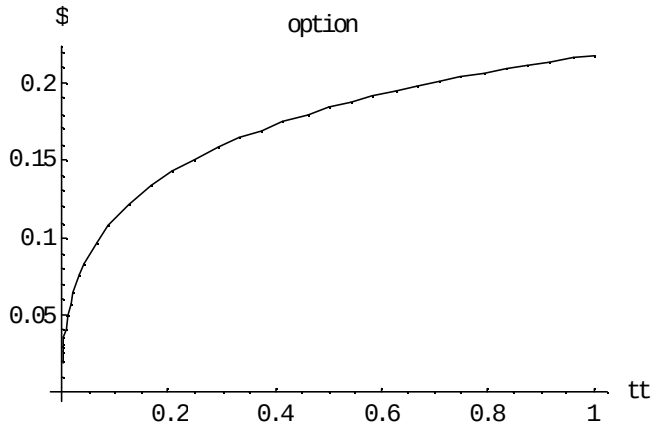


Figure 9.
The value of the freezeout option for *K=0.5* as a function of t
when *r=1o%* and s*=40%.*

When t tends to zero the value of the *B* share approaches the no-freezeout price $0.5 and the value of the option goes to 0. The results are similar to the results of the basic model.[16]

[16] The value of the option in the extended model is about half of its value in the basic model. This is a technical difference due to the fact that the price of the share in the basic model is normalized to one dollar while the B

2. Pricing with An Endogenous Stock Price

In the case that an endogenous stock price is used for pricing, the strike price *K* is set based on the price paid for this share (of type *B* in the second country). We consider aging the case: *K=b*. We show below that when *K* is equal *b* there is no equilibrium price. Set *K=b* and consider the equation for the share *B*. First note that under this condition $z^*=0$. Then the pricing equation becomes:

$$be^{rt} = \int_{-\infty}^{0}\left(b+\frac{z}{2}\right)r(z)dz + b\int_{0}^{+\infty} r(z)dz\,,$$

Or, it can be written as:

$$be^{rt} = b + \int_{-\infty}^{0}\frac{z}{2}\,r(z)dz\,.$$

Note that the left hand side is strictly bigger than *b*, since $t > 0$ and the right hand side is strictly less than *b*, since the first term is *b*, and the second term contains only losses ($z<0$ since the upper limit of integration is 0). Thus, there is no solution and there is no positive equilibrium price for this security.

Intuitively it is clear that the owner of share *A* will exercise it immediately when the company has a profit ($z^*=0$). Whoever will buy the share *B* will either suffer a loss or will have to give up his share in exchange for the price he has originally paid. Nobody will invest a positive amount of money in such a share.

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share in the extended model is about half a dollar.

III. The Model's Implications

Given the result of our option pricing, the debate over the value of the freezeout option becomes a debate over which assumptions best reflect reality. If one assumes that the controlling shareholder has a short horizon of future prices and low probability of knowing them, then the freezeout option has very little value (4%). Given that in freezeout mergers the normal premium paid is substantially greater than the value of the option, the source of this premium is not the exploitation of the minority through the freezeout mechanism.[17] Rather, the premium could come from a new project which the minority has no right to claim[18] or some other source of efficiency.[19] Therefore, the use of the market price and other publicly known information would not distort the fair value determination. Moreover, such a low value of the freezeout option will not justify, from the controlling shareholder point of view, entering inefficient investments or investing in information as to future prices.

However, if one assumes that the controlling shareholder has a long horizon of future prices which she can foresee with a high probability, the freezeout option is very valuable (30%). Given that in some freezeout mergers the premium is lower than the value of the freezeout option, this might reflect an exploitation of the minority through the use of the freezeout mechanism.

It can be argued that the assumptions should differ relative to the specific corporation. In

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[17] *See, e.g.*, Harry DeAngelo, Linda DeAngelo & Edward M. Rice, *Going Private: Minority Freezeouts and Shareholder Wealth*, 27 J. Law & Econ. 367 (1984) (average premium paid to minority shareholders is 56%).

[18] *See*, Hermalin and Schwartz, *supra note 11*.

[19] *See, e.g.*, DeAngelo, DeAngelo & Rice, *supra note 17* (eliminating the costs attendant to the regulation of public ownership); Eli Ofek, *Efficiency Gains in Unsuccessful Management Buyouts*, 49 J. Finance 637 (1994) (reducing agency costs); Frank H. Easterbrook & Daniel R. Fischel, *Corporate Control Transactions*, 93 Yale L.J. 698 (1982) (listing several efficiency gains).

some industries it is more plausible that the controlling shareholder will have longer horizons of future prices or greater probability than in other industries. For instance, in a corporation operating an established supermarket chain, it is reasonable that the controlling shareholder will not have an advantage over market analysts in foreseeing future prices. On the other hand, in the high-tech industry it seem more likely that the controlling shareholder will have greater probability of foreseeing future prices than market analysts.

Nonetheless, it seems unlikely that the controlling shareholder will have an advantage over market analysts. In a mature corporation, for example, there is a greater probability of both foreseeing future prices and foreseeing them for a long horizon, but this effect works for outsiders as well as for insiders. Thus, it is easier for market analysts to erode the insider's advantage by pricing information into today's price. On the other hand, in the high-tech industry, the controlling shareholder has an advantage regarding the probability of knowing future prices, but, given the nature of the industry, her horizon is limited. This suggests that freezeout options will not be very valuable. Indeed, this result is supported by the empirical studies discussed next.

IV. Empirical Support

According to our model, the value of the freezeout option is low, even when its price is based on publicly known information. Thus, it should not be expected that the use of the market price as a measure of fair value would lead to minority shareholders' exploitation or to inefficiency in corporate and controlling owners' decisions.

One way to empirically test the value of the freezeout option may be drawn from a comparison between two kinds of corporations: a majority-owned firm and a diffusely held

firm. The freezeout option is viable only in a majority-owned firm, and the market will reflect its value by discounting the stock price. On the other hand, in a diffusely held firm, the freezeout option does not exist. However, there is a probability that a diffusely held firm will transform into a majority-owned firm -- e.g., through stock accumulation or through a tender to the majority of the stocks -- and the freezeout option will be born. The market will discount the stock price to reflect this probability.

It is reasonable to assume that the discount applied to a majority-owned firm -- in which the existence of the option is certain -- will be greater than the discount applied to a diffusely held firm -- in which there is only a probability that the option will materialized. Thus, if the option has a great value we should find that stocks in majority-owned firms are traded at a discount relative to stocks in diffusely held firms.

Stocks of majority-owned firms, however, do not trade at a discount relative to stocks of diffusely held firms, as found in several studies measuring the impact of large block ownership on firms' market-to-book ratio: the ratio of the market value of the firm to the replacement costs of its assets – a ratio known as Tobin's q. Morck, Shleifer and Vishny found that for 371 Fortune 500 firms, the market-to-book ratio increases when managerial stock holdings went from 0% to 5%, decreases between 5% and 25%, and increases above that. [20] This result suggests that the freezeout option has very little value. If the freezeout option had a great value, we would expect that as managerial holdings increase market-to-book ratio would decrease due to the increased probability that a diffusely held firm would transform into a majority-owned

[20] Randall Morck, Andrei Shleifer & Robert Vishny, *Management Ownership and Market Valuation: An Empirical Analysis*, 20 J. Fin. Econ. 293 (1988).

firm.[21] Similarly, Holderness and Sheehan found *no significant difference* in the book-to-market ratios for paired sample of majority-owned and diffusely held firms.[22]

The studies using Tobin's q are important for our purposes. Investors in a majority owned firm are aware of the freezeout option and will, ex ante, discount the price of the minority stock to a level that should provide them with a return equal to an investment in a diffusely held firm. Therefore, finding equal returns will not be indicative of the freezeout option. The use of Tobin's q avoids this problem by testing discounts relative to an accounting non-market measure – assets' replacement costs. Some of the following studies do not avoid the ex-ante-discounting problem, and their value should be assessed in light of the equal results for the two kinds of firms found in the studies using Tobin's q.

Interestingly enough, the stocks of majority-owned firms are even issued at a premium relative to diffusely held firms. Schipper and Smith, who studied the performance of equity carve-outs announced between 1965 and 1983,[23] found that the initial percentage returns on the stock of the new subsidiaries was much lower than those observed in studies of public offerings generally.[24] That is, in newly issued stocks of a majority-owned firm the issuer could offer a lower discount on its stocks relative to public offerings generally. Although this study measured

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[21] But *see*, John MeConnell & Henri Servaes, *Additional Evidence on Equity Ownership and Corporate Value*, 27 J. Fin. Econ. 595 (1990) (for the one year studied the market-to-book ratio increased until top management owned 40% or 50% of the stock, and declined thereafter).

[22] Clifford Holderness & Dennis Sheehan, *The Role of Majority Shareholders in Publicly Held Corporations: An Exploratory Analysis*, 20 J. Fin. Econ. 317 (1988).

[23] A carved-out occurs when parent firm sells partial ownership interest in a subsidiary to the public. Usually, the parent firm retains at least half of the common stock and thus controls the carved-out subsidiary. A carved-out subsidiary is thus a firm that went public as a majority-owned firm.

[24] Katherine Schipper and Abbie Smith, *A Comparison of Equity Carved-Outs and Equity Offerings: Share Price Effects and Corporate Restructuring*, 15 J. Fin. Econ. 153 (1986).

relative returns – thus susceptible to the ex-ante-discounting problem – the finding of unequal returns in the opposite direction suggests that investors do not discount shares in majority owned firms, due to the freezeout option, relative to diffusely held firms.

Furthermore, we can expect that once a freezeout merger is effected in a majority-owned firm, the premium paid for the minority stocks would be lower relative to the premium paid to shareholders in a merger of a diffusely held firm. The lower premium would be expected for both a firm that went public as a majority-owned firm and a firm that transformed from a diffusely held firm into a majority-owned firm. In fact, in both cases, the discount -- due to the existence of the freezeout option -- reflects this expected low premium. Once the majority owner has paid for the option -- either in the form of discount to newly issued minority stocks or in the form of expenses to create a majority block in a diffusely held firm -- she will wish to make use of this option and pay a lower premium in the freezeout merger.

The empirical findings, however, reveal that premiums paid for the two kinds of firms are substantially similar, supporting the result of our model that the freezeout option has a low value. The equal premium relative to diffusely held firms was found for both firms that went public as a majority-owned firm and firms that transformed from a diffusely held firm into a majority-owned firm. Klein, Rosenfeld and Beranek found that parent firms' announcements of reacquisition of their carved-out subsidiaries are associated with positive abnormal returns for public shareholders which approximate those earned by target firms in arms-length mergers and acquisitions. Additionally, after a parent firm sell-off its interest in the carved-out subsidiary, in most cases minority shareholders are being bought out for the same price.[25] Holderness and

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[25] April Klein, James Rosenfeld & William Beranek, *The Two Stages of An Equity Carve-Out and the Price*

Sheehan paired diffusely held firms and majority-owned firms, and found that minority shareholders in majority-owned firms receive approximately the same premium for their shares as shareholders in diffusely held firms.[26] DeAngelo, DeAngelo and Rice examined Management Buy-Outs – acquisitions that involve informational advantage similar to a freezeout -- and found that the returns to public shareholders were substantially the same whether the buyer had control or not.[27]

Similarly, we would expect the frequency of freezeout mergers and other reorganizations to be greater than the frequency of mergers and other control transactions in diffusely held firms. The motives for mergers and other control transactions in diffusely held firms are substantially the same as in majority-owned firms, while the latter have another motive -- to exercise the freezeout option when the controller receives favorable private information. Indeed, Holderness and Sheehan found that, for paired majority-owned and diffusely held firms over the seven years followed, 36% of the majority shareholders redeemed the minority's shares, while only 29% of the paired firms reorganized over the same period. Similarly, Morck, Shleifer and Vishny found that the probability of a Fortune 500 firm being acquired between 1981 and 1985 increased with the percentage of common stock owned by its top two managers.[28]

The increase in the frequency of reorganizations in majority-owned firms can be due to either a high value of the freezeout option or decreased transaction costs. If the freezeout option

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Response of Parent and Subsidiary Stock, 12 Managerial and Decision Economics, 449 (1991).

[26] Clifford Holderness & Dennis Sheehan, *Constraints on Large-Block Shareholders*, National Bureau of Economic Research, p. 28 (Conference on Concentrated Ownership, June 1998).

[27] DeAngelo, DeAngelo and Rice, *supra note 17* at 393.

[28] Randall Morck, Andrei Shleifer & Robert Vishny, *Characteristics of Targets of Hostile and Friendly Takeover*, in A.J. Auerbach, ed., CORPORATE TAKEOVERS: CAUSES AND CONSEQUENCES 101 (University

has a great value, its exercise should increase the frequency of reorganization. However, even if the freezeout option has a low value, the ownership of a large block of shares makes it easier for the majority shareholder to complete a reorganization relative to a shareholder holding a small fraction of a diffusely held firm. Thus, it is hard to conclude from these studies how much of the difference in the frequency of reorganizations is associated with the value of the option.

A final important caveat should be noted. The two different governance structures have a different mixture of two types of agency problems and thus might have different levels of agency costs that should be taken into account. In a diffusely held firm there is a severe agency problem between shareholders and managers, and a negligible agency problem between majority shareholders and minority shareholders. In a majority owned firm the controlling owner minimizes the effect of the agency problems vis-à-vis the managers but simultaneously aggravate the agency problem between the majority shareholder and the minority shareholders. Indeed, the following empirical studies attempted to reveal the relative efficiency of the two different governance structures. For our purposes, however, it is unnecessary to resolve the question of the relative efficiencies of the differing structures.

Whatever the relative *initial* levels of agency costs of the two differing governance structures are, the empirical studies would still provide us with the supplementary effect of the option. Since the studies reveal that the two governance structures do not have substantially differing levels of efficiency, it is very indicative of the value of the freezeout option. In other words, if the level of agency costs is similar for both structures then the empirical studies suggest that *the freezeout option is not valuable enough to change that result*. If the assumption is that a majority owned firm has higher agency costs relative to a diffusely held

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of Chicago Press, 1988).

firm, then the empirical studies suggest that such effect cannot be identified and even *the freezeout option is not valuable enough to aggravate the problem* to a discernible level. Only when the assumption is that the agency costs of majority owned firms are lower than the agency costs of diffusely held firms, the result of the empirical studies is blurred. Finding equal levels of efficiency suggests that the freezeout option counter the difference in agency costs levels in full, but what was the initial level of agency costs is unclear. With this caveat in mind we next conclude.

V. Summary

The freezeout option allows a majority shareholder to freezeout the minority for a price determined by the majority. The law protects minority shareholder by requiring the majority to pay a "fair price" for the minority shares. Courts determine the fair value to be paid to minority shareholders based on publicly known information. However, the majority shareholder can as well exercise the option when she holds favorable private information. Thus, it is claimed that using publicly known information undervalues minority stocks. Moreover, the initial under-valuation leads to a chain reaction resulting in a substantial market price discounting – a "lemons effect" – and greater under-valuation. Consequently, inefficiency will result. This claim suggests that the freezeout option is very valuable. Others have claimed that market prices in an efficient market are not discounted due to the freezeout option, and thus market prices can be used as a measure of fair value. This claim suggests that the freezeout option has a negligible value.

We presented a model that enabled us to price the freezeout option. Our model indicates that the freezeout option has a low value. This result implies that the use of publicly known

information, including market prices, in determining a fair value for minority stocks will not cause expropriation of minority shareholders and will not distort efficiency.

The result of our model is supported by empirical findings. Stocks of majority-owned firms are not traded at a discount relative to stocks of diffusely held firms. Moreover, in reorganizations and other control transactions, the premium received by minority shareholders in majority-owned firms is similar to the premium paid to shareholders in diffusely held firms.

Appendix

$$\begin{cases} ae^{rt} = \int_{-\infty}^{z^*}\left(a+\frac{z}{2}\right)r(z)dz + \int_{z^*}^{+\infty}(a+b+z-K)r(z)dz \\ be^{rt} = \int_{-\infty}^{z^*}\left(b+\frac{z}{2}\right)r(z)dz + K\int_{z^*}^{+\infty}r(z)dz \\ a+\frac{z^*}{2} = a+b+z^*-K \end{cases}$$

Note that the last two equations can be solved regardless of the first equation. Then in order to find *a* we can use *a+b=1* relationship. Note also that due to the no arbitrage summing the first two equations we get $a+b+m_0 t=e^{rt}$.

$$\begin{cases} be^{rt} = \int_{-\infty}^{z^*}\left(b+\frac{z}{2}\right)r(z)dz + K\int_{z^*}^{+\infty}r(z)dz \\ z^* = 2(K-b) \end{cases}$$

Using simple transformations this can be written as:

$$\begin{cases} be^{rt} = bN\left(\frac{z^*-m}{s}\right) + \frac{1}{2}\int_{-\infty}^{z^*} zr(z)dz + K\left(1-N\left(\frac{z^*-m}{s}\right)\right) \\ z^* = 2(K-b) \end{cases}$$

Or using $\frac{1}{s\sqrt{2p}}\int_{-\infty}^{x} ze^{-\frac{1}{2}\left(\frac{z-m}{s}\right)^2}dz = mN\left(\frac{x-m}{s}\right) - \frac{s}{\sqrt{2p}}e^{-\frac{1}{2}\left(\frac{x-m}{s}\right)^2}$

$$be^{rt} = bN\left(\frac{z^*-m}{s}\right) + \frac{1}{2}mN\left(\frac{z^*-m}{s}\right) - \frac{s}{2\sqrt{2p}}e^{-\frac{1}{2}\left(\frac{z^*-m}{s}\right)^2} + K\left(1-N\left(\frac{z^*-m}{s}\right)\right)$$

Then using $m=m_0 t$ and $s = s_0\sqrt{t}$ we have:

$$be^{rt} = bN\left(\frac{z^*-m_0 t}{s_0\sqrt{t}}\right)+\frac{1}{2}m_0 tN\left(\frac{z^*-m}{s_0\sqrt{t}}\right)-\frac{s_0\sqrt{t}}{2\sqrt{2p}}e^{-\frac{1}{2}\left(\frac{z^*-m_0 t}{s_0\sqrt{t}}\right)^2}+K\left(1-N\left(\frac{z^*-m_0 t}{s_0\sqrt{t}}\right)\right)$$

And further with $m_0 t=e^{rt}-1$, $z^*=2(K-b)$

$$be^{rt} = N\left(\frac{2(K-b)-\left(e^{rt}-1\right)}{s_0\sqrt{t}}\right)\left(b+\frac{\left(e^{rt}-1\right)}{2}\right)-\frac{s_0\sqrt{t}}{2\sqrt{2p}}e^{-\frac{1}{2}\left(\frac{2(K-b)-\left(e^{rt}-1\right)}{s_0\sqrt{t}}\right)^2}+K\left(1-N\left(\frac{2(K-b)-\left(e^{rt}-1\right)}{s_0\sqrt{t}}\right)\right)$$

General Formulas

$$\frac{1}{s\sqrt{2p}}\int_{-\infty}^{x} ze^{-\frac{1}{2}\left(\frac{z-m}{s}\right)^2}dz = mN\left(\frac{x-m}{s}\right)-\frac{s}{\sqrt{2p}}e^{-\frac{1}{2}\left(\frac{x-m}{s}\right)^2}$$

$$\frac{1}{s\sqrt{2p}}\int_{x}^{+\infty} ze^{-\frac{1}{2}\left(\frac{z-m}{s}\right)^2}dz = m\left(1-N\left(\frac{x-m}{s}\right)\right)+\frac{s}{\sqrt{2p}}e^{-\frac{1}{2}\left(\frac{x-m}{s}\right)^2}$$