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Publication Date
2001-04-01

Retirement Savings in an Aging Society: A Case for Innovative Government Debt Management

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April 2001.

Preliminary – Comments Welcome.

Abstract. Aging societies will have to rely increasingly on private savings to finance retirement. The natural savings vehicles, stocks and bonds, are unfortunately lacking key risk-sharing features that are built into public retirement. Innovative government debt management can address this problem. The optimal policy supplies retirees with securities that share the financial risks of aggregate productivity, asset valuation, and demographic shocks across generations. As the population ages, state-contingent government bonds are a better risk sharing tools than pensions, which become too costly, or taxation, which raises time-consistency problems. Wage-indexed and longevity-indexed bonds in particular yield unambiguous efficiency improvements. To the extent that public pensions remain important, plans with wage-indexed defined benefits seem preferable to defined contributions or price-indexed plans. Capital income taxes and pension trust funds can play a supporting role for risk sharing.

* The paper was prepared for the Bundesbank conference on “Aging, financial markets, and monetary policy” in Eltville, Germany, May 4-5, 2001. The paper will also be presented at the CESifo Workshop on Public Pensions, May 2-3, 2001.

1. Introduction

Most retirees, especially in Europe, receive the bulk of their retirement income from the government. As life-expectancy keeps rising and birth rates are declining, publicly-financed retirement becomes increasingly costly, both in absolute terms and as share of national income. (See OECD, 1998; Raffelhüschen, 1998; Disney, 2000 for recent surveys.) While it is debatable how society should respond to these cost pressures, it seems clear that individual savings will be part of the solution. Most retirees will have to rely more on private savings to finance their increasingly long retirement. This paper is about the optimal composition of retirement savings and about the role of fiscal policy in determining the aggregate allocation of risk.[1]

The question how retirement savings should be structured has one quick answer, the laisse faire solution: Let individuals choose their own portfolios of stocks, bonds, and other privately issued securities that best reflect their individual preferences over risks and returns. While appealing on an individual level, the laisse faire answer is seriously incomplete on a macroeconomic level. On aggregate, savers must hold the capital stock and the government debt. Hence, debt policy plays a key role in determining savers' equilibrium portfolios.

The market portfolio available to savers implicitly determines how society allocates aggregate risks over time and over generations. The main sources of aggregate risk are the uncertainty about future productivity growth, uncertainty about population growth, uncertainty about longevity, and the uncertain valuation of capital assets. Productivity uncertainty is naturally shared across cohorts through the correlation of labor income and capital income, though not necessarily in the most efficient way (Bohn, 1998). The other risks are not naturally shared. Population growth raises the marginal product of capital while it reduces the marginal product of labor. Longevity and asset valuation uncertainty mainly affect retirees, not workers. Government policy can reallocate these risks across cohorts, either by changing the market portfolio (through debt management) or through taxes and transfers. The paper will examine the interaction of these policies.

The public policy debate about population aging and pension reform in Europe and in the U.S. has focused on two main options, reforms intended to limit payroll contributions ("privatization") and reforms directed towards partial prefunding.[2] A

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[1] Questions about allocating risk arise at virtually any level of government taxes and transfers. To avoid distracting discussions about the future scale of transfers, I take the income share of intergenerational transfers as given.

[2] I will not consider full funding nor complete privatization plans, because these more radical options seem unrealistic in developed countries with large unfunded systems. See Disney (2000) and McHale (2001) for recent more detailed reviews of pension reform.

cap on payroll taxes necessarily requires benefit cuts, explicit or in disguise, and is often accompanied by tax incentives to encourage private savings. Prefunding involves building up a trust fund that finances part of the contributors' own retirement benefits. If comparably calibrated, both options yield identical expected intergenerational transfers. Apart from questions of corporate control, the key differences involve risk sharing: With prefunding, the publicly-managed trust fund can serve as risk sharing device, e.g., to share stock market uncertainty between cohorts (Bohn, 1999a). With privatization, government influence is more indirect, working through the supply of bonds and potentially through taxation. In either case, the government's problem is to manage the overall portfolio of marketable liabilities (bonds), implicit liabilities (pension promises), and financial assets (trust funds, if any).

The management of the government's portfolio raises three main issues. First, as households rely more on private savings, the structure of net government debt becomes increasingly important. Traditionally, government bonds are considered the typical safe asset. Safe debt does not make the world safer, however, nor does it contribute to risk sharing. To the contrary, a fixed burden of safe government debt in a world with uncertain future incomes makes the after tax income of future generations more volatile. Safe debt provides safety to retirees only by exposing future generations to increased productivity and population uncertainty. Safe debt also fails to protect retirees against longevity risk. A wage-indexed pension system, in contrast, shares all these risk across generations. If the pension replacement rate is reduced due to aging, a natural remedy is to introduce wage-indexed and longevity-indexed bonds that mimic the risk sharing features of public pensions. Moreover, once such bonds are introduced, their supply can be optimized to share risk even better.

Second, the structure of public pensions remains relevant and deserves attention. Should public pensions move to a defined contributions model or maintain defined benefits? And in the latter case, what should be the benefit formula? Defined benefits imply that unexpected shocks must be absorbed by future generations, either directly through variable payroll taxes, or indirectly through debt finance or other taxes. I will argue that a switch to defined contributions would be welfare-reducing.[3] Public pensions with annuitized, wage-indexed, defined benefits have important risk sharing benefits even at a reduced replacement rate. They provide the three most important aggregate risks--uncertainty about growth, uncertainty about population growth, and uncertainty about longevity.

Third, the role of taxes, especially capital income taxes, needs rethinking. The government can in principle implement a perfect sharing of aggregate risks through

[3] If one interprets the baby bust as a demographic shock, the defined benefits principle would call for higher payroll taxes. Reforms that limit contributions and cut benefits to retirees (e.g., recent German reforms) can be interpreted as shifts towards defined contributions. Hence, the issue is relevant even in countries without explicit debate about defined benefits versus defined contributions.

appropriately designed tax-transfers systems--provided they are efficiently and credibly administered. The European welfare state can be interpreted sympathetically as such a system. Capital income taxes are particularly important in this context, because they can reallocate risk in an ex ante non-distortionary way (Zhu, 1992; Chari et al., 1994; Bohn, 1994). Unfortunately, capital income taxes also create troubling time-consistency problems (Fischer, 1980) that discourage savings and limit their usefulness as risk sharing tools. Public debt with explicit contractual contingencies is therefore preferable to state-contingent taxes. Such debt provides a perfect substitute for risk sharing purposes and it is less subject to capital levy problems.

Capital income taxes may have a supporting role if they can be administered in a way that does not discourage savings, i.e., in a non-discretionary manner and accompanied by offsetting savings incentives (Gordon, 1985; Smetters, 2000). Undistorted savings incentives can be achieved in two ways, by making interest income and capital gains tax-exemptor by allowing workers to make pre-tax contributions to special accounts that are taxed at withdrawal (the U.S. model). In the latter case, return risk is shared with future generations through the tax system. This helps to share the underlying productivity, demographic, and asset valuation risks. In addition, the tax deferral implies that a larger stock of debt is available for optimal debt management.

In a setting with pension trust fund, the same risk sharing principles apply to the government's net position. If the trust fund holds government bonds, these bonds cancel out. If the trust fund holds private assets, the government holds a leveraged position. Most interesting is the case of debt-financed stock market investments within a defined benefits pension plan. Then stock market risks and returns are effectively transferred to future generations, i.e., asset pricing uncertainty is shared across generations (Bohn, 1997; 1999a).

The paper is organized as follows. Section 2 presents a simple representative agent overlapping generations (OG) model and explains the main effects of public debt, public pensions, and taxation on the allocation of risk. Section 3 examines population aging and discusses alternative policy responses: business-as-usual (letting public retirement plans grow), reforms that limit payroll taxes, and prefunding. Section 4 examines risk sharing in a heterogenous agents OG model, a stylized "welfare state," and its limitations as the population ages. Section 5 concludes.

2. An Overlapping Generations Perspective on Retirement Finance

Overlapping generations (OG) models provide a natural framework for examining retirement issues. This section presents a stochastic version of Diamond's (1965) two period-life OG model, building on Bohn (1998 and 2001). The model provides a coherent equilibrium framework and is sufficient to discuss life-cycle savings and retirement issues without distracting technical complications. I will start with a laisse faire economy and then introduce government activities one-by-one.

2.1. The Basic Overlapping Generations Economy

In the basic OG model, individuals enter the economy as adults and live for two periods. In their first period, individuals work, consume, pay taxes, and save for retirement. In the second period, individuals are retired, consume, and receive transfers from the government. Generation t consists of N_t identical individuals entering the workforce in period t. The labor supply per worker is normalized to 1, so that N_t is also the aggregate labor supply. In period t+1, generation t is retired. To capture a variable life-expectancy, I model retirement as a fractional period of length λ_t, following Auerbach and Hassett (1998) and Bohn (2001). Individuals maximize a utility function

$$U_t = u(c_t^1) + \rho \quad \lambda_{t+1} \quad u(c_{t+1}^2) \tag{1}$$

where c_t^1 and c_{t+1}^2 are consumption flows (per unit time). Period utility is assumed increasing, concave, and homothetic (see Bohn, 1998 for generalizations); $\rho>0$ is a time preference factor, assumed low enough to guarantee dynamic efficiency.

The aggregate output Y_t is produced with capital and labor,

$$Y_t = K_t^{\alpha} \quad (A_t \quad N_t)^{1-\alpha} \tag{2}$$

where K_t is the economy's capital stock, A_t is productivity, and α (0,1) is the capital share. The assumption of Cobb-Douglas production is standard and motivated by balanced growth. Capital in the next generational period, K_{t+1}, is a function of old capital K_t and new investment spending I_t,

$$K_{t+1} = I_t + v_t^k \quad K_t, \tag{3}$$

where v_t^k is the residual value of old capital.[4]

[4] Fluctuations in v^k_t can be interpreted as stochastic depreciation or in terms of Tobin's q. For the latter, K should be interpreted as the value of capital. If K' is physical capital and $K_{t+1}=K'_{t+1} \; q_t$ is its market value, then (3) is equivalent to $K'_{t+1} = I_t/q_t + v^k_t \; q_{t-1}/q_t \; K'_t = I_t/q_t + v'_t \; K'_t$, where v'_t is one minus the physical depreciation rate; and (2) can be replaced by $Y_t = K'^{\alpha}_t \; N_t^{1-\alpha} \; A'^{1-\alpha}_t$, where A'_t is the underlying physical productivity. If one defines $A_t=A'_t/q_{t-1}^{\alpha/(1-\alpha)}$ as measured productivity, one recovers equation (2). Thus, the parameter $v^k_t = q_t/q_{t-1} \; v'_t$ combines physical depreciation and changes in the market value of capital.

To examine the allocation of risk, I assume the economy faces four fundamental types of risk: uncertain productivity (A), an uncertain population (N), uncertain longevity (λ), and an uncertain value of capital (v^k). To avoid dealing with idiosyncratic risk, I assume that individual within a cohort can share risks perfectly.[5]

The aggregate risks at issue here are quantitatively huge. A one percent change in annual productivity growth, for example, will raise or lower the next generation's income by more than 30% (30 years ahead). Breaks in productivity growth of more than a percent per year have been observed, e.g., during the productivity slowdown of the 1970s and during the recent internet revolution. Variations in birthrates and jumps in longevity have financial effects of a similar magnitude. (After all, this is why we discuss aging.) On a generational scale, these risks are so large that stock market crashes and booms (v^k_t shocks) might look small in comparison.[6] Nonetheless, valuation risk is explicitly modeled because it raises some special issues and because stock market uncertainty is such a well known source of risk.

For this section, I will abstract from all government interventions. This is in the interest of clarity, to lay out the equilibrium allocation of risk in a laisse faire economy.[7] Then the budget constraints and equilibrium conditions are as follows. Workers earn a wage w_t equal to the marginal product of labor, $w_t = (1-\alpha)\ K_{t+1}^{\alpha}\ A_t^{1-\alpha}\ N_t^{-\alpha}$. Workers divide their labor income into consumption c_t^1 and savings s_t, $w_t = c_t^1 + s_t$. On aggregate, savings are invested in capital, $K_{t+1} = N_t\ s_t$. The gross return on savings is the marginal product of capital,

$$R_{t+1}^k = \alpha\ K_{t+1}^{\alpha-1}\ (A_t\ N_t)^{1-\alpha} + v_t^k. \qquad (4)$$

[5] If uncertain longevity includes an idiosyncratic element (survival risk), this implicitly assumes the existence of annuities markets; see Bohn (2001). An economy with heterogenous agents would raise the same aggregate issues; see Section 4.

[6] In the U.S. for example, the recent NASDAQ crash (about 60% decline from March 2000 to March 2001) cost investors about \$3 trillion. If one interprets the post-1970 productivity slowdown as a 1% growth reduction from 1970-1995 (with normal growth resuming after 1995), GDP is $1.01^{25}-1 = 28\%$ or about \$2.8 trillion below the previous growth path. Over a generation (30 years), the income loss is more than \$85 trillion. In this sense, a \$3 trillion stock market loss is a small number.

[7] Some complicating factors and extensions are discussed in Bohn (1998, 2001). Throughout, I abstract from international interactions because they would considerably complicate the analysis (see Baxter and King, 2001 for an analysis) and because aging is a world-wide phenomenon that cannot be avoided by going abroad. In the model, foreign assets can be interpreted as investment opportunities that matter to the extent that they alter the aggregate return on capital, eq. (4) below; they would enter like additional v^k_t shocks. Country risk and informational frictions suggest that international investments have a very limited risk sharing role, especially if the model is applied to Europe as a whole; see Kraay et al (2000) on country risk and Gordon and Bovenberg (1996) on informational issues.

The retirement income $R^k_{t+1}\ s_t$ is allocated to the fractional period λ_{t+1}, yielding a consumption flow of

$$c^2_{t+1} = R^k_{t+1} / \lambda_{t+1} \; s_t . \qquad (5)$$

The dynamics of the OG model are driven by the effective capital-labor ratio $K_t /(A_t \; N_t)$. Let $k_{t-1} = K_t /(A_{t-1} \; N_{t-1})$ denote its predetermined component and let $a_t = \ln(A_t/A_{t-1})$ and $n_t = \ln(N_t/N_{t-1})$ denote the stochastic growth rates of productivity and population. Then the wage can be written as

$$w_t = (1-\alpha) \; k^{\alpha}_{t-1} \; \exp\{(1-\alpha) \; a_t - \alpha \; n_t\} , \qquad (6)$$

showing that wages depend positively on productivity shocks and negatively on the size of the workforce. Similarly,

$$R^k_t = \alpha \; k^{\alpha-1}_{t-1} \; \exp\{(1-\alpha) \; a_t + (1-\alpha) \; n_t\} + v^k_t . \qquad (7)$$

shows that the return on capital responds positively to productivity shocks, population shocks, and shocks to the value of capital. Retiree consumption depends positively on all these shocks, and negatively on longevity.

Overall, productivity shocks are naturally shared between workers and retirees. Population shocks, in contrast, affect retirees and workers in opposite directions. Current longevity and valuation shocks affect only retirees, not workers, while news about future disturbances affect only workers, who may have to adjust their savings. Thus, there is substantial scope for improved risk sharing.

The laisse faire allocation is examined in more detail in Bohn (1998). Since this paper is more concerned with policy reforms in economies with substantial government participation, I will add a government sector before providing a quantitative analysis.

2.2. Public Pensions and Public Debt

Pensions and debt are the two classic intergenerational transfer schemes and therefore central to intergenerational risk sharing. This section adds public pensions, public debt, and a rudimentary tax system to the laisse faire economy.

Public pensions are commonly operated as defined benefit (DB) pay-as-you-go (PAYG) systems, financing formula-fixed pension benefits with current payroll taxes. In a growing economy, taxes and benefits are conveniently stated in terms of the payroll tax rate θ_t and the replacement rate β_t. At time t, real benefits $\beta_t \; w_t$ are paid to N_{t-1} retirees during their lifetime, the fraction λ_t of the period, and financed by taxes $\theta_t \; w_t$ on N_t workers. This implies the basic PAYG constraint

$$\theta_t = \beta_t \; \lambda_t/\exp\{n_t\}, \qquad (8)$$

saying that the tax rate required to finance a given replacement rate is proportional to the retiree-worker ratio $\lambda_t/\exp\{n_t\}$.

The general government budget (taxes, spending, and debt) raises a multitude of new issues. Importantly, taxes can be used a risk sharing devices in ways that can interact with the income and incentive effects. For clarity, this section assumes a very simple tax system, a wage tax designed to maintain a stable debt-GDP ratio. The goal is to present a stylized model of an economy with "limited government" to be contrasted with a stylized "welfare state" in Section 4. The welfare state will be endowed with a much more sophisticated tax system. To anticipate, the limited government setting will provides key insights about the allocation of risk in a welfare state after pension reforms that force the middle class to save privately.

The simple tax system is motivated by two results from tax theory. First, the income effects of taxation depend only on a cohort's generational account (the present value of taxes; see Auerbach et al., 1999), not on the timing of tax payments over the life cycle. Second, taxes that distort savings would be inefficient. Given the revenues that the government wants to extract from some individual, this revenue is most conveniently extracted in form of a wage tax (τ^w_t).[8]

Let end-of-period government debt be D_{t+1}, the gross return on old debt R^d_t, and abstract from government spending.[9] The basic government budget equation is then

$$D_{t+1} = R^d_t \cdot D_t - \tau^w_t \cdot w_t \cdot N_t . \qquad (9)$$

Workers budget equation is now $c^1_t + s_t = w_t \cdot (1-\theta_t - \tau^w_t)$. On aggregate, savers must hold all capital plus government bonds, $N_t \cdot s_t = K_{t+1} + D_{t+1}$. Their retirement consumption is augmented by public pensions and by income from bonds,

$$c^2_{t+1} = R^k_{t+1}/\lambda_{t+1} \cdot K_{t+1}/N_t + R^d_{t+1}/\lambda_{t+1} \cdot D_{t+1}/N_t + \beta_{t+1} \cdot w_{t+1} . \qquad (10)$$

Since taxes are bounded by wage income, a sustainable debt policy requires a bounded debt-income (or debt-wage) ratio. Without much loss of generality, one may assume a debt path consistent with balanced growth, i.e., a stationary time series for the ratio $d_t = D_{t+1}/(N_t \cdot A_t)$. Combined with the budget constraint, this implies

$$c^1_t + K_{t+1}/N_t = w_t \cdot (1-\theta_t) - R^d_t \cdot d_{t-1} \cdot A_{t-1}/\exp\{n_t\} = w_t \cdot (1-\theta^*_t) , \qquad (11)$$

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[8] In this OG model, the inefficiency of taxes on savings is due to homothetic utility; for other arguments, see Chamley (1986), Lucas (1990), and Chari et al. (1994). Given the inelastic labor supply, the wage tax is lump sum; this is convenient, but inessential.

[9] Government spending could be added easily but it would merely complicate the analysis. Shocks to real government spending are not separately modeled because they can be interpreted as negative productivity shocks (see Bohn 1998).

where

$$\theta_t^* = \theta_t + \frac{R_t^d \quad A_{t-1}}{w_t \quad \exp\{n_t\}} \; d_{t-1} \tag{12}$$

summarizes the overall level of intergenerational transfers.[10] This defines the model with basic government sector.

For workers, taxation can increase or decrease the volatility of after-tax income. For retirees, public pensions and the income from selling government debt weaken the role of capital income. Pensions and debt may, however, introduce new sources uncertainty. How risk is shared depends on the form of government debt and on the pension system.

Government debt is commonly viewed as safe claim, a safe income source for retirees (if one abstracts from inflation risk). For retirees, safe debt clearly reduces the volatility of consumption. However, to keep debt over output stationary, debt growth must be held down whenever output growth is low, which requires higher taxes τ^w_t on workers. High output growth, in contrast, allows more debt growth and lower taxes. Algebraically, if $d_{t-1}>0$ and R^d_t is predetermined, eq. (12) shows that the fraction θ^*_t of wages paid to the government depends inversely on wages and population growth. Hence, safe debt magnifies the impact of productivity and population shocks on workers and makes their consumption more volatile. Safe government debt shifts productivity and population risk from retirees to workers.

Innovative debt management is promising in this context because appropriately indexed government bonds would yield a different allocation of risk. If the government issued GDP-indexed debt, for example, the return R^d_t would be proportional to $w_t \exp\{n_t\}$, canceling out the effects of productivity and population growth in (12). This would make retiree income more positively correlated productivity and population growth, however. The wisdom of issuing these or other types of bonds is a question to be examined; it is the central question for optimal debt management in an OG setting.[11]

Public pension plans typically promise defined benefits that are either wage-indexed (e.g. in Germany and France) or indexed to a combination of prices and

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[10] Aggregate transfers to retirees are $N_t \; w_t \; \theta^*_t$, which is (-1) times the retirees generational account. The government budget constraint implies that generational accounts for all generations add up to zero. Hence, $N_t \; w_t \; \theta^*_t$ is also the sum of the generational accounts of generations t+i, summed over i≥0. Either way, θ^*_t has a generational accounting interpretation.

[11] In a representative agent setting, optimal debt management usually focuses on tax distortions; see Barro (1979), Lucas-Stokey (1983), Bohn (1990), Missale (1999). The presentation here is complementary, focusing broad periods and abstracting from tax distortions.

wages (e.g. in the UK and US).[12] Wage-indexed benefits offer a fixed replacement rate $\beta_t=\beta$ and they provide retirement income proportional to the current wage. With wage-indexed pensions, the payroll tax depends on demographic shocks but not on macroeconomic disturbances (see eq. (8)). Importantly, the payroll tax must rise whenever longevity increase and/or workforce growth declines. This demographic link is a key feature of defined benefits pensions; it yields an automatic sharing of longevity and population shocks.

Price-indexed public pensions share longevity risk just like wage-indexed pensions, but risks correlated with wages are shared differently. If real benefits $\beta_{t+1}\ w_{t+1}$ are predetermined at time t, the replacement rate β_{t+1} must responds inversely to all shocks affecting w_{t+1}, e.g., to productivity and population shocks. Due to the PAYG constraint, the tax rate θ_{t+1} must also vary inversely with w_{t+1}, which makes workers after-tax income more volatile than the pre-tax wage. Thus, safe pensions (like safe debt) provide safety to retirees by shifting aggregate risks to future generations of workers.

2.3. The Quantitative Allocation of Risk: Measurement and Welfare

How efficient is the allocation of aggregate risks with and without government intervention? This section will provide a quantitative perspective on risk sharing.

For a formal analysis, explicit assumptions about policy and about the stochastic shocks are needed. This section focuses on risk sharing in a stochastic steady state, as baseline. The stochastic shocks are assumed to follow a vector Markov process and the policy rules are assumed stationary. Then the model variables have a simple Markov structure. Unless noted, I assume that productivity growth, population, and valuation shocks are i.i.d. on a generational time scale; that debt is safe and grows at the same average rate as GDP (d_t is either constant or a function of k_t); and that pensions offer either a constant replacement rate ($\beta_t=\beta$) or a replacement rate that depends only on current shocks. Longevity shocks (λ) are treated as essentially permanent (AR=0.9), because medical innovations that affect retirees longevity also tend to increase workers life expectancy. (Some results for i.i.d. longevity shocks are shown for comparison.)

In terms of efficiency units, the allocation at time t is then a mapping from the vector of disturbances (a_t, n_t, λ_t, v^k_t) and the lagged effective capital-labor ratio k_{t-1} to the model's endogenous variables. As usual in growth models, per-worker variables are obtained as efficiency-unit values multiplied by the productivity trend (A_t); aggregate variables are efficiency units scaled by $N_t\ A_t$. The mapping from

[12] In the UK and US, benefits are price-indexed after retirement and wage-indexed up to the date of retirement; in terms of broad generational periods, this amounts to a mixture of price and wage indexation.

state variables to endogenous variables is generally non-linear and does not have a closed form solution. One can, however, derive the elasticities of the endogenous variables with respect to the Markov state variables. The tables will show elasticity values that are evaluated at a log-linear approximation around the model's deterministic steady state.

For any variable ξ, the elasticity with respect to a state variable s is denoted by $\pi_{\xi,s}$. Steady state values are denoted by symbols without time subscripts. If applied to consumption ($\xi=c^1,c^2$), the elasticities $\pi_{c1,s}$ and $\pi_{c2,s}$ measure the percentage exposure of the cohort's consumption to shocks of type s. The risk exposure of factor incomes and after-tax incomes can be measured similarly by inspecting $\pi_{w,s}$, $\pi_{Rk,s}$, etc. For retirees, after tax income equals consumption. For workers, income and consumption typically move in the same direction, but not always, since savings are endogenous.

As is well known, risk sharing in OG economies is generally not ex ante Pareto efficient (Peled, 1982; Gale, 1990). Though ex ante inefficiency does not necessarily justify government intervention, ex ante efficiency provides a natural criterion for comparing *alternative* policies if one takes as given that governments do intervene (Bohn, 1998). For standard time-separable homothetic preferences, ex ante efficiency has strong implications: The consumption of workers and retirees cohorts should be equally exposed to all types of shocks. That is, the elasticity coefficients $\pi_{c1,s}$ and $\pi_{c2,s}$ should satisfy $\pi_{c1,s} = \pi_{c2,s}$ for all shocks *s*. Bohn (1998) has shown that these consumption elasticities should have the same sign even if the assumption of time-separable homothetic preferences is dropped (e.g., in case of non-expected utility or age-dependent risk aversion). Hence, coefficients pairs $\{\pi_{c1,s}, \pi_{c2,s}\}$ with opposite signs and pairs with zero and non-zero values provide clear indications of inefficiency.

Table 1 displays elasticity coefficients for several numerical versions of the OG economy. The benchmark is an economy with traditional wage-linked defined-benefit pension system and safe debt. The pension is assumed PAYG with a 10% payroll tax that supports a 40.4% replacement rate. The government debt is calibrated to generate an intergenerational transfer of 5% of wages on average, which corresponds to an annual debt/GDP ratio of about 50% (see Bohn, 1999a). Thus, steady state intergenerational transfers are θ^*=15% of wages. The demographic parameters are an average retirement period of λ=1/3 and an average annual population growth of 1%. The macro assumptions are a capital share of α=30%, homothetic utility $u(c)=c^{1-1/\varepsilon}/(1-1/\varepsilon)$ with elasticity of intertemporal substitution ε=0.75, average annual productivity growth of 1.5%, an average real annual return on capital 5%, and an average share v^k/R^k=10% of old capital in the return on capital.[13]

[13] The ε=0.75 elasticity of substitution and the v^k/R^k=10% share of old capital are perhaps worth a comment. The 0.75 elasticity is at the upper end of empirical estimates (Ogaki and Reinhart, 1998) while the v^k/R^k=10% is rather low, less than the share of raw

The above "round" numbers are used to emphasize the conceptual nature of the results. The objective is to understand the principal role of the main fiscal policy instruments and their interaction. This requires strong assumptions, not only about parameter values, but perhaps more importantly, about aspects of reality that are excluded from the model. (Most results below are robust to reasonable parametric variations.) In Table 1, Col.3 shows the allocation of risk in the benchmark case; Col. 2 shows the allocation of risk with the same public pensions but zero debt; and Col. 1 shows the laisse faire allocation with neither debt nor pensions.

In the laisse faire allocation, productivity shocks raise both wages and the return on capital and therefore raise retiree and worker consumption, though not by the same amount. The numerical elasticity values depend on a number of modeling choices discussed in more detail in Bohn (1998). I found that under fairly general conditions, $\pi_{c1,a}$ is well above $\pi_{c2,a}$: Under laisse faire, workers are more exposed to productivity risk than retirees. The parameters here are chosen to be consistent with this finding.

Unexpected growth in the work force reduces the capital-labor ratio and therefore reduces wages while raising the return on capital. The consumption elasticities with respect to n_t-shocks therefore have opposite signs, $\pi_{c1,n}<0$ versus $\pi_{c2,n}>0$. Conversely, a baby bust is a plus for workers and a negative for retirees. Similar linkages arise in larger, multi-period-lives OG models (e.g. Auerbach-Kotlikoff, 1987; De Nardi et. al, 1999) and in representative agent models (e.g. Cutler et al., 1990). Knabb (2001) has carefully examined factor price effects in an OG model with education and endogenous human capital and shown that human capital accumulation would magnify the wage effects. The linkage between population and wages is also consistent with findings in labor economics and in the empirical growth literature (see Bohn, 2001 for references). Overall, this literature emphasizes one key point*: Demographic changes by themselves have significant macroeconomic effects, primarily through factor prices; fiscal policy merely adds to these effects and therefore cannot reasonably be considered in isolation*.

Unexpected longevity shocks have a strong negative impact on retirees, with elasticity $\pi_{c2,\lambda} = -1$. Retiree consumption declines one-for-one if they must divide their savings over a longer life span. Workers are affected much less and only because of the assumed autocorrelation of longevity shocks, which signals workers that they need to save for a longer retirement period. (Workers would be unaffected

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land in wealth. Lower ε and/or higher v^k values would raise the relative exposure of workers to productivity risk under laisse faire and therefore make innovative debt management look better (see Bohn, 1998). The high-ε and low-v^k parametrization is therefore conservative, to avoid exaggeration. The low v^k is also intended to correct for the omission of bequests; namely, if one took as given that certain assets are bequeathed without market transactions (say, illiquid real estate and durable goods), such assets are best excluded for risk sharing purposes and for calibrating v^k.

by i.i.d. longevity shocks, as noted at the bottom of the table.) Finally, retirees bear the entire risk of asset valuation shocks; workers are unaffected.

In the allocation with public pensions (Col.2), the gaps between worker and retiree exposures for all shocks are less than under laisse faire. The wage-indexation of public pension raises retirees exposure to productivity shocks, moving $\pi_{c2,a}$ closed to $\pi_{c1,a}$. The annuitized nature of public pensions reduces retirees exposure to longevity shocks. The defined benefit principle triggers automatically payroll tax adjustments that help share the impact of population shocks (Smith, 1982). This is because population growth reduces pre-tax wages and reduces payroll taxes, and the latter increases the after-tax wage. Increased longevity triggers higher payroll taxes and gives workers a negative exposure to such shocks. Asset valuation risk is still carried entirely by retirees, but their exposure is reduced because capital income is crowded out by pension income.[14] Overall, wage-indexed public pensions have strikingly beneficial risk sharing effects that no discussion of retirement reform should ignore.

When safe public debt is added (Col.3), productivity risk is shifted from retirees to workers, making the gap in elasticities even wider than in the laisse faire economy. The increased intergenerational transfers further reduce the role of valuation shocks. Since population growth reduces the per capita burden of debt, debt reduces the negative exposure of workers to n_t-shocks; but it does not help share longevity shocks. While these effects might appear mixed, one should remember that debt and pensions are substitutable tools of intergenerational redistribution. If the safe debt were replaced by pensions, the productivity and demographic risks would be more evenly shared.

The risk sharing effects of public pensions depend importantly on the wage-indexation. In countries where public pensions are inflation-indexed, i.e., fixed in real terms, the allocation of risk would differ. This is shown in Col.4. Productivity and population shocks are clearly shared less evenly than in Col.3, suggesting that price-indexed pensions are inferior to wage-indexed pensions.

2.4. Innovative Debt Management

The role of alternative types of public debt is demonstrated in Table 2. It shows the effects of replacing safe debt by GDP-indexed debt (Col.2); wage-indexed debt (Col.3); and longevity-contingent debt (Col.4); for comparison, Col.1 reproduces the benchmark setting with safe debt.

Wage- and GDP-indexed debt both improve the sharing of productivity shocks. They differ with respect to demographic shocks. GDP-indexed debt magnifies the

[14] Here the quantitative effect depends on the elasticity of substitution. A higher elasticity would imply slightly less crowding out. The qualitative insights are robust, however.

effect of population shocks on workers because such debt rises in value when the workforce increases and wages fall. Wage-indexed debt, in contrast, is devalued when the workforce increases and therefore helps share population risk. It seems to dominate GDP-indexed debt.[15]

Longevity-indexed debt would help protect retirees against shocks to life expectancy and thereby improve risk-sharing (see Col.4). Longevity-indexed debt would essentially replicate the annuitization of pensions, but through tradable securities and with the potential to share risk in a more flexible way. To implement longevity-indexed debt, one might consider a variety of security designs, depending on practical considerations and microeconomic objectives. One option would be for the government to sell annuities directly to consumers. This would likely trigger adverse selection, which might be undesirable. Adverse selection may, one the other hand, be socially desirable if the government wants to make annuities available to individuals who would be subject to statistical discrimination in the private market. Alternatively, the government could issue bonds with payoffs indexed to aggregate survival statistics. Such bonds might be attractive to insurance companies that sell individual annuities. By providing reinsurance, the government would encourage the development of private annuities markets and leave it to the private sector to cope with adverse selection. Either way, an increased aggregate supply of annuities would improve aggregate risk sharing.

2.5. The Risk-Sharing Role of Capital Income Taxes

The analysis would be incomplete without a comment on the risk sharing role of the tax system (Gordon, 1985; Smetters, 2000).

Since the tax on workers (τ^w_t) was already assumed proportional to wage income, the main extension is to include taxes on capital income (broadly defined, including interest, dividends, capital gains). Taxes on the capital income of retirees can share risk across generations if fluctuations in revenues are used (on the margin) to reduce wage taxes. A limiting case, perfect risk sharing through discretionary taxes and transfers, will be discussed in Section 4 as an interpretation of the welfare state. Here I will merely introduce a fixed-rate capital income tax to illustrate the principal effects and to improve the calibration.

The effects of capital income taxes are illustrated in Table 3. Col.1 reproduces the original benchmark from Section 2.3. Col.2 shows the allocation of risk in an economy with the same government debt and the same net intergenerational transfers (θ^*=15%), but with a 25% marginal tax rate on capital income (instead of

[15] GDP-indexed debt was first proposed by Shiller (1993). Shiller considered agents with infinite horizon planning and debt indexed to the present value of GDP. Since agents here have a finite planning horizon, they are not interesting in claims maturing after their death, suggesting finite-maturity claims that do not expose retirees to new valuation risks.

zero), a 16% payroll tax (instead of 10%), and a 65% replacement rate (instead of 40%). These policy parameters are motivated by German payroll taxes and capital income taxes.[16] Capital income tax receipts are allocated to generations to keep total intergenerational transfers unchanged at 15%. I will refer to this as the European calibration (whereas the Section 2.3 case with 10% payroll tax may be called a U.S. calibration).

The main point of Table 3 is that capital income taxes help to share asset valuation risk, a source of risk that none of the other policy tools was able to address. In addition, the model with larger pension system provides a somewhat better sharing of demographic risk. The similarity of Col.1 and Col.2 highlights the general applicability of the model. Without much loss of generality, the Col.2 calibration is used as starting point for the analysis below.

This summarizes the principal risk sharing effects of government policy in the OG economy. The analysis was placed in a balanced growth setting for convenience. The next section considers demographic change from a different perspective that includes "unbalanced" growth.

3. Retirement Savings in an Aging Society

From a welfare perspective, there are two ways to interpret population aging. Up to now, I have treated longevity and birth rate changes as stochastic shocks that call for an appropriate risk sharing response. This interpretation is natural when a demographic change first occurs and probably sufficient for temporary shocks. In case of permanent shocks, however, a risk sharing response may be difficult to implement in practice. Taken seriously, optimal insurance against permanent shocks would require extremely long-term fiscal transfers, "insurance payments" from/to a current cohort to/from an infinite sequence of future generations. Instead, society may interpret some aspects of demographic change as a known, uninsurable trend. This alternative interpretation of the aging process is pursued in this section.

3.1. Population Aging

Longevity and birth rates, the two source of population aging, have very different time series properties (see Lee and Tuljapurkar, 1998). Increased longevity is a

[16] Mendoza et al., (1994, Table 3) show effective capital income tax rates of about 25% for Germany. German payroll taxes were about 11% of GDP in 1995 (Disney, 2000), i.e., about 16% of wages. The capital income tax is assumed offset by investment incentives sufficient to avoid distorted savings. See Section 4 for a formal exposition of the model with capital income taxes. Mendoza et al. do not clearly distinguish marginal from average taxes (cf. Gordon, 1985), so the 25% value should be interpreted as illustrative.

gradual, monotone process largely driven by improvements in medical technology. It increases individuals need to save for retirement and it puts increasing pressure on retirement systems that guarantee benefits for life. Uncertainty about longevity is mainly about the speed, not the direction. Declining birth rates also contribute to an aging population, but the decline is far from smooth or monotone in most countries. Sometimes birth rates decline rapidly (during baby busts), sometimes the declining trend is interrupted by increases (baby booms). Baby boom and bust phenomena are therefore reasonably interpreted as insurable events that should ideally be covered by intergenerational risk sharing. The longevity trend, in contrast, is more difficult to insure against and arguably represents the more serious demographic problem.

The longevity trend, as a non-stationary phenomenon, also creates new complications for economic analysis because it is inherently inconsistent with balanced growth. Working age families not only have to save more, but to save a larger fraction of their income. To maintain defined-benefits pensions, governments would have to raise increasing tax revenues, not just in line with growth, but rising as percentage of GDP.

Two potentially offsetting trends should be discussed, though they will not be at the center of analysis. First, if longevity is driven by medical technology, one may wonder if the same technological changes also reduce the incidence of disability and enable people to defer retirement. This would raise workers lifetime supply of labor and reduce the retiree-worker ratio. This endogeneity of lifetime work effort is a separate reason why I hesitate to apply pure insurance arguments to the longevity trend. In practice, however, the age of retirement has declined throughout Europe and in the U.S. Even if this is due in part to removable policy distortions (Gruber and Wise, 1999), a longer work life is unlikely to stop the rise in retiree-worker ratios. Hence, I will continue to treat the relative length of the retirement period (λ) as exogenous.

Secondly, lower birth rates reduce the youth-dependency rate. Sinn (2000) has argued that it is only fair to ask households that spend less on children to increase their retirement savings. In Bohn (2001), I have added a childhood period to the two-period OG model and examined demographic risk in a setting that includes youth dependency effects. I find that youth-dependency effects modify the effects of birth rate shocks, but do not overturn the basic results. Hence, this paper omits youth dependency.

For the analysis, I use the same model as in Section 2, but with trends in the demographic variables. Formally, let $\lambda_t = \exp(\bar{\lambda}_t + \hat{\lambda}_t)$ be the sum of a time trend $\bar{\lambda}_t$ and an autocorrelated stochastic component $\hat{\lambda}_t$. Similarly, let $n_t = \bar{n}_t + \hat{n}_t$ be the sum of a time trend $\bar{n}_t$, and a stochastic component $\hat{n}_t$, assumed i.i.d. (I.i.d. is a reasonable approximation for birth rates, but not for longevity.) The demographic trends are inherently non-stationary and the resulting macroeconomic dynamics are inconsistent with balanced growth. The OG setting is fortunately simple enough to accommodate time trends and “unbalanced” growth. Assuming Markov-type shocks and policy rules that are functions of the underlying trends and shocks, the model still has an autoregressive structure, not time-independent but still tractable.

In terms of efficiency units, the equilibrium allocation at time t is a mapping from the capital-labor ratio k_{t-1}, the vector of disturbances (a_t, v_t, $\hat{n}_t$, $\hat{\lambda}_t$), and the time-t and time-(t+1) demographic trend values ($\bar{n}_t$, $\bar{\lambda}_t$, $\bar{n}_{t+1}$, $\bar{\lambda}_{t+1}$), to the endogenous variables. Aging in this context means a rise in mean longevity ($\bar{\lambda}_{t+1}$) and/or a decline in mean workforce growth ($\bar{n}_{t+1}$). This raises two issues: What are reasonable policy responses to the shifting means? What is different about policy in a high-$\bar{\lambda}$ /low-$\bar{n}$ environment compared to the setting of Section 2?

3.2. Business as usual

One policy option is to continue the existing government programs *as is*, i.e., to maintain PAYG retirement benefits and to keep the regular budget on a sustainable path. Most public retirement programs promise defined benefits for life. No change is therefore best interpreted as a constant replacement rate ($\beta_t=\beta$). With PAYG financing, this implies a payroll tax rate $\theta_t = \beta \lambda_t/\exp\{n_t\}$ that increases when λ_t rises and when n_t declines. For the regular budget, a sustainable path is usually interpreted a stable debt-GDP ratio, which I will proxy by $d_t=d$. Overall, business as usual implies a rising total tax rate θ^*_t, i.e., growing intergenerational transfers.

In the public policy discussion, especially among academics, it has become fashionable to dismiss the business-as-usual option as infeasible because it would imply an unbalanced growth path. This sense of unsustainability is perhaps reinforced by the upward trend in retiree medical cost. Increasing taxes may indeed be politically undesirable, as I will assume below. One should be careful about the reasoning, however. From an economic perspective, unbalanced growth is not necessarily unsustainable (see Bohn, 1999b for a model). From a political perspective, public pensions remain hugely popular. Bohn (1999b) has argued that this is not accidental. Starting at about age 40 (for U.S. data), the present value of public pension and medical benefits starts exceeding the present value of future payroll taxes. As the population ages and payroll taxes rise, the critical age where future benefits and taxes balance out will increase. But the age of the median voter rises, too, fast enough that a substantial majority can expect positive net benefits (for U.S. data; the same logic applies to European PAYG systems). Altruism and redistributional arguments provide further support for public pensions. Thus, as a cautionary note, a contraction or demise of public pensions is far from inevitable.

Table 4, Column 1, illustrates the implications of keeping the replacement rate constant as the population ages. The demographic scenario is a drop in mean population growth to $\bar{n}_t = \bar{n}_{t+1} = 0$ combined with an average longevity of $\bar{\lambda}_t = \bar{\lambda}_{t+1} = 0.5$, starting from the policy setting in Section 2.5 (Table 3, Col.2). An unchanged 65% replacement rate now requires an average payroll tax rate of more than 32%. From a risk sharing perspective, this policy looks remarkably good. With the expanded pension system, new demographic shocks are shared almost perfectly. (Workers actually bear slightly more demographic risk than retirees, suggesting that policy is shifting too much risk from retirees to workers.) The sharing of productivity and valuation shocks remains roughly unchanged as compared to Table 3, Col.2.

The good risk sharing properties of this policy should not, however, obscure the key problem. Business-as-usual means sharply higher payroll taxes. In the calibrated example, intergenerational transfers would more than double to θ^*=0.313. In practice, some of the model's "consumption" would be tax-financed public goods, and one would have to add redistributional transfers. Thus, business-as-usual imposes a heavy tax burden on workers.

For examining alternative policy options, I will rule out increased intergenerational transfers. For the sake of economic analysis, assume instead that a political decision is made to hold expected intergenerational transfers constant as the population ages (at θ^*=15% in the numerical examples). How should these limited transfers be structured to provide optimal risk sharing? The next two section will examine two classes of policy alternatives, limiting payroll taxes and partial prefunding. Both can be packaged in ways that turn out to have very different risk sharing implications.

3.3. Limiting Payroll Taxes

Cost cutting has been a key objective of most recent pension reforms, e.g., in Germany, France, and Italy in the 1990s. Since reduced pensions tend to encourage private savings, these reforms can be interpreted as partial "privatization" scenarios. For this paper, the main questions are about the implied changes in the allocation of risk. I will suggest two interpretations, a literal and a functional one.

Taken literally, many recent reforms have included movements away from wage-indexation, e.g., shifts from wage- to price-indexation in Italy and France and from gross wage to net wage indexation in Germany (see McHale, 2001). In each case, the apparent intent was to shift to a slower growing index and thereby to reduce the average replacement rate over time. As noted in Section 2, such changes in indexation have profound effects on the allocation of risk. They remove a mechanism that shares future productivity and population risks.

Table 4, Col.2-3, demonstrates the quantitative effects of all cost-cutting reforms. For the stylized interpretation, I assume that average payroll taxes are held constant to prevent an increase in intergenerational transfers. For θ=16%, $\bar{\lambda}$ =0.5, and $\bar{n}$ =0, the PAYG constraint implies an average replacement rate of β=32%, much less than in the "young" economy of Section 2. Table 4, Col.2 shows the risk-sharing features of a wage-indexed pension system with 32% replacement rate and an average payroll tax rate of 16%. Col.3 illustrates a price-indexed system of the same size. The price-indexed system clearly displays a more uneven generational exposure to productivity and other shocks, confirming the negative results in Section 2.

An alternative, functional interpretation of recent reforms is to focus on their intent, namely to cap payroll taxes. The hallmark of a defined benefit system is that unexpected shocks are not allowed to affect the benefits of current retirees. Benefit changes must be phased in over time. In a country where benefits are adjusted through repeated “reforms” whenever unexpected cost increases occur, the pension system is best interpreted economically as a defined contributions (DC) system, regardless of the statutory benefit formulas.

The risk sharing implications of a DC pension system are shown in Col.4 of Table 4, assuming a fixed payroll tax rate and variable benefits. The DC system clearly fails to share demographic risk. It is inferior in this respect to both DB systems (Col.2-3). The fixed tax rate combined with PAYG implies that benefits are effectively proportional to wages. Hence, the DC system shares productivity risk as well as the wage-indexed DB system, but only because DC makes indexation clauses meaningless. A price-indexed system with DC-type adjustments to demographic shocks would be even worse than a pure DC system.

To conclude, all interpretations of piecemeal, cost-cutting pension reform imply that movements away from wage-indexed defined benefits have adverse effects on risk sharing. If retirement benefits must be reduced, an outright, transparent benefit reduction that leaves a wage-indexed defined benefit system in place seems more efficient than cost cutting through shifts in indexation.

3.4. Partial Prefunding

Now consider the second main reform trend, the movement towards prefunding through a publicly-owned trust fund (e.g. in Sweden, U.S, Canada). The two key questions for a trust fund system are how the fund is invested and who the true beneficiaries are. In accounting terms, a trust fund buildup means that worker pay more in payroll taxes than current retirees receive in transfers. The surplus is invested and helps to pay pension benefits in future periods, i.e., helps to keep future payroll taxes lower.

The question about true beneficiaries is again about defined benefits versus defined contributions. Within a defined contributions system, trust fund returns would accrue to retirees, as if there was no trust fund and as if the retirees were holding the portfolio themselves (Bohn, 1997). The trust fund would be irrelevant for risk sharing. Within a defined benefits system, in contrast, a public trust fund has significant risk sharing because the investments returns effectively accrue to future generations (Bohn, 1997; 1999a).

The economic analysis is most straightforward if the trust fund holds government bonds (or close substitutes, like high-quality corporate bonds). In this case, the fund is economically equivalent to a reduction in net government debt. For given intergenerational transfers θ^*, this allows the replacement rate β to be higher than in the privatization scenarios. Table 5, Col.2, shows the risk sharing implications of a trust fund that holds the entire gross public debt, reduces net debt to zero, and keeps the replacement rate at 42%, above the pure PAYG value of 32%. For comparison, the PAYG allocation is reproduced in Col.1. Since trust funds with different sizes would have proportional effects, the 100%-of-debt case is sufficient to explain the impact of prefunding. Table 5, Col.2 shows that productivity, population, and longevity risks are better shared with partial prefunding than in the pure PAYG case. For the intuition, recall that safe debt shifts productivity risk and population risk from retirees to workers. The trust fund does the reverse and thereby equalizes retiree and worker exposure to a_t- and n_t-shocks. In addition, the increased replacement rate provides more annuitized benefits and helps share longevity risk.

One alternative is to invest public trust fund balances in the stock market, i.e., in claims on capital. Public equity investments are promising in this context because they allow a flexible sharing of valuation risk (Bohn, 1999a), which augments the risk sharing through capital income taxes (Smetters, 2000). This is demonstrated in Table 5, Col.3. Pensions, gross public debt, and the 100%-of-debt size of the trust fund are the same as in Col.2, but the trust fund holds claims on capital. This leaves the gross government debt in the hand of the public, i.e., involves a leveraged equity investment. Since future generations effectively hold part of the capital stock through the trust fund, they have increased exposure to valuation risk and increased exposure to population risk. However, equity investments also shift productivity risk to future generations, which increases the gap between worker and retiree exposures.[17] Innovative debt management provides an even better solution that avoids the tradeoff between valuation and productivity risk. As illustrated in Table 5, Col.4, the adverse shift of productivity risk can be avoided if government debt is wage-indexed instead of safe; population risk would also be better shared.

Overall, Table 5 yields two general and arguably robust lessons about debt management. First, a combination of "risky" trust fund investments (here, equity) combined with "risky" debt (here, wage-indexed) can produce better risk sharing results than simpler alternatives. Second, compared to safe debt, innovative debt management can improve welfare in the direction of taking a "short" position in productivity risk (exemplified by wage-indexed debt) and by taking a "long" position in valuation risk (exemplified by equities).

[17] For calibrated U.S. data, I found that leveraged equity investments yield a net welfare gain despite the adverse shift in productivity risk (Bohn, 1999a) This is a somewhat fragile result, however, because it relies on an estimated covariance of shocks based on only about 4 generational observations (120 years of data).

For comparison, Bohn (1990) used tax smoothing arguments in a Ricardian economy to argue for a government short position in either GDP-indexed bonds or in stocks. Intergenerational risk sharing suggest a refinement. Wage-indexed debt, wage-indexed public pensions, GDP-indexed debt, and a short position in equities all give the government a short position in productivity (a_t). This improves risk sharing *and* is desirable on tax smoothing grounds. With regard to demographic shocks, there is a tradeoff between tax smoothing and risk sharing. Risk-sharing tends to favor defined benefit pensions (best) or wage-indexed debt (second best, because they lack annuitization), both because they yield a long position in "population" (n_t). Tax smoothing, in contrasts, suggests a short position in n_t-shocks, both in Bohn (1990) and in OG model here. (To see the former, note that θ^*_t in (12) would be "smoothed" by a positive correlation between R^d_t and n_t.) Valuation shocks were implicitly treated as noise in Bohn (1990), which reduced the government's optimal equity position without overturning the negative sign. Here, valuation shocks are treated as separate risk factor; the optimal policy is then a long position in equity (in v^k_t) combined with productivity-contingent debt that ensures a net short position in productivity.

In the U.S., proposals to invest government trust funds in the stock market have encountered strong opposition because of corporate control concerns. While European countries may not be as averse to government influence over corporations, the amount of trust fund equity holdings is probably limited in practice. The rest of the paper will therefore focus on privatization scenarios.

The representative agent analysis used so far implicitly assumes that all individuals of a cohort have equal access to financial markets. This is clearly unrealistic for most European countries, and an abstraction even for the U.S. The next section will turn to a heterogenous agent setting.

4. Risk Sharing with Heterogenous Agents

Retirement funding in Europe differs significantly from the laisse faire model. Government pensions are sufficiently generous that much of the population has no need to save privately for retirement. Redistributional policies and the presence of non-savers have potentially significant effects on the allocation of risk. This section develops a stylized "welfare state" OG model to capture these issues.[18] The main points are that a welfare state has potentially excellent risk sharing properties; that risk sharing through welfare-state taxation is difficult to sustain with an aging population; and that population aging is therefore likely to trigger a transition to a

[18] Welfare state is used as label and not in a judgmental way. The general model also applies to the U.S., but the interpretation focuses on a European-style welfare state setting.

setting where most citizens make their own savings decisions and where the risk sharing principles of Sections 2-3 apply.

4.1. A Welfare State OG Model

Consider the same OG model as before except that each generation consists of a continuum of individuals with heterogenous labor productivity.

Individuals are identified by their relative productivity level x. The wage income of a type x individual is x w_t. Let x have a distribution F(x) with unit mean so that w_t is the average wage. For comparison with Sections 2-3, note that heterogeneity per se is inessential if individuals have standard time-additive homothetic preferences. A laisse faire economy with heterogeneity then has the same per-capita allocation as the representative agent OG economy except that individuals with productivity $x \neq 1$ save and consume x times as much as individuals of type x=1. The same applies to an economy with proportional taxes and transfers. Hence, non-proportional redistributive taxes and transfers are critical to obtain qualitatively new results.

A second critical assumption is that individuals cannot borrow against public pensions or other transfers. At some level of public retirement benefits, individuals will stop saving and drop out of financial markets. Their consumption path and their exposure to aggregate risks is determined by the government. In a prototypical welfare state, this applies to bulk of the population. People expect the government to protect them from risk and to provide for their retirement. As I will show, such a welfare state can have a very different allocation of risk than an economy with universal private savings.[19] To be clear about the label: by a welfare state, I mean a system of public pensions and transfers generous enough that most people don't save.

It is beyond the scope of this paper to formalize the limits of income redistribution, to motivate redistribution, or to examine the possible allocations in great detail. The objective is merely to provide a framework to think about aggregate risk sharing in a heterogenous agent environment. The desired level of cross-sectional redistribution is therefore taken as given; the focus is on the aggregate, intertemporal risk sharing implications.

Welfare state governments tend to impose a variety of taxes, e.g., on payrolls, on general labor income, on capital income, and on consumption (or value added).

[19] To emphasize, the key issue is market participation, not heterogeneity or redistribution per se. Regardless of government intervention, if all individuals of a cohort can trade state contingent claims, they will equate their marginal rates of substitutions, i.e., share risk within a cohort. The allocation of risk between cohorts would then resemble the representative agent OG allocation. This section considers the opposite case, an economy where only a minority engages in private retirement savings.

For the worker cohort, let $\theta_t(x)$ be the payroll tax rate, let $\tau_t^I(x, s_t(x))$ be the savings incentives (if any) as fraction of savings $s_t(x)$, and let $\tau_t^1(x)$ be the total of other taxes and transfers as fraction of the wage (negative in case of net transfers). All taxes and transfers may depend on productivity x, either directly or (without explicit modeling) through observable proxies like income. Because savings are the model's key endogenous variable, savings-related taxes and incentives are distinguished from others and allowed to vary with the level of savings. A worker's budget equation is then

$$c_t^1(x) = x\ w_t\ [1 - \theta_t(x) - \tau_t^1(x)] - s_t(x)\ [1 - \tau_t^I(x, s_t(x))] \tag{13}$$

For retirees, let $\beta_{t+1}(x)$ denote the pension replacement rate, let $\tau_{t+1}^2(x)$ represent all other taxes on retirees without savings (e.g., consumption taxes), and let $\tau_{t+1}^R(x, s_t(x))$ be the tax rate on capital income. Then

$$c_{t+1}^2(x) = \beta_{t+1}(x)\ x\ w_{t+1} - \frac{\tau_{t+1}^2(x)}{\lambda_{t+1}}\ x\ w_{t+1} + \frac{R_{t+1}^s(x)}{\lambda_{t+1}}\ s_t(x)\ [1 - \tau_{t+1}^R(x, s_t(x))] \tag{14}$$

is the budget equation of retirees, where $R_{t+1}^s(x)$ is the gross return on savings.

For tractability and to focus on financial market participation, I will make assumptions designed to confront individuals with a discrete choice, to be *non-savers* who accept the age-consumption profile implied by the government's tax-transfer system, or to be *savers* who have full access to financial markets. Financial markets are assumed complete so that savers can and will perfectly pool their risks. (This implicitly determines how $R_{t+1}^s(x)$ varies with x.) All savers are assumed to face the same effective marginal tax rate on all financial investments, to avoiding market segmentation. To produce a discrete choice, my main assumption is that a certain minimum level of savings is required to access to financial markets, namely savings of at least $\sigma\ w_t$, where $\sigma>0$ is exogenous.[20] One may interpret σ as the minimum collateral needed to operate on contingent claims markets or as the minimum equity needed to establish a business. In many countries, only business owners can take advantage of savings incentives such as investment tax credits and accelerated depreciation. Such incentives are especially important if capital income is taxed at high statutory rates, as is common in welfare states, and the government

[20] The minimum is assumed proportional to w_t to maintain balanced growth; $\sigma>0$ is a technical necessity because existence problems would arise if individuals could gain access to complete markets with infinitesimal savings. One may assume that σ is small so that with uniform taxation only a small fraction of individuals would save less than $\sigma\ w_t$.

relies on special incentives ($\tau^I>0$) to avoid gross distortions in capital accumulation. With this motivation, I assume that individuals with savings $s_t(x) \geq \sigma w_t$ face a common tax rate $\tau^R_{t+1}(x, s_t(x)) = \tau^R_{t+1}$ and incentives $\tau^I_t(x, s_t(x)) = \tau^I_t$. This yields a common effective tax rate on savings, $\tau^s_{t+1} = (\tau^R_{t+1} - \tau^I_t)/(1-\tau^I_t) \geq 0$. For savings below σw_t, assume the effective tax rate is high enough to discourage savings.

The population then divides into two groups, non-savers and savers with $s_t(x) \geq \sigma w_t$. Non-savers have a consumption stream that is entirely determined by government policy.[21] Their consumption growth rate is

$$\frac{c^2_{t+1}(x)}{c^1_t(x)} = \frac{w_{t+1}}{w_t} \left(\frac{\beta_{t+1}(x) - \tau^2_{t+1}(x)/\lambda_{t+1}}{1 - \theta_t(x) - \tau^1_t(x)} \right). \tag{15}$$

Typical public pensions offer replacement rates that decline with income. Hence, it is reasonable to assume that the ratio on the r.h.s. of (15) is decreasing in x (at least weakly). For any $\sigma>0$, the ratio of minimum savings over type-x's wage, σ/x, is strictly decreasing in x, which makes individual incentives to save strictly increasing in x. Hence, there are cutoff values x^0_t and $x^+_t \geq x^0_t$ such that individuals with productivity $x \leq x^0_t$ are non-savers, individuals with x $(x^0_t, x^+_t]$ save the minimal amount, and individuals with $x>x^+_t$ save more than σw_t. If σ is small and the cross-state pattern of pension benefits is reasonably efficient, x^+_t will be close to x^0_t, which makes the corner solution $s_t(x)=\sigma w_t$ unimportant. The main point is that individuals sort themselves into savers and non-savers by productivity (or equivalently, by income).

For any value of x, savings incentives are a decreasing function of net retirement transfers $\beta_{t+1}(x)-\tau^2_{t+1}(x)/\lambda_{t+1}$ and of taxes $\theta_t(x)+\tau^1_t(x)$. In a welfare state, pensions and transfers are sufficiently generous and taxes sufficiently high that the cutoff x^0_t lies in the upper end of the income distribution, i.e., that most people don't save.

On aggregate, the heterogenous agent OG economy as follows. Since savers have access to complete markets, their consumption across states of nature is perfectly correlated. The marginal rate of substitution of the unconstrained savers defines a well-defined pricing kernel for valuing all financial assets. Denoting aggregate savings by $S_t = \left(\int_{x \geq x^0_t} s_t(x) dF\right) N_t$, capital market equilibrium requires $K_{t+1} = S_t - D_{t+1}$. This condition combined with savers' first-order optimality

[21] A discussion of private pension plans is beyond the scope of this paper, but they should be mentioned here. Traditional corporate pension plans with defined benefits would also provide an essentially exogenous retirement income and further reduce the incentives to save individually. Defined contribution plans (as increasingly common in the U.S.), in contrast, should be considered a form of individual savings. In a welfare state, pension plans tend to have defined benefits and are usually subject to tight regulations. Hence, there are reasonably treated as government controlled for the purposes of this analysis.

condition for capital investment determines the equilibrium capital stock. The PAYG constraint for pensions is now $(\int \beta_t(x)\ xdF)\ \lambda_t\ \ N_t) = (\int \theta_t(x)\ xdF)\ N_t$. The general government budget constraint is $(\int \tau^1_t(x)\ xdF)\, w_t N_t - \tau^I_t S_t + (\int \tau^2_t(x)\ xdF)\, w_t N_{t-1} + \tau^R_t (\int R^s_t(x)\ s_{t-1}(x)\, dF)\, N_{t-1} + D_{t+1} = R^d_t D_t$. Assuming policy depends only on current x-values, on current aggregate shocks, and on the initial capital-labor ratio, the economy has again a Markov structure with the same aggregate state variables as the representative agent OG model.

For the interpretation, a simplified distributional scenario is perhaps instructive: Suppose there are three groups, some "poor" low productivity types $x=x^L$, a large middle class $x=x^M$, and a minority of "rich" high productivity types $x=x^H$, where $x^L<x^M<x^H$. As starting point, assume pensions are generous enough that neither the poor nor the middle class have sufficient incentives to save ($x^M<x^0_t$ for all t). The government controls their exposure to risk through taxes and transfers. Only the rich save on their own ($x^H>x^+_t$). Their risk exposure is influenced indirectly as in Sections 2-3: They hold all savings, savings must add up to capital plus government debt, and the government controls taxes and the supply of debt securities. The next section will examine optimal risk sharing in this setting.

4.2. Efficient Policies in the Welfare State

This section considers an "ideal" welfare state, a setting where the government allocates aggregate risks perfectly, sets taxes efficiently, and attains redistributional goals at its discretion.

Efficiency (ex ante Pareto optimality) imposes strong restrictions on the timing of taxes and transfers and on capital income taxes. First, since distortions in the capital stock are inefficient, the ideal welfare state should set the marginal tax rate on savings to zero, $\tau^s=0$ (i.e., $\tau^R=\tau^I\geq 0$). Intuitively, whatever revenue the government wants to extract from high income individuals is more efficiently raised through wage taxes or constant consumption taxes rather than capital income taxes.

Second, efficiency calls for equal marginal rates of substitution across income groups. Otherwise, utility could be increased by shifts in tax timing across groups. Unconstrained savers already have a common marginal rate of substitution. The marginal rate of substitution of non-savers is determined by the consumption growth rate, which is government-controlled as shown in equation (15). An efficient tax-transfer system therefore requires that the ratio

$$\frac{\beta_{t+1}(x) - \tau^2_{t+1}(x)/\ \lambda_{t+1}}{1 - \theta_t(x) - \tau^1_t(x)} \qquad (16)$$

is the same for all non-savers and that the resulting marginal rate of substitution matches the marginal rate of substitution of savers. If σ is near zero, this means that non-savers are just on the verge of starting to save (at least if savings are not taxed).[22] For each cohort and productivity level, this determines the time profile of taxes and transfers. Since efficiency requires equal consumption growth for all groups and all states of nature, this growth rate equals the growth rate of aggregate consumption:

$$\frac{c_{t+1}^2(x)}{c_t^1(x)} = (\int c_{t+1}^2(z)\ dF(z))/(\int c_t^1(z)\ dF(z)) \text{ holds for all x.}$$

This aggregation condition has an important implication: *The welfare state with efficient redistribution has exactly the same macroeconomic dynamics as a representative agent OG economy*. Hence, the main insights from Sections 2-3 continue to apply.

Given efficiency, the only new issue is redistribution within a cohort. Infra- and intergenerational distribution can be distinguished by their impact on generational accounts. Policies that change the retirees generational account are intergenerational transfers; all others are infra-generational. For comparison with (12), intergenerational transfers can be expressed in terms net payments to the government as share of the wage, θ^*_t; this is again the negative of retiree's generational account divided by $N_t\ w_t$. Using the government budget equation, θ^*_t can be written as

$$\theta_t^* = (\int x\ \theta_t(x)dF) + (\int x\ \tau_t^1(x)dF) - \tau_t^I\ (\int s_t(x)dF) + \frac{D_{t+1}}{N_t\ w_t} \qquad (17)$$

$$= (\int [\beta_t(x)\lambda_t - \tau_t^2(x)]\ xdF)\ e^{-n_t} - \tau_t^R\ (\int R_t^s(x)\ s_{t-1}(x)dF)\ e^{-n_t} + \frac{R_t^d\ D_t}{N_t\ w_t}$$

This captures the distributional equivalence of payroll taxes, other taxes, and new debt for workers and of pension benefits, taxes, and payments on government debt for retirees. This equivalence combined with the multiplicity of policy instruments in (16) implies that the government has many different ways to implement efficient risk sharing at a given level of inter- and infra-generational redistribution. The redundant instruments can be set arbitrarily.

Table 6, Col.1 shows the efficient aggregate allocation of risk in the calibrated economy of Section 2 (with 1% annual population growth and $\bar{\lambda}$ =1/3). Given the multiplicity of policy instruments, optimal taxes and transfers for type-x agents are best specified in terms of total transfers $\theta^*_t(x)$, the agents' contribution to the

[22] With efficient risk sharing though transfers, individuals have no incentive to save merely to gain access to financial markets. This makes the assumption σ>0 unnecessary. For non-zero σ, efficiency requires a discontinuous downward jump in retirement benefits at x^0_t so that the minimum savings constraint is non-binding, i.e., all savers are unconstrained (x^0_t=x^+_t).

intergenerational transfer in (17). Non-saving workers contribute $\theta^*_t(x)=\theta_t(x)+\tau^1_t(x)$ to the generational account. The optimal elasticities of $1-\theta^*_t(x) = c^1_t(x)/(x\, w_t)$ with respect to the shocks are uniquely determined and shown in Col.2.[23] The same elasticities apply to the net replacement rate of non-saving retirees, to $\beta_t(x)-\tau^2_t(x)/\lambda_t$. Table 6 suggests that efficient taxes on non-savers should increase with productivity, decrease when asset values are high (when tax receipts from capital owners are high), decrease with population growth, and increase with longevity. Note in particular that the efficient risk sharing response to population aging (low n_t, high λ_t) calls for higher taxes, confirming the findings in Section 2.

4.3. Aging, Credibility, and Middle Class Savings

While a welfare state can implement an efficient allocation with perfect risk sharing, it is doubtful that the efficiency benchmark is attained in practice, and it is unlikely that the ideal welfare state can survive population aging.

One problem is that the abundance of discretionary policy interventions raises credibility and commitment issues. In practice, welfare states tend to impose high taxes on capital income. While statutory taxes may overstate the effective tax rates, e.g., if investment is subsidized through tax credits or generous depreciation allowances, any positive taxation will depress capital accumulation and result in first-order welfare losses.[24] Lack of commitment provides a plausible explanation for the observed high capital income taxes. The cost of commitment problems has arguably increased in recent decades due to increased international capital mobility and tax competition.

Aging is a serious problem for the welfare state because the government must finance the *entire* consumption of retirees to maintain control over their exposure to risk. This becomes increasingly costly as the population ages. The PAYG constraint (8) shows that population aging reduces the ratio of replacement rate to payroll tax rate. Unless the adjustment takes place primarily through higher taxes, not reduced benefits, this will reduce the consumption growth rate of non-savers (see (15)-(16)) and therefore motivate the middle class to start saving--and perhaps, to lobby for tax reforms that facilitate retirement savings.

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[23] The elasticities of $\theta^*_t(x)$ with respect to shock are equal to $-(1-\theta^*_t(x))/\theta^*_t(x)$ times the elasticities of $1-\theta^*_t(x)$. I show the elasticities of "1-tax rate" because they does not depend on x, Since $(1-\theta^*_t(x))/\theta^*_t(x)$ varies with x, the elasticities of $\theta^*_t(x)$ vary with x and are sensitive to the level of taxes. Analogous calculations for savers are omitted because they would mix taxes and debt purchases and therefore be difficult to interpret.

[24] Lucas (1990) has argued that distortionary capital income taxes that reduce the capital stock induce quantitatively large welfare losses. The welfare state faces many other problems, of course, e.g., rigidities in making policy adjustments. Rigidities are of second order importance, however, if policy instruments are set optimally on average and the welfare function is flat near the optimum. Hence, I emphasize systematic biases that yield first order welfare losses, e.g., capital income taxes.

Thus, maintaining the welfare state as the population ages requires either huge a increase in redistribution from the rich to the middle class and poor (holding (16) and θ^* constant) or a massive increase in intergenerational redistribution (holding (16) constant by raising θ^*). If higher taxes on the rich are impractical (taxes that can easily be imposed on the rich are probably in effect already, and an increased temptation to tax the rich would reinforce the capital levy problem) and if increased intergenerational redistribution is politically undesirable, this leaves only one solution: The middle class must start to save for its own retirement.[25]

If this solution is accepted, it triggers a quantum leap in the allocation of risk--not just a marginal shift as in the representative agent OG model. Several policy recommendations follow. First, private savings require a credible commitment not to tax the returns. As more people are supposed to save, avoiding capital levy problems becomes increasingly important. This suggests caution about discretionary taxation, especially of retiree incomes. The alternative is to share risk through government bonds with contractually well-defined, non-discretionary indexation clauses. Second, if the poor are a sufficiently small group, reforms that encourage middle class savings will release resources that can be used to maintain government support for the poor. To avoid the Samaritan's dilemma, the government should probably maintain considerable control over the saving of potential welfare recipients. Thus, public pensions will continue to have a role as forced savings mechanism, but only at the bottom of the income distribution.

The need to avoid discretionary retiree taxation has several implications of its own. First, discretion must be constrained for all components of retirees' generational account, otherwise the government could impose indirect capital levies. Credible plans are therefore needed not only for taxes, but also for pension benefits and debt returns.

With regard to pensions, credibility requires clear pension rules, including commitments about their taxation. This might also be useful to minimize labor supply distortions by convincing workers that payroll taxes are in part contributions to a "savings plan" that offers a well-defined return. The results in Sections 3 suggest that a wage-indexed defined benefits plan remains the best choice, especially if the choice is among simple, transparent systems.

With regard to government debt, credibility suggest a move away from nominal bonds that are subject to discretionary monetization and towards securities that are explicitly, contractually indexed to relevant macroeconomic variables. The recent trend towards price-indexed debt (e.g., in the UK, US, Sweden) documents the feasibility of indexation. Unfortunately, price-indexed debt makes no contribution

[25] Another alternative is massive government savings, e.g., in a pension trust fund. To the extent that large, equity-holding trust funds or direct government ownership of capital are politically acceptable, public savings is obviously a substitute for private savings. The same considerations as in Section 3 would apply; hence this option is ignored here.

to risk sharing and mainly eliminates the informal shock absorber role of nominal debt. Since the heterogenous agent economy faces the same aggregate risk sharing problems as the representative agent economy, the same forms of indexation are welfare improving, notably wage-indexation and linkages to longevity. An indexation of government debt to these variables would be preferable to price-indexation.[26]

With regard to capital income taxes, policy makers around the world seem to take one of two approaches. One is to exempt capital income from taxation ($\tau^I = \tau^R = 0$), at least up to a generous limit (e.g., in Germany). The other is to set up special retirement savings plans, exempt the contributions from wage taxes, and subject the withdrawals to normal income taxes ($\tau^I \approx \tau^R > 0$, as exemplified by the U.S 401k and IRA plans). If credible, both approaches allow for approximately undistorted savings choices. I would conjecture that the savings plan approach is more credible, however, because it grants a tax break up front and "back loads" the taxes. The two approaches are also different with regard to risk sharing. The savings plan approach provides an automatic sharing of rate of return risk; in addition (though related in equilibrium) the tax deferral implies a higher government debt that can be used for debt management.[27] A movement towards tax-exempt capital income, in contrast, would eliminate the risk sharing features inherent in the existing income tax system.

4.4. Examples of efficient and nearly-efficient risk sharing

Table 7 provides several scenarios of efficient risk sharing that combine debt management, pension, and tax considerations. Col.1 shows the efficient allocation of risk in the "aged" economy with $\bar{n}=0$ and $\bar{\lambda}=0.5$. The efficient allocation of risk is similar to the allocation in Table 6, but it would be much more costly to implement without middle class savings.[28] Table 7 shows instead how the efficient allocation can be implemented at the original level of intergenerational

[26] As a caveat, note that optimal debt may consist entirely of nominal bonds if the monetary authority is viewed as credible with regard to maintaining low average inflation *and* willing to pursue a state-contingent inflation policy that induces optimal variations the real value of nominal bonds. My interpretation of current trends in monetary policy is that central banks are trying to build credibility by targeting a fixed low inflation rate and would be unwilling to vary inflation in the interest of debt policy. This makes nominal debt equivalent to price-indexed debt and hence inferior to wage- and longevity-indexed debt.

[27] A substitution of labor income taxes by a uniform consumption tax would imply a similar back-loading of taxes and a sharing of return risk. In Europe, the value-added tax can be interpretated in this way, but only if the tax rate is held constant to avoid a tax on savings.

[28] In the calibrated model, payroll tax would have to rise from θ=16% to θ=28% to keep the ratio $\beta/(1-\theta)$ (the relevant part of (16)) constant, i.e., θ^* would have to rise from 15% to 27%, almost as much as in the business-as-usual scenario of Section 3.

redistribution (θ^*=0.15%) via optimal debt policy. Specifically, Col.1 shows the required factor loadings on the debt return R^d_t, assuming debt policy is combined with the same wage-indexed PAYG pension (β=32%) and 25% capital income tax as in Table 4, Col.2.

Some of the factor loadings are clearly high and might create concerns about practical feasibility. To address this, note that the required factor loadings depend on the pension and tax system. Col.2 displays the debt policy parameters when capital accumulation takes place through a retirement savings plan that allows a 25% deduction of contributions and taxes the returns at the same rate. Optimal factor loadings are then much lower. Factor loadings would have to be much higher, on the other hand, if debt policy were accompanied by less-effective pension and tax policies. This is illustrated in Col.3, which assumes a DC pension (capped payroll taxes) and zero capital income taxes.

Another way to avoid policies that rely on high factor loadings is simply to constrain the factors loadings. Table 8 presents two examples; in both, factor loadings are limited to ±1.0 for productivity, population, and longevity, and to zero for valuation risk (i.e., ruling out claims on capital). Col.1 again considers debt policy in a setting with wage-indexed PAYG pension and 25% capital income tax (as in Table 4, Col.2 and Table 7, Col.1). Col.2 displays a non-nested alternative, an optimal debt policy with the same constraints on factor loadings, but in a setting with equity-holding trust fund (as in Table 5) instead of a retirement savings plan. This alternative yields better sharing of valuation risk but a less efficient sharing of productivity risk. Even with the limited factor loadings, both policy settings yield better risk sharing than any of the "simple" policies discussed in Section 3. For comparison, Col.3 shows the "capped taxes" scenario from Table 4, Col.4, my preferred interpretation of piecemeal pension reform.

All these debt policies can be implemented in a heterogenous-income environment. Assuming the poor do not save under either scenario, the government will still have to set their taxes and benefits optimally, as discussed in the previous section. For the middle class and the rich, the government has to do no more than to issue the appropriately indexed government debt. Equilibrium asset prices will then direct savers to the efficient allocation. Savers' portfolios do not have to be identical for this to work. For example, if pensions are a larger share of middle class than of rich retirees incomes, middle class retirees will naturally hold assets that provide hedges against the state-contingent features (if any) of the pension formula. Thus, efficient risk sharing can be implemented in a market setting that does not impose unreasonable informational requirements on the government.

5. Concluding Comments

Population aging in a welfare state is likely to trigger a transition to a policy setting where most households will participate in financial markets. The government supply of savings instruments will then play a critical role in allocating risk. To gain an overall perspective on risk sharing and on the role of innovative debt management, it is instructive to compare the allocation of aggregate risk with piecemeal reform/capped contributions (Table 8, Col. 3) to perfect risk sharing and to the final privatization and prefunding scenarios in Table 8, Col. 1-2.

To move from capped contributions to perfect risk sharing, three elements interact: Public debt management, public pensions, and capital income taxes. Debt management alone would require an aggressive policy with high (perhaps implausibly high) factor loadings on public debt. Pension policy can help by maintaining a wage-indexed defined benefit system that shifts demographic and productivity risks in the right direction. (In other words, if aging makes the original replacement rates too expensive, cost cutting should be done in a way that maintains wage-indexed defined benefits.) Capital income taxes can also help to share risk, though their role is limited because of time-consistency problems. Particularly promising are retirement savings plans that provide tax breaks up front (say, tax-deductible savings) and subject the withdrawals to taxation. Pension policy provides additional degrees of freedom if pensions are partially prefunded and the trust fund is invested in capital assets.

Regardless of the specific pension and capital income tax setting, government bonds indexed to wages and demographic variables offer a flexible set of instruments to share risk. Table 8 provides two stylized examples that combine defined benefits pensions and optimally-indexed (but leverage constrained) public debt with either a retirement savings plan (Col.1) or a partially prefunded pension system holding equities (Col.2). Both settings eliminate much of the inefficiencies of a safe debt/capped payroll taxes scenario. Both rely on government debt indexed to productivity growth (positively), population growth (negatively), and longevity (positively). Optimal public debt management should supply securities to savers that reflects these risk exposures. In practice, wage-indexed and/or longevity indexed debt would go in the right direction.

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Table 1. The Allocation of Risk in a basic OG Economy with Public Debt and Pensions

Policies: **Risk/Exposure** Column:	**Laisse Faire** (No public pension, no debt) (1)	**Economy with DB Pension only** (No debt) (2)	**Benchmark** (DB Pension and Safe Debt) **(3)**	**Price-indexed Pensions** (4)
Productivity (a_t)				
Workers $\pi_{c1,a}$	0.717	0.717	**0.763**	0.838
Retirees $\pi_{c2,a}$	0.599	0.634	**0.591**	0.479
Risk sharing:	Gap = 0.118	Gap = 0.083	**Gap = 0.172**	Gap = 0.359 Increased/Worse
Asset Value (v^k_t)				
Workers $\pi_{c1,v}$	0.000	0.000	**0.000**	0.000
Retirees $\pi_{c2,v}$	0.144	0.094	**0.076**	0.076
Risk sharing:	All Risk on Retirees	All Risk on Retirees	**All Risk on Retirees**	= Benchmark
Workforce (n_t)				
Workers $\pi_{c1,n}$	-0.283	-0.178	**-0.129**	-0.162
Retirees $\pi_{c2,n}$	0.599	0.463	**0.431**	0.479
Risk sharing:	Different signs: Gap = 0.882	Different signs: Gap = 0.641	**Different signs: Gap = 0.560**	Gap = 0.641 Increased/Worse
Longevity (λ_t)				
Workers $\pi_{c1,\lambda}$	-0.389	-0.384	**-0.388**	-0.388
Retirees $\pi_{c2,\lambda}$	-1.000	-0.829	**-0.840**	-0.840
Risk sharing:	Gap = 0.611	Gap = 0.445	**Gap = 0.452**	= Benchmark
Memo: λ_t = i.i.d.				
Workers $\pi_{c1,\lambda}$	0.000	-0.105	**-0.108**	-0.108
Retirees $\pi_{c2,\lambda}$	-1.000	-0.829	**-0.840**	-0.840
Risk sharing:	All Risk on Retirees	Gap = 0.724	**Gap = 0.732**	= Benchmark

Legend: Table entries show elasticities of worker and retiree consumption with respect to macroeconomic disturbances in the calibrated economy of Section 2.3. Col.1 shows the laisse faire allocation without debt and pensions. Col.2 adds a wage-linked defined-benefit PAYG pension system and Col.3 (the benchmark) adds safe government debt. Col.4 replaces the wage-linked pension system by a price-indexed pension system.

Table 2. Policy Alternatives

Policies: **Risk/Exposure** Column:	**Safe Debt** (For comparison =Table 1, Col.3) (1)	**GDP- indexed Debt** (2)	**Wage-indexed Debt** (3)	**Longevity-indexed Debt** (4)
Productivity (a_t)				
Workers $\pi_{c1,a}$	0.763	**0.725**	**0.725**	0.763
Retirees $\pi_{c2,a}$	0.591	**0.647**	**0.647**	0.591
Risk sharing:	Gap = 0.172	**Gap = 0.078** **Reduced/Better**	**Gap = 0.078** **Reduced/Better**	= Benchmark
Asset Value (v^k_t)				
Workers $\pi_{c1,v}$	0.000	0.000	0.000	0.000
Retirees $\pi_{c2,v}$	0.076	0.076	0.076	0.076
Risk sharing:	All Risk on Retirees	= Benchmark	= Benchmark	= Benchmark
Workforce (n_t)				
Workers $\pi_{c1,n}$	-0.129	**-0.167**	**-0.113**	-0.129
Retirees $\pi_{c2,n}$	0.431	**0.487**	**0.407**	0.431
Risk sharing:	Different signs: Gap = 0.560	**Gap = 0.654** **Increased/Worse**	**Gap = 0.521** **Reduced/Better**	= Benchmark
Longevity (λ_t)				
Workers $\pi_{c1,\lambda}$	-0.388	-0.388	-0.388	**-0.415**
Retirees $\pi_{c2,\lambda}$	-0.840	-0.840	-0.840	**-0.760**
Risk sharing:	Gap = 0.452	= Benchmark	= Benchmark	**Gap = 0.345** **Reduced/Better**
Overall Assessment:	=Benchmark	Mixed Effects	More Efficient	More Efficient

Legend: The table entries show elasticities of worker and retiree consumption with respect to macroeconomic disturbances for different policies. Except for the specified policy alternative, the assumptions of Section 2.3 apply. The comparisons in Col.2-4 are relative to the benchmark setting from Table, Col.3, which is reproduced in Col.1. Entries that differ from the benchmark are shown in **bold**, to highlight the changes.

Table 3. The Role of Capital Income Taxes

Setting:	Initial Benchmark	Modified:
Risk/Exposure	(For comparison =Table 1, Col.3)	**with capital income taxes**
Column:	(1)	(2)
Productivity (a_t)		
Workers $\pi_{c1,a}$	0.763	0.770
Retirees $\pi_{c2,a}$	0.591	0.579
Risk sharing:	Gap = 0.172	Gap = 0.191
Asset Value ($v^k{}_t$)		
Workers $\pi_{c1,v}$	0.000	0.008
Retirees $\pi_{c2,v}$	0.076	0.064
Risk sharing:	All Risk on Retirees	Risk is shared: Gap = 0.055
Workforce (n_t)		
Workers $\pi_{c1,n}$	-0.129	-0.057
Retirees $\pi_{c2,n}$	0.431	0.324
Risk sharing:	Different signs: Gap = 0.560	Different signs: Gap = 0.381
Longevity (λ_t)		
Workers $\pi_{c1,\lambda}$	-0.388	-0.421
Retirees $\pi_{c2,\lambda}$	-0.840	-0.744
Risk sharing:	Gap = 0.452	Gap = 0.324

Legend: Table entries show elasticities of worker and retiree consumption with respect to macroeconomic disturbances. Col.1 reproduces the original benchmark allocation from Section 2.3 (=Table 1, Col.3). Col.2 shows elasticity values for the modified economy with 25% capital income taxes described in Section 2.5.

Table 4. Basic Responses to an Aging Population

Given:	Rising Taxes (θ^* = 31.3%)	Holding Intergenerational Transfers Constant (θ^* = 15%)		
Policies: **Risk/Exposure** Column:	"Business as usual" (1)	Wage-indexed DB Pension (2)	Price-indexed DB pension (3)	Capped taxes (DC Pension) (4)
Productivity (a_t)				
Workers $\pi_{c1,a}$	0.787	0.788	0.905	0.788
Retirees $\pi_{c2,a}$	0.591	0.525	0.358	0.525
Risk sharing:	Gap = 0.197	Gap = 0.263	Gap = 0.546 Worse than Col.2	= Col.2
Asset Value ($v^k{}_t$)				
Workers $\pi_{c1,v}$	0.014	0.019	0.019	0.019
Retirees $\pi_{c2,v}$	0.063	0.112	0.112	0.112
Risk sharing:	Gap = 0.049	Gap = 0.093	= Col.2	= Col.2
Workforce (n_t)				
Workers $\pi_{c1,n}$	0.214	-0.045	-0.095	-0.212
Retirees $\pi_{c2,n}$	0.188	0.287	0.358	0.525
Risk sharing:	Gap = 0.026	Gap = 0.332	Gap = 0.454 Worse than Col.2	Gap = 0.737 Worse than Col.2
Longevity (λ_t)				
Workers $\pi_{c1,\lambda}$	-0.658	-0.523	-0.523	-0.466
Retirees $\pi_{c2,\lambda}$	-0.597	-0.762	-0.762	-1.000
Risk sharing:	Gap = 0.061	Gap = 0.239	= Col.2	Gap = 0.534 Worse than Col.2
Overall assessment:	Reduced gaps with high θ^*	Less efficient than Col.1	Less Efficient than Col.2	Less Efficient than Col.2

Legend: The table entries show elasticities of worker and retiree consumption with respect to macroeconomic disturbances under different policy assumptions. All columns describe the "aged" economy with zero population growth and 50% retiree-worker ratio. Col.1 shows the "business as usual" scenario described in Section 3.1. Columns 2-4 show the alternatives described in Section 3.2.

Table 5. The Role of Pension Trust Fund Investments

Policy: **Risk/Exposure** Column:	**Pure PAYG Pension** (For comparison =Table 4, Col.2 (1)	**Trust Fund in Gov. Bonds Debt = Safe** (2)	**Trust Fund in Equities Debt = Safe** (3)	**Trust Fund in Equities Debt = Wage-indexed** (4)
Productivity (a_t)				
Workers $\pi_{c1,a}$	0.788	0.741	0.773	0.735
Retirees $\pi_{c2,a}$	0.525	0.577	0.534	0.586
Risk sharing:	Gap = 0.263	Gap = 0.165 Better than PAYG	Gap = 0.238 Worse than Col.2	Gap = 0.148 Smallest Gap/Best
Asset Value (v^k_t)				
Workers $\pi_{c1,v}$	0.019	0.020	0.030	0.030
Retirees $\pi_{c2,v}$	0.112	0.112	0.099	0.099
Risk sharing:	Gap = 0.093	Gap = 0.092 Like PAYG	Gap = 0.069 Better than Col.2	Gap = 0.069 Like Col.3/Best
Workforce (n_t)				
Workers $\pi_{c1,n}$	-0.045	0.008	0.039	0.055
Retirees $\pi_{c2,n}$	0.287	0.213	0.170	0.148
Risk sharing:	Gap = 0.332	Gap = 0.205 Better than PAYG	Gap = 0.131 Better than Col.2	Gap = 0.093 Smallest Gap/Best
Longevity (λ_t)				
Workers $\pi_{c1,\lambda}$	-0.523	-0.535	-0.535	-0.535
Retirees $\pi_{c2,\lambda}$	-0.762	-0.688	-0.688	-0.688
Risk sharing:	Gap = 0.239	Gap = 0.153 Better than PAYG	Gap = 0.153 Same as Col.2	Gap = 0.153 Like Col.3/Best
Overall assessment:		Better than PAYG	Better than PAYG	Combines best of Col.2 and Col.3

Legend: The table entries show elasticities of worker and retiree consumption with respect to macroeconomic disturbances under different policy assumptions. All columns describe the "aged" economy with zero population growth and 50% retiree-worker ratio. Column 1 shows the basic PAYG/defined benefits policy from Table 3, Col.2, for comparison. Col.2-4 assume a wage-indexed defined benefit pension system with partial prefunding, as described in Section 3.2. The columns differ according to the assumed investment policy, as noted.

Table 6. Efficient Risk sharing and Taxes in the Welfare State

Elasticities for: **Risk/Exposure** Column:	Efficient consumption (1)	Tax factors $1-\theta^{*}_{t}(x)$ of workers (2)
Productivity (a_t)		
Workers $\pi_{c1,a}$	0.691	-0.009
Retirees $\pi_{c2,a}$	0.691	
Risk sharing:	Efficient	
Asset Value (v^k_t)		
Workers $\pi_{c1,v}$	0.031	+0.031
Retirees $\pi_{c2,v}$	0.031	
Risk sharing:	Efficient	
Workforce (n_t)		
Workers $\pi_{c1,n}$	0.097	+0.397
Retirees $\pi_{c2,n}$	0.097	
Risk sharing:	Efficient	
Longevity (λ_t)		
Workers $\pi_{c1,\lambda}$	-0.503	-0.503
Retirees $\pi_{c2,\lambda}$	-0.503	
Risk sharing:	Efficient	

Legend: Col.1 shows the consumption elasticities with respect to shocks when taxes and transfers are used efficiently to share all risks perfectly. Col.2 shows the elasticities of $1-\theta^{*}_{t}(x)$ with respect to shocks that are required to implement perfect risk sharing. The elasticities of $\theta^{*}_{t}(x)$ with respect to the shocks would have the opposite signs.

Table 7. Optimal Debt Management with an Aged Population: Perfect risk sharing

Policy Setting: **Risk/Exposure** Column:	Optimal Debt combined with DB pension & standard taxes (1)	Optimal Debt combined with tax-deductible retirement savings (2)	Optimal Debt combined with DC pension & untaxed savings (3)
Productivity (a_t)			
Workers $\pi_{c1,a}$	0.681		
Retirees $\pi_{c2,a}$	0.681		
Risk sharing:	Efficient	= Col.1	= Col.1
Asset Value (v^k_t)			
Workers $\pi_{c1,v}$	0.057		
Retirees $\pi_{c2,v}$	0.057		
Risk sharing:	Efficient	= Col.1	= Col.1
Workforce (n_t)			
Workers $\pi_{c1,n}$	0.092		
Retirees $\pi_{c2,n}$	0.092		
Risk sharing:	Efficient	= Col.1	= Col.1
Longevity (λ_t)			
Workers $\pi_{c1,\lambda}$	-0.569		
Retirees $\pi_{c2,\lambda}$	-0.569		
Risk sharing:	Efficient	= Col.1	= Col.1
Debt Management:			
Factor loadings:			
Productivity $\pi_{Rd,a}$	2.095	1.129	1.755
Valuation $\pi_{Rd,v}$	-0.737	-0.397	-1.107
Population $\pi_{Rd,n}$	-2.626	-1.415	-6.167
Longevity $\pi_{Rd,\lambda}$	2.601	1.401	5.801

Legend: The table entries show the elasticities of worker and retiree consumption with respect to macroeconomic disturbances with optimal debt policy, and the associated debt policy parameters. Col.1 shows the efficient allocation in the "aged" economy and debt policy parameters with pension and tax policy as in Table 3, Col.2 (DB 32% replacement rate, 25% capital income tax). Col.2 is like Col.1 but with a retirement savings plan that allows a 25% deduction on contributions; the tax deferral is debt-financed. Col.3 has the same setting as in Col.1, but with DC pension as in Table 3, Col.4 and tax-exempt capital income.

Table 8. Optimal Debt Management with Constrained Factor Loadings

Policy Setting: **Risk/Exposure** Column:	Constrained Optimal Debt & Tax-deductible retirement savings (1)	Constrained Optimal Debt & Trust fund holding capital (2)	"Capped taxes" & Safe Debt (For comparison =Table 4, Col.4) (3)
Productivity (a_t)			
Workers $\pi_{c1,a}$	0.693	0.731	0.788
Retirees $\pi_{c2,a}$	0.663	0.609	0.525
Risk sharing:	Gap = 0.03 Almost efficient	Gap = 0.122	Gap = 0.263
Asset Value (v^k_t)			
Workers $\pi_{c1,v}$	0.019	0.029	0.019
Retirees $\pi_{c2,v}$	0.112	0.099	0.112
Risk sharing:	Gap = 0.093	Gap = 0.07 Better than Col.1	Gap = 0.093
Workforce (n_t)			
Workers $\pi_{c1,n}$	0.052	0.089	-0.212
Retirees $\pi_{c2,n}$	0.149	0.096	0.525
Risk sharing:	Gap = 0.097 Almost efficient	Gap = 0.007 Almost efficient	Gap = 0.737
Longevity (λ_t)			
Workers $\pi_{c1,\lambda}$	-0.556	-0.559	-0.466
Retirees $\pi_{c2,\lambda}$	-0.624	-0.613	-1.000
Risk sharing:	Gap = 0.068 Almost efficient	Gap = 0.055 Almost efficient	Gap = 0.534
Debt Management:			
Factor loadings:	Constrained to [-1,1]	Constrained to [-1,1]	Zero (Safe Debt)

Legend: Col.1-2 show the allocation of risk with constrained optimal debt policy. Col.3 shows the "capped taxes" scenario of Table 4, Col.4 for comparison. Col.1 assumes DB pensions and a retirement savings plan with 25% deduction on contributions (as in Table 7, Col.2). Col.2 substitutes a pension fund holding equity (as is Table 5, Col.3), for the savings plan. The constrained optimal factor loadings are +1 on a_t and on λ_t and –1 on n_t in both Col.1 and Col.2.