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**Publication Date**

1986

DIVISION OF AGRICULTURE AND NATURAL RESOURCES  
UNIVERSITY OF CALIFORNIA

Working Paper No. 404

RISK AVERSION IN A DYNAMIC TRADING GAME

by

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January 1986



## RISK AVERSION IN A DYNAMIC TRADING GAME

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## ABSTRACT

The effect of risk aversion on Nash equilibrium trade restrictions is studied using numerical methods. An increase in a nation's level of risk aversion can lead to either an increase or decrease in its equilibrium restriction and either an increase or decrease in its rival's restriction. The linear quadratic dynamic game is generalized to include risk aversion.

RISK AVERSION IN A DYNAMIC TRADING GAME

"Across the Pacific, the Japanese are officially very worried about the rumpus in America over trade. Privately, however, they think it is a cheap negotiating ploy. . . . the Americans are always saying they have no more time; yet the earth still turns."

The Economist, March 30, 1985

## 1. INTRODUCTION

The belief is prevalent (among the public and policymakers) that, by adopting an aggressive posture, a nation can obtain concessions from trading partners. Recent U. S. attempts to use agricultural export subsidies to influence the Common Agricultural Policy of the European Community (EC) is an example of such a strategy. The generality of the circumstances in which aggression induces a conciliatory response has probably been exaggerated. This issue is investigated by using a numerical solution to a dynamic stochastic noncooperative Nash game and examining the sensitivity of the long-run equilibrium to changes in players' levels of risk aversion. Attention is focused on the parameters of risk aversion for two reasons.

The first reason is that aggressive positions tend, at least in the popular imagination, to be associated with greater risk. This may occur because aggression invites retaliation and often involves threats which are painful to carry out and costly to repudiate. More simply, an aggressive trade policy may leave a nation more exposed to exogenous uncertainty. In the examples considered below, where large trading nations use tariffs and export taxes, one nation's welfare is inversely related to the level of its rival's policy; therefore, the level of that policy provides an index of the level of aggression. The analysis suggests that a nation's level of risk aversion may be either positively or negatively related to the level of its equilibrium policy and, hence, its level of aggression.

The second reason for concentrating on the effect of a nation's level of risk aversion is that this defuses the question of the credibility of policies. These policies are determined endogenously given levels of risk

aversion. The latter can be regarded as structural parameters. The level of risk aversion is one aspect of a nation's reputation. The analysis gives an indication of the value of that characteristic. The issue of how a nation communicates its degree of risk aversion, which is analogous to the question of how a nation develops a reputation, is not considered. Here there is no uncertainty about the motives of one's rival as in the papers by Kreps and Wilson (1982) and Sobel (1985); the uncertainty is with respect to the evolution of the environment.

Even in situations where lower risk aversion leads to more bellicose behavior, the relation between one nation's risk aversion and its rival's equilibrium policy is unclear. This is the more interesting question because of the popular belief that a reputation for toughness may persuade rivals to be cooperative. The conclusion is that, even where a decrease in risk aversion corresponds to a reputation for toughness (a more aggressive policy), rivals may choose not to moderate their own policies. This is not a particularly surprising result (reaction functions need not be monotonic), but it appears to be frequently overlooked by negotiators.

The game analyzed here involves the determination of Nash equilibrium tariffs/export taxes by a group of large nations. Johnson's (1954) paper is the classic article on this subject. Recent additions to the literature include the papers by Hamilton and Whalley (1983), Thursby and Jensen (1983), and Karp and McCalla (1983). Hamilton and Whalley extend Johnson's paper by considering equilibrium policies under more general demand and supply conditions. Thursby and Jensen examine the sensitivity of the equilibrium to players' conjectural variations; previous studies assumed a zero conjectural variation. Karp and McCalla use Kydland's (1976) dynamic programming solution

to the linear-quadratic (LQ) discrete time dynamic game; they obtain equilibrium tariffs/taxes for a partial equilibrium model where supply is determined by lagged price and the government's objectives are the discounted streams of future social surplus.

This paper uses the linear-exponential-Gaussian (LEG) dynamic game for the numerical examples. This has the same form as the LQ game except that the quadratic objective function is exponentiated so players can be regarded as maximizing the expected value of the utility of a payoff which is quadratic in the state and controls. The utility function has constant absolute risk aversion (CARA). The random term is additive and Gaussian. This is a generalization of the LEG control problem solved by Jacobson (1973). If all players are risk neutral or the variance of the random term goes to 0, the game collapses to the LQ game solved by Kydland (1975) and Pindyck (1977). Since tractable dynamic stochastic games are something of a rarity, the solution of this game holds some intrinsic interest (see Castanon and Athans (1976) and Clemhout and Wan (1985) for other tractable games; Clemhout and Wan (1979) and Jorgensen (1982) survey applications of dynamic games).

The reason for the choice of a game that accommodates stochastics is apparent from the previous discussion. There are two reasons for including dynamics. The obvious one is that, in many markets, dynamics play a critical role; this is especially true, for example, in agricultural markets where supply decisions are made before price is known. Therefore, a dynamic model provides a more accurate description of reality than does a static model.

A second and for the present purposes a more compelling reason for including dynamics is to circumvent the problem associated with conjectural variations. The equilibrium to a static game presumably represents a long-run

equilibrium. The common assumption of zero conjectural variations provides an implausible adjustment mechanism and, also, an implausible equilibrium; agents never learn from past mistakes. The notion of consistent conjectural variations (Bresnahan (1981) and Perry (1982)) used to select a particular conjectural variation and, hence, a particular equilibrium has not been entirely successful. The reason is that the imposition of "consistency" narrows the field but does not lead to a unique conjectural variation unless an additional restriction, such as linearity, is imposed (Kamien and Schwartz (1983); see, also, Laitner (1980) on the question of rational conjectures). Conjectural variations are used to tell a dynamic story with a static model but have no role in a dynamic model. At each stage in the dynamic game, a zero conjectural variation for that stage is most reasonable since rivals can only react to departures from the equilibrium in subsequent stages. In a dynamic game, the notion of subgame perfectness (Selten, 1975) seems the appropriate way to determine the equilibrium. The dynamic programming solution to the game results in subgame perfectness.

The next section presents the LEG game and the solution. The following section describes numerical examples which illustrate the remarks made above concerning the effect of a player's level of risk aversion. A conclusion follows.

## 2. THE PROBLEM AND SOLUTION

Player  $i$ 's objective is to maximize

$$E_t J_i(t, x_{t-1}; \sigma_i) = E_t \sigma_i \exp\left[\frac{\sigma_i}{2} \sum_{s=t}^n x_s' Q_{s,i} x_s\right], \quad (1)$$



for  $i = 1, 2, \dots, p$  where  $p$  is the number of players,  $\sigma_i = \pm 1$ , and  $E_t$  is the expectation at time  $t$ . The state equation is:

$$x_s = A_s x_{s-1} + \sum_{i=1}^P B_{s,i} u_{s,i} + \Gamma_s e_s \quad (2)$$

where  $u_{s,i}$  is the  $i$ th player's control vector in period  $s$ ,  $e_s \sim N(0, \Sigma_s)$ , and the vector  $x$  has been augmented to include the stacked vector of controls  $(u'_{s,1}, u'_{s,2} \dots u'_{s,p})' = u_s$  and the element 1. Thus, the quadratic form of  $x$  includes linear terms and interaction among the various players' controls. The matrices,  $Q_{s,i}$ ,  $A_s$ ,  $B_{s,i}$ , and  $\Gamma_s$ , are of appropriate dimension;  $\Sigma_s$  is nonsingular, with inverse  $P_s$ . (Equation (2) may be the first-order representation of a higher order difference equation.)

A set of Nash controls at time  $t$ , given state  $x_{t-1}$ , consists of a set of control rules,  $G_{s,i}^*(x_{s-1})$ , for  $s = t, t+1, \dots, n$  and  $i = 1, 2, \dots, p$  such that no player can unilaterally deviate from his control rule without decreasing his expected utility. As in the LQ dynamic game and the LQ and LEG control problems,  $G_{s,i}^*$  is linear in  $x_{s-1}$ . It will be apparent from the solution that giving the players access to the history of the state prior to  $x_{s-1}$  does not alter the control rules. This is not true when the state is imperfectly observed (Speyer et al. (1974)).

The following definitions are used:

$$B_s = [B_{s,1} | B_{s,2} | \dots | B_{s,p}]; \quad (B^* \tilde{W})_s = [\tilde{W}_{s,1} B_{s,1} | \tilde{W}_{s,2} B_{s,2} | \dots | \tilde{W}_{s,p} B_{s,p}]'$$

where  $\tilde{W}_i$  is defined below;  $G_s = [G'_{s,1} | G'_{s,2} | \dots | G'_{s,p}]'$ ; and  $u_s$  was defined above as the stacked vector of controls.

The Nash controls are:

$$u_s = G_s x_{s-1} \quad (3)$$

where

$$G_s = -[(B^* \tilde{W})_s B_s]^{-1} (B^* \tilde{W})_s A_s \quad (4)$$

where  $\tilde{W}_{s,i}$  is obtained by solving

$$\tilde{W}_{s,i} = W_{s,i} + \sigma_i W_{s,i} \Gamma_s (P_s - \sigma_i \Gamma_s' W_{s,i} \Gamma_s)^{-1} \Gamma_s' W_s \quad (5)$$

and  $W_i$  is given by

$$W_{s-1,i} = Q_{s-1,i} + A_s' \{W_s - (B^* \tilde{W})_s' [(B^* \tilde{W})_s B_s]^{-1} B_s' \tilde{W}_{s,i} - \tilde{W}_{s,i} B_s [(B^* \tilde{W})_s B_s]^{-1} (B^* \tilde{W})_s \quad (6a)$$

$$+ (B^* \tilde{W})_s' [(B^* \tilde{W})_s B_s]^{-1} B_s' \tilde{W}_i B_s [(B^* \tilde{W})_s B_s]^{-1} (B^* \tilde{W})_s \} A_s$$

$$W_{n,i} = Q_{n,i} \quad (6b)$$

The expected value of the game to player i at time t is:

$$\sigma_i f_{t,i} \exp \frac{\sigma_i}{2} (x_{t-1}' W_{t,i} x_{t-1})$$

where

$$f_{s-1,i} = f_{s,i} |I - \sigma_i \Sigma_{s-1} \Gamma_{s-1}' W_{i,s-1} \Gamma_{s-1}|^{-1/2} \quad (7a)$$

$$f_{n,i} = 1. \quad (7b)$$

The derivation of the above equations follows Jacobson's solution to the LEG control problem. Rather than go through the algebra, a brief discussion of the method is given. At period  $n$ , the dynamic game is a single-period game. Integrate player  $i$ 's objective function over the normal density and complete the square to remove the expectation operator. The result is a nonstochastic game which is easily solved for the set of control rules,  $G_n$ . The control vector,  $u_n$ , is replaced by  $G_n x_{n-1}$ ; and the process is repeated at stage  $n - 1$ .

The second-order conditions are that  $B'_{s,i} \tilde{W}_{s,i} B_{s,i}$  be negative definite for all  $s, i$  and that  $(B^* \tilde{W})_s B$  be nonsingular (cf. 4). These are analogous to the second-order conditions given by Kydland (1975), and the interpretation is the same. An additional necessary condition is that  $|I - \sigma \sum_s \Gamma'_s W_{i,s} \Gamma_s| > 0$  to ensure that the solution is bounded. These conditions are evident from the derivation.

Giving the players at  $t$  access to the history of the state prior to  $t - 1$  does not change the equilibrium rules. This can be seen from (4). Suppose that the  $n+1$ th element of  $x$  is a lagged value of another element of  $x$ . Then the  $n+1$ th column of  $A$  is the null vector so that rules and, consequently, the expected payoff at each period is unchanged.

A slight modification to the algorithm is required for problems that involve a discount factor. At any stage  $t$ , the present value of the future stream of benefits (the quadratic form in 1), is discounted to time  $t$ , not to time 0. Failure to do this implies that the risk-aversion coefficient changes over time. Define the discount rate as  $\beta$ . The second term on the right side of (6a) is multiplied by  $\beta$ , and the first term,  $Q_{s-1,i}$ , is not discounted.

The LEG differential game can be easily solved with methods analogous to those used by Jacobson for the LEG continuous time control problem. This provides a generalization of Starr and Ho's (1969) results. This solution and the analysis of the symmetric game are provided in Karp (1985b).

### 3. THE ROLE OF RISK AVERSION IN A DYNAMIC TRADING GAME

Numerical examples are used to shed some light on the effect of an agent's risk aversion on equilibrium policies. The basic model is taken from Karp and McCalla who describe the data; this model provides a simple representation of the world market for coarse grains. It is chosen not for the insight it offers into the coarse grains market, which is minimal, but because it incorporates parameter values lying within a plausible range for many traded commodities. The world is divided into three regions: the United States--a net exporter of coarse grains--and the EC and the Rest of World (ROW)--both net importers. Average quantities produced and consumed and average domestic prices from the 1970s and, also, previously estimated supply and demand elasticities were used to synthesize domestic linear supply and demand functions for the three regions. Domestic supply in period  $t$  is a function of domestic price in  $t - 1$ . Setting world excess demand to 0 results in a first-order difference equation for world price. A random term,  $\epsilon \sim N(0, .1)$ , was added to this equation. Domestic price in the EC (United States) differs from world price by a unit tariff (export tax).<sup>1</sup> The ROW behaves passively; any trade restrictions it imposes are assumed to be already incorporated in its supply and demand functions.

Average quantities from the 1970s are given in Table I. Although the EC accounts for over a third of the world's net imports, it is small relative to

Table I

Quantity used to synthesize functions for small EC scenario

	United States	European Communities	Rest of world
	10 million metric tons		
Supply	14.6	1.4	16.1
Demand	11.0	2.7	18.4

the United States. The domestic supply and demand elasticities for the United States and the EC<sup>2</sup> are similar, but the difference in size implies that a U. S. export tax has a significantly greater effect on world price than does an EC tariff of the same magnitude. The effect of the asymmetry is investigated by decreasing the size of ROW and making corresponding increases in the size of the EC; that is, the quantities used to compute ROW's supply and demand curves are reduced, and those that are used to compute the EC's curves are increased so that average world quantity (at the historical price) remains the same. This changes all coefficients in the difference equation for world price and not just the impact of a particular EC policy. It also alters the coefficients of the EC objective function.

The scenario in which the domestic supply and demand functions for the EC and ROW were calculated using the quantities in Table I is referred to as "small EC." Two other scenarios are discussed below: "medium EC" and "large EC." In the former, average EC demand (supply) is increased by 5(4); in the latter, average EC demand (supply) is increased by 16.1 (18.4). Since ROW quantities were reduced by corresponding amounts, the large EC scenario essentially eliminates ROW. Define the equilibrium impact multiplier of a nation's trade restriction as the derivative of current price with respect to the nation's restriction, given that the levels of current and lagged restrictions are equal; this multiplier is the sum of the coefficients on the current and lagged restriction in the price equation. The multipliers for the United States in the cases where the EC is small, medium, and large are .82, .79, and .72, respectively. The multipliers for the EC in the three cases are -.11, -.34, and -.91, respectively. These provide one index of the region's relative power.

The single-period payoff for the EC and the United States is the sum of domestic consumer and producer surplus and tariff/tax revenues. The total payoff for each country is the present value of the stream of its future single-period payoffs; the discount rate was set at .909. The total payoff of region  $i$  was scaled by a constant  $r_i$  and exponentiated. The switch,  $\sigma_i$ , was set at  $\pm 1$ ;  $\sigma_i = 1(-1)$  implies constant absolute risk preference (aversion) of  $r_i$ . The horizon,  $n$ , was set at 25 periods.

In all cases discussed, the sufficient conditions for optimality and existence were satisfied. This restricts the range of risk parameters. Table II reports results for the situation where both countries are risk averse. The small EC scenario was also analyzed with risk-preferring countries. Although small levels of risk preference did not alter the results in Table II, it was possible to reverse many of them by choosing sufficiently large levels of risk preference. This situation is of little economic interest and is not considered further.

Two types of statistics were calculated. The first-period control rules of the finite horizon game, which approximate the stationary rules of the infinite horizon game, were used to calculate the long-run expected equilibrium world price, U. S. tax, and EC tariff. There are the stationary values of the undisturbed dynamic system under the Nash equilibrium control rules. The discussion treats these long-run equilibrium values rather than the control rules directly because of the simpler interpretation of the former. For example, if an increase in country  $i$ 's risk aversion causes a decrease in long-run equilibrium trade restriction of both countries, then it is reasonable to claim that, as country  $i$  behaves more cautiously (less aggressively), its rival follows suit.

Table II  
Changes in statics resulting from changes in risk aversion

	Price <sup>a</sup>	U. S. tax <sup>a</sup>	European Community tariff <sup>a</sup>	United States		European Community	
				Expected <sub>b</sub> welfare	Variance of welfare <sup>b</sup>	Expected <sub>b</sub> welfare	Variance of welfare <sup>b</sup>
<u>Small EC</u>							
Effect of increase in U. S. risk aversion	+	-	-	-	-	+	-
Effect of increase in EC risk aversion	+	+	-	+	-	-	-
<u>Medium EC</u>							
Effect of increase in U. S. risk aversion	-	-	-	-	-	+	-
Effect of increase in EC risk aversion	?	-	-*	+	-	-	-
<u>Large EC</u>							
Effect of increase in U. S. risk aversion	?	-	-	+	-	+	?
Effect of increase in EC risk aversion	-	-	+	?	-	+	-

<sup>a</sup>Long-run expected values.

<sup>b</sup>Present value of a 10-year stream using long-run equilibrium values as initial conditions and Nash control rules.

NOTE: The symbols +, -, and ? indicate that the change is positive, negative, or positive for some values and negative for others, respectively. The risk-aversion parameters were varied between .01 and .2. The symbol -\* is negative except when the United States is very risk averse; then the opposite holds.



The first-period control rules were then used to calculate the expected value and variance of each country's payoff over a 10-year period using the stationary price and tariff/tax as an initial condition. This provides another indication of how a change in equilibrium, caused by a change in one government's degree of caution, affects its own and its rival's welfare. The expected value of the payoff was calculated using a recursive formula, and the variance was obtained using numerical approximations to the moment-generating function. Karp (1985a) discusses this method and compares the approximation to exact calculation of the variance.

When the EC is small, the following results hold (columns 1-3 and rows 1 and 2 of Table II):

1. An increase in risk aversion by either country causes its own expected long-run equilibrium (hereafter, "equilibrium") trade restriction to decrease.
2. An increase in U. S. risk aversion causes (a) EC equilibrium restriction to fall and (b) equilibrium price to rise.
3. An increase in EC risk aversion causes (a) U. S. equilibrium restriction to rise and (b) equilibrium price to rise.

Result 1 was expected. Previous simulation studies (Karp (1985a)) indicate that, for the optimal tariff problem (rather than the game), an increase in risk aversion leads to a drop in trade restrictions. For the control problem, a more risk-averse nation lowers its trade restriction to lessen the instability of world price. Although there is a tendency for this to also hold in a game, rows 4 and 6 indicate that it need not. For the large EC scenario (and medium EC with the United States very risk averse), an increase in EC risk aversion leads to an increase in the EC equilibrium tariff. This is an

instance where the intuition obtained from an optimization problem does not carry over to the games analog. A given change in equilibrium may be associated with either an increase or decrease in a country's risk aversion, depending on the other parameters of the game (compare columns 1-3 of rows 2 and 6 in Table II). Although in some respects the game is a very simple one, the determination of the equilibrium is still a black box.

The contrast between 2(a) and 3(a) is somewhat surprising. As the United States becomes more cautious and, hence, less aggressive, the EC follows suit. The United States, however, takes advantage of a more cautious EC by increasing its trade restriction. A possible explanation, suggested above, is the relative dominance of the United States caused by its greater size (its larger impact multiplier). The results of the medium EC and large EC scenarios support this hypothesis. In those scenarios, both nations become more accommodating as their rival becomes more risk averse. This has to be interpreted cautiously since greater risk aversion by the EC does not necessarily imply a lower level of trade restriction as noted above. There is a tendency for the two countries to lower their restriction in response to their rival's trade liberalization, but this is no more than a tendency.

Equilibrium price may either rise or fall as a result of a nation becoming more risk averse. When the EC is small, price must rise as it becomes more risk averse since its tariff decreases and U. S. tax increases; the opposite holds when the EC is large. In other cases, the change in price depends on the relative magnitude of the changes in trade restrictions. World price is not a sufficient statistic for a nation's welfare, but it is very meaningful to policymakers. Therefore, it is of interest to point out that, by appearing to be more cautious and reducing trade barriers, an exporting nation may

induce its rival to make more than offsetting reductions, leading to an increase in the price of exports.

The second set of statistics consists of the expected value and variance of the present value of a 10-year stream of social surplus. As the risk-aversion parameter squared goes to 0, the CARA utility function is asymptotic to the mean-variance criterion. Therefore, for small levels of risk aversion, a positive relation between a nation's mean and variance is expected (as its own level of risk aversion is altered). This need not hold for large levels of risk aversion where higher than second moments become important.

The results for the small and medium EC are very intuitive. As a nation becomes more risk averse, it acts to stabilize price and its welfare and decreases the expected value of welfare. Its rival benefits from the increased stability which decreases the variance and increases the expected value of its welfare. Using mean and variance of welfare as indices, the conclusion from the small and medium EC scenarios is that a nation prefers to face a cautious rival. As the previous discussion pointed out, this conclusion does not hold if world price is used as an index.

The large EC scenario shows that the above intuitive results can be reversed. For example, an increase in U. S. (EC) risk aversion leads to an increase in U. S. (EC) expected welfare and a decrease in the variance of its own welfare. An increase in one country's risk aversion may lead to a decrease in its opponent's expected welfare or an increase in its opponent's variance of welfare. Thus, using mean and variance as indices, there are situations where a country may not prefer to face a cautious rival; similarly, there are situations where a country may prefer to convince its rival that it is very risk averse.

Table III suggests the magnitude of the changes. The first column gives the statistics of the free-trade equilibrium for the three scenarios, and the second two columns give the statistics of the Nash game when both countries have the same level of risk aversion. Using world price as an index of welfare, the U. S. gains and the EC loses when the region switches from free trade to a Nash equilibrium. The same is true for the small and medium EC scenarios when the mean and variance of the payoff are used as indices. The United States achieves substantial reductions in the variance of its payoff under both Nash regimes. When the EC is large, the expected U. S. payoff decreases as does the variance of the EC payoff under the Nash regimes. The historical price (lagged price) used to synthesize the domestic supply and demand functions was 1.126 (1.1); therefore, in the large EC scenario, the ROW becomes a net exporter.

#### 4. CONCLUSION

The effect of risk aversion on equilibrium trade policies was studied using numerical examples and a LEG noncooperative Nash dynamic game. The level of risk aversion was chosen because it is an important characteristic of a nation's reputation. Two questions were posed: Does greater risk aversion cause a nation to choose a less-aggressive policy? At a noncooperative Nash equilibrium, is a decrease in the level of one nation's trade restriction met by an increase or decrease in a rival's policy? The questions were addressed within the confines of a very restrictive model, the chief features of which are an LQ structure with additive Gaussian noise and a CARA utility function. Even with such a restrictive model, the answer to both questions is ambiguous; different conclusions are obtained from a range of reasonable parameter values.

Table III  
 Statistics for different scenarios<sup>a</sup>

	Game		
	Free trade	Risk aversion of both countries = .05	Risk aversion of both countries = .15
<u>Small EC</u>			
Price	1.164	1.278	1.256
U. S. tax	0	.268	.218
Expected U. S. payoff <sup>b</sup>	- 18.11	- 11.09	- 11.46
Variance of U. S. payoff	136.90	25.74	22.18
EC tariff	0	.079	.061
Expected EC payoff <sup>b</sup>	- 36.67	- 38.43	- 37.91
Variance of EC payoff	2.11	2.87	2.74
<u>Medium EC</u>			
Price	1.204	1.304	1.285
U. S. tax	0	.286	.235
Expected U. S. payoff <sup>b</sup>	- 13.47	- 10.26	- 10.27
Variance of U. S. payoff	80.63	25.7	21.99
EC tariff	0	.1326	.117
Expected EC payoff <sup>b</sup>	- 87.57	- 90.27	- 89.39
Variance of EC payoff	7.76	10.95	9.09
<u>Large EC</u>			
Price	1.315	1.435	1.365
U. S. tax	0	.463	.348
Expected U. S. payoff <sup>b</sup>	- 2.97	- 8.28	- 8.20
Variance of U. S. payoff	42.00	23.71	18.81
EC tariff	0	.149	.185
Expected EC payoff <sup>b</sup>	-211.06	-217.56	-214.84
Variance of EC payoff	24.32	24.9	13.71

<sup>a</sup>The three scenarios are small, medium, and large ECs. For each scenario, the case of free trade and the case where both countries have same levels of risk aversion (.05 or .15) and play the Nash game are considered.

<sup>b</sup>Expected payoff is negative because constant is deleted from social surplus function.

NOTE: Unit of prices = \$100 per metric ton.

This is important because policymakers are likely to select negotiating positions under the impressions that a more aggressive posture is riskier and that, when credible, a more aggressive policy leads to concessions on the part of rivals. As is usually the case, one must be circumspect in extracting general policy recommendations from the model. The determination of actual trade policies is a far more subtle and complex process and has more in common with a bargaining problem than a noncooperative game.

The problems of making an aggressive posture (or threat point) credible and the dangers of politically (or psychologically) motivated retaliation are well recognized. The fact that an aggressive stance may not be desirable even when these problems and dangers are assumed away provides an argument in favor of adopting a conciliatory posture in negotiations.

NOTES

1. The players are referred to as the "U. S." and the "EC" throughout. Given the inadequacy of the empirical base, these designations are intended only to suggest a large exporter and importer of agricultural products. In practice, the United States obviously does not attempt to use an optimal export tax, although certain policies (e.g., the loan rate) may inhibit U. S. exports.

2. These were .3 and -.4 for the United States and .25 and -.5 for the EC.

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