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HOW MANY CAKE-EATERS?
Chouette, on a du monde à dîner!

by
Pascal Favard

and
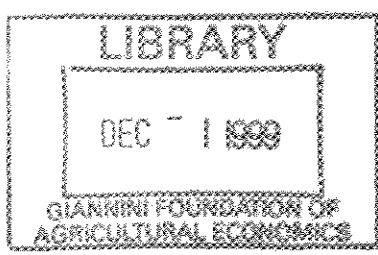
Larry Karp

How many cake-eaters?

Chouette, on a du monde à dîner!

Pascal Favard[*]and Larry Karp[†]

Abstract

We use a cake-eating model with a non-renewable resource and a backstop technology to describe the effect of migration of poor workers into a rich country with surplus labor. Migrants receive a large transfer from natives. If future migration is anticipated, natives' flow of utility increases discontinuously at the time of migration. Migration at time 0 may cause the initial flow of natives' utility to be higher. However, the present discounted value of the stream of per capita utility falls. Thus, when migration occurs, it may benefit the current generation of natives, although it harms other generations.

Key words: migration, nonrenewable resources, cake-eating

JEL Classification numbers: F22, Q3

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1 Introduction

The late 20'th century has seen several episodes of large population movements from poor to rich regions. There have also been several cases of the integration of labor markets in countries with different levels of development. Although migration and the integration of labor markets have many distinctive features, they have in common that relatively poor workers join a richer population. This feature has led to serious political debates within the richer/host country, particularly when migrants receive transfers from their hosts.

We study the effects, on the per capita utility in the host country, of a combined exogenous increase in population and a transfer to the newcomers. For brevity we refer to the additional workers as migrants, although we can also think of them as being the workers in the poor integration partner. In our model, migration (combined with the transfer) reduces the steady state level and the present discounted stream of natives' welfare. In that sense, the model confirms the view that migration harms natives. However, migration is likely to benefit the generation alive at the time that it occurs. This result is counter-intuitive, since it might seem that the generation alive when migration occurs would bear the brunt of the change.

Important examples of migrations include the flows from North Africa and the Mideast into Europe, and from Latin America into North America. Migration sometimes causes an abrupt increase in the labor pool. For example, in 1962 nearly a million French colonists returned from Algeria, increasing the French labor force by 2 percent (Hunt [14]). In the mid 1970s, refugees from Mozambique and Angola increased the Portuguese population by nearly

7 percent (Carrington and de Lima [8]). The 1980 Cuban exodus from the port of Mariel increased Miami's labor force by 7 percent (Card [7]). In some cases one country absorbs the population of another, even though the amount of physical movement of labor is small. The German unification is the most extreme recent example of this kind of integration. The adhesion of South European countries to the European Union were similar but less extreme events.

We assume that all agents have the same productivity, and that migrants are endowed only with labor-power. These simplifying assumptions are not essential, but they enable us to analyze the intertemporal effects of migration[1] . We also assume that prior to migration, the home country has an excess supply of labor, in a sense which we make precise below. Thus, migrants bring a factor of production which is in excess supply.

The equilibrium we study is determined by a social planner who maximizes the present discounted stream of the sum of per capita utilities, including the utility of migrants. Equivalently, the equilibrium is determined by a competitive market, and migrants receive an equal share of society's total capital. Since this assumption is important to our results, and is not literally true, it requires discussion.

In some cases migrants receive a portion of social capital in the form of transfers. For example, the French colonists returning from Algeria were entitled to the same social benefits as other French citizens. The integration of East and West Germany, and the adhesion of

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[1] In many cases, migrants have relatively low levels of education and compete with poorly educated natives. A large empirical literature (e.g. Borjas et al. [5]) measures the differential effects of migrants on different segments of the native population. Migrants also bring cultural variety (art, music, cuisine) that can increase natives' utility.

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Southern European countries to the European Union involved smaller flows of labor, but large transfers.

Regardless of the actual size of the transfers, there is widespread popular belief that they have been and will continue to be large. In campaigning for re-election, Chancellor Kohl vowed that integration would not require higher taxes. This promise reflected the fear that integration would require large transfers, which would lower welfare in West Germany. Similarly, the (perceived) need to make large transfers has been one impediment to enlargement of the European Union. In some cases the actual transfer may be small or even negative, but the native population believes it is large. In California, the popular belief that immigration has resulted in a large drain on the public purse fueled State Proposition 187, which sought to eliminate this transfer for illegal immigrants[2] . Thus, our assumption that migrants receive an equal share of social capital exaggerates what in some cases is true, and in many cases is believed.

If there are no transfers, then the assumption of a social planner who redistributes wealth overstates the economic cost of migration borne by the existing population. Our model can then be viewed as a worst-case scenario (from the perspective of natives). Even in this case, migration has surprising effects.

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[2] A study by Huddle [13] estimates that the net social cost of immigrants (the value of transfers and immigrants' consumption of public services minus their tax payments) was $65 billion in 1996. Fix and Passel [12] calculate that migrants' net social cost is negative. Vernez and McCarthy's review of the literature [19] reports that the estimates of net social cost per immigrant range from $1,600 to –$1,400. Smith and Edmonston [17], using New Jersey and California data, estimate that the social cost of immigrants has been between $15 and $20 billion per year. Borjas [4] provides a useful review of this and other empirical issues related to migration.

There have also been attempts to determine the social cost of immigrants in countries other than the U.S.. Straubhaar [18] estimates that in 1990 immigrants made net contributions to the Swiss fiscal budget. Baker and Benjamin [2] find that in Canada immigrants are less likely than natives to receive welfare benefits.

Our main assumptions [(i) agents are homogeneous, (ii) migrants are endowed only with their labor power, (iii) there is abundance of labor in the host economy prior to migration, and (iv) migrants receive substantial transfers from the host country] correspond to a simple but widely held view. This view is obviously not favorable to the case for migration. It is no surprise that we find that the present value of the stream of natives' discounted per capita utility falls with migration.

However, migration has surprising effects on the intertemporal distribution of utility. Migration might increase the per capita flow of utility of the generation alive at the time it occurs, even though it certainly decreases the present value of the future stream of per capita utility. If future migration is anticipated, then there is a jump in per capita utility at the time migration occurs.

Our results are relevant for the political economy of migration, since they imply that the current generation might be too willing (from the standpoint of future generations) to admit migrants. Also, if a certain level of migration is inevitable, the current generation prefers that it happen sooner rather than later. On a more abstract level – or for readers who dispute the plausibility of our basic assumptions – our analysis contributes to the understanding of cake-eating models.

Our version of the cake-eating model builds on the work of Kemp and Long [15] and Amigues et al. [1], but these papers do not investigate the effects of population growth. There is a large theoretical literature on migration, recently surveyed by Wong [20], Chapter 14. Most of this literature assumes that migrants receive no transfers, and that the host

country welfare does not include migrants' welfare. In this case, for a small open economy, migration typically increases welfare in the host country, and (in the presence of non-traded goods) decreases the welfare of the agents who remain in the source country. Much of the theoretical literature therefore focuses on emigration policies. The theoretical literature on illegal immigration (e.g. Bond and Chen [3], Djajic [9], and Ethier [10]) compares the welfare effects of different methods of controlling immigration.

The next section describes the model. The following two sections present the results.

2 The model

We first describe the technology, and then the economic objective.

2.1 The technology

The economy consists of N identical agents who obtain utility from consumption of a homogenous good and from leisure. Each agent is endowed with one unit of leisure. The economy owns a stock of a nonrenewable resource, $Y(t)$ and a fixed nondepreciable stock of capital $\bar{x}$.

There are two ways of acquiring the consumption good. Labor can be combined with a non-renewable natural resource or with capital using Leontieff technologies[3] . To distinguish these two activities, we refer to the first as *extraction* and the second as *production*. We choose units so that one unit of the consumption good obtained by *production* requires one

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[3] The Leontieff assumption implies that unit labor costs in production are constant for rates of production $x(t) \leq \bar{x}$, and infinite otherwise. We could have used a more general convex technology, but this would have added complications without insights. A more interesting extension allows the capacity constraint to be endogenous. We discuss this extension briefly in the Appendix.

unit of capital, and one unit of the good obtained by *extraction* requires one unit of the nonrenewable resource. Define $y(t)$ as the rate of extraction of the resource, and $x(t) \leq \bar{x}$ as the amount of capital used. The extraction technology requires μ units of labor per unit of output, and the production activity requires η units of labor per unit of output. When society extracts at rate $y(t)$ and uses $x(t)$ units of capital, aggregate consumption is $y(t) + x(t)$ and aggregate employment is $\mu y(t) + \eta x(t)$. Per capita consumption is $c(t) = \frac{y(t)+x(t)}{N}$ and per capita leisure is $l(t) = 1 - \frac{\mu y(t)+\eta x(t)}{N}$.

Extraction requires less labor to create a unit of the consumption good, relative to production, so $\mu < \eta$. We can think of the consumption good as representing "embodied energy", which can be produced either from exhaustible natural resources (e.g. oil) or by using an unlimited resource such as sunlight together with a fixed stock of capital. It is cheaper to obtain energy using oil rather than sunlight.

The stock of the nonrenewable resource is eventually exhausted, but while a positive amount remains, the flow of extraction is unbounded. However, the flow of production is bounded at every point in time since it requires capital, which is in fixed supply. At a point in time society's ability to extract oil is – for all practical purposes – unbounded, but it's ability to use solar power is constrained by limited capital.

2.2 The economic problem

Per capita utility is $U\left(\frac{y(t)+x(t)}{N}, 1 - \frac{\mu y(t)+\eta x(t)}{N}\right) = U(c(t), l(t))$. Utility is increasing and concave in consumption and leisure, with $U_{cl} \geq 0$. Consumption and leisure are "essential goods": their marginal utilities become infinite at levels near 0.

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We assume that the initial resource stock $Y(0)$ and the capital stock $\bar{x}$ are small enough that the constraint $x(t) \leq \bar{x}$ is binding on the entire optimal trajectory. This assumption means that the economy would be willing to sacrifice leisure to obtain more of the consumption good, but is constrained from doing so because it cannot increase production, and can increase current extraction only at the cost of reducing future extraction. Labor is not a constraining factor; in this sense, labor is in excess supply. Define $\alpha(t)$ as the shadow value of the constraint $x(t) \leq \bar{x}$ (i.e., the rental rate of capital on the optimal trajectory): $U_c^* - \eta U_l^* \equiv \alpha(t)$, where a "$*$" indicates optimality. We restate the assumption as[4]

Assumption 1: $\alpha(t) \equiv U_c^* - \eta U_l^* > 0, \forall\ t \geq 0$.

Given that the initial resource stock is finite, there will be a finite time T at which it is exhausted. Thereafter the economy relies exclusively on production. In view of Assumption 1, $\bar{x}$ is the optimal level of production after T. For a discount rate of r, the present discounted value of social welfare at T is $\bar{U} \equiv \frac{N}{r} U(\frac{\bar{x}}{N}, 1 - \frac{\eta \bar{x}}{N})$.

At time $t = 0$, given a resource stock $Y(0)$, the planner wants to maximize the present discounted value of the total per capita utility of the N agents. Here we write N as a constant parameter, but later we treat it as a function of time. Using Assumption 1 to eliminate the constraint $x(t) \leq \bar{x}$, we write the problem as

$$J(Y_0; N) = \max_{T, \{y(t)\}_{t=0}^{T}} \int_0^T N e^{-rt} U\left(\frac{\bar{x} + y(t)}{N}, 1 - \frac{\eta \bar{x} + \mu y(t)}{N} \right) dt + e^{-rT}\bar{U}$$

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[4] Amigues et al [1] show that a necessary and sufficient condition for Assumption 1 is that the initial resource stock, $Y(0)$, and the level of $\bar{x}$ are sufficiently small.

subject to

$$Y(0) = Y_0 = \int_0^T y(t)dt \tag{1}$$

and the non-negativity constraint $y(t) \geqslant 0$.

The first order condition is:

$$U_c(c^*(t), l^*(t)) \square \mu U_l(c^*(t), l^*(t)) = \lambda e^{rt} \tag{2}$$

where (the endogenous constant) λ is the shadow value at time 0 of the resource stock.

3 An anticipated future increase in population

We begin by showing that an increase in population lowers both the present discounted future stream of per capita utility and the steady state per capita utility. An additional worker obtains an equal share of society's wealth and contributes his labor power. Since labor is not the constraining factor of production, natives (the inframarginal agents) lose more than they gain. This result sets the stage for a discussion of the intertemporal flow effects of a population increase.

The present discounted value of social welfare is $J(Y; N)$, and the present discounted value of the stream of per capita utility is $\frac{J(Y;N)}{N}$. In the appendix we establish[5]

Proposition 1 $\frac{J(Y;N)}{N}$ *is a decreasing function of* N.

Proposition 1 says that for a given stock of the resource, migration decreases the stream of per capita utility. However, if the social planner knows that migration will occur in the

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[5] This proof uses the "dynamic envelope theorem". For a general discussion of this theorem, see Caputo [6] and LaFrance and Barney [16].

future, there will be a change in the extraction trajectory and a change in the resource stock at the time migration occurs. For this case, we have the following corollary.

Corollary 1 *Suppose that at time 0 the social planner anticipates that migration will occur at time $\tau > 0$. This anticipation causes an adjustment of the extraction trajectory that lowers the present discounted value of the future stream of per capita utility of agents alive at time 0 (the natives).*

It is also obvious that the steady state per capita utility, $U(\frac{\bar{x}}{N}, 1 \square \frac{n\bar{x}}{N})$, is a decreasing function of N. Although the model contains no surprises with respect to either the steady state level of, or the value of the stream of per capita utility, the effects on utility at a point in time are unexpected. Here we consider the situation where the social planner knows at time 0 that there will be a discrete increase in population at an exogenous time $\tau > 0$. We show that the flow of per capita utility increases discontinuously at $t = \tau$. This result improves our intuition about the cake-eating problem and also leads to a conjecture that we verify in a simpler setting.

We refer to the situation where N is constant as the "reference case". Since the future population change reduces the value of the stream of welfare, and since the social planner wants to smooth utility, he adjusts the program so that generations over $[0, \tau)$ bear some of the cost. This adjustment delays extraction, relative to the reference case. This delay requires an increase in the shadow value of the resource and a corresponding decrease in $y(t)$[6]. Over $[0, \tau)$ the only change, relative to the reference case, is that the extraction profile $y(t)$ shifts down. Consequently, the flow of utility over $[0, \tau)$ is less than in the reference case.

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[6] The proof of Proposition 1 shows formally that $\frac{d\lambda}{dN} > 0$.

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Continuity of λ at τ and equation (2) imply that the marginal utility of extraction, $U_c \square \mu U_l$ is also continuous at τ[7] . In the standard cake-eating problem without leisure, continuity of marginal utility implies that per capita consumption, and therefore per capita utility, are also continuous at τ. The introduction of leisure implies that per capita utility *cannot* be continuous at τ.

Define the indicator function $I(t) = \left\{ \begin{array}{l} 0 \text{ for } t < \tau \\ 1 \text{ for } t \geq \tau \end{array} \right\}$ and let $N_t = \tilde{N} + I(t)\Delta$, where $\tilde{N}$ and Δ are positive constants. The population jumps by the amount Δ at time τ and is otherwise constant[8] . Denote the optimal extraction a moment before and a moment after τ as $y^{\square}$ and y^{+}.

In the appendix we establish the following

Proposition 2 *At time τ there is a discontinuous increase in per capita consumption and/or in per capita leisure; neither variable falls. There is also a discontinuous increase in per capita utility. That is, $c^{+} \geq c^{\square}$, and $l^{+} \geq l^{\square}$ with at least one inequality holding strictly; and $U(c^{+}, l^{+}) > U(c^{\square}, l^{\square})$.*

Here we sketch a geometric proof for Proposition 2, using Figure 1[9] . This figure shows the extraction levels that would maintain continuity in per capita consumption (the graph $c^{+} = c^{\square}$) and the extraction levels that would maintain continuity in per capita leisure (the graph $l^{+} = l^{\square}$). The shaded area shows the extraction levels for which $c^{+} \geq c^{\square}$ and $l^{+} \geq l^{\square}$.

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[7] If marginal utility were discontinuous at τ it would be possible to reallocate extraction between $\tau^{\square}$ and τ^{+} in such a way as to increase per capita utility.

[8] The increase in population, Δ, can be large. However, we assume that it is not so large that it causes a regime change. Specifically, the constraint $y \geq 0$ is not binding over $[0, \tau)$; extraction remains positive over this interval. See Favard [11] for details.

[9] In this sketch (unlike in the formal proof) we assume that U is strictly concave in c and l. Therefore the weak inequalities in the proposition are replaced by strict inequalities. Allowing weak concavity merely complicates the exposition.
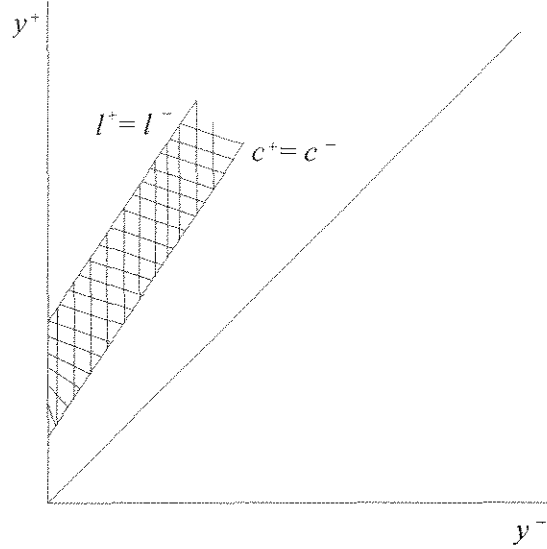
Figure 1: Extraction when the population increases

Above the line $l^+ = l^\square$ consumption increases and leisure falls at τ. Below the line $c^+ = c^\square$ leisure increases and consumption falls.

In order to maintain continuity of the marginal utility of extraction, extraction must increase at τ. To show this, define $v(y, N) \equiv U(c, l)$, so the first order condition is $v_y(y, N) = \lambda$. Since $v_{yN} > 0$ and $v_{yy} < 0$, y must increase when N increases, i.e., $y^+ > y^\square$. Therefore, we can restrict attention to the region above the 45^0 line in Figure 1.

Consider a value of y^+ that satisfies $c^+ \leq c^\square$; at this value $l^+ \equiv \bar{l} = 1 \square \frac{\mu\left[c^+\left(\bar{N}+\Delta\right)\square\bar{x}\right]\square\eta\bar{x}}{\bar{N}+\Delta} > l^\square$. Here, the marginal utility of extraction is $U_c(c^+, \bar{l})\square\mu U_l(c^+, \bar{l}) > U_c(c^\square, l^\square)\square\mu U_l(c^\square, l^\square) = \lambda$. The inequality uses the facts that $\frac{\partial(U_c\square\mu U_l)}{\partial l} > 0$ and $\frac{\partial(U_c\square\mu U_l)}{\partial c} < 0$. Hence, to maintain continuity of marginal utility we must increase y^+ (since $v_{yy} < 0$). Consequently, it must be the case that $c^+ > c^\square$. A parallel argument establishes that $l^+ > l^\square$.

The surprising result is that when the population increases, both per capita consumption
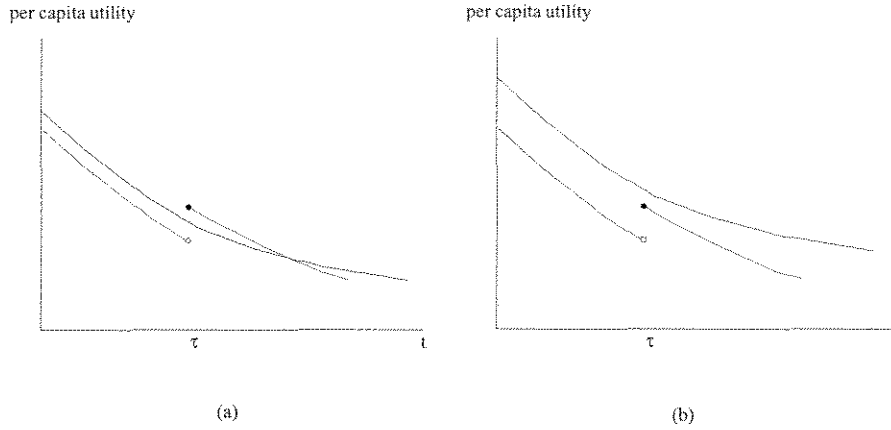
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Figure 2: Trajectories of per capita utility

and leisure increase[10] . The social planner smooths the marginal utility of consumption by decreasing extraction (relative to the reference case) prior to time τ. This decrease requires an increase in the asset price, λ. Since this asset price anticipates the increase in population at time τ, it must be continuous at τ. However, the population increases discontinuously at τ. Thus, at τ society has more workers than it had a moment before, but faces the same price of the resource. Since extraction requires less labor than production ($\mu < \eta$), natives shift some of their labor from production into extraction. They consume more than before τ, while working less[11] .

Figure 2 shows two trajectories of per capita utility in the reference case (the solid lines), where N is constant, and trajectories in cases where the population jumps at time τ (the dashed lines). Proposition 2 assures us that per capita utility jumps up at time τ. Figure

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[10] More precisely, neither decreases and at least one increases. If U is strictly concave in both c and l, then both strictly increase.

[11] If $\mu > \eta$ Proposition 1 is reversed: there is a discontinuous drop in per capita consumption, leisure and utility at τ. We do not consider this case, because it seems empirically less interesting.

2a illustrates the case where this jump is large enough that per capita utility is higher than in the reference case (during an interval after τ) – a possibility we have not yet confirmed. Figure 2b shows the other possibility, where the trajectory of per capita utility remains below the reference trajectory. If the case illustrated in Figure 2a occurs, it means that the population increase has different qualitative effects on different generations. The generation alive just after the increase in population has a higher flow of utility (relative to the reference case). The anticipation of the population increase causes earlier generations to decrease their extraction of the resource, leading to a larger stock at time τ.

4 A population increase at the initial time

Proposition 2 led to the conjecture that the generation alive at the time of an exogenous population increase may benefit from the change. Here we confirm that possibility by considering an increase in population at the initial time: $\tau = 0$. When $\tau > 0$, generations prior to τ bear some of the costs of the population increase, and bequeath a larger resource stock to generation τ (relative to the reference case). This larger stock makes it more likely that per capita utility at τ is higher than in the reference case. When $\tau = 0$ it is obviously not possible to shift the cost to previous generations. The special case $\tau = 0$ therefore provides a challenging test for our conjecture, because it eliminates one mechanism that tends to make utility higher at the time of the population increase.

Regardless of when the population increases ($\tau = 0$ or $\tau > 0$) the shadow value of the resource, λ, must increase. This increase tends to reduce extraction, and therefore to reduce

the current flow of welfare. When $\tau > 0$, the higher value of λ induced by the population increase is not offset by any other change over $[0, \tau)$, so the flow of welfare over that interval unambiguously decreases (relative to the reference case). We saw from Proposition 2 that at time τ the higher population provides an offsetting effect: the larger stock of labor causes natives to shift from production to extraction, allowing them to increase both consumption and leisure.

When the population increases at $\tau = 0$, the two counteracting forces occur at the same time. The population increase causes λ to rise, which promotes a reduction in extraction and a loss in the flow of utility. However, the increased stock of labor causes each worker to spend relatively more time on extraction. Since extraction (compared to production) requires less labor per unit of consumption, this shift increases leisure, promoting an increase in utility. Either of these two effects might dominate, so the flow of welfare at time 0 might increase or decrease.

Define $y^*(t) \equiv y(t, \lambda, N)$ as the optimal extraction policy [the solution to equations (1) and (2)]. We have

Proposition 3 *A necessary and sufficient condition for a population increase at time 0 to increase the flow of utility at time $t \geq 0$ is*

$$\frac{dy^*(t)}{dN} > \frac{\overline{x}}{N} \frac{\alpha(t)}{\lambda} + \frac{y^*(t)}{N}. \tag{3}$$

Proof.

$$\frac{dU(\frac{\overline{x}+y^*(t)}{N}, 1 \square \frac{\eta \overline{x}+\mu y^*(t)}{N})}{dN} > 0$$

$$\Leftrightarrow \square \frac{1}{N^2}\left\{(\overline{x} + y^*(t))\, U_c \square (\eta \overline{x} + \mu y^*(t))\, U_l\right\} dN + \frac{1}{N}\left\{U_c \square \mu U_l\right\} dy > 0$$

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$$\Leftrightarrow \Box \frac{1}{N} \left\{ \bar{x}\alpha(t)e^{rt} + y^*(t)\lambda e^{rt} \right\} dN + \lambda e^{rt} dy > 0.$$

Rearranging the last inequality implies equation (3) . $\blacksquare$

The left side of equation (3) is a total derivative; it includes the direct effect (on equilibrium extraction) of a change in N as well as the indirect effect, via the change in λ. The appendix provides the formula for $\frac{dy^*}{dN}$. Since this formula involves the entire trajectory of the optimal path, it cannot be easily interpreted. However, for a particular example, it is easy to determine whether equation (3) is satisfied.

In order to illustrate that migration at $t = 0$ might increase the initial flow of welfare, we use the separable utility function $U(c,l) = \beta c + 2\sqrt{l}$, where $\beta > 0$ is a constant[12] . Rather than choosing an initial stock $Y(0)$, we choose $\lambda = 1$ and adjust the initial stock in obtain a fixed shadow value, $\lambda = 1$. We can then easily evaluate equation (3) and also check that Assumption 1 is satisfied.

For the parameters $r = \bar{x} = \mu = 0.5$ and $\eta = N = 1$, it is straightforward to show that $\alpha > 0$ and $T > 0$ if and only if β is in the feasible range $1.71 < \beta < 2$[13] . As we increase β, we increase the initial stock to maintain the equality $\lambda = 1$. For β in the feasible range, equation (3) is satisfied (i.e. migration increases the initial flow of per capita utility) if and only if $1.8 \leq \beta < 2$. Migration always lowers per capita consumption and increases leisure in our example. For high values of β (and correspondingly high stocks, to insure

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[12] For this example, $U_c \equiv \beta$, contrary to our earlier assumption that the marginal utility of consumption is infinite near 0. Here we need restrictions on the value of β to insure that consumption and extraction remain positive.

[13] The choice $r = 0.5$ is consistent with a 5% discount rate if we choose one unit of time to equal approximately 10 years.

$\lambda = 1$) migration increases leisure by enough to increase the initial flow of utility. For $1.71 < \beta < 1.8$, migration at time 0 lowers the initial flow of per capita utility.

5 Conclusion

We have studied a cake-eating problem that includes leisure and a backstop technology, which in equilibrium is used to capacity. At some time $\tau \geq 0$ there is an anticipated increase in the population. We used this model to analyze the effects of migration into a rich labor-surplus economy (or the effects of the integration of labor markets). We assumed that the new workers receive an equal share of social capital.

Migration lowers the present discounted stream of per capita utility – and the steady state per capita utility – but has unexpected intertemporal effects. When migration occurs in the future, there is a positive jump in per capita utility at the time the workers enter. The generation that precedes the migration subsidizes the generation that follows it. When migration occurs at the initial time, it may increase the initial per capita flow of utility. Migration increases the resource price and reduces per capita consumption. However, the natives spend relatively more time working in the "extraction activity", and less time in the "production activity". If the resulting increase in the leisure more than offsets the lower consumption, their utility rises. Using a numerical example, we showed that this case can certainly occur.

A popular view of migration holds that even if it has long run benefits, it imposes short run costs on the current generation of natives, which needs to make transfers to the migrants.

Owing to the particular assumptions of our model, migration never has long run benefits, either in the steady state or with respect to the present value of the stream of per capita utility. However, contrary to the popular view, migration might benefit the generation alive at the time it occurs.

This conclusion has two political economy implications. The first is that the current generation might be too willing to accept migrants, from the perspective of national welfare. This might happen if migrants benefit the current generation of natives, but harm the stream of future generations.[14] The second implication is that if migration is bound to occur, it is in the interests of the current generation that it occur sooner rather than later. These implications are interesting because they run counter to conventional wisdom.

As we emphasized in the Introduction, this model has a built-in bias against migration, since it views migrants as bringing only their appetites and labor power to a labor surplus economy; it ignores their other contributions to society. It is worth repeating that the anti-migrant implication (Proposition 1) is an obvious artifact of our restrictive assumptions, and is therefore not particularly useful. On the other hand, we think that the greater understanding of the intertemporal effects of migration (summarized in Propositions 2 and 3) is useful.

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[14] The empirical literature (e.g. [17]) stresses the opposite possibility: there are short run cost of educating migrants' children, and long run benefits as these children become productive. Our theoretical model does not address — and therefore does not contradict — this possibility. Instead, we focus on a less obvious mechanism through which migration has different short and long run effects.

6   Appendix

The appendix consists of three parts: proofs of Propositions 1 and 2 and Corollary 1, details

for the example in Section 4, and a discussion of the more general model with endogenous

capital.

6.1   The proofs

Proof. (Proposition 1)   We write the solution to equations (1) and (2) as the function

$y(t, \lambda, N)$. Taking partial derivatives of equation (2) implies

$$\frac{\partial y}{\partial t} = N \frac{\lambda r e^{rt}}{U_{cc} + \mu^2 U_{ll} \square 2\mu U_{cl}} < 0 \tag{4}$$

$$\frac{\partial y}{\partial \lambda} = \frac{1}{\lambda r} \frac{\partial y}{\partial t} < 0 \tag{5}$$

$$\frac{\partial y}{\partial N} = \frac{1}{N} \frac{(\overline{x} + y)U_{cc} + \mu(\eta \overline{x} + \mu y)U_{ll} \square ((\mu + \eta)\overline{x} + 2\mu y)U_{cl}}{U_{cc} + \mu^2 U_{ll} \square 2\mu U_{cl}} > 0. \tag{6}$$

We take the differential of equation (1), using the optimality condition $y(T, \lambda, N) = 0$, to

obtain

$$d\lambda \int_0^{T(\lambda,N)} \frac{\partial y(t, \lambda, N)}{\partial \lambda} dt + dN \int_0^{T(\lambda,N)} \frac{\partial y(t, \lambda, N)}{\partial N} dt = 0,$$

which implies

$$\frac{d\lambda}{dN} = \square \frac{\int_0^{T(\lambda,N)} \frac{\partial y(t, \lambda, N)}{\partial N} dt}{\int_0^{T(\lambda,N)} \frac{\partial y(t, \lambda, N)}{\partial \lambda} dt} \geqslant 0. \tag{7}$$

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We write the dynamic programming equation for the social planner's problem and divide by N to obtain

$$\frac{rJ(Y(t),N)}{N} = \max_{y(t)} \left\{ U\left(\frac{\bar{x}+y(t)}{N}, 1 - \frac{\eta\bar{x}+\mu y(t)}{N}\right) - \frac{\partial J(Y(t),N)}{\partial Y(t)}\frac{y(t)}{N}\right\}.$$

The function $\frac{J(Y;N)}{N}$ is the present discounted value of the per capita utility of a single agent, when the size of the population is N. Differentiating both sides by N, using the envelope theorem, implies

$$\implies \frac{\partial \frac{rJ(Y(t),N)}{N}}{\partial N} = -\frac{\bar{x}+y(t)}{N^2}U_c + \frac{\eta\bar{x}+\mu y(t)}{N^2}U_l - \frac{d\lambda}{dN}\frac{y(t)}{N} + \lambda\frac{y(t)}{N^2},$$

where we define $\lambda = J_Y(Y;N)$, and $\frac{d\lambda}{dN} = J_{Y,N}$. Using equation (2), we rewrite this equality as

$$\frac{\partial \frac{rJ(Y(t),N)}{N}}{\partial N} = -\frac{\bar{x}+y(t)}{N^2}U_c + \frac{\eta\bar{x}+\mu y(t)}{N^2}U_l - \frac{d\lambda}{dN}\frac{y(t)}{N} + (U_c - \mu U_l)\frac{y(t)}{N^2}$$

$$= -\frac{\bar{x}}{N^2}\left(U_c - \eta U_l\right) - \frac{d\lambda}{dN}\frac{y(t)}{N}$$

$$= -\alpha(t)\frac{\bar{x}}{N^2} - \frac{d\lambda}{dN}\frac{y(t)}{N} < 0.$$

The last inequality follows from equation (7) and Assumption 1. ∎

We now prove Corollary 1

Proof. (Corollary 1) Suppose that the social planner anticipates that at time τ, $\Delta > 0$ migrants will arrive. The present discounted value of per capita utility *in the absence of migration* (N is constant) satisfies

$$\frac{J(Y_0;N)}{N} = \max\left\{\int_0^\tau e^{-rs}U\left(c_s,l_s\right)ds + e^{-r\tau}\frac{J(Y_\tau;N)}{N}\right\} > \\ \max\left\{\int_0^\tau e^{-rs}U\left(c_s,l_s\right)ds + e^{-r\tau}\frac{J(Y_\tau;N+\Delta)}{N+\Delta}\right\} \tag{8}$$

The equality in (8) is obtained by dividing the social planner's maximization problem by the constant N (which does not change the optimal trajectory); the inequality follows from Proposition 1.

Now consider the case where the planner anticipates that migration will occur at τ. Dividing this planner's objective by the constant N, we obtain an equivalent objective (i.e., one that leads to the same optimal extraction trajectory):

$$\max \left\{ \int_0^\tau e^{-rs} U\left(c_s, l_s\right) ds + e^{-r\tau} \frac{J\left(Y_\tau; N+\Delta\right)}{N} \right\}. \tag{9}$$

Denote the solution to the problem in (9) as $c_s^{**}, l_s^{**}, Y_\tau^{**}$. Denote the present discounted stream of per capita utility of natives in the regime where the anticipated migration occurs as $V(Y; N, \Delta)$. We have

$$\begin{aligned} V(Y; N, \Delta) &\equiv \int_0^\tau e^{-rs} U\left(c_s^{**}, l_s^{**}\right) ds + e^{-r\tau} \frac{J(Y_\tau^{**}; N+\Delta)}{N+\Delta} \\ &< \max \left\{ \int_0^\tau e^{-rs} U\left(c_s, l_s\right) ds + e^{-r\tau} \frac{J(Y_\tau; N+\Delta)}{N+\Delta} \right\} \end{aligned} \tag{10}$$

The inequality in (10) follows from the fact that the triple $c_s^{**}, l_s^{**}, Y_\tau^{**}$ is the solution to (9). This triple does not maximize expression in brackets in (10), since the denominator of the second term of the maximand is $N + \Delta$ rather than N [as in equation (9)].

Equations (8) and (10) imply

$$\frac{J(Y_0; N)}{N} > \int_0^\tau e^{-rs} U\left(c_s^{**}, l_s^{**}\right) ds + e^{-r\tau} \frac{J\left(Y_\tau^{**}; N+\Delta\right)}{N+\Delta} = V(Y; N, \Delta),$$

i.e., the present discounted value of per capita utility in the absence of migration is higher than the present discounted value of the per capita utility of natives when the population increases by Δ at time τ. ∎

To prove Proposition 2 we use the following lemma.

Lemma 1

$$\frac{\Delta}{\bar{N}} \left(\frac{\eta \bar{x}}{\mu} + y^\square \right) \geq y^+ \square\, y^\square \geq \frac{\Delta}{\bar{N}} \left(\bar{x} + y^\square \right).$$

Proof. We again use the function $y(t, \lambda, N_t)$, the solution to equation (2), where $N_t = \bar{N} + I(t)\Delta$. We simplify notation by writing N_t as N. We rewrite the partial derivative $\frac{\partial y}{\partial N}$ given in equation (6) as

$$\frac{\partial y}{\partial N} = \frac{(\bar{x} + y)}{N} + \frac{\eta \,\square\, \mu}{\mu} \frac{\bar{x}}{N} \mu \gamma(y), \text{ with } \gamma(y) \equiv \frac{\mu U_{ll} \,\square\, U_{cl}}{U_{cc} + \mu^2 U_{ll} \,\square\, 2\mu U_{cl}},\ 0 \leq \mu\gamma \leq 1, \quad (11)$$

which implies

$$\frac{\bar{x} + y}{N} \leq \frac{\partial y}{\partial N} \leq \frac{\eta \bar{x} + \mu y}{\mu N}. \quad (12)$$

At time $t = \tau$, t and λ are fixed, but N changes, so we can write

$$y^+ \,\square\, y^\square = \int_{\bar{N}}^{\bar{N}+\Delta} \frac{\partial y(t, \lambda, N)}{\partial N} dN.$$

Using equation (12) we have

$$\int_{\bar{N}}^{\bar{N}+\Delta} \frac{\bar{x} + y(N)}{N} dN \leq y^+ \,\square\, y^\square \leq \int_{\bar{N}}^{\bar{N}+\Delta} \frac{\eta \bar{x} + \mu y(N)}{\mu N} dN \quad (13)$$

where we abbreviate $y(t, \lambda, N)$ as $y(N)$.

Define

$$z(\bar{N} + \Delta) \equiv y^\square + \int_{\bar{N}}^{\bar{N}+\Delta} \frac{\eta \bar{x} + \mu z(N)}{\mu N} dN.$$

Thus, $z(N)$ is the solution to the differential equation

$$\frac{dz}{d\Delta} = \frac{\eta \bar{x} + \mu z}{\mu \left(\bar{N} + \Delta \right)}, \quad (14)$$

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with the boundary condition $z(\bar{N}) = y^\square$. We can solve the differential equation (14) to obtain $z(\bar{N} + \Delta) = \frac{\bar{N} + \Delta}{\bar{N}} y^\square + \frac{\Delta \eta \bar{x}}{\bar{N} \mu}$.

In view of the second inequality in equation (13), we have $y^+ \leq z(\bar{N} + \Delta)$ for $\Delta > 0$. This inequality and the solution to equation (14) imply

$$y^+ \,\square\, y^\square \leq z(\bar{N} + \Delta) \,\square\, z(\bar{N}) = \frac{\Delta}{\bar{N}} \left(\frac{\eta \bar{x}}{\mu} + y^\square \right)$$

which establishes the first inequality in the statement of Lemma 1. The proof of the second inequality is parallel, so we omit the details. ∎

We now prove Proposition 2.

Proof. (Proposition 2) Per capita consumption and leisure depend on aggregate extraction, $y(t)$, and the population, $N(t)$. Using the definition of consumption, we have

$$y^+ \,\square\, y^\square \geq \frac{\Delta}{\bar{N}} \left(\bar{x} + y^\square \right) \iff \frac{y^+ + \bar{x}}{\bar{N} + \Delta} \geq \frac{y^\square + \bar{x}}{\bar{N}} \iff c^+ \geq c^\square .$$

Using the definition of leisure, we have

$$\frac{\Delta}{\bar{N}} \left(\frac{\eta \bar{x}}{\mu} + y^\square \right) \geq y^+ \,\square\, y^\square \iff \frac{\mu y^+ + \eta \bar{x}}{\bar{N} + \Delta} \geq \frac{\mu y^- + \eta \bar{x}}{\bar{N}} \iff l^+ \geq l^\square .$$

In view of Lemma 1, we conclude that $c^+ \geq c^\square$ and that $l^+ \geq l^\square$. Since these two inequalities cannot both hold as equalities, either consumption or leisure must be strictly higher at τ^+. Since utility is increasing in both its arguments, we obtain the third inequality in the Proposition. ∎

6.2 The Example

□ Derivation of $\frac{dy^*}{dN}$. We substitute equations (5) and (11) into (7) to obtain

$$\frac{d\lambda}{dN} = \frac{\dfrac{Y_0 + \bar{x}T(\lambda, N)}{N} + (\eta \,\square\, \mu)\dfrac{\bar{x}}{N}\int_0^{T(\lambda, N)} \gamma\left(y^*(t)\right) dt}{\dfrac{1}{\lambda r}y(0, \lambda, N)}. \tag{15}$$

The total effect of a marginal change in population on the instantaneous extraction rule is

$$\frac{dy}{dN} = \frac{\partial y(t, \lambda, N)}{\partial \lambda}\frac{d\lambda}{dN} + \frac{\partial y(t, \lambda, N)}{\partial N}. \tag{16}$$

We can use equations (5), (11) and (7) to simplify equation (16).

□ Simplifying the necessary and sufficient conditions for a welfare increase. Using equations (5), (11), (7) and (16), we can rewrite (3) as

$$P(t) \equiv \frac{\bar{x}}{N}\left(1 \,\square\, \frac{\alpha(t)}{\lambda} + (\eta \,\square\, \mu)\,\gamma(y^*(t))\right) >$$

$$\frac{\lambda r e^{rt}}{[2\mu U_{cl} \,\square\, U_{cc} \,\square\, \mu^2 U_{ll}]\, y^*(0)}\left[Y(0) + \bar{x}T + (\eta \,\square\, \mu)\,\bar{x}\int_0^T \gamma(y^*(t))dt\right] \equiv Q(t). \tag{17}$$

□ The equation for T. In order to evaluate condition (17) we need the equation for $T(\lambda, N)$. Optimality requires that $y(T, \lambda, N) = 0$. This condition and equation (2) imply

$$T(\lambda, N) = \frac{\ln\left(U_c(\bar{x}/N, 1 \,\square\, \eta\bar{x}/N) \,\square\, \mu U_l(\bar{x}/N, 1 \,\square\, \eta\bar{x}/N)\right) \,\square\, \ln\lambda}{r}. \tag{18}$$

□ Calculations. For the utility function $U(c, l) = \beta c + 2\sqrt{l}$, we have $U_{cc} = U_{cl} = 0$, $U_l = l^{\square 0.5}$, $U_{ll} = \square\frac{l^{\square 1.5}}{2}$, and $\gamma(y^*(t)) = \frac{1}{\mu}$. We normalize by setting $N = 1$ and we choose $\lambda = 1$, so that the initial stock $Y(0)$ is determined by the parameters. We set

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β	$Y(0)$	T	α	P	Q
1.72	0.00046	0.0256	0.28	0.86	0.98
1.8	0.021	0.178	0.2	0.9	0.89
1.9	0.078	0.0352	0.1	0.95	0.77
1.99	0.15	0.5	0.01	0.99	0.68

Table 1: Endogenous values for example

$\eta = 1$, $\bar{x} = r = \mu = 0.5$. In order for equation (18) to have a positive solution, and for Assumption 1 to be satisfied, we require that $\beta \in (1.71, 2)$. The following table presents the endogenous values of several variables (at $t = 0$) for different values of β.

A larger value of β requires a higher initial stock, in order to maintain $\lambda = 1$. The time to exhaustion, T, is correspondingly higher, and the rental rate on capital, α, is lower.

6.3 Endogenous capital

Here we briefly consider the case where capital is endogenous. We do not attempt to show formally that Propositions 1 - 3 hold in this more general setting, but we explain why the intuition behind those propositions remains valid.

Suppose that society can increase the stock of capital, $\bar{x}$. As in the text, we suppose that production of one unit of the consumption good requires one unit of capital and η units of labor: the flow of production is $x(t)$ when society uses $x(t)$ units of capital and $\eta x(t)$ units of labor. As before, we have the constraint which is implied by the finite stock of capital and the Leontieff production function:

$$x(t) \leq \bar{x}(t). \tag{19}$$

The increase in the stock of capital equals investment, which is proportional to production plus extraction minus consumption (capital does not depreciate)

$$\frac{d\bar{x}}{dt} = \rho\left(x(t) + y(t) - Nc(t)\right) \tag{20}$$

where $\frac{1}{\rho}$ equals the number of units of the consumption good needed to obtain an additional unit of capital, $\bar{x}$. We obtain our model in the text by setting $\rho = 0$. If the equilibrium comparative statics are continuous in ρ (as we expect in a model of this sort), then all of our results would carry over to the case where ρ is small.

We suppose that investment must be non-negative, i.e. it is not possible to eat capital:

$$x(t) + y(t) - Nc(t) \geq 0. \tag{21}$$

Capital is useful as a factor of production, not as a means of storing value.

The social planner's control problem now contains two state variables, $Y(t)$ and $\bar{x}(t)$, with the associated costate variables $\lambda(t)$ and $\alpha(t)$. The Hamiltonian of the planner's problem is

$$H = NU\left(c, 1 - \frac{\mu y + \eta x}{N}\right) - \lambda y + \alpha\left(x + y - Nc\right) + \theta_1\left(\bar{x} - x\right) + \theta_2\left(x + y - Nc\right) \tag{22}$$

where θ_1 and θ_2 are the constraint multipliers associated with the constraints (19) and (21). The first order conditions include (2) and

$$\eta U_l + \theta_1 - \theta_2 = \alpha. \tag{23}$$

The inability to eat capital reduces the shadow value of capital, since $\theta_2(t) \geq 0$.

The introduction of endogenous capital leads to several new possibilities. For example, $\alpha(t) > 0$ now requires only that the constraint (19) is binding at some time in the future, not necessarily at the current time. We therefore replace Assumption 1 with

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Assumption 1*. Capital is always fully employed, i.e. $\theta_1(t) > 0$ for all t.

The endogeneity of capital provides an additional method of smoothing consumption. The per capita cost of a higher population is therefore smaller (relative to the case of fixed capital) but is still positive. When migration occurs at $\tau > 0$, the shadow value of the resource rises and consumption over $[0, \tau)$ falls as before. However, since it is possible to convert extraction to capital, leisure does not necessarily rise (or does not rise by as much as in the case with fixed capital). Utility over $[0, \tau)$ still falls (relative to the reference case).

In the simplest case, inequality (21) is binding after the jump at τ. In this case, the intuition for Proposition 2 still holds. When (19) and (21) are binding, there is a single free variable. Continuity of λ and equation (2) still imply that per capita utility jumps up at τ. This example illustrates the importance of the assumption that capital provides a means of production, not a store of value. If it were possible to eat capital (i.e. if we removed constraint (21)) then at time τ there would be two free variables. In that case, continuity of the marginal utility of extraction would require continuity of the marginal utilities of both consumption and leisure.

When the population increases at time 0 agents reallocate labor time from production to extraction, so their leisure tends to increase. However, the possibility of investing increases the incentive to defer consumption. The investment opportunity also makes leisure less attractive. Thus, we conjecture that if it is easy to convert the consumption good to capital (i.e. if ρ is large) the increase in population at time 0 is less likely to increase the flow of utility at time 0.

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