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Publication Date
2004-10-01

Firm Reputation and Horizontal Integration[*]

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Abstract

We study effects of horizontal integration on firm reputation. In an environment where customers observe only imperfect signals about firms' effort/quality choices, firms cannot maintain reputations of high quality and earn quality premium forever. Even when firms are choosing high quality/effort, there is always a possibility that a bad signal is observed. In this case, firms must give up their quality premium, at least temporarily, as punishment. A firm's integration decision is based on the extent to which integration attenuates this necessary cost of maintaining a good reputation. Horizontal integration leads to a larger market base for the merged firm and may allow better monitoring of the firm's choices, hence improving the punishment scheme for deviations. On the other hand, it gives the merged firm more room for sophisticated deviations. We characterize the optimal level of integration and provide sufficient conditions under which nonintegration dominates integration. We show that the optimal size of the firm is smaller when (1) trades are more frequent and information is disseminated more rapidly; or (2) the deviation gain is smaller than the honesty benefit; or (3) customer information about firm choices is more precise.

Keywords: Reputation; Integration; Imperfect Monitoring; Theory of the Firm

JEL Classification: C70; D80; L14

1 Introduction

Reputation has long been considered critical for firm survival and success in the business world. Since the seminal work of Kreps (1990), the idea of firms as bearers of reputation has become increasingly important in the modern development of the theory of the firm. For example, Tadelis (1999, 2002), Mailath and Samuelson (2001), and Marvel and Ye (2004) develop models of firm reputation as tradable assets and study the market equilibrium for such reputation assets. Klein and Leffler (1981) and Horner (2002) analyze how competition helps firms build good reputations when their behavior is not perfectly monitored by customers. These studies provide very useful insights into how firm reputation can be built, maintained and traded. However, for reputation to be a defining feature in the theory of the firm, an important question needs to be answered: How does firm reputation affect the boundaries of the firm?[1]

In this paper we build a simple model to study the effects of horizontal integration on firm reputation. We consider an environment where firm products are experience goods in the sense that customers cannot observe product quality at the time of purchase, but their consumption experience provides noisy public information about product quality (e.g., consumer ratings).[2] Absent proper incentives, firms will tend to shirk on quality to save costs, making customers reluctant to purchase. Using a model of repeated games with imperfect monitoring, we show that as long as firms care sufficiently about the future, they can establish reputations of high quality and earn quality premium while building customers loyalty.[3] However, unlike the case with perfect monitoring, firm reputation can be sustained only if the public signal about a firm's choices is above a certain cut-off point in every period. With positive probability the public signal will fall below the cut-off point, in which case firm reputation will be lost: either customers will never buy again or the firm must pay large financial penalties to win back previous customers.

We then consider the situation where several firms, each serving an independent market,

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[1]The boundary of the firm question was first raised by the classical work of Coase (1937). Several influential theories have been proposed to answer the question, for example, Alchian and Demsetz (1972), Williamson (1985), and Hart (1995). Holmstrom and Roberts (1998) offer a review and critique of these theories.

[2]Professional services, food services, and consumer durable goods are standard examples of experience goods.

[3]Our analysis is an application of the theory of repeated games with imperfect monitoring. For important contributions in this area, see, e.g., Green and Porter (1984), Radner (1985), Abreu, Pearce, and Stacchetti (1986, 1990), Abreu, Milgrom, and Pearce (1991), Fudenberg, Levine and Maskin (1994), Athey, Bagwell, and Sanchirico (2002), and many others.

merge into one large firm.[4] To focus on reputation, we assume away any technological economies (or diseconomies) of scale or demand linkages across markets. Horizontal integration leads to a larger market base for the merged firm and may allow better monitoring of the firm's choices, hence improving the punishment scheme for deviations. On the other hand, horizontal integration gives the merged firm more room for sophisticated deviations. With imperfect monitoring of firms' quality choices, these effects on reputation building give rise to meaningful trade-offs for horizontal integration. We characterize the optimal level of integration and provide sufficient conditions under which nonintegration is optimal. We show that the optimal size of the firm is smaller (or, non-integration is more likely to dominate integration) when (1) trades are more frequent and information is disseminated more rapidly; or (2) the deviation gain is relatively smaller; or (3) the information customers have about firms' choices is more precise.

The results of our paper can shed light on patterns of horizontal integration observed in the real world. For example, horizontal integration such as franchising is very common in industries that mainly provide services to travelers, such as hotels and car rentals. In these industries, customers interact with firms relatively infrequently, which corresponds to low discount factors in our model. As our results show, for low discount factors, independent firms cannot build reputation effectively by themselves, and horizontal integration can improve on reputation building. Similarly, in industries that provide services to both travelers and locals such as taxicabs and convenience stores, horizontal integration (either as franchising like the Seven-Eleven stores, or mergers of taxicab companies) seems to be quite common, though less as common as in travel industries.

For another example, chains are more common in the fast food sector than they are among high end restaurants. Fast food restaurants provide more homogenous products than high end restaurants, thus their profit margins are on average smaller than high end restaurants. Our results suggest that if payoffs from maintaining a good reputation are greater relative to deviation gains (e.g. high profit margin restaurants), non-integration is more likely to dominate integration.[5] While other explanations are certainly possible in these examples, our theory pro-

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[4]Note that we consider horizontal integration of firms that produce similar products. Our model does not directly apply to firms with multiple product lines that are obviously of different quality levels, e.g., Toyota's wide range of models, from Tercel to Lexus.

[5]The pattern of horizontal integration in the food industry is also consistent with the previous point: compared to fast food restaurants (especially those along highways or in airports), high end restaurants are more focused on

vides a new perspective and offers new insight into horizontal integration that potentially can be tested with real world data. In fact, in a recent empirical paper that analyzes reputation incentives for restaurant hygiene, Jin and Leslie (2004) find evidence consistent with our theory. For example, they find that restaurant chains are more likely to be found in tourist locations. Moreover, "regions where independent restaurants tend to have relatively good quality hygiene, the incremental effect on hygiene from chain affiliation is lower."

To understand the basic ideas, consider first the case of non-integration where a long-lived firm serves its customers (who are short- or long-lived and anonymous) in a market. In each period, the firm chooses a price level and whether to exert high or low effort/quality. Customers decide whether to buy without observing the firm's effort/quality choice. If they buy, a noisy public signal about the firm's choice is generated from customer experience. Focusing on public strategy equilibria in which strategies depend on the history of public signals only, we show that the best equilibria for the firm have a nice stationary feature. Reputation will be sustained, that is, the firm charges a high price and provides high effort/quality and customers trust the firm and buy its products if and only if the public signal is above a certain cut-off point. Thus, the lower is the cut-off point, the longer the firm's reputation will be sustained in expectation. We show that a firm's ability to maintain its reputation depends on several intuitive factors, such as the discount factor, the relative magnitude of the deviation gain to the honesty payoff, and the informativeness of signals.

Now suppose several independent firms merge into one single firm. The integrated firm makes effort/quality decisions in the production process and allocates products to the markets it serves. Customers in each market observe some noisy signal about the firm's product quality in all the markets. We first demonstrate that in the best equilibrium, only the aggregate or average signal matters. More precisely, as long as the average signal is above a certain cut-off point, the firm provides high effort/quality and customers in all markets buy from the firm. If the average signal is below the cut-off point, either customers in all markets desert the firm forever, or the firm pays large financial penalties to customers in all markets.[6]

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serving local communities.

[6]This is consistent with the observation that customers usually care about public signals about a firm's aggregate choices or overall performance such as its product quality ranking and rating of consumer satisfaction. Public signals about each branch's choice may not be available or too noisy to be useful. For example, it can prove very difficult to discern accounting records for each of the firm's divisions since there are numerous ways to allocate

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For a firm that serves any given number of markets, we characterize the best reputation equilibrium. The optimal degree of horizontal integration, i.e., the optimal firm size, is the size for which the lowest cut-off point is obtained so that reputation lasts the longest, thus the expected profit per market is maximized. In our model, integration has three effects on reputation: a positive size effect, a negative deviation effect, and a positive information effect.

First, large firm size helps reputation building by providing more severe punishments (e.g., shutting down the whole firm) for a fixed magnitude of deviation (e.g., choosing low quality in only one market). The merged firm has more to lose if it loses the trust of its customers, giving it stronger incentives to maintain reputation.[7]

The second effect of integration on reputation is that the merged firm has more opportunity for deviation than independent firms. A firm that serves n markets can deviate in any $m \leq n$ markets, and thus has to satisfy n incentive constraints to maintain its reputation. This, of course, impedes reputation-building. We show that under mild conditions, the single-market deviation constraint is the only binding constraint. As firm size increases, it becomes more difficult to prevent the one-market deviation, decreasing reputation-building.

The last effect of integration on reputation, the information effect, is that integration may allow information aggregation across markets and thus make it easier for customers to monitor a firm serving a large number of markets. Since an integrated firm's reputation is contingent on the public signal averaged over the markets it serves, its reputation mechanism depends on the informativeness of the average signal. Consequently, as long as the processes that generate the public signals are not perfectly correlated across markets, the average signal will be more informative for larger firms.[8] Thus reputation building is easier for large firms, implying that the information aggregation effect tends to increase the optimal size of the firm.

These three effects of horizontal integration on reputation present meaningful trade-offs

costs and revenues within the firm. But even in cases where it is possible to get signals of product quality in each market, our result suggests that it is sufficient to look at the aggregate signal about the firm's overall quality.

[7]To be more precise, it is not firm size per se that matters. If an independent firm expands so that its payoffs in all contingencies simply scale up, its incentives to build reputation will not be affected at all. When two independent firms merge into one, what is important is that the big firm makes joint decisions for both branches and its customers understand this. Hence if it appears that the firm has cheated somewhere, all its customers everywhere will punish the firm by desertion.

[8]In the model, for concreteness, we suppose that the "production uncertainty" of an integrated firm is common shock whereas the "taste uncertainty" is independent across the markets it serves.

for firm size. When the positive size effect and information aggregation effect dominate the negative deviation effect, then we expect firms to optimally choose a greater degree of horizontal integration. Otherwise, non-integration will likely dominate integration and the optimal size of the firm will be smaller.

When the discount factor is large, future payoffs are important and can serve as effective punishments for deviations. Independent firms can sustain reputation quite effectively in this case, hence the large losses an integrated firm faces for deviation (size effect) do not greatly improve reputation-building incentives. Thus, it is more likely for non-integration to dominate integration for large discount factors. Because discount factors are larger when trades are more frequent and information is disseminated more rapidly, our model predicts that the optimal size of the firm will be smaller in such situations. When the deviation gain is large relative to the honesty payoff, independent firms cannot sustain a reputation for very long. In this case, the size effect of integration can help reputation-building significantly by increasing punishments for deviations. Thus, for relatively large deviation gains and small honesty payoffs, integration is more likely to dominate non-integration. In contrast, when customers' information about the firm's choices is more precise (reducing the relative benefits of information aggregation), independent firms can maintain reputation quite effectively. Furthermore, it is more difficult to "detect" and punish small deviations in larger firms, diminishing the size effect. Thus, with more precise signals, the optimal size of the firm will be smaller.

Our paper builds on the idea of the "firm as bearer of reputation" in the existing literature, e.g., Kreps (1990), Tadelis (1999, 2002), Mailath and Samuelson (2001), and Marvel and Ye (2004). These papers investigate whether and how firm reputation can be traded, but do not address the question about the boundaries of the firm. Our paper is also closely related to the literature on multimarket contacts, e.g., Bernheim and Whinston (1990) and Matsushima (2001). Bernheim and Whinston (1990) show that in the perfect monitoring setting, two firms may find it easier to collude if they interact in multiple markets in which they have uneven competitive positions than if they interact in a single market. Matsushima (2001) considers the setting of imperfect monitoring and proves that two firms can approach perfect collusion when the number of market contacts goes to infinity. Neither of these papers consider the issue of the boundaries of the firm. In terms of motivations, our paper is perhaps most closely related to Andersson (2002),

who studies the effects of firm scope on its reputation in a model with perfect monitoring.[9] Producing multiple products may increase the firm's total profits (relative to independent firms producing those products), because pooling the incentive constraints in the multiple markets may allow the firm to increase its prices. In our model, firm choices are imperfectly monitored, and we consider integration of symmetric markets.[10] From the trade-offs between the positive size and information effects and the negative deviation effect, we develop a theory of optimal firm size. In contrast, while Bernheim and Whinston (1990), Matsushima (2001) and Andersson (2002) all show that firm size may matter, firm profit can only increase monotonically in firm size.[11]

The rest of the paper is organized as follows. The next section presents the model. Then in Sections 3 and 4, we characterize the best reputation equilibrium of the game for the firm under non-integration and integration, respectively. Comparing the equilibrium outcomes in the two cases, in Section 5 we obtain the main results about the optimal firm size and examine how it is affected by the model's parameters. Concluding remarks are in Section 6.

2   The Model

There are n separate markets, in each of which a long-lived firm sells its products to its customers. Time is discrete and the horizon is infinite. Customers in each market are identical, and the firms and their respective markets are symmetric. To focus on reputation, we assume away any technological and demand linkages across markets, such as economies of scale or scope, demand spillovers, or competition across markets. In each period, the firm in each market and its customers play the following stage game. At the beginning of a period, the firm, who we assume has price-setting power, sets the price p for the period.[12] Then the customers decide

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[9]The applications Andersson (2002) considers are brand extensions or "umbrella branding" whereby a firm produces different kinds of products under one brand (e.g., Porsche watches). For a recent contribution and a summary of the literature, see Cabral (2000).

[10]In both Bernheim and Whinston (1990) and Andersson (2002), firm size will have no effect if all markets are symmetric.

[11]Fishman and Rob (2002) study a model of investment in reputation in which firms' product qualities are perfectly observed by some customers, and show that bigger and older firms have better reputations.

[12]Our analysis and the main results of the paper will not be affected significantly if the firm does not have price-setting power.

whether to purchase from the firm. If they do not buy from the firm, both the customers and the firm get a payoff of zero. If they decide to buy from the firm, their payoffs depend on the firm's product quality. The firm decides whether to exert high effort e_h (or, provide high quality) or exert low effort e_l (or, provide low quality), where e_h and e_l are both real numbers and $e_h > e_l$. The firm incurs an effort/quality cost of c_h (c_l) for providing high (low) quality, where $c_h > c_l$. The customers' expected benefit is v_h if the firm chooses e_h and is v_l if the firm chooses e_l, where $v_h > v_l$. Given p, the stage game is depicted below in the normal form. Equivalently one can think of an extensive form game in which customers move first with their purchase decisions.

		Firm	
		Low	High
Customers	$Don't\ Buy$	$0\quad,\quad 0$	$0\quad,\quad 0$
	Buy	$v_l - p\ ,\ p - c_l$	$v_h - p\ ,\ p - c_h$

We assume $v_h - c_h > 0 > v_l - c_l$: high effort/quality is more efficient than no trade, which in turn is more efficient than low effort/quality. Since $c_h > c_l$, e_l (weakly) dominates e_h for the firm. Hence, for any price $p > v_l$, the unique equilibrium outcome is $(Don't\ Buy, Low)$, resulting in payoffs $(0, 0)$. The outcome $(Buy, High)$ is the first best efficient in terms of total surplus and Pareto-dominates $(Don't\ Buy, Low)$ for $p \in (c_h, v_h)$. However, this efficient outcome is not attainable without reputation effects. Our stage game is in the spirit of Kreps (1990), who highlights the firm's incentive problem in a one-sided Prisoners' Dilemma game.

We suppose that in each market there are a large number of identical customers that are anonymous to the firm in the market. That is, the firm in each market can only observe the aggregate distribution of customers' behaviors but not individual customer behavior. Since an individual customer's behavior is not observable, each customer will maximize his current period payoff. Customers would also maximize current period payoffs if they purchase the products only once (i.e., short-lived customers).[13] Thus another interpretation of our model is that the customers are short-lived.

If effort in each period were publicly observable, it would be straightforward to show that the efficient outcome (Buy, High) can be supported when future is sufficiently important to the

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[13]Our assumption that customers maximize current period payoffs implies that the folk theorem result of Fudenberg, Levine and Maskin (1994) does not apply.

firm. Let δ be the firm's discount factor. It can be easily checked that for any price $p \in (c_h, v_h]$, the first best is attainable if and only if $\delta \geq (c_h - c_l)/(p - c_l)$. To maximize its profit, the firm will set price $p = v_h$.

In most cases, however, it may be natural that effort cannot be perfectly observable, especially for experience goods. First, given the firm's effort (e.g., investment in quality control, personnel, training, and procedures), there are unavoidable uncertainties (e.g., machine malfunctioning, human errors) in production processes that introduce random shocks into product qualities. Alternatively, customers or analysts may have to infer the firm's effort choices, such as whether it hires high caliber (and expensive) consultants or uses reliable (and expensive) parts, through information from its public (e.g., accounting) records. But such information and the inferences based on this information are usually quite noisy. Second, when the firm's products or service cannot be evaluated in isolation (e.g., an intermediate input), noise can introduce error in gauging its quality. For example, it is typically very difficult to estimate the value added of a consulting project as numerous other factors affect a client's revenue. This feature is shared by most professional services (e.g., law, medical services). Third, when customers purchase the products just once (i.e., short-lived), experiences of the current period customers may be communicated to future customers only with substantial noise (e.g., consumer on-line ranking/comments). In all such cases, the firm's effort/quality choices can only be imperfectly observed by customers at the end of each period.

Given these observations, we consider an environment in which a firm's effort is not public information, but rather a noisy public signal $y \in \mathbb{R}$ of the firm's effort choice becomes available at the end of each period in each market, to all the players in all markets. Conditional on the firm's effort/quality choices, signals are independently and identically distributed across markets and across periods. For concreteness and simplicity, we will focus on the case of linear normal information structure, that is, the public signal y is given by the firm's effort level plus a mean-zero normally distributed noise component. Many of our results can be extended to a general information structure, where the signal is drawn from a distribution function $F(y|e)$ with a positive density function $f(y|e)$ that satisfies the strict Monotone Likelihood Ratio Property in the sense of Milgrom (1981).

Specifically, for market $j \in \{1, 2, ..., n\}$, $y_j = e_j + \eta_j + \theta_j$, where η_j and θ_j are independent mean-zero normally-distributed noise terms with a variance of σ_η^2 and σ_θ^2, respectively. In this

formulation, η_j represents the noise in the production process ("production noise"), and θ_j represents the noise from the consumption process ("taste noise"). Then we can think of $q_j = e_j + \eta_j$ as the product quality, and $y_j = q_j + \theta_j$ as a noisy signal of q_j. Depending on the firm's effort choice, the customers' expected benefit is then $v_h = E[v(q_j)|e_j = e_h]$ or $v_l = E[v(q_j)|e_j = e_l]$. In the special case of no production noise (η_j is known to be zero), the firm chooses production quality directly ($e_j = q_j$), but customers only observe a noisy signal of product quality y_j. In the special case of no taste noise (θ_j is known to be zero), customers observe perfectly the product quality $y_i = q_i$, but not the firm's effort e_i. In many parts of our analysis, the distinction between production and taste noises is not important except for interpretations and applications. Then it will be convenient to define $\epsilon_j = \eta_j + \theta_j$ as the noise in the signal of the firm's effort, where ϵ_j is a mean-zero normally distributed random variable with a variance of $\sigma^2 = \sigma_\eta^2 + \sigma_\theta^2$. For ease of exposition, we will call e_j "effort" and y_j "signal", while keeping in mind that the model allows multiple interpretations. Also, the subscript j, denoting market, will be omitted when there is no risk of confusion.

Following Green and Porter (1984) and Fudenberg, Levine and Maskin (1994), we focus on perfect public (pure strategy) equilibria of the game. In a perfect public equilibrium, players' strategies depend only on the past realizations of the public signals. For periods $t = 2, 3, ...$, the public history H_t of the game is the sequence of the signal realizations $\{y_i\}_{i=1}^{t-1}$ and price $\{p_i\}_{i=1}^{t-1}$. The customers will base their period t decisions on (h_t, p_t). The firm's pricing decision in period t depends on h_t and its effort/quality decision in period t on (h_t, p_t).

We will characterize the perfect public equilibria of the game that yield the greatest average payoff for the firm, first for the non-integration case in which firms are independent, and then for the integration case, in which firms merge into one big firm. Since firms make decisions about integration or disintegration to maximize their value, by comparing the best equilibrium outcomes in the non-integration and integration cases, we derive conditions under which integration is better than non-integration or vice versa. We simply call a perfect public equilibrium that yields the greatest average payoff for the firm a "best equilibrium".

3 Best Equilibrium for the Non-Integration Case

We start with the non-integration case. In the non-integration case, firms are independent decision makers, so the public signal in one market will not affect the other markets at all, even if

it is observable to the participants in the other markets. Since firms and markets are symmetric, we focus on a representative firm and its market.

As is typical in repeated games, there can be many perfect public equilibria in our game, many of which can involve complicated path-dependent strategies. However, it turns out that the best equilibrium in our game has a very simple structure. Define a *cut-off trigger strategy equilibrium* as follows: the firm and its customers play (Buy, $High$) in the first period and continue to choose (Buy, $High$) as long as y stays above some threshold $\widetilde{y}$, and play the stage game Nash equilibrium ($Don't\ Buy$, Low) forever once y falls below the threshold $\widetilde{y}$. The following lemma, which is similar to Proposition 3 of Abreu, Pearce, Stacchetti (1987), shows that we can focus on this type of equilibrium without loss of generality.

Lemma 1 *A best equilibrium for the firm is either a cut-off trigger strategy equilibrium with $p = v_h$ in every period, or the repetition of the stage game Nash equilibrium ($Don't\ Buy$, Low).*

Proof: See the Appendix.

Note that in our game, the firm's pricing decision can be treated separately from its quality decision and customers' purchase decision. Since customers maximize their current period payoffs, they will purchase if and only if the price is not greater than their expected valuation (i.e., v_h or v_l, depending on their expectation of the firm's quality choice). Thus, by rational expectation, the firm will charge a price that equals customers' expected valuation given its quality choice. When the firm's reputation is good and is to provide high effort in the current period, it sets price $p = v_h$. If the firm loses its reputation and is to choose low effort forever, then its price has to be at least c_l to cover the cost. But when customers believe that it will choose low effort, the highest price acceptable to customers is v_l, which is not sufficient to cover c_l by our assumption. Therefore, whenever customers expect the firm to choose low effort, the equilibrium outcome is no trade and price is trivially indeterminate. Since the firm's optimal pricing decision is quite straightforward, we focus our analysis on its quality decisions and reputation building.

Let us fix some terminology and notation. We will sometimes call a stationary *cut-off trigger strategy equilibrium* "reputation equilibrium". The periods in which the firm's reputation is good and can thus earn quality premia are called the "reputation phase"; otherwise they are called the "punishment phase". Let π be the firm's expected profit averaged over all periods in the best equilibrium. Define $r = p - c_h \left(= v_h - c_h \right)$ to be the firm's current period payoff if it

exerts high effort ("honesty payoff"), and $d = c_h - c_l$ to be the cost differential of high and low efforts. If the firm chooses low effort, its current period payoff is $p - c_l = r + d$, so d is the firm's gain from deviation.

Let $\tilde{y}$ be the cut-off signal used in the equilibrium. Since the public signal is given by $y = e + \epsilon$, where $\epsilon \sim N(0, \sigma^2)$, the probability of reputation continuing conditional on effort e is $1 - F(\tilde{y}|e) = 1 - \Phi\left(\frac{\tilde{y}-e}{\sigma}\right)$. Then the firm's average payoff in the equilibrium, π, must satisfy the following value recursive equation:

$$\pi = (1-\delta)r + \delta(1 - F(\tilde{y}|e_h))\pi = (1-\delta)r + \delta\left(1 - \Phi\left(\frac{\tilde{y}-e_h}{\sigma}\right)\right)\pi \tag{1}$$

Equation 1 says that the firm's per period value in the equilibrium is the sum of its current period profit averaged out over time, $(1-\delta)r$, plus the expected average value from continuation, $\delta(1 - F(\tilde{y}|e_h))\pi$.

For the firm to be willing to choose e_h, the incentive compatibility constraint requires

$$\pi \geq (1-\delta)(r+d) + \delta(1 - F(\tilde{y}|e_l))\pi = (1-\delta)(r+d) + \delta\left(1 - \Phi\left(\frac{\tilde{y}-e_l}{\sigma}\right)\right)\pi \tag{2}$$

Any pair of $(\pi, \tilde{y})$ that satisfies both Equations (1) and (2) gives rise to an equilibrium in which the firm will choose high effort every period and customers continue to purchase as long as $y \geq \tilde{y}$.

Lemma 2 *The IC constraint of Equation (2) must be binding in the best reputation equilibrium.*

Proof: Consider any cut-off trigger strategy equilibrium with $(\pi, \tilde{y})$ such that Equation (2) holds as a strict inequality. Then we can decrease $\tilde{y}$ without affecting the IC constraint. However, as is clear from Equation (1), this reduces the value of $F(\tilde{y}|e_h)$, thus increases π. Contradiction. Q.E.D.

By Lemma 2, we can solve for the cut-off point $\tilde{y}$ and the firm's average profit π in the best reputation equilibrium (if it exists) from Equation (1) and Equation (2) as an equality. After some manipulation of terms we obtain

$$(1-\delta)d = \delta\left[F(\tilde{y}|e_l) - F(\tilde{y}|e_h)\right]\pi = \delta\left[\Phi\left(\frac{\tilde{y}-e_l}{\sigma}\right) - \Phi\left(\frac{\tilde{y}-e_h}{\sigma}\right)\right]\pi \tag{3}$$

This equation simply says that the current period gain from deviation averaged out over time (the LHS) equals the expected loss of future profit from deviation (the RHS).

It is convenient to focus on the normalized signal $k = \frac{y - e_h}{\sigma}$ instead of the signal y. Abusing notation slightly, we shall call k the public signal. Let $\tau = d/r$ be the ratio of the deviation gain to the honesty payoff, and $\triangle = e_h - e_l$ be the effort differential. Using Equation (1) to eliminate π from Equation (3), we obtain the following *"fundamental equation"*:

$$G(k) \equiv \frac{\Phi(k + \frac{\triangle}{\sigma}) - \Phi(k)}{\tau} - \Phi(k) = \frac{1 - \delta}{\delta} \tag{4}$$

If there is a solution $\tilde{k}$ to the fundamental equation (4), then from Equation (1) there exists a reputation equilibrium with the equilibrium payoff π given by

$$\pi = \frac{(1 - \delta)r}{1 - \delta[1 - \Phi(\tilde{k})]} \tag{5}$$

Clearly π is a decreasing function of $\tilde{k}$. Hence the smallest solution to Equation (4) constitutes the cut-off (normalized) signal in the best reputation equilibrium. Thus we have

Proposition 1 *There exists a reputation equilibrium if and only if the fundamental equation (4) has a solution. If that is the case, then the smallest solution is the cut-off point in the best equilibrium. The firm's value in the best equilibrium is given by (5).*

By Proposition 1 and Lemma 1, the existence of reputation equilibria hinges on whether Equation (4) has a solution. It is called the fundamental equation because its smallest solution determines the best cut-off point, which in turn determines the best equilibrium payoff for the firm through Equation (5). The characterization of the firm's average expected profit, or value, in Proposition 1 resembles that of Abreu, Milgrom and Pearce (1991), who study symmetric public strategy equilibria in repeated partnership games. It can be verified that Equation (5) is equivalent to

$$\pi = r - \frac{d}{\Phi(\tilde{k} + \frac{\triangle}{\sigma})/\Phi(\tilde{k}) - 1}$$

As in their model, here the firm's value equals its honesty payoff r minus an incentive cost (the second term of RHS) that depends on the deviation gain d and the likelihood ration $\Phi(\tilde{k} + \frac{\triangle}{\sigma})/\Phi(\tilde{k})$, which measures how easily the public signal can reveal deviations.

It can be verified that the $G(k)$ is maximized at

$$k^* = -\frac{\triangle}{2\sigma} - \frac{\sigma \ln(1+\tau)}{\triangle}$$

Let $\bar{\delta}$ be the discount factor to satisfy $\frac{1-\bar{\delta}}{\delta} = G(k^*)$. In the Appendix, we show that the function $G(k)$ has the shape as shown in Figure 1. Clearly Equation (4) has either no solution or two solutions, depending on whether δ is above or below $\bar{\delta}$.[14] When there are two solutions, the smaller solution $\tilde{k}$ is the cut-off point in the best equilibrium. Thus we have the following result.

Figure 1: Graphic Illustration of the Fundamental Equation
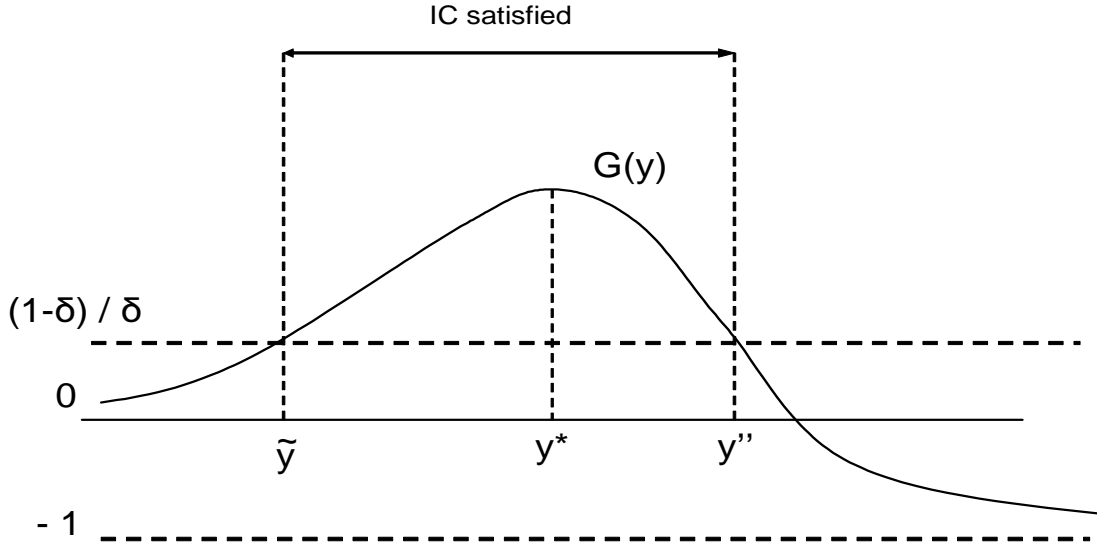
Proposition 2 *There exists a reputation equilibrium if and only if $\delta \geq \bar{\delta}$. When $\delta > \bar{\delta}$, the cut-off point $\tilde{k}$ for the best equilibrium is decreasing in δ and $\triangle$, and increasing in τ and σ; the firm's average payoff is increasing in r, δ and $\triangle$, and decreasing in d and σ.*

Proof: See the Appendix.

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[14]In a degenerate case, it has one solution for one particular $\bar{\delta}$.

Proposition 2 says that as long as the firm cares sufficiently about the future, reputation can be built in equilibrium in our model of imperfect monitoring. However, note that compared with the case of perfect monitoring (observable effort choices), reputation works less well in two aspects. First, reputation can break down with a positive probability (indeed almost surely in the long run) even on the equilibrium path as in Green and Porter (1984). This is necessary to give the firm an incentive to stick to good behavior. In the case of perfect monitoring, no actual punishment is incurred in motivating the firm to choose high effort, since any deviation is perfectly detected. Second, the requirement of a minimum discount factor to sustain reputation is greater in the case of imperfect monitoring than in the case of perfect monitoring. From Equation 4, it is easy to see that $(1 - \bar{\delta})/\bar{\delta} = G(k^*) < 1/\tau = r/d$. Hence, $\bar{\delta} > d/(d + r)$, which is the minimum discount factor required to sustain reputation in the case of perfect monitoring. This implies that for some range of δ, cooperation can be supported in equilibrium with perfect monitoring but not with imperfect monitoring.

Proposition 2 also establishes the comparative statics for the best equilibrium, which are all intuitive. It says that reputation will more likely be sustained if (i) the firm cares more about the future (greater δ); (ii) the public signal is more sensitive to the firm's effort choice (greater $\triangle$); (iii) the gain from deviation in relative terms is smaller (smaller τ), or (iv) the public signal is less noisy (smaller σ). Note that in Equation (5), a smaller cut-off point leads to a higher average payoff (with r kept constant). All comparative statics follow from this simple observation.

Renegotiation – Proof Equilibrium

So far we have assumed that the stage Nash equilibrium ($Don't\ Buy, Low$) is played forever in the punishment stage. However, both the firm and the customers have strong incentives to renegotiate and continue with their relationship even if the signal falls below the cut-off point (especially considering that the firm did not do anything wrong on the equilibrium path). In other words, the above equilibrium is not renegotiation-proof. Here we would like to note that the firm's minmax payoff 0 may be implemented in another equilibrium that is renegotiation-proof, in which the firm offers a large discount to the customers by drastically cutting its price.

Consider the following strategy. If the public signal falls below the cut-off point $\tilde{k}$, in the next period the firm offers customers a (very low) price p' and continues to provide high effort, and customers continue to buy from the firm. If the public signal in the punishment period is above another cut-off point k', then the firm is "redeemed," and can switch back to the reputation

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phase, charging $p = v_h$ in the next period. Otherwise, the firm stays in the punishment phase offering the low price p'.[15] Since the firm provides high effort in every period in both reputation and punishment phases, any such reputation equilibrium is efficient.

For this punishment scheme together and reputation phase described above to constitute an equilibrium, we need to have

$$(1 - \delta)(p' - c_h) + \delta \left(1 - \Phi(k')\right) \pi = 0 \tag{6}$$

where π is the firm's average value as characterized in Proposition 1. The first term, $(1-\delta)(p'-c_h)$, is the firm's (negative) profit per period in the punishment phase averaged over periods; while the second term is the discounted expected future profit if it redeems itself, which occurs with probability $1 - \Phi(k')$. Therefore this condition states that p' and k' should be chosen so that the firm's expected payoff is zero once it is in the punishment phase.

In addition, the firm must be willing to provide high effort in the punishment phase, which requires the following incentive constraint:

$$(1 - \delta)(p' - c_h) + \delta \left(1 - \Phi(k')\right) \pi \geq (1 - \delta)(p' - c_l) + \delta \left(1 - \Phi(k' + \frac{\triangle}{\sigma})\right) \pi$$

Or, recalling that $d = c_h - c_l$, we have

$$(1 - \delta)d \leq \delta \left(\Phi(k' + \frac{\triangle}{\sigma}) - \Phi(k')\right) \pi \tag{7}$$

This simply says that the gain from a one period deviation to low effort during the punishment phase is less than the loss of future profit from low effort which serves to reduce the probability of switching back to the high price and high profit of the reputation phase.

Note that the IC constraint of equation (7) is identical to that of the reputation phase, Equation (2) or (3). Thus, (7) is satisfied if and only if the cut-off point k' is not less than the smaller solution and not greater than the larger solution to the fundamental equation (4). For any such k', if there exists a price p' satisfying equation (6), then we have an efficient renegotiation-proof equilibrium

Two remarks are in order here. First, there may be multiple (k', p') that satisfy the above two conditions (6) and (7). Clearly, the larger k' is, the higher p' is. Second, to satisfy equation (6) may require a negative price p', which may not be feasible in many contexts. By Equation

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[15]This is similar to a "stick and carrot" equilibrium (Abreu, 1988).

(6), p' can be made as high as possible when k' is the largest among those which satisfy (7). The largest such k' is $k' = -\left(\widehat{k} + \frac{\triangle}{\sigma}\right)$, where $\widehat{k}$ is the cut-off point in the reputation phase, due to the symmetry of normal distribution. Thus, there exists $(k', p'(\geq 0))$ that satisfy (6) and (7) if and only if

$$-(1 - \delta)c_h + \delta\Phi(\widehat{k} + \frac{\triangle}{\sigma})\pi \leq 0 \qquad (8)$$

This inequality is satisfied when δ is large enough. To summarize, we have the following result.

Proposition 3 *There exist $k' \in \mathbb{R}$ and $p' \in \mathbb{R}^+$ that satisfy (6) and (7) if and only if (8) is satisfied. Moreover, (8) is satisfied when δ is sufficiently large.*[16]

Proof: See the Appendix.

This renegotiation-proof best reputation equilibrium exhibits a particular kind of price dynamic. The price dynamics of a best reputation equilibrium characterized in this section is shown in Figure 2 below. An example of such price dynamics is an airline company who just had a bad incident (e.g., a plane crash). Even if the incident can be purely bad luck, the company typically offers large discounts to "win back" customers; and such discounts are phased out over time as customers "regain" confidence in the company.

This is similar to the price dynamics of Green and Porter (1984), the first to construct public strategy equilibria with punishment phases in a model of imperfect monitoring. In their equilibrium construction of a repeated duopoly model with stochastic demand, firms continue to collude until the price drops below a threshold level, then they play the Nash Cournot equilibrium for a fixed number of periods before reverting back to the collusive phase. Our construction of the best equilibrium with efficient punishment phases differs from theirs because the firm and customers in our model can use prices to transfer utilities, achieving the efficient outcome at every history.

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[16]It can also be shown that if this "stick and carrot" strategy cannot implement the minmax payoff 0 for the firm, then there is no efficient renegotiation-proof equilibrium that can do so.

Figure 2: Price Dynamics
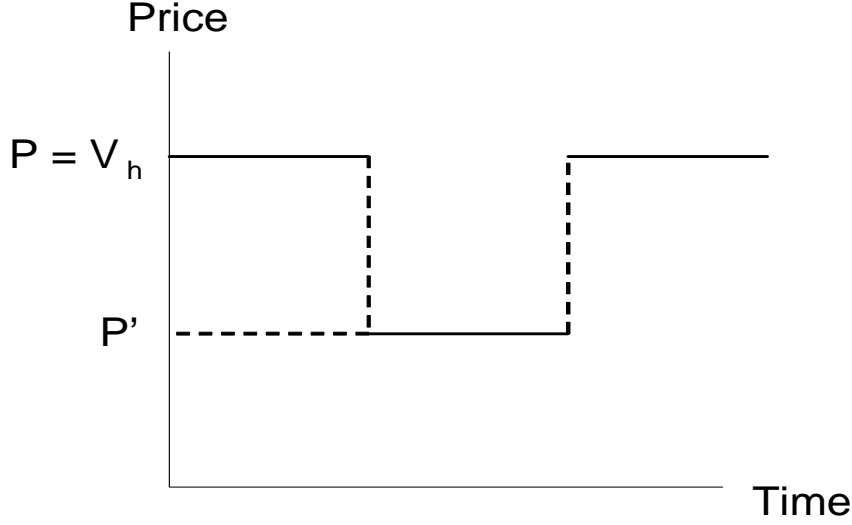
4 Best Equilibrium for the Integration Case

Now we analyze the integration case in which n firms merge into one big firm. To focus on the effects of integration on reputation building, we assume away economies or diseconomies of scale. We suppose that once integrated, the big firm makes price and quality decisions in all markets in a centralized way. That is, the big firm first chooses a price vector $(p_1, p_2, ..., p_n)$. Then we suppose that it adopts a common technology and chooses an effort vector of $(e_1, e_2, ..., e_n)$. This effort vector results in a quality vector of $(q_1, q_2, ..., q_n)$, where $q_i = e_i + \eta$ and η is a common production noise component whose distribution is determined by the firm's production technology. To facilitate comparison with the non-integration case, we suppose that η is a mean-zero normally distributed random variable with a variance of σ_η^2, exactly as in the non-integration case. If η's distribution in the integration case is different from η_i's in the non-integration case, then certain economies or diseconomies of scales are implied in the production process of the merged firm. Moreover, our analysis extends to the case in which q_i has some additional idiosyncratic shocks across markets (e.g., due to human errors), e.g., $q_i = e_i + \eta + \xi_i$,

where ξ_i are i.i.d. mean-zero random variables and $\eta + \xi_i$ has the same distribution as η_i. Our formulation seems to be consistent with the common practice of firms having centralized quality controls and resource allocations. For example, to protect firm image and maintain quality standards, franchised firms and chains typically have centralized supply systems and closely monitor branches and stores for quality controls. Similarly, large professional service firms typically have centralized human resource departments that oversee hiring in the whole firm to maintain quality standards of new employees.

After the production process, the merged firm can choose to ship its products to the n markets in whatever way it likes. While large firms allocate resources across markets and divisions all the time (e.g., hiring quality control personnel and sending them to individual branches, forming a team of consultants from different cities, or shipping products from centralized warehouses or production facilities to different markets), the firms' decision processes are usually not, at least not perfectly, observable to customers.[17] To capture this feature, we suppose that there are a large number of customers in each market and q_j is the average quality of products in market j. Then by allocating products across markets, the merged firm can achieve any profile of average qualities in the n markets, $(q_1', q_2', ..., q_n')$, as long as $\sum_1^n q_j' = \sum_1^n q_j$.

After customers in each market consume the products, a public signal $y_j = q_j' + \theta_j$ is generated. The noise term, θ_j, represents the taste shocks in different markets, and are assumed to be independent. We suppose that the signal profile $(y_1, y_2, ..., y_n)$ is observable to customers in all markets. Denote the aggregate public signal by $Y = \sum_1^n y_j$, and the average signal by $y_n = Y/n$, where the subscript n denotes the size of the firm. Then $y_n = \bar{e}_n + \eta + \bar{\theta}_n$, where $\bar{e}_n = \sum_1^n e_j/n$ is the average effort of the firm and $\bar{\theta}_n = \sum_1^n \theta_j/n$ is the average taste noise across the n markets. Note that since $e_j = e_h$ or e_l, the average effort of the firm $\bar{e}$ takes values of $e_h - \frac{j}{n}\triangle$, where $j = 0, 1, ..., n$ and $\triangle = e_h - e_l$. Also note that because of the independence of the θ_j's, $\bar{\theta}_n$ is a mean-zero normal random variable with a variance of σ_θ^2/n. We define the noise in the average signal as $\bar{\epsilon}_n = \eta + \bar{\theta}_n$, so $\bar{\epsilon}_n$ is a mean-zero normal random variable with a variance of $\bar{\sigma}_n^2 = \sigma_\eta^2 + \sigma_\theta^2/n$.

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[17]Because of reasons related to coordination, information and influence activities, it is difficult to isolate divisions or branches from the interventions of the headquarters or influences of other divisions, even if such independence is desirable. Milgrom and Roberts (1992, p568-576) discuss the advantages and disadvantages of horizontal integration, and present some interesting case studies such as "IBM and EDS" (page 576) that illustrate the difficulties of maintaining independence for divisions in multidivisional firms.

As in the non-integration case, we focus on the best (pure) perfect public equilibria for the integrated firm. In particular, we focus on symmetric equilibria where the merged firm chooses e_h and equalizes quality across all the markets ($q_i' = \frac{\sum_1^n q_j}{n}$, $i = 1, ..., n$). Later we show that it is indeed optimal to chooses e_h in all n markets instead of choosing e_h in only $l < n$ markets, thus the only assumption we make is equal quality across all markets. This is a reasonable assumption, and it seems unlikely that the firm can achieve greater expected payoff by generating unequal quality distribution across markets.[18]

Under integration, our model of moral hazard features multidimensional efforts and multiple signals. This leads to two particular complications in the integration case. First, firms can choose many possible configurations of efforts. Thus we need to deal with many incentive constraints. Second, customers in each market can base their purchase decisions on many possible configurations of signals. In general, it is much harder to characterize a range of signal profiles where customers should react when multiple signals are present. However, this n-dimensional problem can be reduced to a single dimensional problem in the following way. As we show below, the average signal $y_n = \frac{\sum_{j=1}^n y_j}{n}$ can serve as a sufficient statistic for disciplining the firm's behavior. We define a stationary *cut-off trigger strategy equilibrium* as before: an equilibrium in which (Buy, $High$) is played in every market while the average signal y_n stays above some threshold $\widetilde{y}_n$, and the play switches to the punishment phase in every market once y_n falls below the threshold $\widetilde{y}_n$. The following result greatly simplifies our analysis.

Proposition 4 *Within the class of perfect public equilibria described above (with equal effort and equal quality across markets), a best (non-trivial) equilibrium is a cut-off trigger strategy equilibrium with $p = v_h$ in all the markets, which can be characterized by a cutoff point $\widetilde{y}$ with respect to the average signal $\frac{\sum_{j=1}^n y_j}{n}$.*[19]

Proof: See the Appendix.

In the non-integration case, customers' purchase decisions and firms' effort decisions depend on the realization of the signal in their own market and are independent of those in other markets, even though they can observe signals in other markets. In the integration case, the decisions

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[18]That is, even though we do not prove it here, we conjecture that in the best equilibria for the firm, the firm should set $q_i' = \frac{\sum_1^n q_j}{n}$, $i = 1, ..., n$.

[19]The non-integration case (Lemma 1) is a special case ($n = 1$).

of customers in one market are related to the decisions of customers in other markets. Since the merged firm is a single decision-maker in all markets, the most effective way to maintain reputation is to use the harshest possible punishment when punishment is called for, which is to punish the firm simultaneously in all markets. Proposition 4 shows that in the best equilibrium, independent of the signal configuration in the n markets, this punishment is used only when the average/aggregate signal falls below a cut-off point. This is consistent with the observation that typically people pay attention to aggregated information about a big firm's overall performance (instead of disaggregated information about its performance measure in each market), such as its product quality ranking and rating of consumer satisfaction.

We now characterize the best reputation equilibrium in which the integrated firm chooses high effort in all n markets. For $j = 0, 1, ..., n$, denote $F_{nj} = F_n(y_n|\bar{e} = e_h - \frac{j}{n}\triangle)$ as the distribution function of y_n when the big firm chooses low effort in j of n markets. Its best payoff per period is given by the following value recursive equation

$$\Pi = (1 - \delta)\, nr + \delta(1 - F_{n0}(\tilde{y}_n))\Pi \qquad (9)$$

where $\tilde{y}_n$ is the cut-off point in the best equilibrium and $F_{n0} = \Pr\{y < \tilde{y}_n|\bar{e} = e_h\}$ is the probability of termination when the firm chooses high effort in all markets.

For the integrated firm serving n markets, it has n possible deviations by providing low effort in $m = 1, 2, ..., n$ of the n markets. The IC constraint associated with the mth deviation is

$$\Pi \geq (1 - \delta)(nr + md) + \delta(1 - F_{nm}(\tilde{y}_n))\Pi \qquad (10)$$

where $F_{nm} = \Pr\{y_n < \tilde{y}_n|\bar{e} = e_h - \frac{m}{n}\triangle\}$, the probability of the termination of the reputation phase when the firm chooses low effort in m of the n markets.

To facilitate comparisons, define $\pi_n = \Pi/n$ as the firm's value per market. Then Equations (9) and (10) can be rewritten as

$$\pi_n = (1 - \delta)\, r + \delta(1 - F_{n0}(\tilde{y}_n))\pi_n \qquad (11)$$

$$\pi_n \geq (1 - \delta)(r + \frac{m}{n}d) + \delta(1 - F_{nm}(\tilde{y}_n))\pi_n \qquad (12)$$

Any pair of $(\pi_n, \tilde{y}_n)$ that satisfies Equation (11) and all the IC constraints of Equation (12) gives rise to a reputation equilibrium in which the big firm chooses high effort in all markets

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and customers buy its products as long as the average signal y_n is above $\tilde{y}_n$. As before, the best equilibria feature the smallest cut-off point $\tilde{y}_n$ that satisfies Equation (11) and all the IC constraints of Equation (12). Similar to Lemma 2, it can be shown that one of the IC constraints must be binding at the smallest cut-off point. To solve for the smallest cut-off point $\tilde{y}_n$, we need to determine which IC constraint is binding.

Suppose the mth IC constraint is binding in the best equilibrium. Parallel to Proposition 1 and Equation (4), the cut-off point in the best equilibrium is the smaller solution to the following fundamental equation:

$$G_{n,m}\left(y\right) \equiv \frac{F_n\left(y|e_h - \frac{m}{n}\triangle\right) - F_n\left(y|e_h\right)}{\tau\frac{m}{n}} - F_n\left(y|e_h\right) = \frac{1-\delta}{\delta} \tag{13}$$

The effects of horizontal integration on reputation-building can be clearly seen from Equation (13). Observe that the denominator of the first expression on the left hand side is the ratio of deviation gain of m deviations to the total honesty payoff in n markets. For any fixed m, a larger n means that the punishment for deviations is relatively greater, which will tend to increase the left hand side of Equation (13). This size effect of horizontal integration helps reputation-building by lowering the equilibrium cut-off point $\tilde{y}_n$. On the other hand, a larger n reduces $F_n\left(y|e_h - \frac{m}{n}\triangle\right) - F_n\left(y|e_h\right)$, the numerator of the first expression on the left hand side of Equation (13), because deviations in a fixed number of markets are more difficult to detect when the total number of markets n is larger. This will tend to increase the equilibrium cut-off point $\tilde{y}_n$, thus making reputation-building less effective. Moreover, a larger n also means that the merged firm has more sophisticated deviations to contemplate, that is, the number of IC constraints grows with n.

Let's call (12) the $1-$market deviation constraint when $m = 1$.[20] With our information structure $y_n = \bar{e}_n + \bar{\epsilon}_n$, where $\bar{\epsilon}_n$ follows $N(0, \bar{\sigma}_\epsilon^2)$, we have the following result.

Proposition 5 *Suppose $0.5\triangle^2/\sigma_\eta^2 \leq \ln(1+\tau)$. For any n, only the 1-market deviation constraint is binding in the best equilibrium for the firm.*

Proof: See the Appendix.

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[20]Note that since the firm allocates products across markets evenly, product quality in every market becomes lower if the firm deviates to low effort in one of the n markets.

Proposition 5 gives a sufficient condition under which the 1-market deviation IC constraint is the most difficult to satisfy and hence must be binding in the best equilibrium. Roughly speaking, the condition says that the signal is not very informative about the firm's effort. In such cases, smaller deviations are much more difficult to detect than are larger deviations, thus making the 1-market deviation the most demanding to satisfy. The condition in Proposition 5 is far from necessary, implying Proposition 5 holds much more generally. We can show that only the 1-market deviation constraint binds as long as the optimal cut-off point is reasonably small, which is necessarily the case, for example, when δ is large. In the rest of the paper, we assume that only the 1-market deviation is binding in the best equilibrium for any n.

By Proposition 5, the optimal cut-off point in the best equilibrium $\tilde{y}_n$ is the smaller solution to the following fundamental equation:

$$G_{n,1}\left(y\right) \equiv \frac{F_n\left(y|e_h - \frac{1}{n}\triangle\right) - F_n\left(y|e_h\right)}{\tau\frac{1}{n}} - F_n\left(y|e_h\right) = \frac{1-\delta}{\delta} \tag{14}$$

And the merged firm's value per market under integration is

$$\pi_n = \frac{(1-\delta)r}{1 - \delta[1 - F_n(\tilde{y}_n|e_h)]}$$

With the information structure $y_n = \bar{e}_n + \bar{\epsilon}_n$, the fundamental equation of (14) becomes

$$G_n\left(k\right) \equiv \frac{\Phi\left(k + \frac{\triangle}{n\bar{\sigma}_n}\right) - \Phi\left(k\right)}{\tau\frac{1}{n}} - \Phi\left(k\right) = \frac{1-\delta}{\delta} \tag{15}$$

where $k = \frac{y - e_h}{\bar{\sigma}_n}$. Note that when $n = 1$, $\bar{\sigma}_1 = \sigma$, and Equation (15) becomes Equation (4). Let $\tilde{k}_n(\delta, \tau, \triangle, \bar{\sigma}_n)$ be the smaller of the two solutions to Equation (15). It is easy to see that $\tilde{k}_n(\delta, \tau, \triangle, \bar{\sigma}_n)$ has all the properties of $\tilde{k}(\delta, \tau, \triangle, \sigma)$, the smaller of the two solutions to Equation (4). That is, $\tilde{k}_n(\delta, \tau, \triangle, \bar{\sigma}_n)$ is increasing in τ and $\bar{\sigma}_n$, and decreasing in δ and $\triangle$.

With this transformation of variables, the probability of reputation termination in the best equilibrium, $F_n(\tilde{y}_n|e_h)$, is simply $\Phi(\tilde{k}_n)$. In summary, the best equilibrium for the firm serving n markets can be characterized as follows.

Proposition 6 *There exists a reputation equilibrium for the firm serving n markets as long as $\delta \geq \bar{\delta}_n$ for some $\bar{\delta}_n$. The optimal cut-off point in the best equilibrium for the firm, $\tilde{k}_n$, is the smaller solution to the fundamental equation (15). The merged firm's value per market is*

$$\pi_n = \frac{(1-\delta)r}{1 - \delta[1 - \Phi(\tilde{k}_n)]}$$

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It is increasing in r, δ and $\triangle$, and decreasing in d and $\bar{\sigma}_n$.

From Proposition 6, the construction of the best equilibrium under integration parallels nicely with that under non-integration. We exploit this in the next section to investigate the optimal size of horizontal integration and to conduct comparative statics.

Finally, we assumed that the firm chooses high efforts in all n markets in the reputation phase. One remaining question is whether the firm can be better off in a reputation equilibrium in which it does not choose high effort in all n markets in the reputation phase. The following result shows that this is not the case.

Proposition 7 *If there is a reputation equilibrium in which the firm chooses high effort in $n_1 < n$ markets in the reputation phase, then there is another reputation equilibrium in which the firm chooses high effort in all n markets and achieves greater profit.*

Proof: See the Appendix.

The idea of Proposition 7 is that if customers anticipate that the firm does not choose high effort in all n markets, they are not willing to pay as high as v_h. Since the firm's profit margin is lower, it has smaller incentives to maintain reputation, thus requiring a higher cut-off point. Since the honesty payoff is lower and the probability of reputation termination is higher, the firm's value per market is lower when it does not choose high effort in all markets than when it does.

5   The Optimal Degree of Horizontal Integration

In the two preceding sections we derived the best equilibria under non-integration and integration (of n markets). We say that non-integration dominates integration if and only if an independent firm's value is greater than the value per market of the large firm serving n markets. By Equation (1), an independent firm's value under non-integration is

$$\pi = \frac{(1-\delta)r}{1-\delta[1-F(\tilde{y}|e_h)]} = \frac{(1-\delta)r}{1-\delta[1-\Phi(\tilde{k})]}$$

By Proposition 6, it is clear that the comparison of non-integration and integration depends on the probability of reputation termination in equilibrium under non-integration, $\Phi(\tilde{k})$, and under integration, $\Phi(\tilde{k}_n)$. Hence, for any given $n > 1$, non-integration dominates integration ($\pi \geq \pi_n$) if and only if $\tilde{k} \leq \tilde{k}_n$.

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Since Equation (4) is a special case of Equation (14), a more general question is: what is the optimal degree of horizontal integration? Or, in other words, what is the optimal size of the firm? Conceptually, the answer is straightforward. For all $n = 1, 2, ...$, let n^* be such that $\tilde{k}_n$ is smallest. Then the optimal size of the firm is simply n^*. If $n^* = 1$, then non-integration is optimal. If $n^* > 1$, then a firm serving n^* markets can best maintain reputation.

Proposition 8 *For any δ, $\tilde{k}_n$ is bounded from below.*

Proof: See the Appendix.

Proposition 8 says that the maximum profit level, r, possible under perfect monitoring, cannot be approximated even if the size of the firm is allowed to go to infinity. This is in sharp contrast with Matsushima (2001), who shows that as firm size increases, the best equilibrium outcome in his duopoly collusion model approaches perfect collusion. Our result differs from Matsushima because there exists common production noise, η, which does not vanish with information aggregation even when $n \to \infty$.[21]

In fact, we can prove a stronger result.

Proposition 9 *Non-integration is optimal when (i) σ_θ is sufficiently small; or (ii) δ is sufficiently close to one; or (iii) τ is sufficiently small.*

Proof: See the Appendix.

Proposition 9 gives several sufficient conditions under which non-integration is optimal ($n^* = 1$). In the first case when the idiosyncratic taste noise is not important, the information aggregation benefit from integration is gone, so non-integration is optimal. In the last two cases, reputation can be maintained quite effectively for firms of all sizes in the sense that the cut-off point can be set at a low level. In such cases, the marginal benefit of the size effect from having more severe punishments is less important. In addition, it is much more difficult to detect small deviations in larger firms. As a result, integration brings less benefits but more costs, thus it is dominated by non-integration.

Proposition 9 is in sharp contrast with Bernheim and Whinston (1990), Matsushima (2001), Andersson (2002), and Fishman and Rob (2002), all of which imply that the bigger, the better.

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[21]As long as there exists some common noise component, we can allow idiosyncratic production noise as well.

The optimal size of the firm can be bounded in our model because as the firm size increases, the positive size and information effects become less and less important, but the negative deviation effect becomes more and more significant (it is more demanding to "detect" one-market deviations for larger firms).

Proposition 10 *The optimal degree of integration n^* is non-increasing in δ.*

Proof: See the Appendix.

Proposition 10 shows that as the discount factor increases, the optimal size of the firm will decrease (at least weakly) and non-integration is more likely to dominate integration. The intuition behind this result is roughly as follows. As δ increases, the future payoffs are more important and hence punishments for deviations are larger. This implies that firms of all sizes can maintain reputation more effectively. That is, the equilibrium cut-off points to continue cooperative actions can be set at low levels. Relatively speaking, the positive size effect of integration is less important in the sense that the marginal benefits of increasing punishments for deviation through integration become smaller. On the other hand, since the equilibrium cut-off points are low, the negative deviation effect of integration becomes more important because low cut-off points make it more difficult to detect a small deviation of a large firm. These forces together imply that as δ increases, the optimal size of the firm will not be larger.

Proposition 11 *The optimal degree of integration n^* is non-decreasing in τ.*

Proof: See the Appendix.

Proposition 11 shows that as τ decreases, the optimal size of the firm will decrease (at least weakly) and non-integration is more likely to dominate integration. Since $\tau = d/r$, it means that a smaller deviation gain, d, or a greater honesty payoff, r, will favor smaller firms and non-integration. The intuition behind this result is similar to that of Proposition 10. A smaller τ means less incentive to deviate and thus smaller or independent firms can build reputation more effectively. Consequently, the marginal benefits of the size effect of integration become less important, while the negative deviation effect of integration is more severe. Therefore, the smaller is τ, the smaller is the optimal size of the firm.

Next we consider how the informativeness of a signal affects the optimal degree of integration. We say that the public signal is uniformly more informative if $\triangle$ is larger (keeping other parameters constant) or if both σ_θ and σ_η are smaller while their ratio is kept the same.

Proposition 12 *Suppose δ is sufficiently close to one. The optimal degree of integration, n^*, is non-increasing when the public signal becomes uniformly more informative.*

Proof: See the Appendix.

Proposition 12 shows that for large δ, as the public signal becomes more informative about the firm's effort/quality choices, the optimal size of the firm will decrease (at least weakly) and non-integration is more likely to dominate integration. The intuition behind this result is as follows. When δ is large, a firm of any size can maintain reputation quite effectively, that is, the optimal cut-off point can be set quite low. When the public signal becomes more informative, small firms benefit more than larger firms because a small deviation by a larger firm can be "detected" only slightly better with more informative signals. Thus, while more informative signals make firms of all sizes better, the negative deviation effect of integration makes smaller firms benefit more. Therefore, the more informative the public signal, the smaller the optimal size of the firm.

6 Conclusion

In this paper, we build a simple model of firm reputation in which customers can only imperfectly monitor firms' effort/quality choices, and then use the model to study the effects of horizontal integration on firm reputation. Our analysis leads to a reputation theory of the optimal size of the firm. The comparative statics results of the optimal size of the firm can shed new light on patterns of horizontal integration in the real world.

For concreteness, we focus on the linear normal information structure in our analysis. However, the qualitative results of the model should hold in more general information structures. In particular, Section 3 will be mostly unchanged in a general information structure. Proposition 4 holds for any information structure and Proposition 5 is true for linear models with more general distribution functions. When only the one-market deviation is binding, a comparison of firm size depends on the optimal cut-off points of the average signal that is necessary for the

firms of each size to sustain reputation. Under reasonable conditions, for smaller y, the n^* that maximizes the function $G_{n,1}(y)$ will be smaller. That is, it is more difficult to "detect" a one market deviation by a larger firm if the cut-off point is in the lower tail of the signal distribution. Given these conditions, results similar to Propositions 10 and 11 should hold assuming more general distribution functions.

This paper has focused on the moral hazard aspects of firm reputation. As is common in this type of model, the firm maintains good reputation on the equilibrium path until a bad realization of the public signal, from which point the firm enters the punishment phase in which either customers desert the firm or the firm pays large financial penalties. This kind of equilibrium behavior has some unattractive features. First, firm reputation is relatively constant and has no real dynamics. Second, the reversion from good reputation to punishment phases, which is necessary to provide incentives to maintain reputation, depends heavily on coordination of beliefs between the firm and its customers. In equilibrium, punishments are triggered purely by bad luck, not by bad behavior on the firm's part. In addition, when punishments take the form of a permanent end to the relationship, they are not renegotiation-proof.

To deal with the above shortcomings, we demonstrated that there exists an efficient renegotiation-proof equilibrium instead of Nash reversion. However, these issues may be addressed more suitably by introducing adverse selection into the model. Recent contributions by Mailath and Samuelson (2001), Horner (2002) and Tadelis (2002) have made important progress in that direction. Introducing adverse selection into our model may not only generate richer reputation dynamics and serve to relax belief coordination requirements, but also may address interesting questions such as: does larger firm size help good-type firms build reputation? Can good-type firms use size to separate themselves from bad types? These questions are left as topics for future research.

7 Appendix

Proof of Lemma 1:

First suppose that $(Don't\ Buy,\ Low)$ is played in the first period in the best equilibrium. Then it is optimal to play the same equilibrium from the second period on, because the continuation game is isomorphic to the original game. This implies that one best equilibrium is to play $(Don't\ Buy,\ Low)$ every period independent of history (with price being set high enough) and the best equilibrium payoff is 0. Note that the repetition of $(Don't\ Buy,\ Low)$ is the only equilibrium outcome (modulo different price dynamics) which achieves such equilibrium payoff.

Suppose that the best equilibrium achieves more than $(0,0)$. It is easy to show that neither $(Buy,\ Low)$ nor $(Don't\ Buy,\ High)$ is the first period outcome of the equilibrium which maximizes the firm's payoff. Therefore $(Buy,\ High)$ with $p \leq v_h$ should be the outcome of the first period of such equilibrium.

Now we show that such a best equilibrium for the firm must be a *cut-off trigger strategy equilibrium* with $p = v_h$.[22] Let $V^* > 0$ be the best equilibrium payoff for the firm, $p^* (\leq v_h)$ be the equilibrium first period price, and u^* be the mapping which maps each public signal y to the equilibrium continuation payoff $u^* (y) \in [0, V^*]$. Let U be the set of all measurable functions $u : \mathbb{R} \to [0, V^*]$. Then the following inequalities hold.

$$
\begin{aligned}
V^* \quad \leq \quad & \max_{u \in U} (1 - \delta) (p^* - c_h) + \delta E\left[u(y)\,|e_h\right] \\
& s.t.\ (1 - \delta) (p^* - c_h) + \delta E\left[u(y)\,|e_h\right] \geq (1 - \delta) (p^* - c_l) + \delta E\left[u(y)\,|e_l\right]
\end{aligned}
$$

where the first inequality comes from the fact that the true set of continuation equilibrium payoffs may not be able to take all the values between 0 and V^*.

However, it is not difficult to show that the (essentially unique) solution $\tilde{u} \in U$ for this optimization problem satisfies $\tilde{u}(y) = 0$ for $y \in (-\infty, \widetilde{y})$ and $\tilde{u}(y) = V^*$ for $y \in [\widetilde{y}, \infty)$ for some $\widetilde{y}$ by the MLRP. Then since both V^* and 0 are feasible equilibrium payoffs, the maximized value of this optimization problem can indeed be achieved as an equilibrium payoff by the following cut-off trigger strategy starting at State 1;

$State\ 1 \quad : \quad$ Play $High$ with $p = p^*$ and move to State 2 if and only if $y \in (-\infty, \widetilde{y})$

---

[22]The best equilibrium payoff exists because the equilibrium payof set is compact.

State 2   :   Play *Low* with $p > v_l$ and stay at state 2.

Since $\tilde{u} \in U$ is (essentially) the unique solution to the above optimization problem, the equilibrium continuation payoff function u^* to achieve V^* must be $\tilde{u}$ (almost everywhere). Finally, there is no restriction on p^* as long as $p^* \leq v_h$. Thus the optimal price level should be set $p^* = v_h$ in every period at State 1   *Q.E.D.*

Proof of Proposition 2: It is easy to check that $\lim_{k \to -\infty} G(k) = 0$ and $\lim_{k \to \infty} G(k) = -1$. Furthermore, $G(k)$ is unimodal (pseudoconcave). To see this, note that

$$G'(k) = \frac{\phi(k + \frac{\triangle}{\sigma}) - \phi(k)}{\tau} - \phi(k)$$

It is easy to check that $G'(k^*) = 0$ has a unique solution at

$$k^* = -\frac{\triangle}{2\sigma} - \frac{\sigma ln(1 + \tau)}{\triangle}$$

Since the normal distribution satisfies the strict monotone likelihood ration property (Milgrom, 1981; Riley, 1988), it must be that $G'(k) > 0$ for $k \in (-\infty, k^*)$ and $G'(k) < 0$ for $k \in (k^*, \infty)$. Hence k^* maximizes $G(k)$ and $G(k^*) > 0$. Thus, $G(k)$ is unimodal (pseudoconcave).

Since $(1 - \delta)/\delta$ is strictly decreasing in δ and goes to zero as δ goes to one, Equation (4) has a solution for all $\delta \geq \bar{\delta}$, where $\bar{\delta}$ satisfies $G(k^*) = (1 - \delta)/\delta$. For all $\delta > \bar{\delta}$, there are two solutions for Equation (4). By Proposition 1, the cut-off point in the best equilibrium corresponds to the smaller solution for Equation (4), and its equilibrium payoff is given by Equation (5). For $\delta > \bar{\delta}$, all the comparative statics results are verified immediately from Figure 1.   *Q.E.D.*

Proof of Proposition 3: We only show that (8) is satisfied when δ is sufficiently large. (8) is equal to

$$\frac{\delta \Phi(\widehat{k} + \frac{\triangle}{\sigma}) \pi}{1 - \delta} \leq c_h$$

First we can show that $\frac{\Phi(\widehat{k} + \frac{\triangle}{\sigma})}{1 - \delta} \to \tau$ as $\delta \to 1$ as follows:

$$\begin{aligned}
\lim_{\delta \to 1} \frac{\Phi(\widehat{k} + \frac{\triangle}{\sigma})}{1 - \delta} &= \lim_{\delta \to 1} -\phi(\widehat{k} + \frac{\triangle}{\sigma}) \cdot \frac{\partial \widehat{k}}{\partial \delta} \\
&= \lim_{\delta \to 1} \frac{\phi(\widehat{k} + \frac{\triangle}{\sigma})/\delta^2}{\frac{\phi(\widehat{k} + \frac{\triangle}{\sigma}) - \phi(\widehat{k})}{\tau} - \phi(\widehat{k})} \quad \text{(by Equation 4)} \\
&= \lim_{\delta \to 1} \frac{\phi(\widehat{k} + \frac{\triangle}{\sigma})/\phi(\widehat{k})}{\frac{\phi(\widehat{k} + \frac{\triangle}{\sigma})/\phi(\widehat{k}) - 1}{\tau} - 1}
\end{aligned}$$

Since $\widehat{k} \to -\infty$, thus $\phi(\widehat{k} + \frac{\triangle}{\sigma})/\phi(\widehat{k}) \to \infty$ as $\delta \to 1$, we have

$$\lim_{\delta \to 1} \frac{\Phi(\widehat{k} + \frac{\triangle}{\sigma})}{1 - \delta} \to \tau$$

Then, since $\pi \to r$ as $\delta \to 1$, we have

$$\lim_{\delta \to 1} \frac{\delta \Phi(\widehat{k} + \frac{\triangle}{\sigma})\pi}{1 - \delta} = d = c_h - c_l < c_h$$

$$Q.E.D.$$

Proof of Proposition 4: Take any nontrivial equilibrium in this class of equilibrium, in which e_h is chosen in all the markets (and price is below v_h). Throughout most of the proof, we assume that quality is always equally distributed $(\widetilde{z}_i = \overline{z})$. We will prove in the end that it is indeed the best response of the firm.

Let V^* be the best equilibrium payoff per market for the firm among such equilibria and u^* be the mapping which maps each public signal profile $y = (y_1, ...y_n)$ to the equilibrium continuation average payoff $u^*(y) \in [0, V^*]$. As in Lemma 1, V^* satisfies the following

$$
\begin{aligned}
V^* \quad \leq \quad & \max_{u \in U} (1 - \delta)(p - c_h) + \delta E[u(y)|e_0] \\
& s.t. (1 - \delta)(p - c_h) + \delta E[u(y)|e_0] \geq (1 - \delta)\left\{ p - \frac{(n - m)c_h + mc_l}{n} \right\} + \delta E[u(y)|e_m] \\
& \text{for all } e_m \text{ and } m.
\end{aligned}
$$

where e_m stands for an effort profile of $(n - m)$ e_h and m e_l. Note that p is an average price across the markets, but every price should be set to $p = v_h$ as it does not affect the optimization problem at all as in the non-integration case. Note also that different $m-$deviations correspond to different constraints because different signals may be interpreted in asymmetric ways (which turns out not to be the case).

Let λ^{e_m} be a multiplier for the incentive constraint with respect to e_m. The Kuhn-Tucker conditions for this optimization problem are

$$1 + \sum_{e_m} \lambda^{e_m} \left(1 - \frac{f(y|e_m)}{f(y|e_0)} \right) \quad > \quad 0 \Rightarrow u^*(y) = V^*$$

$$1 + \sum_{e_m} \lambda^{e_m} \left(1 - \frac{f(y|e_m)}{f(y|e_0)} \right) \quad < \quad 0 \Rightarrow u^*(y) = 0$$

$$1 + \sum_{e_m} \lambda^{e_m} \left(1 - \frac{f(y|e_m)}{f(y|e_0)} \right) \quad = \quad 0 \Rightarrow u^*(y) \in [0, V^*]$$

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This implies that the best equilibrium exhibits a bang-bang property as in Lemma 1. For the best equilibrium, the same best equilibrium is played after y is in some region and the stage game Nash equilibrium is played forever in every market when y is in the other region. Characterizing such regions in the n dimensional signal space can be complicated as we don't know which constraints are binding (which λ^{e_m} is 0). In the following we show that we can reduce this n dimensional problem to a 1 dimensional problem and obtain a simple characterization of such regions thanks to our assumptions of normality and linearity of signals.

Let $\bar{e}_m$ be the average effort level of e_m. Since quality is equally distributed across markets by assumption, we can derive $\frac{f(y|e_m)}{f(y|e_0)}$ as follows:

$$
\begin{aligned}
\frac{f(y|e_m)}{f(y|e_h)} &= \frac{\int \Pi_{i=1}^n \Pr(y_i|\bar{z}) \Pr(\bar{z}|e_m) d\bar{z}}{\int \Pi_{i=1}^n \Pr(y_i|\bar{z}) \Pr(\bar{z}|e_0) d\bar{z}} \\
&= \frac{\int \Pi_{i=1}^n \phi\left(\frac{y_i-\bar{z}}{\sigma_\varepsilon}\right) \phi\left(\frac{\bar{z}-\bar{e}_m}{\sigma_\eta}\right) d\bar{z}}{\int \Pi_{i=1}^n \phi\left(\frac{y_i-\bar{z}}{\sigma_\varepsilon}\right) \phi\left(\frac{\bar{z}-e_h}{\sigma_\eta}\right) d\bar{z}} \\
&= \frac{\int \exp\left(-\frac{1}{2}\sum_{i=1}^n \frac{(y_i-\bar{z})^2}{\sigma_\varepsilon^2}\right) \phi\left(\frac{\bar{z}-\bar{e}_m}{\sigma_\eta}\right) d\bar{z}}{\int \exp\left(-\frac{1}{2}\sum_{i=1}^n \frac{(y_i-\bar{z})^2}{\sigma_\varepsilon^2}\right) \phi\left(\frac{\bar{z}-e_h}{\sigma_\eta}\right) d\bar{z}} \\
&= \frac{\int \exp\left(\frac{\bar{z}\sum_{i=1}^n y_i - n\bar{z}^2}{\sigma_\varepsilon^2}\right) \phi\left(\frac{\bar{z}-\bar{e}_m}{\sigma_\eta}\right) d\bar{z}}{\int \exp\left(\frac{\bar{z}\sum_{i=1}^n y_i - n\bar{z}^2}{\sigma_\varepsilon^2}\right) \phi\left(\frac{\bar{z}-e_h}{\sigma_\eta}\right) d\bar{z}}
\end{aligned}
$$

Thus this ratio only depends on the sum of public signals $\sum_{i=1}^n y_i$. Let $\bar{y}$ be the average signal $\frac{\sum_{i=1}^n y_i}{n}$. Then we can think of $\bar{y}$ effectively as the only public signal, thus reducing n dimensional signals into a 1 dimensional signal.

Given an average effort level $\bar{e}$, $\bar{y}$ is a normally distributed random variable with mean $\bar{e}$ and variance $\sigma_\eta^2 + \frac{\sigma_\varepsilon^2}{n}$. Hence

$$
\frac{f(y|e_m)}{f(y|e_0)} = \frac{\phi\left(\frac{\bar{y}-\bar{e}_m}{\sqrt{\sigma_\eta^2+\frac{\sigma_\varepsilon^2}{n}}}\right)}{\phi\left(\frac{\bar{y}-e_h}{\sqrt{\sigma_\eta^2+\frac{\sigma_\varepsilon^2}{n}}}\right)}
$$

Note that this is a decreasing function of $\bar{y}$ for any $e_m \neq e_h$. Therefore, *whichever constraints are binding*, there should be a cutoff point $\tilde{y}$ which characterizes the optimal solution such that $u^*(y) = V^*$ for $\bar{y} > \tilde{y}$ and $u^*(y) = 0$ for $\bar{y} < \tilde{y}$.

Finally we need to check that the firm indeed has an incentive to distribute quality equally

in equilibrium, as we initially assumed. This is obvious because the firm's future payoff only depends on $\bar{y}$, which cannot be affected at all by the way to distribute $\sum_{i=1}^{n} z_i$ to $\widetilde{z}$. Therefore distributing quality equally $(\widetilde{z}_i = \frac{\sum_{i=1}^{n} z_i}{n})$ is indeed optimal for the firm. $\hfill Q.E.D.$

Proof of Proposition 5: For any n, let $k = \frac{y - e_h}{\bar{\sigma}_n}$. Let $\widetilde{k}_{nm}$ be the smaller solution to

$$G_{nm}(k) \equiv \frac{\Phi\left(k + \frac{m\triangle}{n\bar{\sigma}_n}\right) - \Phi(k)}{\tau \frac{m}{n}} - \Phi(k) = \frac{1 - \delta}{\delta} \tag{16}$$

Just as $G(k)$ in Proposition 2, $G_{nm}(k)$ is unimodal and is maximized at

$$k_{nm}^* = -\frac{1}{2}\frac{m\triangle}{n\bar{\sigma}_n} - \frac{n\bar{\sigma}_n}{m\triangle}\ln(1 + \tau\frac{m}{n})$$

Lemma 3 *Suppose* $\ln(1 + \tau) \geq 0.5\triangle^2/\sigma_\eta^2$. *Then for any n and for any $m \leq n$, $k_{nm}^* + \frac{m\triangle}{n\bar{\sigma}_n} \leq 0$.*

Proof: Let $x = m/n \in (0, 1]$. Define $\kappa_{nm}(x)$ as

$$\kappa_{nm}(x) = k_{nm}^* + \frac{m\triangle}{n\bar{\sigma}_n} = \frac{1}{2}\frac{x\triangle}{\bar{\sigma}_n} - \frac{\bar{\sigma}_n}{x\triangle}\ln(1 + x\tau)$$

Under the assumption that $\ln(1 + \tau) \geq 0.5\triangle^2/\sigma_\eta^2$, we have

$$\kappa_{nm}(x = 1) = \frac{1}{2}\frac{\triangle}{\bar{\sigma}_n} - \frac{\bar{\sigma}_n}{\triangle}\ln(1 + \tau) < 0$$

Also, as $x \to 0$,

$$\lim_{x \to 0} \kappa_{nm} \to -\lim_{x \to 0}\frac{\tau\bar{\sigma}_n}{\triangle(1 + x\tau)} = -\frac{\tau\bar{\sigma}_n}{\triangle} < 0$$

Note that

$$x^2\kappa_{nm}'(x) = \frac{1}{2}\frac{x^2\triangle}{\bar{\sigma}_n} + \frac{\bar{\sigma}_n}{\triangle}\ln(1 + x\tau) - \frac{\bar{\sigma}_n}{\triangle}\frac{\tau x}{1 + x\tau}$$

Let $\mu(x)$ be the RHS expression. Then $\mu(x = 0) = 0$. Moreover,

$$
\begin{aligned}
\mu'(x) &= \frac{x\triangle}{\bar{\sigma}_n} + \frac{\tau\bar{\sigma}_n}{\triangle(1 + x\tau)} - \frac{\tau\bar{\sigma}_n}{\triangle}\frac{1}{(1 + x\tau)^2} \\
&= \frac{x\triangle}{\bar{\sigma}_n} + \frac{\tau\bar{\sigma}_n}{\triangle(1 + x\tau)}(1 - \frac{1}{1 + x\tau}) > 0
\end{aligned}
$$

So, $\mu(x) > 0$ and hence $\kappa'_{nm}(x) > 0$ for all $x \in (0, 1]$. It follows that $\kappa_{nm}(x) < 0$ for all $x \in (0, 1]$. $\hfill Q.E.D.$

Since $\tilde{k}_{nm} \leq k^*_{nm}$, Lemma 3 implies that for any n and for any $m \leq n$, $\tilde{k}_{nm} + \frac{m\triangle}{n\bar{\sigma}_n} \leq 0$. Note that

$$
\begin{aligned}
\frac{\partial G_{n,m}}{\partial m} &= \frac{\frac{m\triangle}{n\bar{\sigma}_n}\phi\left(k + \frac{m\triangle}{n\bar{\sigma}_n}\right) - \Phi\left(k + \frac{m\triangle}{n\bar{\sigma}_n}\right) + \Phi(k)}{\tau\frac{m^2}{n}} \\
&= \frac{\triangle}{m\tau\bar{\sigma}_n}\left[\phi\left(k + \frac{m\triangle}{n\bar{\sigma}_n}\right) - \phi(\bar{k})\right]]]]
\end{aligned}
$$

where $\bar{k} \in (k, k + \frac{m\triangle}{n\bar{\sigma}_n})$. Since for any n and for any $m \leq n$, $\tilde{k}_{nm} + \frac{m\triangle}{n\bar{\sigma}_n} \leq 0$. This implies $\phi\left(k + \frac{m\triangle}{n\bar{\sigma}_n}\right) > \phi(\bar{k})$ in the relevant range, and thus $G_{nm}(k)$ is increasing in m. Therefore, only the 1-shot deviation constraint is binding in the best equilibria for any n. $\hfill Q.E.D.$

Proof of Proposition 7: Since we assume that quality is equally distributed, we only need to focus on the average signal as before. In the reputation equilibrium in which the firm chooses high effort in $n_1 < n$ markets, the maximum price it can charge in the reputation phase is $p_1 = [n_1 v_h + (n-n_1)v_l]/n < v_h$, and its "honesty" payoff is $r_1 = p_1 - [n_1 c_h + (n-n_1)c_l]/n < v_h - c_h = r$. In the reputation phase, the cut-off signal is $\tilde{y}_{n_1}$, and the probability of reputation termination is $\Phi(\tilde{k}_{n_1})$, where $\tilde{k}_{n_1} = [\tilde{y}_{n_1} - e_{n_1}]/\bar{\sigma}_n$ is the normalized cut-off point and $e_{n_1} = [n_1 e_h + (n-n_1)e_l]/n$. Let π_{n_1} be the firm's value per market. Similar to equations (11) and (12), the equilibrium conditions are, for all $m \leq n_1$

$$
\begin{aligned}
\pi_{n_1} &= (1-\delta)r_1 + \delta\left(1 - \Phi(\tilde{k}_{n_1})\right)\pi_{n_1} \\
\pi_{n_1} &\geq (1-\delta)(r_1 + \frac{m}{n}d) + \delta(1 - \Phi(\tilde{k}_{n_1} + \frac{m\triangle}{n\bar{\sigma}_n}))\pi_{n_1}
\end{aligned}
$$

These equilibrium conditions are exactly the same as those for the equilibria in which the firm chooses high effort in all n markets, except that $r_1 < r$ and there are only n_1 constraints. Exactly as in Proposition 5, only the one-shot IC constraint is binding in the equilibria in which the firm gets the highest payoff. It follows that $\tilde{k}_{n_1}$ is the smaller solution to the fundamental equation (15), except with a larger $\tau_1 = d/r_1$. Thus, $\tilde{k}_{n_1} > \tilde{k}_n$. From Proposition 6, since $r_1 < r$ and $\tilde{k}_{n_1} > \tilde{k}_n$, the firm's value per market, π_{n_1}, is smaller than π_n. $\hfill Q.E.D.$

Proof of Proposition 8: We ignore the issue of integer values and treat n as a continuous variable. Since $\bar{\sigma}_n^2 = \sigma_\eta^2 + \sigma_\theta^2/n$, it can be verified that

$$
\begin{aligned}
\frac{d\frac{1}{n\bar{\sigma}_n}}{dn} &= -\frac{1}{n^2\bar{\sigma}_n}\left(1 - \frac{0.5\sigma_\theta^2}{n\bar{\sigma}_n^2}\right) = -\frac{1}{n^2\bar{\sigma}_n} + \frac{0.5\sigma_\theta^2}{n^3\bar{\sigma}_n^3} \\
\frac{d^2\frac{1}{n\bar{\sigma}_n}}{dn^2} &= \frac{2}{n^3\bar{\sigma}_n} - \frac{2\sigma_\theta^2}{n^4\bar{\sigma}_n^3} + \frac{3\sigma_\theta^4}{4n^5\bar{\sigma}_n^5}
\end{aligned}
$$

When n goes to infinity, we have

$$
\begin{aligned}
\lim_{n\to\infty} G_n(k) &\longrightarrow \lim_{n\to\infty} \frac{\phi\left(k + \frac{\triangle}{n\bar{\sigma}_n}\right) d\frac{\triangle}{n\bar{\sigma}_n}/dn}{-\tau\frac{1}{n^2}} - \Phi(k) \\
&= \frac{\triangle\phi(k)}{\tau}\lim_{n\to\infty}\left[\frac{1}{\bar{\sigma}_n} - \frac{0.5\sigma_\theta^2}{n\bar{\sigma}_n^3}\right] - \Phi(k) \\
&= \frac{\triangle\phi(k)}{\tau\sigma_\eta} - \Phi(k)
\end{aligned}
$$

Hence, when n goes to infinity, the fundamental equation (15) becomes

$$
\frac{\triangle\phi(k)}{\tau\sigma_\eta} - \Phi(k) = \frac{1-\delta}{\delta}
$$

The solution $\tilde{k}$ for this equation (if it exists) is clearly finite. Hence, $\lim_{n\to\infty}\tilde{k}_n > -\infty$.

$$Q.E.D.$$

Proof of Proposition 9: Differentiating the LHS of Equation (15) with respect to n gives

$$
\begin{aligned}
\tau\frac{\partial G_n(k)}{\partial n} &= \Phi\left(k + \frac{\triangle}{n\ \bar{\sigma}_n}\right) - \Phi(k) + n\phi\left(k + \frac{\triangle}{n\bar{\sigma}_n}\right)\frac{d\frac{\triangle}{n\bar{\sigma}_n}}{dn} \\
&= \Phi\left(k + \frac{\triangle}{n\ \bar{\sigma}_n}\right) - \Phi(k) - \phi\left(k + \frac{\triangle}{n\bar{\sigma}_n}\right)\frac{\triangle}{n\bar{\sigma}_n}\left(1 - \frac{0.5\sigma_\theta^2}{n\bar{\sigma}_n^2}\right)
\end{aligned}
$$

This can be rewritten as

$$
\frac{\tau n\bar{\sigma}_n^2}{\phi\left(k + \frac{\triangle}{n\bar{\sigma}_n}\right)\frac{\triangle}{n\bar{\sigma}_n}}\frac{\partial G_n(k)}{\partial n} = \triangle\bar{\sigma}_n\frac{\Phi\left(k + \frac{\triangle}{n\ \bar{\sigma}_n}\right) - \Phi(k) - \phi\left(k + \frac{\triangle}{n\bar{\sigma}_n}\right)\frac{\triangle}{n\bar{\sigma}_n}}{\phi\left(k + \frac{\triangle}{n\bar{\sigma}_n}\right)(\frac{\triangle}{n\bar{\sigma}_n})^2} + 0.5\sigma_\theta^2 \qquad (17)
$$

As $\sigma_\theta \to 0$, the last term goes to zero. Also, $\triangle \bar{\sigma}_n \geq \triangle \sigma_\eta > 0$ for all σ_θ and all n. Let $x = \frac{\triangle}{n\bar{\sigma}_n} \in (0, \frac{\triangle}{\sigma})$. We show that for all x (hence, for all n and all σ_θ), $\exists \xi < 0$ such that

$$\rho(k, x) = \frac{\Phi(k+x) - \Phi(k) - \phi(k+x)x}{\phi(k+x)x^2} < \xi < 0$$

Note first that $\rho(k, x) < 0$ for all $x > 0$, because the numerator equals $(\phi(\vec{k}) - \phi(k+x))x < 0$, where $\vec{k} \in (k, k+x)$ (assuming that all the cut-off points are small enough). As $x \to 0$, it can be verified that $\rho(k, x) \to k/2 < 0$. Furthermore, one can show that $\partial\rho/\partial x$ has the same sign as

$$-2[\Phi(k+x) - \Phi(k) - \phi(k+x)x] + x(k+x)[\Phi(k+x) - \Phi(k)]$$

This function takes a value of zero when $x = 0$ and has a derivative of $(k+x)[\Phi(k+x) - \Phi(k) - \phi(k+x)x] + x[\Phi(k+x) - \Phi(k)] > 0$. Thus, $\rho(k, x)$ is increasing in x. Let $\xi = \rho(\frac{\triangle}{\sigma}) < 0$. Then ρ is uniformly bounded above by some $\xi < 0$ (for any small enough k). Therefore, for each fixed k, $\frac{\partial G_n(k)}{\partial n} < 0$ as $\sigma_\theta \to 0$ for all n.

Let k_n^* be the solution of fundamental equation for $G_n(k) = \frac{1-\delta}{\delta}$. If σ_θ is small enough, $G_1(k_1^*) > G_n(k_1^*)$, therefore $k_1^* < k_n^*$ for $n = 2, 3.....$ It follows that non-integration is optimal.

Now consider the case of $k \to -\infty$. Note that as $k \to -\infty$, for all $x \in (0, \frac{\triangle}{\sigma})$,

$$\begin{aligned} \lim_{k\to\infty} \rho(k, x) &= \lim_{k\to\infty} \frac{\Phi(k+x) - \Phi(k)}{\phi(k+x)x^2} - \frac{1}{x} \\ &= \lim_{k\to\infty} \frac{1 - \exp(kx + 0.5x^2)}{-(k+x)x^2} - \frac{1}{x} \\ &= -\frac{1}{x} \end{aligned}$$

Hence, for any n and for a sufficiently small $\underline{k}_n$, since $x = \frac{\triangle}{n\bar{\sigma}_n}$, the RHS of Equation (17) goes to

$$-\triangle\bar{\sigma}_n \frac{1}{x} + 0.5\sigma_\theta^2 = -n\bar{\sigma}_n^2 + 0.5\sigma_\theta^2 < 0$$

Furthermore, it can be verified that $\partial\rho/\partial k$ has the same sign as

$$(k+x)[\Phi(k+x) - \Phi(k)] + \phi(k+x) - \phi(k)$$

This function goes to zero as $k \to -\infty$, and has a derivative of $\Phi(k+x) - \Phi(k) - x\phi(k) > 0$. Thus, $\rho(k, x)$ is increasing in k.

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Take $\underline{k}_1$. We know that the RHS of Equation (17) is negative at $x = \frac{\triangle}{\bar{\sigma}}$ as long as $k \leq \underline{k}_1$. Since ρ is increasing in x, the RHS of Equation (17) must be negative for all $x < \frac{\triangle}{\bar{\sigma}}$ when $k = \underline{k}_1$. Moreover, since ρ is increasing in k, this must be true for all x as long as $k \leq \underline{k}_1$.

Therefore, for sufficiently small k, it must be that G_n is decreasing in n so that $G_1(k) > G_n(k)$ for all $n > 1$. Since $\tilde{k}_n$ goes to $-\infty$ for all n when either δ is sufficiently close to one or τ is sufficiently small, Non-integration is optimal ($n^* = 1$) as before in either of the cases. $Q.E.D.$

Proof of Proposition 10: We first prove a very useful lemma. Define $G_0(k; t, x) = \frac{\Phi(k+x) - \Phi(k)}{t} - \Phi(k)$.

Lemma 4 Let $0 < t' < t$ and $0 < x' < x$. There exists a $\hat{k}$ such that $G_0(k; t', x') \leq G_0(k; t, x)$ if and only if $k \leq \hat{k}$.

Proof of Lemma 4: Define $w(k; t, x, t', x') = G_0(k; t, x) - G_0(k; t', x')$. Note that as $k \to -\infty$, both $G_0(k; t, x)$ and $G(k; t', x')$ go to zero; and as $k \to \infty$, both $G(k; t, x)$ and $G(k; t', x')$ go to -1. Thus, $w(k)$ goes to zero as $k \to -\infty$ and as $k \to \infty$.

Differentiating w gives

$$
\begin{aligned}
w'(k) &= \frac{\phi(k+x) - \phi(k)(1+t)}{t} - \frac{\phi(k+x') - \phi(k)(1+t')}{t'} \\
&= \frac{\phi(k)}{t}[e^{-\frac{x^2}{2}}e^{-kx} - 1 - \frac{t}{t'}e^{-\frac{(x')^2}{2}}e^{-kx'} + \frac{t}{t'}]
\end{aligned}
$$

Let us call the expression in the bracket above $\eta(k)$. Since $x > x'$, $\eta(k) \to \infty$ as $k \to -\infty$. When $k \to \infty$, $\eta(k) \to t/t' - 1 > 0$. Furthermore, we have

$$
\eta'(k) = -xe^{-\frac{x^2}{2}}e^{-kx} + x'\frac{t}{t'}e^{-\frac{(x')^2}{2}}e^{-kx'} = e^{-kx}\left[-xe^{-\frac{x^2}{2}} + x'\frac{t}{t'}e^{-\frac{(x')^2}{2}}e^{k(x-x')}\right]
$$

Setting $\eta'(k) = 0$ we obtain

$$
\bar{k} = \frac{ln(xt') - ln(x't) - 0.5x^2 + 0.5(x')^2}{x - x'}
$$

It is clear that $\eta'(k) < 0$ for $k < \bar{k}$ and $\eta'(k) > 0$ for $k > \bar{k}$. So $\eta(k)$ reaches its minimum at $\bar{k}$. If $\eta(\bar{k}) \geq 0$, then $\eta(k) > 0$ for all $k \neq \bar{k}$. This implies that $w'(k) = \phi(k)\eta(k)/t$, $w'(k) > 0$

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for all $k \neq \bar{k}$ and $w'(k) = 0$ at $k = \bar{k}$. Thus $w(k)$ is always strictly increasing except at $\bar{k}$ as a reflection point. But this contradicts the fact that $w(k)$ goes to zero as $k \to -\infty$ and as $k \to \infty$. Therefore, it must be that $\eta(\bar{k}) < 0$.

Since $\eta(-\infty) = \infty$ and $\eta(\infty) = t/t' - 1 > 0$, there exist k_1 and k_2, $k_1 < \bar{k} < k_2$, such that $\eta(k) = 0$. So $\eta(k) > 0$ for $k \in (-\infty, k_1) \cup (k_2, \infty)$ and $\eta(k) < 0$ for $k \in (k_1, k_2)$. Since the sign of $w'(k)$ is identical to that of $\eta(k)$, $w(k)$ is strictly increasing for $k \in (-\infty, k_1) \cup (k_2, \infty)$ and strictly decreasing $k \in (k_1, k_2)$. Therefore, there must exist a unique $\hat{k} \in (k_1, k_2)$ such that $w(k) = 0$. Moreover, $w(k) > 0$ for $k < \hat{k}$ and $w(k) < 0$ for $k > \hat{k}$. $\hspace{2cm} Q.E.D.$

Consider any $n < n'$. Let $t = \tau/n$, $x = \triangle/(n\bar{\sigma}_n)$, and $t' = \tau/n'$, $x' = \triangle/(n'\bar{\sigma}_{n'})$. Then, $t > t'$ and $x > x'$. Since $G_n(k) = G_0(k; t, x)$ and $G_{n'}(k) = G_0(k; t', x')$, by Lemma 4, there exists a $\hat{k}_{nn'}$ such that $G_n(k) > G_{n'}(k)$ if and only if $k < \hat{k}_{nn'}$.

Now suppose for some δ, the optimal degree of integration is $n^*(\delta)$. This means that at $\tilde{k}_{n^*(\delta)}$, $G_{n^*(\delta)} \geq G_{n'}$ for all n'. Therefore, for $n' > n^*(\delta)$, it must be that $\tilde{k}_{n^*(\delta)} < \hat{k}_{n^*(\delta)n'}$; and for $n' < n^*(\delta)$, it must be that $\tilde{k}_{n^*(\delta)} > \hat{k}_{n^*(\delta)n'}$.

Consider an increase in δ to $\delta_1 > \delta$. For any n, $\tilde{k}_n$ decreases in δ. In particular, $\tilde{k}_{n^*(\delta)}(\delta_1)$ is smaller than $\tilde{k}_{n^*(\delta)}(\delta)$. Note that δ does not effect $\hat{k}_{nn'}$ at all. Hence, for $n' > n^*(\delta)$, we have $\tilde{k}_{n^*(\delta)}(\delta_1) < \hat{k}_{n^*(\delta)n'}$, so $G_{n^*(\delta)} \geq G_{n'}$ at $\tilde{k}_{n^*(\delta)}(\delta_1)$. Therefore, the optimal degree of integration at δ_1, $n^*(\delta_1)$, cannot be greater than $n^*(\delta)$. For all $n' < n^*(\delta)$, if $\tilde{k}_{n^*(\delta_1)} > \hat{k}_{n^*(\delta)n'}$, then we must have $n^*(\delta_1) = n^*(\delta)$. Otherwise, if $\tilde{k}_{n^*(\delta_1)} < \hat{k}_{n^*(\delta)n'}$ for some $n' < n^*(\delta)$, then $G_{n^*(\delta)} < G_{n'}$ at $\tilde{k}_{n^*(\delta_1)}$, which implies that $n^*(\delta_1)$ must be smaller than $n^*(\delta)$. $\hspace{2cm} Q.E.D.$

Proof of Proposition 11: The proof is similar to that of Proposition 10. Suppose for some τ, the optimal degree of integration is $n^*(\tau)$. Consider a decrease in τ to $\tau_1 < \tau$. Since G_n is decreasing in τ, then $\tilde{k}_n$ is increasing in τ. Hence, $\tilde{k}_{n^*(\tau)}(\tau_1)$ is smaller than $\tilde{k}_{n^*(\tau)}(\tau)$. From the proof of Lemma 4, for any n and n', $\hat{k}_{nn'}$ depends on the ratio of t/t' but not on t nor t'. Since $t/t' = n/n'$ is independent of τ, for any n and n', $\hat{k}_{nn'}$ is independent of τ. That is, the relative positions of G_n are independent of τ. Therefore, a decrease in τ is like an increase in δ. The same argument in the proof of Proposition 10 applies. $\hspace{2cm} Q.E.D.$

Proof of Proposition 12: We only need to consider an increase in $\triangle$. The argument is identical for a decrease in σ_θ and σ_η of the same proportions. Suppose for some $\triangle$, the optimal degree of integration is $n^*(\triangle)$. Consider an increase in $\triangle$ to $\triangle_1 > \triangle$. Since G_n is increasing in $\triangle$, $\tilde{k}_n$ is

decreasing in $\triangle$. Hence, $\tilde{k}_{n^*(\triangle)}(\triangle_1)$ is smaller than $\tilde{k}_{n^*(\triangle)}(\triangle)$. From the proofs of Propositions 10 and 11, it suffices to show that for any n and $n' > n$, $\hat{k}_{nn'}$ is nondecreasing in $\triangle$.

Since $G_n = G_{n'}$ at $\hat{k}_{nn'}$, $\hat{k}_{nn'}$ is the solution to $w(k) = 0$, where

$$w(k) = n \left[\Phi \left(k + \frac{\triangle}{n \ \bar{\sigma}_n} \right) - \Phi (k) \right] - n' \left[\Phi \left(k + \frac{\triangle}{n' \ \bar{\sigma}'_n} \right) - \Phi (k) \right]$$

Exactly as in Lemma 4 and Propositions 10, w crosses zero only once at $\hat{k}_{nn'}$ as it is decreasing. So to show $\hat{k}_{nn'}$ is nondecreasing in $\triangle$, we only need to show that $w(k)$ is increasing in $\triangle$ at $\hat{k}_{nn'}$, or $\partial w / \partial \triangle > 0$ at $\hat{k}_{nn'}$.

Since

$$\frac{\partial w}{\partial \triangle} = \frac{1}{\bar{\sigma}_n} \phi \left(k + \frac{\triangle}{n \ \bar{\sigma}_n} \right) - \frac{1}{\bar{\sigma}'_n} \phi \left(k + \frac{\triangle}{n' \ \bar{\sigma}'_n} \right)$$

$\partial w / \partial \triangle > 0$ if $R(n) = \frac{1}{\bar{\sigma}_n} \phi \left(k + \frac{\triangle}{n \ \bar{\sigma}_n} \right)$ is decreasing at $\hat{k}_{nn'}$. It can be verified that

$$\frac{\partial R}{\partial n} = \frac{\phi \left(k + \frac{\triangle}{n \ \bar{\sigma}_n} \right)}{n^2 \bar{\sigma}_n^3} \left[\triangle \left(k + \frac{\triangle}{n \ \bar{\sigma}_n} \right) \left(\frac{\sigma_\eta^2}{\bar{\sigma}_n} + \frac{0.5\sigma_\theta^2}{n\bar{\sigma}_n} \right) + 0.5\sigma_\theta^2 \right]$$

It is easy to see that $\frac{\partial R}{\partial n} < 0$ for all n if $k < -\frac{0.5\sigma_\theta^2}{\sigma_\eta \triangle} - \frac{\triangle}{\sigma}$. If $\hat{k}_{nn'} < -\frac{0.5\sigma_\theta^2}{\sigma_\eta \triangle} - \frac{\triangle}{\sigma}$, then the proposition holds. Suppose $\hat{k}_{nn'} \geq -\frac{0.5\sigma_\theta^2}{\sigma_\eta \triangle} - \frac{\triangle}{\sigma}$. For large δ, $\tilde{k}_{n^*(\triangle)}(\triangle_1)$ and $\tilde{k}_{n^*(\triangle)}(\triangle)$ are smaller than $-\frac{0.5\sigma_\theta^2}{\sigma_\eta \triangle} - \frac{\triangle}{\sigma}$. Since $\hat{k}_{nn'}$ is independent of δ, then for large δ, $\hat{k}_{nn'}$ is greater than $\tilde{k}_{n^*(\triangle)}(\triangle_1)$ and $\tilde{k}_{n^*(\triangle)}(\triangle)$, so the claim of the proposition is true. $\qquad Q.E.D.$

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