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## **The Value of Private Information**

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Abstract

## **The Value of Private Information**

We study the consumption-investment problem of a CRRA agent who possesses private information about the future prospects of a stock. We compute the value of the information to the agent by comparing the utility equivalent with and without the information of the agent. The value of private information to the agent depends linearly on the wealth of agents and decreases with both the propensity to intermediate consumption and the risk aversion. Agents with low coefficients of relative risk aversion value private information more highly. Consistent with the empirical literature, the optimal portfolio holdings of informed agents are correlated with expected returns on the risky asset.

# 1 Introduction

This paper studies the optimal investment and consumption problem of a risk averse agent with private information about a stock. Using the solution to this problem, we calculate the value of private information to that agent. Our analysis applies both to insider-managers, who could receive private signals about their companies in the course of their employment, and to outside agents who may obtain private signals through investment analysis, possibly at a cost.

Specifically, we consider a continuous time model where an informed agent possesses a constant relative risk aversion (CRRA) preference, that is, the informed agent has a power utility function over intermediate consumption and terminal wealth. We solve the optimization problem in closed form. By comparing the utility equivalent of the agent with and without the information, we compute the value of the private information to the agent.

The value of information is proportional to the agent's wealth under CRRA utility. Therefore, the more wealthy the agent, the more valuable is the information in dollar terms. This implies that inside information is more valuable to wealthy executives, as opposed to employees lower in the corporate hierarchy.

We find that the value of private information depends on the propensity for intermediate consumption. Informed agents with greater consumption propensities find private information to be less valuable. Further, for agents with a low elasticity of intertemporal substitution, the propensity to consume negatively influences their expected initial holding in the risky asset. An agent with low risk aversion, however, wishes to save for the future, and is expected to hold more stock initially to consume relatively more

in the future as the propensity to consume increases. In addition, we find that the value of private information is decreasing in the risk aversion of informed agents. Less risk averse agents take a more aggressive position in the stock, increasing the value of private information.

For the special case of log utility (where  $\gamma$ , the power of the CRRA utility function, equals unity), the consumption to wealth ratio for an informed equals that for an uninformed, so that information does not affect the intermediate consumption. In general, the ratio of the consumption-to-wealth ratio for the informed over the uninformed depends on  $\gamma$ . When  $\gamma > 1$ , increased precision of private information causes the informed to consume more (as a fraction of wealth) relative to the uninformed. When  $\gamma < 1$ , the informed are more patient and greater precision causes them consume less as a proportion of wealth (relative to the uninformed), in order to more fully exploit private information through time.

While we are able to calculate the optimal portfolio holdings of the agent as well as the value of private information in a fairly general setting which includes the possibility of intermediate consumption, we do not consider transaction costs such as taxes and bid-ask spreads. More importantly, we assume that the trading by the agent does not affect the market price.<sup>1</sup> This is justified if the trading volume by the informed agent is very small compared to the total trading volume.<sup>2</sup> If the full-fledged equilibrium could

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<sup>1</sup>This assumption is in the spirit of the analysis of Kahl, Liu, and Longstaff (2003), which explores the cost of lockup periods within a continuous time setting.

<sup>2</sup>There is evidence (Cornell and Sirri, 1992, Meulbroek, 1992, and Chakravarty and McConnell, 1999) which indicates that insider trades may not have as strong an effect on the market price as is suggested by strategic models of insider trading. While we are hesitant to use this literature as justification for our assumption that the informed agent is small enough to not affect the market price, the evidence does suggest that many insiders make trading decisions under the assumption that they are atomistic.

be solved, our calculation would correspond to the limiting case where the fraction of informed agents goes to zero, so that the agent becomes atomistic. Alternatively, the value calculated in our paper can be viewed as an upper bound to the value of private information.

Our analysis is related to the literature of investment analysts. There is a large market for financial advice, and investment analysts are widely followed by the popular press. Extensive empirical studies have been performed on the analyst industry.<sup>3</sup> The incentives of investment analysts to provide advice depend on the value of analyst advice to investors, because that value is linked to the compensation that analysts receive for providing the advice. Understanding of the value of private information and how that information alters portfolio holdings could potentially led further light on the analyst industry.

Our paper is also relevant to the extensive literature which focuses on the incentives of insiders to hold stock in their company.<sup>4</sup> One strand of this body of work links insider holdings to future price movements (e.g., Seyhun, 1986, 1991, Hadlock, 1998), and another considers the link between insider-manager ownership and measures of firm valuation (e.g., Jensen and Murphy, 1976, Demsetz and Lehn, 1985, Core and Guay, 1999, and McConnell and Servaes, 1990). However, a dynamic analysis of how a risk averse insider allocates wealth between different assets, including the equity of his own company, is an issue about which not much is known. Understanding this

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<sup>3</sup>A partial list of such studies is Barber and Loeffler (1993), Bhushan (1989), Dimson and Marsh (1984), Stickle (1995), Womack (1996), Barber, Lehavy, McNichols, and Trueman (2001), Jegadeesh, Kim, Krische, and Lee (2004), Jegadeesh and Kim (2003), Hong, Kubik, and Solomon (2000), Clement and Tse (2004), and Malloy (2004).

<sup>4</sup>From an empirical standpoint, Bhushan (1989) finds that in the cross-section, insiders own about 23% the outstanding stock of companies on average.

topic is important, because it also is key to comprehending the theoretical linkage between incentives and stock ownership. In particular, potentially shed richer insights could be shed on that linkage if a primary incentive to hold company's stock, namely, the possibility of superior information about the firm's cash flows, is understood in a theoretical setting.

Finally, our paper is related to the literature on under-diversification. Surprisingly, we find in a calibrated exercise that the holding in the individual stock may approach as high as 80% even for relatively low values of the precision. This offers a theoretical rationale for why corporate executives may not be as well-diversified as conventional theory would suggest. The paper is also related to the literature on what may appear to be excessive holdings in private investment (Moskowitz and Vissing-Jorgensen, 2002), familiar stocks (Huberman, 2001) and the literature on home bias (Brennan and Cao, 1997, Kang and Stulz, 1997). In each of these cases, strong information about a company or an asset class' performance prospects may cause portfolios to appear considerably underdiversified. The under-diversification, as we show, can be a rational response to superior (positive) information about assets' future prospects.<sup>5</sup> We also show that the change in the portfolio holdings are positively correlated with future expected returns on the risky asset. This evidence is consistent both with the literature that relates insider and institutional holdings to future returns.<sup>6</sup>

The value of private information, of course, has been studied extensively in earlier literature, but largely in the CARA (constant absolute risk aversion)-normal setting.

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<sup>5</sup>The finding of high holdings of an individual stock applies in the case of a positive signal. Even on average, however, short-selling constraints could impede a symmetric negative position (see, e.g., Hong and Stein, 2003, Ofek and Richardson, 2003, and Lamont and Jones, 2002). Hence, portfolios of agents with private investors may appear to be under-diversified in the cross-section.

<sup>6</sup>See Rozeff and Zaman (1988), Seyhun (1992), and Gompers and Metrick (2001).

Research on this topic dates from the seminal arguments of Hayek (1945) and Hirshleifer (1971). Work in this area has experienced tremendous growth since the development of the pathbreaking model of Grossman and Stiglitz (1980), which provides a tractable closed-form solution to the expected utility of informed agents, as well as the equilibrium proportion of agents that choose to become informed. This model has been extended to a number of interesting scenarios, for example, the buying and selling of information (Admati and Pfleiderer, 1987, 1990), multiple securities (Admati, 1985), diverse information (Verrecchia, 1982, Diamond and Verrecchia, 1981, Hellwig, 1980).<sup>7</sup>

The Grossman and Stiglitz (1980) model and its extensions have undoubtedly yielded numerous valuable insights. However, the specific combination of the utility function and the normal distribution that is imposed for tractability has restricted the generality of conclusions that can be drawn.<sup>8</sup> There have been numerical approximations to calculating the value of information for more general utility functions, but again, in a static model, and by using approximations to the equilibrium (Bernardo and Judd, 2000, Peress, 2003). More general preferences could potentially yield richer economic insights and also help relating the model to empirical quantities. For example, one feature of the basic Grossman and Stiglitz (1980) model is that the analysis is done in terms of price levels, not returns. Thus, the calculation of return moments is impeded, because returns are ratios of normally distributed variables. As an empirical matter, however, returns are the quantity of interest in a cross-sectional setting, and CRRA utility specifications allow the primitive to be returns rather than prices. Second, the Grossman

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<sup>7</sup>For an interaction between insider trading and corporate investment in the context of the Grossman and Stiglitz (1980) setting, see Leland (1992), Ausubel (1991), and Manove (1989).

<sup>8</sup>The models of the Kyle (1985) class (e.g., Admati and Pfleiderer, 1988, Foster and Viswanathan, 1996) have also adopted risk-neutrality as a special preference structure, and the versions with risk aversion (e.g., Subrahmanyam, 1991) have used the same CARA utility function as Grossman and Stiglitz (1980).



and Stiglitz (1980) setting allows for the calculation of dollar holdings, not percentage portfolio holdings, but again in comparing securities and agents, the proportional holdings are of greater interest.

Previous research has also examined dynamic extensions of the Grossman and Stiglitz model (Wang, 1994, Brown and Jennings, 1989, Hirshleifer, Subrahmanyam, and Titman, 1994, Vives, 1995). Other than the difference in the preference structure, another distinction between our paper and these other papers relates to the timing of information arrival. While the agent in our paper receives the signal at only the beginning of the period, those in these other papers receive signals at multiple dates.

The rest of the paper is organized as follows. Section 2 describes the basic model. Section 3 provides some graphical comparative statics. Section 4 considers the analytical solution for two special cases. Section 5 concludes.

## 2 The Model

We consider the consumption-investment problem of an agent who represents a firm's insider (possibly an executive in the company). The agent has a finite investment horizon  $T < \infty$ . We will assume that the agent has a constant relative risk aversion utility over the intermediate consumption and the final period wealth

$$U = E_0 \left[ \int_0^T \alpha^\gamma e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt + e^{-\rho T} \frac{W_T^{1-\gamma}}{1-\gamma} \right], \quad (1)$$

where  $c_t$  and  $W_t$  represent consumption and wealth, respectively, at time  $t$ . The parameter  $\gamma$  is a measure of risk aversion as well as an inverse measure of the elasticity of intertemporal substitution, while  $\alpha$  represents the agent's propensity to consume at intermediate time-points. We further assume that there are two risky assets: a market

and a stock. The agent allocates wealth across three assets: his own risky stock, the market portfolio, and a risky asset.

Since the insider is atomistic, he does not influence the market price. However, other agents trade the risky assets and determine prices. The market value of the market portfolio,  $P_t$ , follows the process

$$dP_t = P_t((r + \mu)dt + \sigma_B dB_t),$$

whereas that of the individual stock,  $S_t$  evolves according to

$$dS_t = S_t((r + \beta\mu)dt + \beta\sigma_B dB_t + \sigma_z dZ_t),$$

where  $r$  is the riskfree rate. We assume that  $B_t$  and  $Z_t$  are two independent standard Brownian motions. It can be seen that the diffusion processes for the stock and the market portfolio are correlated through the common term involving  $dB_t$ , and  $dZ_t$  represents stock-specific, or unique risk. It follows from the Brownian motion specification that the stock price at time  $T$  is

$$S_T = S_0 e^{(r + \beta\mu - \frac{1}{2}(\beta^2\sigma_B^2 + \sigma_z^2))T + \beta\sigma_B B_T + \sigma_z Z_T}.$$

We assume that the agent receives a private signal about the diffusion process  $Z_t$ . Specifically, the agent observes a signal  $L$  about  $Z_T$ ,

$$L = Z_T + \epsilon,$$

where  $\epsilon$  is a standard normal random variable with volatility  $\sigma_\epsilon$ .

We can use established mathematical techniques to define a Brownian motion that traces the evolution of the informed agent's expectation. Using the "enlargement of

filtration” method (see for example, Pikovsky and Karatzas, 1996), we know that

$$\hat{Z}_t = Z_t - \int_0^t \frac{(L - Z_s)}{(T - s) + \sigma_\epsilon^2} ds$$

is a standard Brownian motion in the information set of the agent. The original Brownian motion  $Z_t$  is an Ornstein-Uhlenbeck process in the information set of the agent,

$$dZ_t = \frac{(L - Z_t)}{T - t + \sigma_\epsilon^2} dt + d\hat{Z}_t.$$

Let us define a quantity we term the “spread” as  $\Lambda_t \equiv L - Z_t$ . This quantity satisfies

$$d\Lambda_t = \frac{-\Lambda_t}{T - t + \sigma_\epsilon^2} dt - d\hat{Z}_t.$$

Note that the spread has a mean of 0 and a time-varying mean reversion coefficient of  $\frac{1}{T-t+\sigma_\epsilon^2}$ . The mean reversion is highest at  $t = T$ .

The spread  $\Lambda_t$  can be expressed as a weighted average of past  $d\hat{Z}_t$  realizations,

$$\Lambda_t = \frac{T - t + \sigma_\epsilon^2}{T + \sigma_\epsilon^2} L - \int_0^t \frac{T - t + \sigma_\epsilon^2}{T - u + \sigma_\epsilon^2} d\hat{Z}_u.$$

For the agent, the evolution of the stock price given by

$$\frac{dS_t}{S_t} = \left( r + \beta\mu + \frac{\Lambda_t\sigma_z}{T - t + \sigma_\epsilon^2} \right) dt + \beta\sigma_B dB_t + \sigma_z d\hat{Z}_t. \quad (2)$$

It is clear from the above expression that the instantaneous expected return on the stock, conditional on the information signal, is directly related to  $\Lambda_t$ .

Since  $\Lambda_t$  is determined given the paths of  $dB_t$  and  $d\hat{Z}_t$  up to time  $t$ ,  $d\hat{Z}_t$  is determined by  $\frac{dS_t}{S_t}$  and  $\frac{dP_t}{P_t}$ :

$$d\hat{Z}_t = \frac{1}{\sigma_z} \left( \frac{dS_t}{S_t} - \beta \frac{dP_t}{P_t} \right) - \frac{\Lambda_t}{T - t + \sigma_\epsilon^2} dt.$$

We adopt the standard stochastic control approach to solve the asset allocation problem of the informed agent. Let  $\phi_t$  and  $\phi_t^m$  denote the time  $t$  proportional holdings in the stock and the market, respectively. We follow Merton (1971) in defining the indirect utility function  $J$  by

$$J(W, V, t) = \max_{c_t, \phi_t, \phi_t^m} E_t[U(W_T)]$$

The wealth dynamics are given by

$$\begin{aligned} dW_t &= W_t \left( r + \mu\phi_t^m + \left( \beta\mu + \frac{\Lambda_t\sigma_z}{T-t+\sigma_\epsilon} \right) \phi_t \right) dt - c_t dt \\ &\quad + W_t (\phi_t^m \sigma_B dB_t + \phi_t (\beta\sigma_B dB_t + \sigma_z d\hat{Z}_t)) \\ &= W_t \left( r + \mu(\phi_t^m + \beta\phi_t) + \frac{\Lambda_t\sigma_z}{T-t+\sigma_\epsilon} \phi_t \right) dt - c_t dt \\ &\quad + W_t ((\phi_t^m + \beta\phi_t)\sigma_B dB_t + \phi_t \sigma_z d\hat{Z}_t) \\ &= W_t \left( r + \mu\varphi_t^m + \frac{\Lambda_t}{T-t+\sigma_\epsilon} \varphi_t \right) dt - c_t dt + W_t (\varphi_t^m \sigma_B dB_t + \varphi_t d\hat{Z}_t), \end{aligned}$$

where  $\varphi_t^m = \phi_t^m + \beta\phi_t$  and  $\varphi_t = \sigma_z\phi_t$ . Note from above that the expected evolution of the wealth of the individual depends on the filtration parameter  $\Lambda$ , which represents the amount of information the agent has at any given time. This indicates that  $\Lambda$  is expected to play a key role in determining the individual's portfolio holdings, and it is to this issue we will now turn.

From the Hamilton-Jacobi-Bellman equation, we obtain the following:

$$\begin{aligned} \max_{c, \varphi^m, \varphi} \quad & \alpha^\gamma e^{-\rho t} \frac{c^{1-\gamma}}{1-\gamma} + \frac{\partial}{\partial t} J + W \left( \left( r + \mu\varphi^m + \varphi \frac{\Lambda}{T-t+\sigma_\epsilon^2} \right) - c \right) J_W \\ & + \frac{1}{2} (\varphi^{m2} \sigma_B^2 + \varphi^2) W^2 J_{WW} - \frac{\Lambda}{T-t+\sigma_\epsilon^2} J_\Lambda + \frac{1}{2} J_{\Lambda\Lambda} - W\varphi J_{W\Lambda} = 0, \end{aligned}$$

with the terminal condition

$$J(t, W_T, \Lambda) = e^{-\rho T} \frac{W_T^{1-\gamma}}{1-\gamma}.$$

We solve for the optimal portfolio strategy by conjecturing that the indirect utility function should have the form

$$J(t, W_t, \Lambda_t) = e^{-\rho t} \frac{W_t^{1-\gamma}}{1-\gamma} f^\gamma(t, \Lambda_t).$$

The first order condition for consumption  $c$  is given by

$$\alpha^\gamma c^{-\gamma} = W^{-\gamma} f^\gamma,$$

so that

$$c = \alpha \frac{W}{f}. \quad (3)$$

As can be seen from the above expression, the consumption of the agent is a known proportion of current wealth. We discuss the specific relation between consumption and the parameter  $\alpha$  a bit later in the paper.

The first order conditions for the portfolio weights are

$$\mu W J_W + \varphi^m \sigma_B^2 W^2 J_{WW} = 0; \quad (4)$$

$$\frac{\Lambda}{T-t+\sigma_\epsilon^2} W J_W + \varphi W^2 J_{WW} - W J_{W\Lambda} = 0. \quad (5)$$

This gives

$$\varphi_t^m = \frac{\mu}{\gamma \sigma_B^2}; \quad (6)$$

$$\varphi_t = \frac{\Lambda}{\gamma(T-t+\sigma_\epsilon^2)} - (\ln f)_\Lambda. \quad (7)$$

It can be seen from above that the optimal holding in the stock depends directly on the current  $\Lambda$ . The bigger is  $\Lambda$ , the greater is the value of information and the more aggressive is the position taken in the stock.

The appendix proves the following proposition.

**Proposition 1** *The function  $f$  in the  $J$  function is given by*

$$f(t, \Lambda; T) = \alpha \int_t^T e^{a(t;s,T) + \frac{1}{2}b(t;s,T)\Lambda_t^2} ds + e^{a(t;T,T) + \frac{1}{2}b(t;T,T)\Lambda_t^2}, \quad (8)$$

where  $a$  and  $b$  are given by

$$\begin{aligned} \gamma a(t; s, T) &= \left( -\rho(1 - \gamma) \left( r + \frac{\mu^2}{2\gamma\sigma_B^2} \right) \right) (s - t) + \frac{1}{2} \ln \left( \frac{T - t + \sigma_\epsilon^2}{T - s + \sigma_\epsilon^2} \right) \\ &\quad - \frac{1}{2} \gamma \ln \left( \frac{s - t}{\gamma(T - s + \sigma_\epsilon^2)} + 1 \right), \\ \gamma b(t; s, T) &= \frac{1 - \gamma}{(T - t + \sigma_\epsilon^2)} \frac{s - t}{[s - t + \gamma(T - s + \sigma_\epsilon^2)]}. \end{aligned}$$

The optimal portfolio weight is given by

$$\varphi_t^* = \left( \frac{1}{T - t + \gamma\sigma_\epsilon^2} + \frac{\alpha \int_t^T (b(t; T, T) - b(t; s, T)) e^{a(t;s,T) + \frac{1}{2}b(t;s,T)\Lambda_t^2} ds}{\alpha \int_t^T e^{a(t;s,T) + \frac{1}{2}b(t;s,T)\Lambda_t^2} ds + e^{a(t;T,T) + \frac{1}{2}b(t;T,T)\Lambda_t^2}} \right) \Lambda_t.$$

Note that the  $J$  function and the optimal portfolio weight do not depend on  $\beta$  because the effect of  $\beta$  can be undone taking an offsetting position in the market. Since the risk-free asset absorbs the residual holdings, the holdings of the individual stock are not sensitive to  $\beta$ .

We can also obtain an interpretation of the parameter  $\alpha$  in the utility function represented by (1). Specifically, note from (3) that

$$\frac{\partial c}{\partial W} = \frac{\alpha}{f}.$$

It can be seen from (8) that  $\alpha/f$  is increasing in  $\alpha$  (since  $a$  and  $b$  do not involve  $\alpha$ ), so that  $\alpha$  is a measure of the propensity to consume at intermediate time points. Later, we will see how changing  $\alpha$  influences the holdings of the informed agent.

Our next proposition is the following:

**Proposition 2** *1. The value of private information is positive.*

2. *Informed agents with  $\gamma < 1$  ( $\gamma > 1$ ) will consume less (more) than uninformed agents with the same values of  $\gamma$ .*

The first part of the proposition is intuitive. The second part formalizes the notion that an agent with  $\gamma < 1$  will quickly try to take advantage of the opportunity offered by inside information and invest more in the stock, and thus consume less. On the other hand, an agent with  $\gamma > 1$  is more conservative and is more likely to spread trading on inside information over the whole period; thus he will consume more. In the next section, we present some graphical comparative statics for the general case, and analyze two special cases in Section 4.

### 3 Comparative Statics

Figure 1 plots the consumption to wealth ratio (at time 0) of an informed agent relative to that of an uninformed one as a function of the risk aversion  $\gamma$ . The parameter values used are  $\alpha = 1$ ,  $\Lambda = 0.5$ , and  $\sigma_\epsilon^2 = 0.09$ . The figure confirms Proposition 2 by showing that the informed consume relatively less than the uninformed for small  $\gamma$ , while the reverse is true for large  $\gamma$ . These results can be explained by noting that the parameter  $\gamma$  in the utility function is inversely related to the elasticity of intertemporal substitution. For small  $\gamma$ , the agent has a stronger tendency to substitute intertemporally, and consequently the consumption-to-wealth ratio is low. The reverse is true for high  $\gamma$ .

Figure 2 presents the same quantity as in Figure 1 for as a function of the noisiness of private information ( $\sigma_\epsilon^2$ ). The figure shows that for log utility ( $\gamma = 1$ ), the consumption to wealth ratio for an informed equals that for an uninformed. In this

case, myopia dictates that the informed is only concerned about the one-step ahead investment opportunity. On the other hand, for  $\gamma > 1$ , the bigger the precision, the more the informed consume relative to the uninformed. In this case, the elasticity of intertemporal substitution is low, so that the informed wish to postpone less consumption into the future. For  $\gamma < 1$ , the informed are more patient and choose to consume less to exploit private information later on in the trading process.

We now show the time zero expectation of the holding in the stock for an informed agent (Figure 3). As can be seen, the bigger the risk aversion, the smaller is the holding. In addition, as time elapses, the holding in the risky stock decreases because there is less and less private information to exploit. The figure indicates that less risk averse insiders will choose to hold more stock in their companies. In Figure 4, the expected holding is plotted as a function of the propensity to consume,  $\alpha$ . For highly risk averse informed agents, the propensity to consume negatively influences their holding in the risky asset. In this case, the agent wishes to hold less stock at time 0 and consume more if  $\alpha$  is large. An agent who is less risk averse than log utility, however, has a greater tendency to postpone consumption for the future, and in this case holds more stock to consume relatively more in the future as  $\alpha$  increases. Note that an informed agent with low risk aversion ( $\gamma = 0.5$ ) initially chooses to invest more than 100% of his wealth in the stock. The notion is that the agent takes a more aggressive position to consume more in the future when the risk aversion is low.

In Figure 5, we present the holdings in the three assets, the stock, the market, and the risk free asset as a function of the noisiness parameter  $\sigma_\epsilon^2$ . We find that as the information becomes more imprecise, the holdings in the risk-free asset decrease, while the holdings in the other two asset categories increase. This finding is intuitive. What is



surprising is that the proportion allocated to the individual stock can approach as high as 80% even for moderate values of  $\sigma_\epsilon^2$ . The paper is thus related to the literature on investing in the familiar (Huberman, 2001) as well as that on home bias (Brennan and Cao, 1997, Kang and Stulz, 1997). In each of these cases, strong positive information about a company or an asset class' performance prospects may cause portfolios to appear considerably underdiversified. The under-diversification, as we show, can be a rational response to superior (positive) information about assets' future prospects.

Note that beyond a certain threshold for  $\sigma_\epsilon^2$ , the holding in the risky stock dips below the holding in both the market as well as the risk-free asset. Thus, insiders with highly imprecise information will place greater emphasis on diversification than on holdings in their own stock. A prediction of this part of the analysis is that for companies where good information is hard to come by, such as the high tech sector, will have better-diversified insiders.<sup>9</sup>

We present the ex post value of information (i.e., after realization of the private signal) in Figure 6 as a function of the time horizon. The parameter values used are the same as those used for Figure 1. As can be seen, the value of information first increases, and then decreases in the time horizon. The intuition is that increasing the time horizon has two effects: there are more opportunities to trade, but it is also more risky to hold a position in the stock. Hence, for small values of  $T$ , the former effect dominates, whereas for large values of  $T$ , the latter feature takes over. The figure also indicates that information is more valuable for less risk averse agents. This is because agents with low risk aversion are able to take a more aggressive position in the stock.

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<sup>9</sup>One way to obtain diversification of private wealth is to go public. Our analysis thus suggests that insiders in companies with highly uncertain cash flows will be more prone to indulge in IPOs.

Finally, in Figure 7, we plot the ex ante value of information (before realization of the signal) as a function of the consumption parameter  $\alpha$ , using the same parameters as those in Figure 4. As can be seen, the greater is the propensity to consume, the smaller is the value of information. In addition the value of information is greater for low risk aversion. The drop in the value of information as a function of  $\alpha$  is steeper for the low risk aversion ( $\gamma = 0.5$ ) case. In this case, the agent wishes to exploit private information by consuming relatively less and saving more for the future. A high  $\alpha$  shifts relatively more consumption early on in the trading process and thus sharply reduces the ex ante value of information. The basic notion is that agents who wish to consume more at intermediate time points find long-term information to be less valuable.

## 4 Special Cases

In this section, we consider two special cases that yield closed-form solutions. These help us build further intuition on the problem.

### 4.1 Logarithmic Utility

The expressions for the case of logarithmic utility ( $\gamma = 1$ ) can be further simplified, as shown in the appendix. In this case the investor's utility can be written as

$$\begin{aligned}
 U &= \lim_{\gamma \rightarrow 1} E_0 \left[ \int_0^T \alpha^\gamma e^{-\rho t} \frac{c_t^{1-\gamma} - 1}{1-\gamma} dt + e^{-\rho T} \frac{W_T^{1-\gamma} - 1}{1-\gamma} \right] \\
 &= E_0 \left[ \int_0^T \alpha e^{-\rho t} \ln(c_t) dt + e^{-\rho T} \ln(W_T) \right]
 \end{aligned} \tag{9}$$

In this case the consumption to wealth ratio will not depend on the signal and will be given by

$$\frac{c}{W} = \frac{\alpha}{f_1}$$

where

$$f_1 = \frac{\alpha}{\rho} \left[ 1 + \left( \frac{\rho}{\alpha} - 1 \right) e^{-\rho(T-t)} \right]$$

From the first order conditions, the portfolio holdings are given by

$$\varphi_t^m = \frac{\mu}{\sigma_B^2}; \tag{10}$$

$$\varphi_t = \frac{\Lambda}{T - t + \sigma_\epsilon^2}. \tag{11}$$

The appendix also demonstrates that the value of information remains positive in this case as well.

As can be seen the consumption-to-wealth ratio is a non-stochastic function of the various parameters that do not involve the information signal. The myopic behavior implied by logarithmic utility dictates that the agent ignore the long-term value of the private signal in designing his optimal consumption policy. From (11), we see, however, that the holdings of the risky stock depends directly on  $\Lambda$ . From (2), therefore, the expected return on the stock is correlated with the informed agent's holding of the risky asset. This accords with the empirical literature (e.g., Seyhun, 1986, 1991, Hadlock, 1998), which indicates that insider holdings predict future stock returns.

## 4.2 The Case of No Intermediate Consumption

Consider the case where  $\alpha = 0$ . In this case the ratio of the utility equivalent of the informed agent to that of the uninformed is given by

$$R(t) = \frac{\left(\frac{T-t}{\sigma_\epsilon^2} + 1\right)^{\frac{1}{2(1-\gamma)}}}{\left(\frac{T-t}{\gamma\sigma_\epsilon^2} + 1\right)^{\frac{\gamma}{2(1-\gamma)}}} \exp\left(\frac{(T-t)\Lambda_t^2}{2(T-t+\sigma_\epsilon^2)(T-t+\gamma\sigma_\epsilon^2)}\right).$$

Note that  $R > 1$  even if  $\Lambda = 0$ . Knowing that  $Z_T$  will equal  $Z_t$  is still more valuable than knowing nothing. The increased value is due to trading before  $T$ . Even though  $\Lambda_t$  may be zero today, the future spread  $\Lambda$  may become non-zero and the informational advantage can thereby be exploited between  $t$  and  $T$ . The value of information depends positively on  $\Lambda_t^2$ , which is intuitive.

Using the explicit expression above, we can verify the following. In the case where there is no information, i.e.,  $\sigma_\epsilon \rightarrow \infty$ ,  $R(t) \rightarrow 1$ . When  $\sigma_\epsilon \rightarrow 0$ , so that the signal is completely precise,  $R(t) \rightarrow \infty$ . It is interesting to note that in this case, the optimal portfolio weight  $\phi_t^* = \frac{1}{\sigma_z(T-t)}$ , is independent of the risk aversion of the agent. Without the risk faced by trading on an imprecise information signal, agents of all risk aversion levels choose to hold the same fraction of the risky asset.

At time 0,  $R(t)$  becomes

$$R(0) = \frac{\left(\frac{1}{\sigma_\epsilon^2} + 1\right)^{\frac{1}{2(1-\gamma)}}}{\left(\frac{1}{\gamma\sigma_\epsilon^2} + 1\right)^{\frac{\gamma}{2(1-\gamma)}}} \exp\left(\frac{L^2}{2(1+\sigma_\epsilon^2)(1+\gamma\sigma_\epsilon^2)T}\right).$$

As can be seen, the ratio of the utility equivalents is related to the square of the signal. Also, since only a fraction  $\phi_0^*$  of the initial wealth is responsible for generating the differential in utility equivalents represented in  $R(0)$ , the net proportion increase in the

stock holding is

$$R_s(0) = (R(0) - 1)/\phi_0^* = \frac{\left(\frac{1}{\sigma_\epsilon^2} + 1\right)^{\frac{1}{2(1-\gamma)}}}{\left(\frac{1}{\gamma\sigma_\epsilon^2} + 1\right)^{\frac{\gamma}{2(1-\gamma)}}} \exp\left(\frac{L^2}{2(1 + \sigma_\epsilon^2)(1 + \gamma\sigma_\epsilon^2)T}\right) \frac{T(1 + \gamma\sigma_\epsilon^2)\sigma_z}{L}.$$

We now compute the ex-ante value of the information. The ratio of the utilities from being informed and uninformed is given by

$$R(t)^{1-\gamma} = \frac{\left(\frac{T-t}{\sigma_\epsilon^2} + 1\right)^{\frac{1}{2}}}{\left(\frac{T-t}{\gamma\sigma_\epsilon^2} + 1\right)^{\frac{\gamma}{2}}} \exp\left(\frac{(1-\gamma)(T-t)\Lambda_t^2}{2(T-t+\sigma_\epsilon^2)(T-t+\gamma\sigma_\epsilon^2)}\right).$$

Noting that at time  $t$ ,  $\Lambda_t$  has a mean of zero and a variance of  $T-t+\sigma_\epsilon^2$ ,

$$\begin{aligned} E[R(t)^{1-\gamma}] &= \int_{-\infty}^{\infty} \frac{\left(\frac{T-t+\sigma_\epsilon^2}{\sigma_\epsilon^2}\right)^{\frac{1}{2}}}{\left(\frac{T-t+\gamma\sigma_\epsilon^2}{\gamma\sigma_\epsilon^2}\right)^{\frac{\gamma}{2}} \sqrt{2\pi(T-t+\sigma_\epsilon^2)}} e^{\left(\frac{(1-\gamma)(T-t)\Lambda_t^2}{2(T-t+\sigma_\epsilon^2)(T-t+\gamma\sigma_\epsilon^2)} - \frac{\Lambda_t^2}{2(T-t+\sigma_\epsilon^2)}\right)} d\Lambda_t \\ &= \frac{(\gamma\sigma_\epsilon^2)^{\frac{\gamma}{2}}}{\sqrt{2\pi}\sigma_\epsilon(T-t+\gamma\sigma_\epsilon^2)^{\frac{\gamma}{2}}} \int_{-\infty}^{\infty} \exp\left(-\frac{\gamma}{2(T-t+\gamma\sigma_\epsilon^2)}\Lambda_t^2\right) d\Lambda_t \\ &= \left(\frac{T-t}{\gamma\sigma_\epsilon^2} + 1\right)^{\frac{1-\gamma}{2}} \end{aligned} \quad (12)$$

Hence the ex-ante value of information at any given time  $t$  is

$$R_v = \left(E[R(t)^{1-\gamma}]\right)^{\frac{1}{1-\gamma}} = \sqrt{1 + \frac{T-t}{\gamma\sigma_\epsilon^2}}. \quad (13)$$

As can be seen, the ex ante value of information is always greater than 1. At any time  $t$ , it is increasing in the ratio of the variance of the brownian motion of the stock return  $(T-t)$  over the variance of the signal  $(\sigma_\epsilon^2)$  and decreasing in the risk aversion. Even if the signal noise has greater variance than the underlying stock the additional information is still valuable, because the signal helps reduce uncertainty about the stock's terminal value.

## 5 Conclusion

In this paper, we attempt to answer the following questions: (i) How valuable is private information to a risk averse informed investor? (ii) What parameters govern the amount of stock the investor chooses to hold in a company vis-à-vis the market portfolio? (iii) How are the holdings of informed agents related to expected stock returns? (iv) How does the propensity for intermediate consumption between receipt of information and the date of information revelation affect the holdings of the informed agent as well as the value of private information? We address the preceding issues by solving the consumption-investment problem of an atomistic agent with CRRA preferences in a continuous time setting. This leads to an expression for the ex ante value of private information for such an agent. Such information is worth more, the greater is the wealth of agents, unlike in the case of CARA preferences, in which case, the value of information is independent of wealth. In addition, less risk averse agents take a more aggressive position in the stock for which they have private information, which also causes private information to be more valuable to such agents.

Since we explicitly model the propensity to consume at intermediate time points, we are able to examine how consumption alters the value of private information. Informed agents who have greater propensities to consume at intermediate times find long-term private information to be less valuable since they are less able to fully obtain the long-term benefits of trading on such information. We also examine how the propensity to consume interacts with the coefficient of relative risk aversion. An informed agent who is less risk averse than log utility wishes to save for the future, and is expected to hold more stock initially to consume relatively more in the future as the propensity

to consume increases. When risk aversion is high, however, the propensity to consume negatively influences the expected initial holding in the risky asset, because the need for intermediate consumption is dominant in this case.

When the CRRA parameter  $\gamma$  exceeds unity, the greater the precision of information, the more the informed consume relative to the uninformed. In this case, they consume more because of the lower elasticity of intertemporal substitution. For  $\gamma < 1$ , the informed are more patient and choose to consume less as a proportion of wealth (relative to the uninformed), in order to exploit private information later on. In the case of log utility, the consumption to wealth ratio for an informed equals that for an uninformed. In this case, myopia dictates that the informed is only concerned about the one-step ahead investment opportunity. We also find that agents with private information about an investment opportunity may appear to be substantially overinvested in that opportunity, which sheds light on the under-diversification phenomenon documented in various settings. Further, insider holdings in the risky asset are related to future expected returns on that stock, which is consistent with the analyses of Seyhun (1986, 1992), and Rozeff and Zaman (1988).

There are aspects of our analysis that could be extended to other settings. First, adapting our framework explicitly to multiple, correlated assets would be interesting and allow for predictions about insider holdings in related stocks, possibly those in the same industry. Second, while this is a difficult analytical issue, a solution to the full rational expectations setting where the insider is not atomistic remains elusive. A search for such a solution is clearly a predominant part of the agenda for future work on the subject.

## Appendix

**Proof of Proposition 1:** The HJB equation can be rewritten as

$$\begin{aligned} & \alpha f^{-1} - \rho + \gamma f^{-1} \frac{\partial}{\partial t} f + r(1 - \gamma) - \alpha(1 - \gamma) f^{-1} + \frac{1}{2}(1 - \gamma) \frac{\mu^2}{\gamma \sigma^2} \\ & + \frac{(1 - \gamma)\gamma}{2} \left( \frac{\Lambda}{\gamma(T - t + \sigma_\epsilon^2)} - (\ln f)_\Lambda \right)^2 - \frac{\Lambda}{T - t + \sigma_\epsilon^2} \gamma f^{-1} f_\Lambda \\ & + \frac{1}{2}(\gamma f^{-1} f_{\Lambda\Lambda} + \gamma(\gamma - 1) f^{-2} f_\Lambda^2) = 0, \end{aligned}$$

or

$$\begin{aligned} & \alpha\gamma - \rho f + \gamma \frac{\partial}{\partial t} f + r(1 - \gamma)f + \frac{1}{2}(1 - \gamma) \frac{\mu^2}{\gamma \sigma^2} f \\ & + \frac{1 - \gamma}{2\gamma} \left( \frac{\Lambda}{T - t + \sigma_\epsilon^2} \right)^2 f - (1 - \gamma) \left( \frac{\Lambda}{T - t + \sigma_\epsilon^2} \right) f_\Lambda \\ & - \frac{\Lambda}{T - t + \sigma_\epsilon^2} \gamma f_\Lambda + \frac{1}{2} \gamma f_{\Lambda\Lambda} = 0, \end{aligned}$$

$$\begin{aligned} & \alpha\gamma - \rho f + \gamma \frac{\partial}{\partial t} f + r(1 - \gamma)f + \frac{1}{2}(1 - \gamma) \frac{\mu^2}{\gamma \sigma^2} f \\ & + \frac{1 - \gamma}{2\gamma} \left( \frac{\Lambda}{T - t + \sigma_\epsilon^2} \right)^2 f - \left( \frac{\Lambda}{T - t + \sigma_\epsilon^2} \right) f_\Lambda + \frac{1}{2} \gamma f_{\Lambda\Lambda} = 0, \end{aligned}$$

The PDE can be written as

$$\alpha\gamma + \mathcal{L}f(t, \Lambda; T) = 0; \tag{14}$$

$$f(T, \Lambda; T) = 1, \tag{15}$$

where

$$\begin{aligned} \mathcal{L}f &= -\rho f + \gamma \frac{\partial}{\partial t} f + r(1 - \gamma)f + \frac{1}{2}(1 - \gamma) \frac{\mu^2}{\gamma \sigma^2} f \\ &+ \frac{1 - \gamma}{2\gamma} \left( \frac{\Lambda}{T - t + \sigma_\epsilon^2} \right)^2 f - \left( \frac{\Lambda}{T - t + \sigma_\epsilon^2} \right) f_\Lambda + \frac{1}{2} \gamma f_{\Lambda\Lambda}. \end{aligned} \tag{16}$$



**Proposition 3** Suppose that  $g(t, \Lambda; s, T)$  satisfies

$$\mathcal{L}g(t, \Lambda; s, T) = 0; \quad (17)$$

$$g(s, \Lambda; s, T) = 1, \quad (18)$$

then

$$f(t, \Lambda; T) = \alpha \int_t^T g(t, \Lambda; s, T) ds + g(t, \Lambda; T, T).$$

Proof. It is obvious that  $\alpha \int_T^T g(t, \Lambda; s, T) ds + g(T, \Lambda; T, T) = g(T, \Lambda; T) = 1$  so that the terminal condition is satisfied. Furthermore,

$$\begin{aligned} & \mathcal{L} \left( \alpha \int_t^T g(t, \Lambda; s, T) ds + g(t, \Lambda; T, T) \right) \\ &= -\alpha \gamma g(t, \Lambda; t, T) + \alpha \int_t^T \mathcal{L}g(t, \Lambda; s, T) ds + \mathcal{L}g(t, \Lambda; T, T) - \alpha \gamma, \end{aligned} \quad (19)$$

where the first term is from  $\gamma \frac{\partial}{\partial t}$  on the lower integration limit.

Now we need to solve the following PDE

$$\begin{aligned} & -\rho g(t, \Lambda; s, T) + \gamma \frac{\partial}{\partial t} g(t, \Lambda; s, T) + r(1 - \gamma)g(t, \Lambda; s, T) \\ & + \frac{1}{2}(1 - \gamma) \frac{\mu^2}{\gamma \sigma^2} g(t, \Lambda; s, T) + \frac{1 - \gamma}{2\gamma} \left( \frac{\Lambda}{T - t + \sigma_\epsilon^2} \right)^2 g(t, \Lambda; s, T) \\ & - \left( \frac{\Lambda}{T - t + \sigma_\epsilon^2} \right) g_\Lambda(t, \Lambda; s, T) + \frac{1}{2} \gamma g_{\Lambda\Lambda}(t, \Lambda; s, T) = 0; \\ & g(s, \Lambda; s, T) = 1. \end{aligned} \quad (20)$$

Let  $g(t, \Lambda; s, T) = e^{a(t; s, T) + \frac{1}{2}b(t; s, T)\Lambda_t^2}$ . This reduces to the following ODE

$$\begin{aligned} & -\rho + \gamma \frac{\partial}{\partial t} a + r(1 - \gamma) + \frac{1}{2}(1 - \gamma) \frac{\mu^2}{\gamma \sigma^2} + \frac{1}{2} \gamma b = 0; \\ & \gamma \frac{\partial}{\partial t} b + \frac{1 - \gamma}{\gamma} \left( \frac{1}{T - t + \sigma_\epsilon^2} \right)^2 - \frac{2b}{T - t + \sigma_\epsilon^2} + \gamma b^2 = 0; \\ & a(s; s, T) = 0; \\ & b(s; s, T) = 0. \end{aligned}$$

Let  $d = (T - t + \sigma_\epsilon^2)\gamma b$  and  $\tau = \ln(T - t + \sigma_\epsilon^2)$ . We have

$$-\frac{\partial}{\partial \tau}d + \frac{1-\gamma}{\gamma} + \left(1 - \frac{2}{\gamma}\right)d + \frac{d^2}{\gamma} = 0.$$

The solution is given by

$$\begin{aligned} \gamma a(t; s, T) &= \\ &\left(-\rho + (1-\gamma)\left(r + \frac{\mu^2}{2\gamma\sigma_B^2}\right)\right)(s-t) + \frac{1}{2}\ln\left(\frac{T-t+\sigma_\epsilon^2}{T-s+\sigma_\epsilon^2}\right) - \frac{1}{2}\gamma\ln\left(\frac{s-t}{\gamma(T-s+\sigma_\epsilon^2)} + 1\right); \\ \gamma b(t; s, T) &= \frac{1-\gamma}{(T-t+\sigma_\epsilon^2)} \frac{s-t}{[s-t+\gamma(T-s+\sigma_\epsilon^2)]}. \end{aligned}$$

The function  $f$  is given by

$$f(t, \Lambda; T) = \alpha \int_t^T e^{a(t;s,T) + \frac{1}{2}b(t;s,T)\Lambda_t^2} ds + e^{a(t;T,T) + \frac{1}{2}b(t;T,T)\Lambda_t^2}. \quad (21)$$

The optimal portfolio weight is given by

$$\begin{aligned} \varphi_t^* &= \left( \frac{1}{\gamma(T-t+\sigma_\epsilon^2)} - \frac{\alpha \int_t^T b(t; s, T) e^{a(t;s,T) + \frac{1}{2}b(t;s,T)\Lambda_t^2} ds + b(t; T, T) e^{a(t;T,T) + \frac{1}{2}b(t;T,T)\Lambda_t^2}}{\alpha \int_t^T e^{a(t;s,T) + \frac{1}{2}b(t;s,T)\Lambda_t^2} ds + e^{a(t;T,T) + \frac{1}{2}b(t;T,T)\Lambda_t^2}} \right) \Lambda_t \\ &= \left( \frac{1}{\gamma(T-t+\sigma_\epsilon^2)} - b(t; T, T) + \frac{\alpha \int_t^T (b(t; T, T) - b(t; s, T)) e^{a(t;s,T) + \frac{1}{2}b(t;s,T)\Lambda_t^2} ds}{\alpha \int_t^T e^{a(t;s,T) + \frac{1}{2}b(t;s,T)\Lambda_t^2} ds + e^{a(t;T,T) + \frac{1}{2}b(t;T,T)\Lambda_t^2}} \right) \Lambda_t \\ &= \left( \frac{1}{T-t+\gamma\sigma_\epsilon^2} + \frac{\alpha \int_t^T (b(t; T, T) - b(t; s, T)) e^{a(t;s,T) + \frac{1}{2}b(t;s,T)\Lambda_t^2} ds}{\alpha \int_t^T e^{a(t;s,T) + \frac{1}{2}b(t;s,T)\Lambda_t^2} ds + e^{a(t;T,T) + \frac{1}{2}b(t;T,T)\Lambda_t^2}} \right) \Lambda_t. \end{aligned}$$

□

**Proof of Proposition 2:** First, suppose that  $\gamma < 1$ , then, it can be easily proved that  $a(t) > 0$  and  $b(t) > 0$ .

$$f(t, \Lambda; T) = \alpha \int_t^T e^{a(t;s) + \frac{1}{2}b(t;s)\Lambda_t^2} ds + e^{a(t;T) + \frac{1}{2}b(t;T)\Lambda_t^2} > \alpha \int_t^T e^{a_0(t;s)} ds + e^{a_0(t;T)} = f_0(t; T).$$

Therefore,

$$R = \left( \frac{f(t, \Lambda; T)}{f_0(t; T)} \right)^{\frac{\gamma}{1-\gamma}} > 1.$$

Second, consider the case  $\gamma > 1$ , in this case, it can be easily proved that  $a(t) < 0$  and  $b(t) < 0$ .

$$f(t, \Lambda; T) = \alpha \int_t^T e^{a(t;s) + \frac{1}{2}b(t;s)\Lambda_t^2} ds + e^{a(t;T) + \frac{1}{2}b(t;T)\Lambda_t^2} < \alpha \int_t^T e^{a_0(t;s)} ds + e^{a_0(t;T)} = f_0(t; T).$$

Therefore,

$$R = \left( \frac{f(t, \Lambda; T)}{f_0(t; T)} \right)^{\frac{\gamma}{1-\gamma}} > 1.$$

For proving part 2, note that when  $\gamma < 1$ ,  $f(t, \Lambda; T) > f_0(t; T)$ . Therefore,  $\frac{\alpha W}{f} < \frac{\alpha W}{f_0}$ . When  $\gamma > 1$ ,  $f(t, \Lambda; T) < f_0(t; T)$ . Therefore,  $\frac{\alpha W}{f} > \frac{\alpha W}{f_0}$ . Thus the informed agent with  $\gamma < 1$  will consume a greater fraction of his wealth than the agent with  $\gamma > 1$ .  $\square$

**The Case of Logarithmic Utility:** In the case of  $\gamma = 1$ , we have logarithmic utility, and the utility function can be written as

$$\begin{aligned} U &= \lim_{\gamma \rightarrow 1} \mathbb{E}_0 \left[ \int_0^T \alpha^\gamma e^{-\rho t} \frac{c_t^{1-\gamma} - 1}{1-\gamma} dt + e^{-\rho T} \frac{W_T^{1-\gamma} - 1}{1-\gamma} \right] = \mathbb{E}_0 \left[ \int_0^T \alpha e^{-\rho t} \ln c_t dt + e^{-\rho T} \ln W_T \right] \\ &= \lim_{\gamma \rightarrow 1} \mathbb{E}_0 \left[ \int_0^T \alpha^\gamma e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} dt + e^{-\rho T} \frac{W_T^{1-\gamma}}{1-\gamma} \right] - \int_0^T \frac{\alpha^\gamma e^{-\rho t}}{1-\gamma} dt - \frac{e^{-\rho T}}{1-\gamma}. \end{aligned}$$

So

$$\begin{aligned} J &= \lim_{\gamma \rightarrow 1} \frac{W_t^{1-\gamma}}{1-\gamma} \left( \int_t^T \alpha e^{a(t;s) + \frac{1}{2}b(t;s)\Lambda_t^2} ds + e^{a(t;T) + \frac{1}{2}b(t;T)\Lambda_t^2} \right)^\gamma - \int_t^T \frac{\alpha^\gamma e^{-\rho(s-t)}}{1-\gamma} ds - \frac{e^{-\rho(T-t)}}{1-\gamma} \\ &= g(t) \ln W + h(t, \Lambda). \end{aligned}$$

Using Taylor expansion at  $\gamma = 1$  and denoting

$$f_1 = f(\gamma = 1) = \alpha \int_t^T e^{-\rho(s-t)} ds + e^{-\rho(T-t)}.$$

The indirect utility is

$$J = - \left( f_1 \ln f_1 + \frac{\partial f}{\partial \gamma}(\gamma = 1) \right) + f_1 \ln W + \alpha \ln \alpha \int_t^T e^{-\rho(s-t)} ds.$$

Noting that

$$\begin{aligned}
\lim_{\gamma \rightarrow 1} a(t, s; T) &= -\rho(s-t) + (1-\gamma) \left( r + \frac{\mu^2}{2\sigma_B^2} - \rho \right) (s-t) \\
&+ \frac{1}{2}(1-\gamma) \left[ \ln \left( \frac{T-t+\sigma_\epsilon^2}{T-s+\sigma_\epsilon^2} \right) - \frac{s-t}{T-t+\sigma_\epsilon^2} \right] \\
&= -\rho(s-t) + (1-\gamma)A_0 + (1-\gamma)A_1 \\
\lim_{\gamma \rightarrow 1} b(t, s; T) &= (1-\gamma) \frac{s-t}{(T-t+\sigma_\epsilon^2)^2} = (1-\gamma)B_1 \\
\frac{\partial f}{\partial \gamma}(\gamma=1) &= \alpha \int_t^T \left( -A_0(t, s; T) - A_1(t, s; T) - \frac{1}{2}B_1(t, s; T)\Lambda^2 \right) e^{-\rho(s-t)} ds \\
&+ \left( -A_0(t, T; T) - A_1(t, T; T) - \frac{1}{2}B_1(t, T; T)\Lambda^2 \right) e^{-\rho(T-t)},
\end{aligned}$$

results in the indirect utility being

$$\begin{aligned}
J &= f_1 \ln W - f_1 \ln f_1 + \alpha \int_t^T \left( A_0(t, s; T) + A_1(t, s; T) + \frac{1}{2}B_1(t, s; T)\Lambda^2 \right) e^{-\rho(s-t)} ds + \\
&+ \left( A_0(t, T; T) + A_1(t, T; T) + \frac{1}{2}B_1(t, T; T)\Lambda^2 \right) e^{-\rho(T-t)} + \alpha \ln \alpha \int_t^T e^{-\rho(s-t)} ds.
\end{aligned}$$

A similar result can be obtained by directly solving the HJB equation for the log utility case under the conjecture that  $h(t, \Lambda) = a(t) + b(t)\Lambda^2$ .

The indirect utility for an informed investor is

$$J_0 = f_1 \ln W - f_1 \ln f_1 + \alpha \int_t^T A_0(t, s; T) e^{-\rho(s-t)} ds + A_0(t, T; T) e^{-\rho(T-t)} + \alpha \ln \alpha \int_t^T e^{-\rho(s-t)} ds.$$

The value of information in this case will be

$$\ln(R) = \frac{\alpha \int_t^T \left( A_1(t, s; T) + \frac{1}{2}B_1(t, s; T)\Lambda^2 \right) e^{-\rho(s-t)} ds + \left( A_1(t, T; T) + \frac{1}{2}B_1(t, T; T)\Lambda^2 \right) e^{-\rho(T-t)}}{f_1},$$

and is always positive.  $\square$

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Figure 1  
Consumption to wealth ratio of informed over that of the uninformed ( $\alpha=1, \Lambda=0.5, \sigma_{\varepsilon}^2=0.09$ )

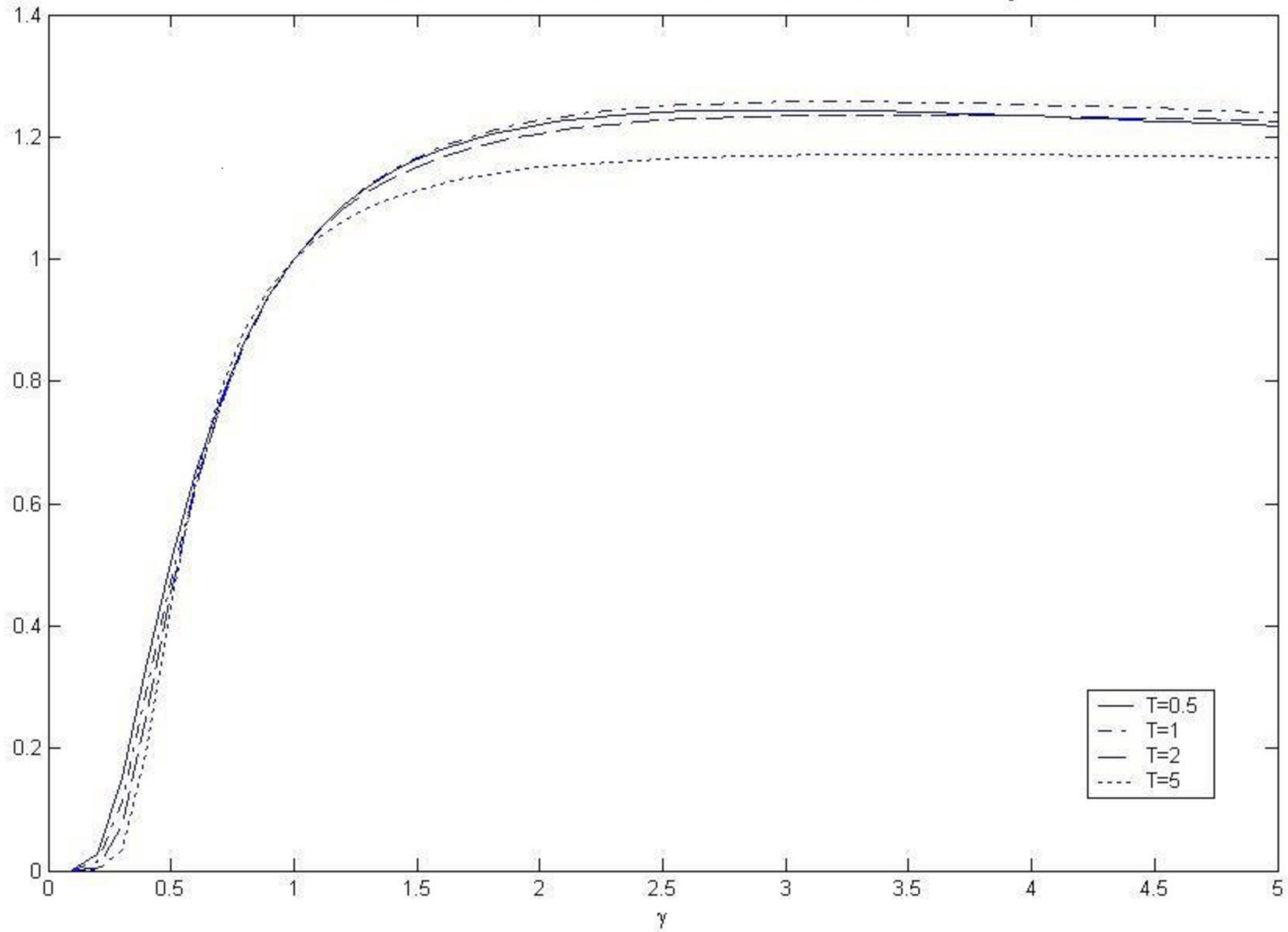


Figure 2

Consumption to wealth ratio of informed over that of the uninformed ( $\alpha= 1, \Lambda=0.5, T= 1$ )

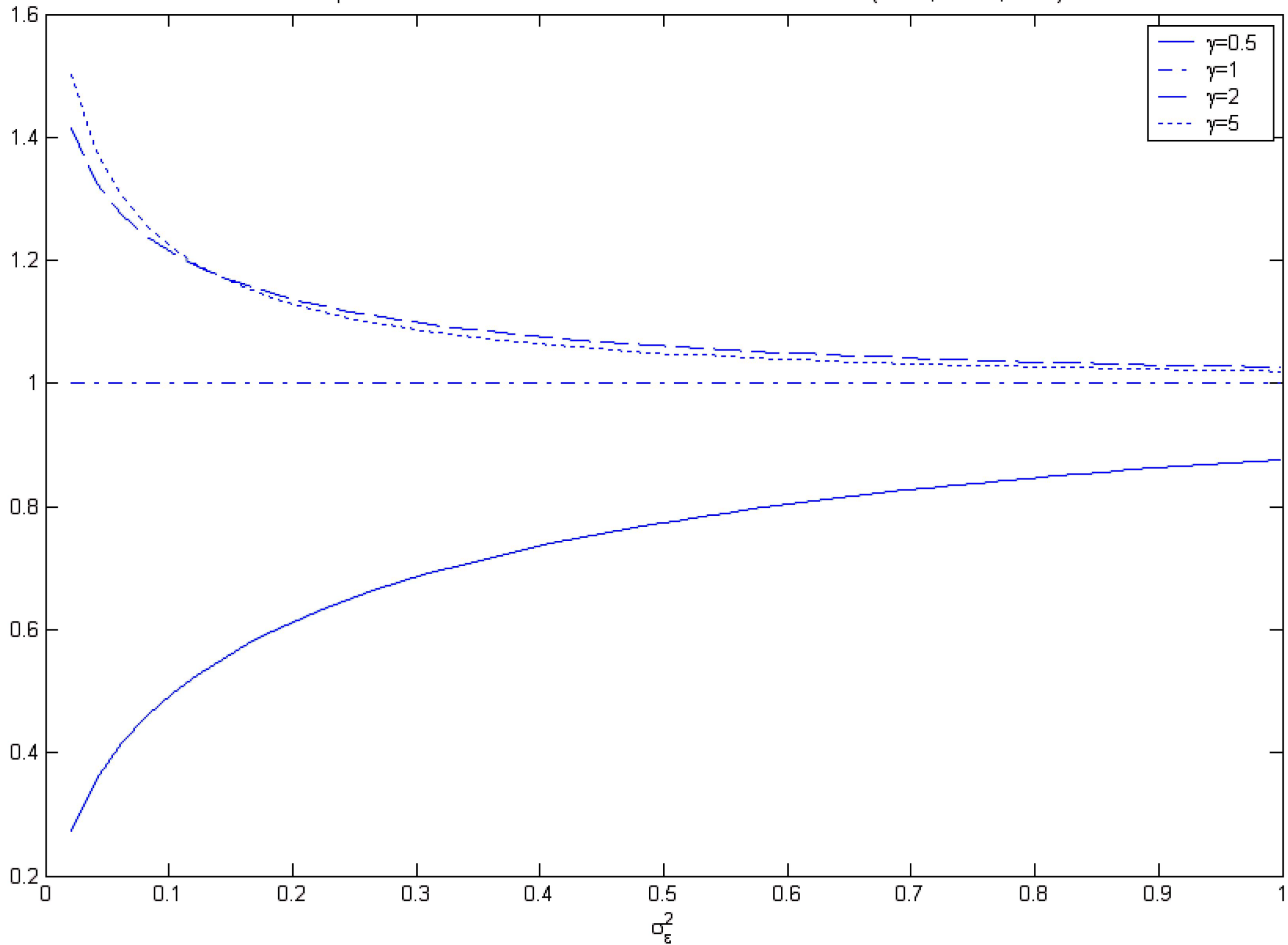


Figure 3  
Time-zero expectation of holding at t, conditioned on L ( $\alpha= 0, \sigma_{\xi}^2=0.09, L=0.5, T= 1$ )

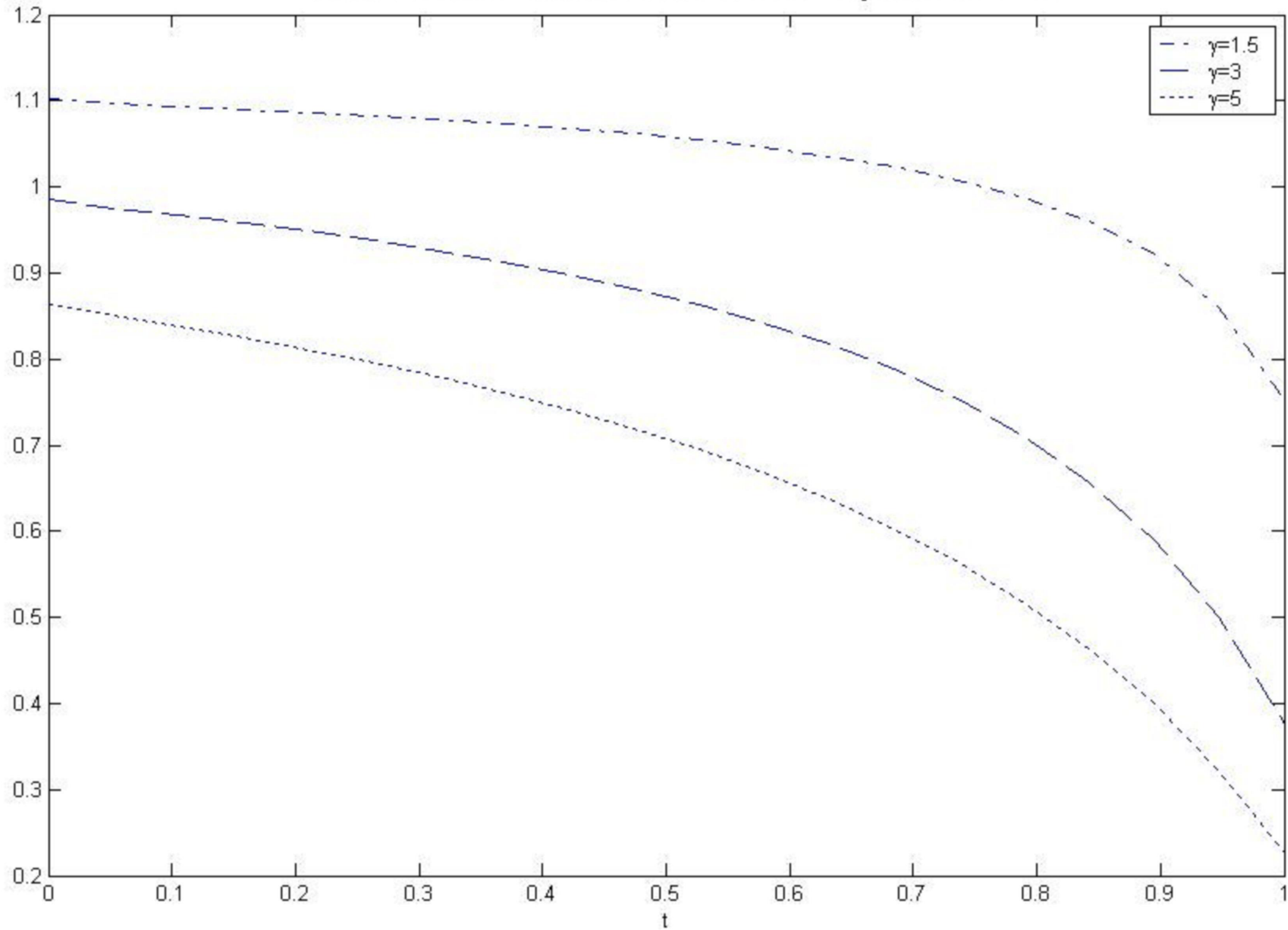


Figure 4  
Holding in stock as function of  $\alpha$  ( $\sigma_{\xi}^2=0.09$ ,  $T-t=1$ ,  $\Lambda=0.5$ )

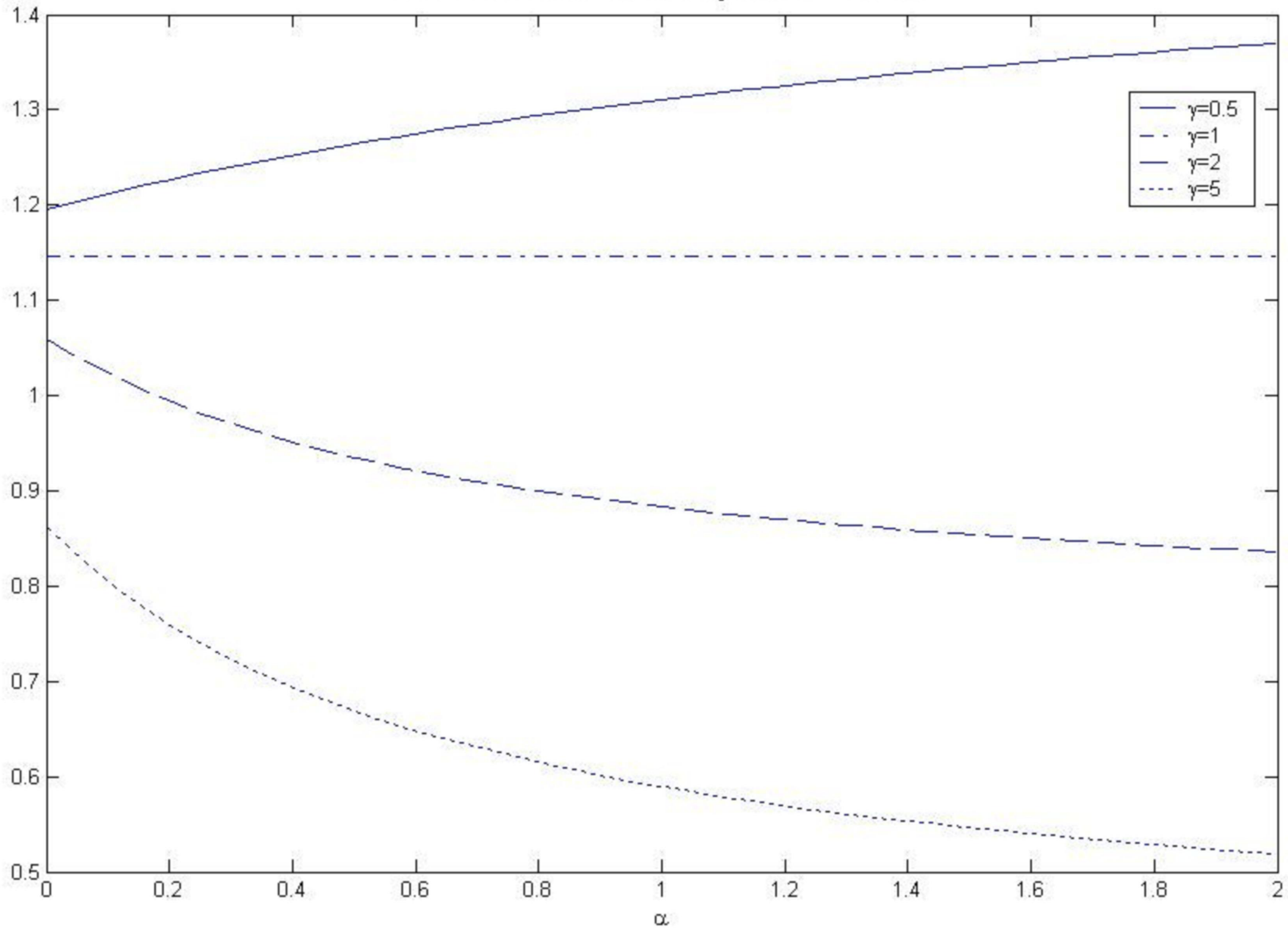


Figure 5  
Holdings in stock, market and risk tree ( $\alpha=1, \Lambda=0.5, \sigma_m^2=0.04, \sigma_s^2=0.2, \mu=0.06, r=0.04$ )

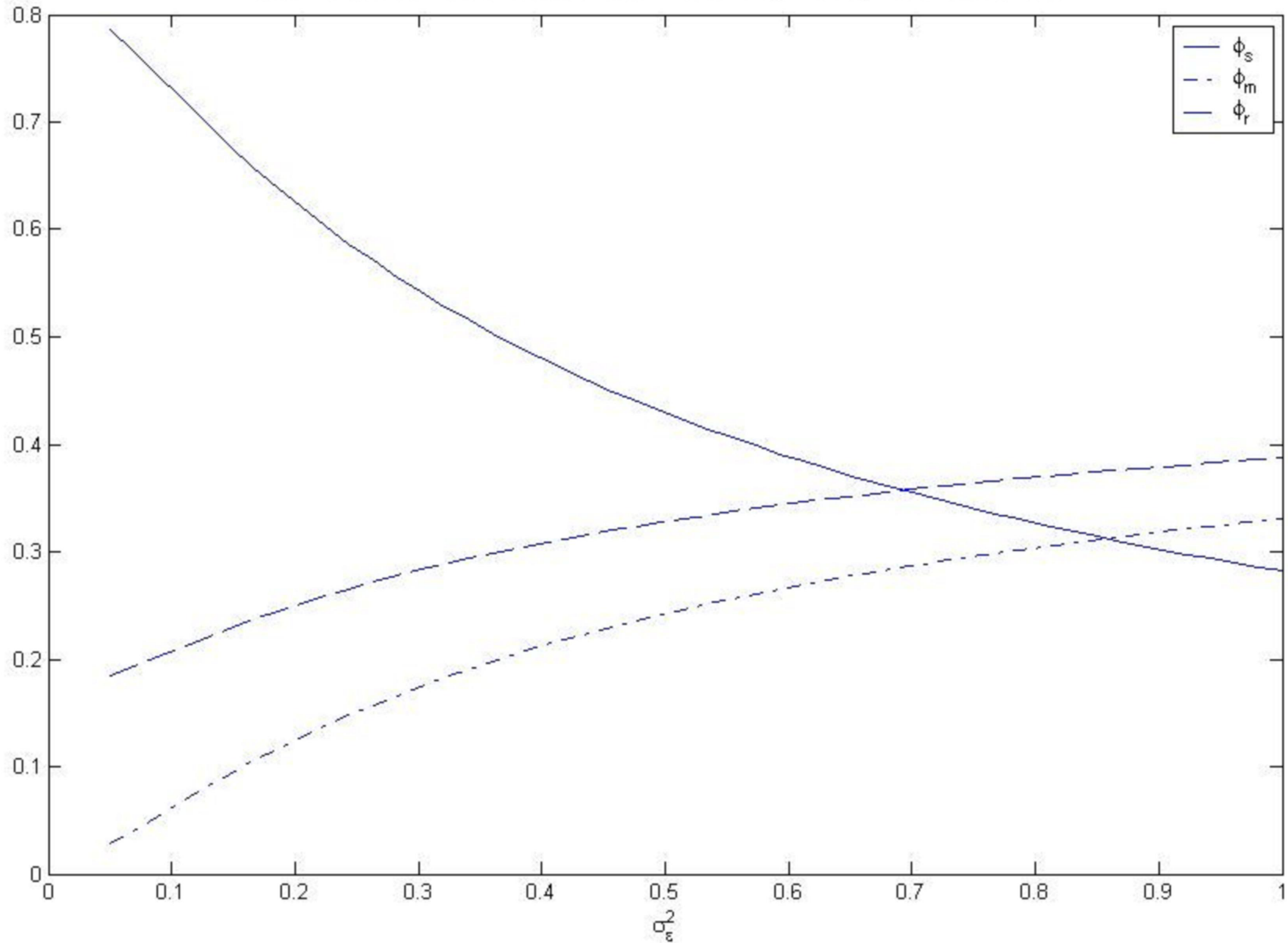


Figure 6  
Ex-post value of information ( $\alpha=1, \Lambda=0.5, \sigma_{\varepsilon}^2=0.09$ )

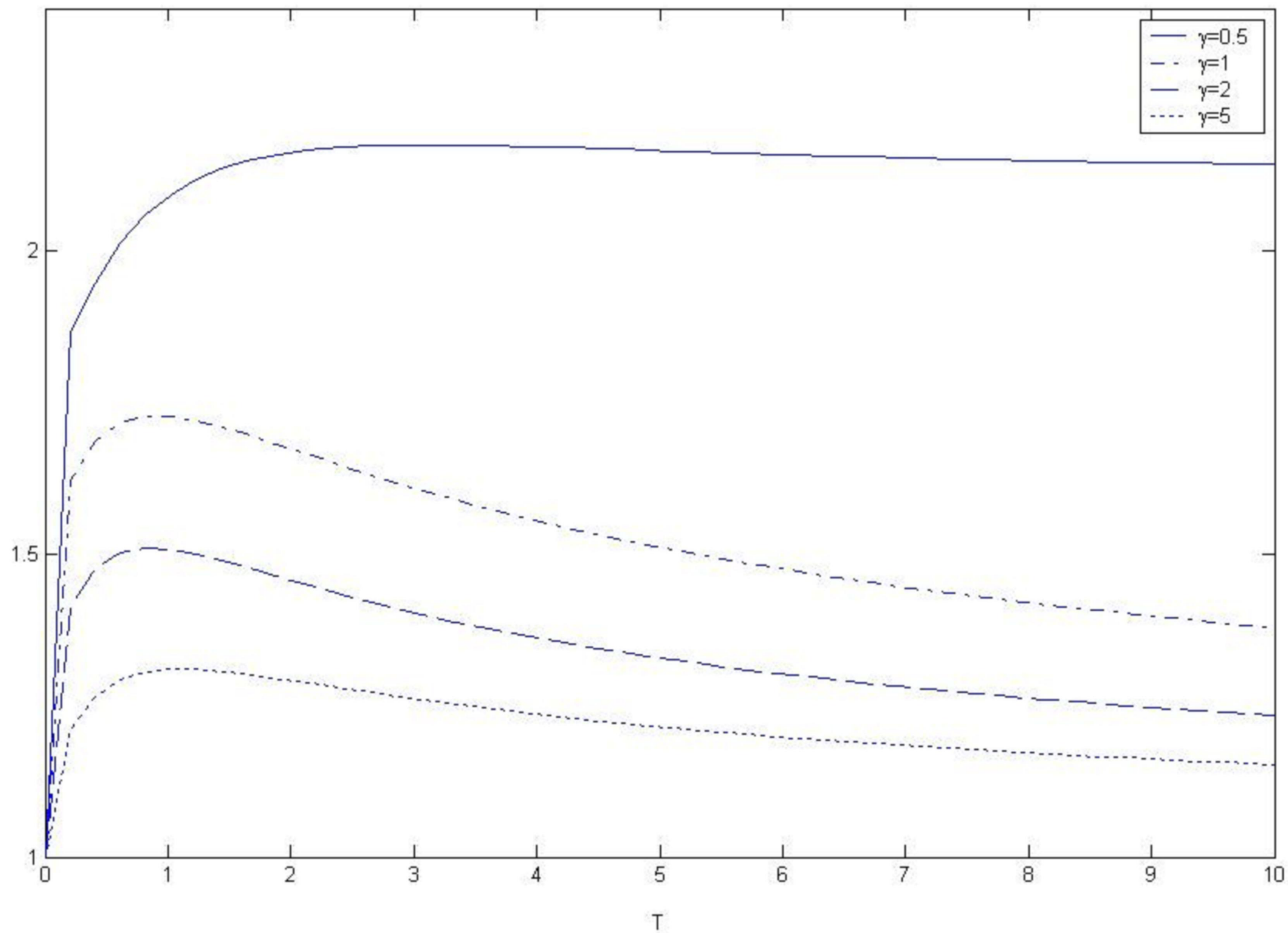


Figure 7  
Ex-ante value of information as function of  $\alpha$  ( $\sigma_{\epsilon}^2=0.09$ ,  $T-t=1$ )

