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DYNAMIC DUMPING

by

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ABSTRACT

A low-cost foreign firm lowers its initially high price--dumping if necessary--until it drives the higher cost domestic firms out of business, whereupon it raises its price. At no time, however, does the foreign firm predate (price below its marginal cost). Tariffs, quotas, and other policies that mandate a minimum number of domestic firms do not qualitatively change the price path (high price, low price, and limit price). The optimal tariff in this dynamic analysis is lower than the optimal tariff in a static analysis (to allow consumers to take advantage of the low-price period).

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Dynamic Dumping

I. Introduction

A dynamic model is used to show why a low-cost foreign firm "dumps," thereby driving domestic firms out of business, and then raises its prices. The model also demonstrates that the foreign firm never finds it profitable to predate (price below marginal cost). By contrast, earlier static and dynamic models of dumping do not explain the complex pricing patterns alleged or observed in recent years. For example, as the Economist (April 5, 1986, p. 82) reports,

> "To a man the American ragtrade believes that Crompton was driven out of business by Japanese dumping. They say that the day after Crompton filed for Chapter 11 bankruptcy the Japanese raised their prices on rival goods by 50 percent."

Similar charges have been made in many industries such as steel, computer chips, automobiles, television sets, glass, glue, baby strollers, paper, nylon yard, shoes, wigs, vinyl, transformers, and various agricultural products.

Previous studies have attributed dumping to adjustment costs or non-competitive behavior. Although these two explanations lead to some of the same market outcomes, they are not observationally equivalent. Most non-competitive behavior models explain different prices in foreign and domestic markets but do not explain why prices are first lowered and then raised. Most adjustment cost arguments can explain cyclical patterns in pricing but not the increase in prices as domestic firms are driven from the market.

Based on our dynamic model, we obtain six chief results. First, the dominant optimal policy of the foreign firm is to price discriminate (dump) and then raise its price to a limit price; that is, an unregulated dominant firm will drive the domestic fringe firms out of business. Second, the foreign firm will not predate (price below its costs).

Third, preventing the foreign firm from pricing below its home price plus transportation costs will have a quantitative but not qualitative effect on its behavior unless the policy prevents it from selling in the importing country. Indeed, under such a regulation, the foreign firm will start with an even higher price, then lower its price below the costs of the domestic firms, and then limit price.

Fourth, preventing the foreign firm from driving out a certain number of domestic firms does not affect the qualitative price behavior of the dominant firm (the price starts high, then falls, and then rises to a limit level) but does lower the welfare of the domestic consumers. Fifth, a standard quota will keep the foreign firm from driving out all the domestic firms; however, the dominant firm will still initially price high, then lower its price to drive out some of the domestic firms, and then raise its price to a limiting level (where the quota just binds).

Sixth, an optimal tariff that maximizes domestic welfare (consumer surplus plus domestic fringe profits plus tariff revenues) will be lower in a dynamic model than in a corresponding static model. A lower tariff results in lower prices, which increases consumer surplus by more than tariff revenues and lowers fringe profits.

In section II we discuss the law and the literature on dumping. Our basic dynamic model is presented in section III. Predation is discussed in section IV, and the effect of constraints on the minimum number of domestic firms (a "quantity constraint") is examined in section V. In section VI a no-dumping rule is analyzed. The effects of a quota are considered in section VII, and simulations of tariffs and quantity constraints are presented in section VIII where it is shown that the optimal dynamic tariff is very different from the optimal static model tariff. The paper ends with conclusions in section IX.

II. The Law and the Literature

The General Agreement on Tariffs and Trade defines dumping as the selling of products at "less than normal value of the products." A price is less than normal value if it is less than the comparable price in the exporting country or, in the absence of such a domestic price, is less than either the highest comparable price for the product in a third country or the cost of production in the exporting country plus a reasonable additional amount for selling costs and profit. Where such dumping causes or treatens "material injury" to an industry of the importing country, that country may impose an antidumping duty.

Since 1921, the United States has defined dumping as "sales at less than fair value" (LTFV), but that definition is extremely complex. Many hundreds of allegations of dumping have been made in the United States in the last couple of decades alone. In only 51 percent of the U. S. complaints was there a finding of LTFV pricing and in only 27 percent of the complaints was there a finding of an injury to the industry, so one can assume that allegations of dumping are much more frequent than cyclical or strategic dumping.[1]

The simplest explanation for the alleged price patterns requires neither adjustment costs nor strategic behavior. When a foreign firm has a cost advantage and prices its goods the same at home as abroad, domestic firms and their lawyers will accuse the foreign firm of dumping. From the point of view of the public, domestic jobs are lost, and the charge of unfair competition may seem justified.

The transaction costs and strategic behavior arguments in the literature are designed to explain cyclical or persistent patterns of dumping. For example, transaction cost arguments may explain the alleged dumping of steel and computer chips at prices below costs as cyclical phenomena. Firms in those industries have high fixed costs and low marginal costs, so adjustment to a worldwide glut is naturally very slow.

In Ethier's (1982) adjustment cost model, a homogeneous good called steel is produced in both the United States and Japan using two factors of production. One factor, managers, is never laid off; while the other, labor, may be laid off. This rather arbitrary rule captures the different social rules of layoffs in Japan where adjustment costs (the manager-to-labor ratio) are higher.

Wages are fixed for all factors across two states of nature. In the good state of nature, firms make pure profits, while in the bad state they lose money; so they break even on average. In the United States, layoffs occur during the bad state of nature, whereas in Japan firms produce more than the home market can absorb where the rules of the model prevent layoffs. To unload this excess production, the Japanese firms dump in the U. S. market at prices below their costs.

In Bernhardt's (1984) dumping model, adjustment costs are explicitly quadratic in the difference between planned production and sales. While also allowing for some price discrimination, Berhardt's (1984) model allows the probability of dumping to come arbitrarily close to one. In adjustment cost models, prices can be below costs as long as costs are not defined to include explicit or implicit adjustment costs. These models give a satisfactory explanation for dumping that is correlated with demand but follows no other temporal pattern.[2]

In adjustment cost models, low-price selling is not motivated by the desire to sell at a high price later. Such a story requires strategic behavior. The more traditional dumping literature dating back to Viner (1923) emphasizes pure price discrimination. For price discrimination to pay, the demand facing the exporting firm must be different in the two countries.

Eichengreen (1982) provides several plausible explanations of why the elasticities may differ. For example, there may be a competitive fringe in one country but not in the other (an explanation used below).

Brander and Krugman (1983) consider the case of monopolistic competition between a domestic and a foreign firm. Transportation costs cause the costs of each firm to be higher abroad than at home. In a Cournot equilibrium, each firm dumps in the market of the other firm.

In Brander and Spencer (1981), a foreign firm is a Stackelberg leader. Where potential domestic firms have Cournot reactions, the optimal policy for the foreign firm may be to price low enough to deter entry. In Brander and Spencer (1985), a foreign country may use tariffs to create a similar Stackelberg result.

Das and Nihio use Gaskin's model of a dominant firm with a myopic competitive fringe to model trade. When the foreign firms have market power, they get Brander and Spencer's result that the optimal tariff equals the revenue-maximizing tariff. In common with Gaskin's model, they also find that the dominant firm does not drive all fringe firms out of business.

None of these static models explain why the price of the dumped good will first fall and then rise. Using a dynamic analysis, we model an industry with a low-cost, foreign dominant firm and a domestic competitive fringe with rational expectations (perfect foresight). The dominant firm uses a dynamic open-loop strategy to maximize the present value of its profits where it chooses its price path at time zero. That is, the dominant firm cannot change its plans after they have been made, although fringe firms may enter or exit the industry at any time.[3] Although complete ex post inflexibility in plans is unlikely, a national plan to strengthen an industry, such as MITI's role in the Japanese semiconductor industry, can approach commitment. Putting the capital stock in place or announcing product innovations well in advance of their arrival in the market are also ways for the dominant producer to commit to increasing production.

III. A Dynamic Model of a Discriminating Monopolist

A low-cost Japanese firm sells in both its home market and the United States. In its domestic market it is a monopolist, while in the United States it faces a competitive fringe of higher cost firms. The constant average cost to the Japanese firm of producing and selling in its domestic market is c^*. It cost of producing, shipping to, and selling in the United States is $c = c^* + j$, where j is the transportation cost. The U. S. firms can each produce a single unit of output at constant cost $\overline{p}$, where $\overline{p} > c$. The total U. S. output (and the number of firms) is x.

If unregulated by either the Japanese or U. S. government, the Japanese firm sets a monopoly price, p_m^* in Japan, to maximize its profits,

$$p_m^* = \max_{p_m^*} \ (p^* - c^*) \ f^*(p^*), \tag{1}$$

where $f^*(p^*)$ is the demand facing the firm in Japan.[4] This price is obviously constant over time.

In contrast, the Japanese firm will charge a time-varying price in the U. S. market. There, the dominant firm chooses a price path over time, $p(t)$, to maximize the functional,

$$V = \int_0^\infty [p(t) - c]\, [f(p) - x(t)] e^{-rt}\, dt, \qquad (2)$$

where V is the present value of the profit stream of the dominant firm, f(p) is the U. S. market demand curve, r is the discount rate of all firms, and [f(p) - x(t)] is the residual demand facing the dominant firm.

This functional is maximized subject to the growth of the fringe, $\dot{x}(t)$. The initial number of fringe firms is given:

$$x(0) = x_0. \qquad (3)$$

Fringe firms enter or exit the industry based on their rational expectations about the present discounted value of their future profits, y(t). Expectations are rational if expected profits equal actual profits. Because the cost of entry depends upon the speed of entry, fringe firms do not enter instantaneously. The rate of entry is a constant proportion, k, of the rationally expected present value of profits:[5]

$$\dot{x}(t) = ky(t) \qquad (4)$$

$$y(t) = \int_t^\infty [p(s) - \bar{p}]\, e^{-r(s-t)}\, ds. \qquad (5)$$

Because the number of fringe firms cannot be negative, the dominant firm also faces a state constraint:

$$x(t) \geq 0. \qquad (6)$$

Before we can state the problem of the dominant firm as one solvable by the maximum principle, we must reduce equation (5) to a form that does not include an integral by differentiating it with respect to time:

$$\dot{y} = ry + \bar{p} - p. \qquad (7)$$

The dominant firm which faces rational fringe firms chooses a price path at time $t = 0$ so as to maximize (2) subject to (3), (4), (6), and (7). The necessary conditions for a solution to the problem of maximizing subject to a state constraint (6) are given by Jacobson, Lele, and Speyer (1971).

The problem may be restated as a Lagrangean which is formed by taking the usual Hamiltonian,

$$H = (p - c)\,(f - x)\,e^{-rt} + zky + v(ry + \bar{p} - p), \tag{8}$$

where z and v are the costate variables corresponding to the state variables, x and y, and adjoining the product of a multiplier (w) and the state constraint, $[x(t) \geq 0]$:

$$L = H + wx. \tag{9}$$

As with the usual Lagrangean methods, optimality requires that $wx = 0$. The other necessary conditions are (a) the equations of motion, (3), (4), and (7); (b) the adjoint equations, $\dot{z} = -L_x$ and $\dot{v} = -L_y$; (c) the transversality conditions, $\lim_{t=\infty} zx = \lim_{t=0} zx = \lim_{t=\infty} vy = \lim_{t=0} vy = 0$; (d) the maximum principle; and (e) the necessary conditions at the "jump time," τ (which, as shown below, is finite), at which x becomes zero. For our problem, there are three jump-time conditions which are discussed below.

We construct an optimal solution in three steps: First, we describe the interior solution where there are competitive firms. Second, we describe the corner solution where these fringe firms are driven from the market. Finally, we link the two types of solutions.

A. Interior Solution

Because H and L are identical when $x > 0$, the usual Hamiltonian methods suffice to construct an interior solution. The necessary conditions include (3), (4), (7), the adjoint equations,

$$\dot{z} = (p - c)\, e^{-rt}, \tag{10}$$

$$\dot{v} = -zk - vr, \tag{11}$$

and the maximum principle which implies

$$H_p = [(f - x) + (p - c)\, f']\, e^{-rt} - v = 0. \tag{12}$$

By substituting, the necessary conditions can be reduced to a single second-order differential equation. First, we solve (12) for v and differentiate with respect to time to obtain

$$\dot{v} = -rv + e^{-rt}[2f'\dot{p} + (p - c)\, f''\dot{p} - ky]. \tag{13}$$

We then equate equations (11) and (13) to eliminate $\dot{v}$,

$$\dot{p} = \frac{ky - zke^{rt}}{[2f' + (p - c)\, f'']}, \tag{14}$$

and differentiate with respect to time to obtain

$$\dot{z} = -rz - \frac{e^{-rt}}{k} \{[3f'' + (p - c)\, f''']\, (\dot{p})^2 + [2f' + (p - c)\, f'']\, \ddot{p} - k\dot{y}\}. \tag{15}$$

Substituting into (15) for $\dot{z}$ from (10), for z from (14), and for $\dot{y}$ from (7), we obtain

$$\frac{1}{k}\,[2f' + (p - c)\,f'']\,\ddot{p} + \frac{1}{k}\,[3f'' + (p - c)\,f''']\,(\dot{p})^2 - \frac{r}{k}\,[2f' + (p - c)\,f'']\,\dot{p} + 2p - c - \bar{p} = 0. \tag{16}$$

Where demand is linear, $f(p) = a - bp$, equation (16) is a second-order ordinary differential equation,

$$\ddot{p} - r\dot{p} - \frac{k}{b}\,p + \frac{k(\bar{p} + c)}{2b} = 0, \tag{17}$$

whose general solution is

$$p = \alpha_1 e^{\gamma_1 t} + \alpha_2 e^{\gamma_2 t} + \frac{\bar{p} + c}{2}, \tag{18}$$

where

$$\gamma_1 = \frac{1}{2}\left(r - \sqrt{r^2 + 4k/b}\right)$$

$$\gamma_2 = \frac{1}{2}\left(r + \sqrt{r^2 + 4k/b}\right).$$

Equation (18) represents the solution to the dynamic limit-price problem where there are fringe firms in the market. It depends upon two unknown parameters, α_1 and α_2. These parameters are determined from the conditions joining the interior and corner solutions where $x(t) = 0$.

The corner is reached in finite time since internal solutions that last forever and meet the transversality conditions eventually have strictly negative values of x. Not all values of α_1 and α_2 are compatible with the transversality conditions. If α_2 (the constant associated with the positive root) is positive, the price will grow without bound. Because $\dot{z} = (p - c)\,e^{-rt}$, z will also grow without bound. Since equations (4) and (5) guarantee that x will also grow without bound, the transversality condition that $\lim_{t\to\infty} zx = 0$ is violated. A similar argument can be made for $\alpha_2 < 0$. Therefore, interior trajectories that last forever must have $\alpha_2 = 0$.

By substituting for p from (18) into (5), integrating to obtain y, substituting for y in (4), and then integrating again with respect to t, we obtain

$$x(t) = x_0 + \alpha_1 b - \alpha_1 b e^{\gamma_1 t} + \frac{k(c - \bar{p})}{2r} t. \tag{19}$$

Because $\gamma_1 < 0$, eventually the last term on the right-hand side dominates; and, since $c - \bar{p} < 0$, $\lim_{t\to\infty} x < 0$. Thus, an optimal policy starting with $x > 0$ will eventually drive x to zero at some time, τ. As a result, interior trajectories that last forever are impossible. Hence, α_2 may be nonzero. Because the constraint is eventually reached, we now consider corner solutions.

B. Corner Solution

When x is zero and remains zero for an open time interval, y will equal 0 and p will equal $\bar{p}$. As a result, once x becomes zero on an open interval, it will remain zero forever.

Continuity may be used to prove that, if $x = 0$ on an open interval, $p = \bar{p}$. Let x be zero from τ_1 to τ_2 (since x is continuous, it is zero at τ_1 and τ_2 as well). Since for $\tau_1 \leq t \leq \tau_2$,

$$x(t) = x(\tau_1) + \int_{\tau_1}^{t} ky\, ds, \tag{20}$$

y is certainly zero. Similarly,

$$y(t) = y(\tau_1) + \int_{\tau_1}^{t} e^{-rs}(p - \bar{p})\, ds, \tag{21}$$

so $y(t) = 0$ implies that $p(t) = \bar{p}$, $\tau_1 \leq t \leq \tau_2$.

The principle of optimality establishes the final claim that once a corner solution is reached it will continue indefinitely. If it is optimal to set $p(t) = \bar{p}$ when $x(t) = y(t) = 0$ at $t = \tau_1$, it will be optimal to set $p = \bar{p}$ at any other time when $x(t) = y(t) = 0$. Since $x(t) = y(t) = 0$ for a corner solution and that implies $p(t) = \bar{p}$, it follows that $x(t + \varepsilon) = y(t + \varepsilon) = 0$ for small ε, so the corner solution will last indefinitely. Thus, an optimal program that begins with competitive fringe firms will consist of one interior segment of finite length followed by a corner segment of infinite length where $p(t) = \bar{p}$ and $x(t) = 0$.

C. Linking the Interior and Corner Solutions

Matching the interior to the corner solution determines α_1, α_2, and τ. The definition of x, the continuity of the Hamiltonian, and the transversality condition at time zero give three equations to determine α_1, α_2, and τ. This section derives each in turn.

Substituting for p from equation (18) in the definition of $\dot{x}$ in (3) and integrating between 0 and τ gives our first condition:

$$x(\tau) = x_0 + \alpha_1 b \left(1 - e^{\gamma_1 \tau}\right) + \frac{\alpha_1 k}{r\gamma_2}\left(e^{-\gamma_2 \tau} - e^{\gamma_1 \tau}\right)$$

$$+ \alpha_2 b \left(1 - e^{\gamma_2 \tau}\right) + \frac{\alpha_2 k}{r\gamma_1}\left(e^{-\gamma_1 \tau} - e^{\gamma_2 \tau}\right) \tag{22}$$

$$+ \frac{k(\bar{p} - c)}{2r^2}\left(1 - e^{-r\tau} - r\tau\right) = 0$$

where the second equality follows because $x(\tau) = 0$ by the definition of τ.

The continuity of the Hamiltonian implies that $p(\tau) = \bar{p}$. Since H for given x, y, z, and v has a unique maximum in p, H is said to be regular which implies that p is continuous at τ:

$$p(\tau) = \alpha_1 e^{\gamma_1 \tau} + \alpha_2 e^{\gamma_2 t} + \frac{c + \bar{p}}{2} = \bar{p} \tag{23}$$

[Jacobson, Lele, and Speyer (1971), p. 272].

The last condition comes from noting that $v(0) = 0$ because $y(0)$ is free and $v(0)\, y(0) = 0$ by a transversality condition. Since $H_p = 0$ by the maximum principle, $a - bp(0) - x_0 - bp(0) + bc = 0$. [Notice that $p(0)$ is the short-run, profit-maximization price.] Substituting for $p(0)$ from (18) and rearranging gives

$$\alpha_1 + \alpha_2 = \frac{a - x_0 - b\bar{p}}{2b}. \tag{24}$$

Equations (22), (23), and (24) determine α_1, α_2, and τ.

D. The Price Path

The price path depends on the parameters of the system. Heuristically, if x_0 is relatively small (given the other parameters), the price starts high, falls below $\overline{p}$, and then rises to $\overline{p}$. Alternatively, if x_0 is relatively large, the price starts below $\overline{p}$ and then rises to $\overline{p}$.

In either case, as $t \to \tau$, $p(t)$ must approach $\overline{p}$ from below for the dominant firm to drive the fringe firms out of the market. After all fringe firms are driven out, price must remain at $\overline{p}$ or new entry would occur. Figure 1 shows a path where x_0 is initially small.[6]

In the interior, the price path is described by equation (18). The path depends on α_1 and α_2. From equation (24) and the associated argument, $\alpha_1 + \alpha_2 > 0$. Since price must approach $\overline{p}$ from below at t near τ, $\dot{p}(t) = \alpha_1\gamma_1 e^{\gamma_1 t} + \alpha_2\gamma_2 e^{\gamma_2 t} \geq 0$. It follows that $\alpha_2 \leq 0$ is impossible. Since α_1 and α_2 cannot both be negative, were $\alpha_2 \leq 0$, α_1 would have to be positive; but that would imply that $\alpha_1\gamma_1$ and $\alpha_2\gamma_2$ would both be negative so that $\dot{p} \leq 0$, which is a contradiction. Thus, $\alpha_2 > 0$, $\alpha_1 + \alpha_2 > 0$, and $\alpha_1 \lesseqgtr 0$. There are two possible price paths as shown in figure 2.

IV. Dumping and Predation

Price discrimination must occur with certainty since p^* is a constant while $p(t)$ varies over time. If dumping is defined as pricing in the United States below the Japanese price plus the shipping costs, $p(t) < p^* + j$, dumping is likely--at least at certain times. Indeed, if $p(0) = (a + bc - x_0)/2b < p^*$, $p(t)$ will be less than p^* at all times.

While dumping is likely, predation (pricing below costs) does not occur. As figure 1 shows, $p(t)$ is always above $(p + c)/2$. Furthermore, since $p > c$, it follows that $(p + c)/2 > c$. Hence, $p(t)$ is always above c. Thus, while dumping will occur with certainty, predation will never occur.

Figure 1

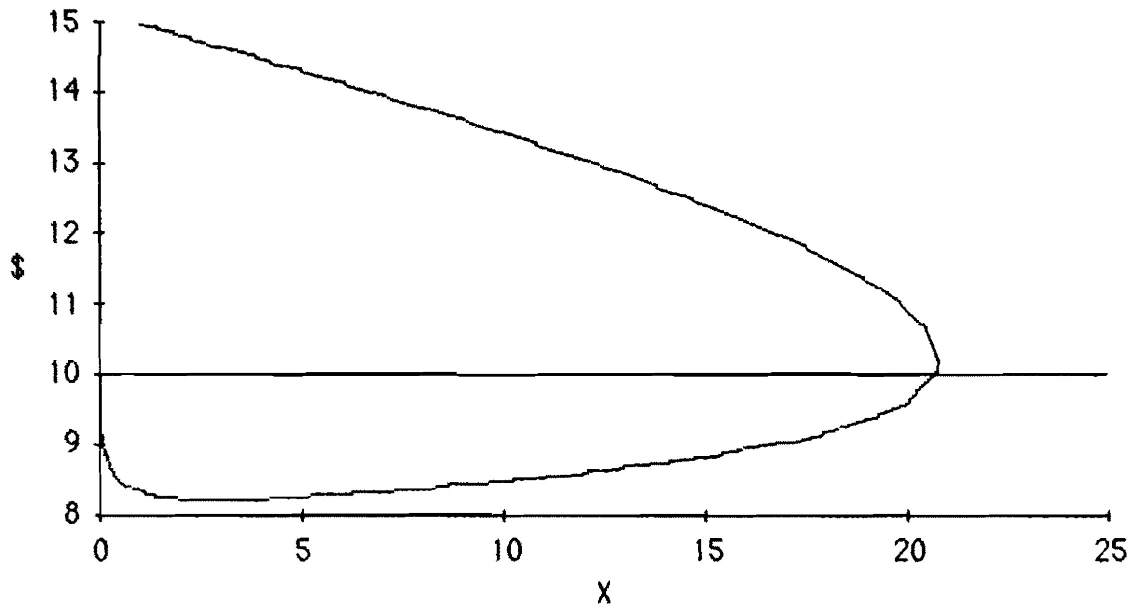
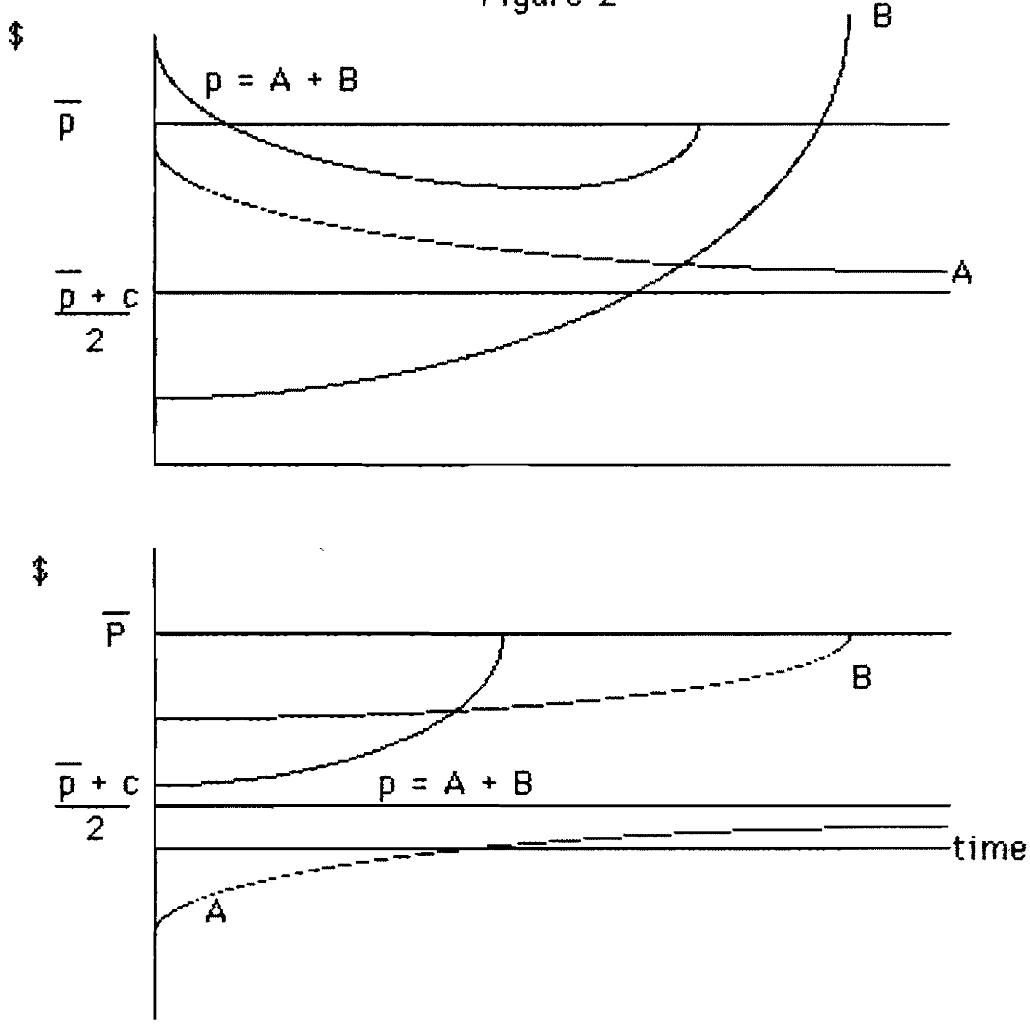


Figure 2



$$A = \alpha_1 e^{\gamma_1 t} + (\bar{p} + c)/2$$

$$B = \alpha_2 e^{\gamma_2 t}$$

As one professor of law put it, antipredation should be the "proper touchstone of antidumping law" (Barceló, 1979, p. 65).[7] Except on narrow nationalistic grounds (see section VIII below), preventing low-cost firms from undercutting high-cost firms is not a sound policy.

V. Quantity Constraints

Suppose the government constrains the dominant firm so that it cannot drive all the fringe firms out of the market. We start by assuming that the government requires that $x \geq \underline{x}$, where $\underline{x}$ is the minimum number of fringe firms. Presumably, this constraint is enforced by the use of a quota (or the threat of a large tariff).

Where entry is instantaneous ($k \to \infty$), this policy clearly lowers welfare as the dominant firm is the low-cost producer and the market price is unaffected or rises. Where entry is gradual, however, this policy may benefit consumers and possibly increase welfare (defined as consumer surplus plus profits of all firms).

The model is too complicated to solve analytically for conditions such that welfare rises. We have failed to find a case through simulations where consumer surplus rises when $\underline{x}$ is set greater than zero; that is, it appears that such interventions always hurt consumers (see the simulations below).

VI. The No-Dumping Rule

Suppose the U. S. government imposes a no-dumping rule that requires the Japanese firm to set its price in the United States no lower than its costs or its price in Japan. As we have already shown that a low-cost dominant firm will not set its price below c (and, hence, not below its costs in the United

States), we now examine a restriction that prevents it from setting its price in the United States below its price in Japan.

The dominant firm no longer can price discriminate unless it wishes to set a higher price in the United States. In other cases, it will use the same price in both countries. Its profit-maximizing problem becomes

$$\max_{p,p^*} \int [(p - c)(f - x) + (p^* - c^*) f^*] e^{-rt} dt \tag{25}$$

$$\text{s. t. } p^* + j \leq p, \text{ equations (3), (4), (6), and (7).}$$

Where p_m^* maximizes $(p^* - c^*)$ and $p_m^* + j < p$, we have the original unconstrained problem. We now consider the case where $p_m^* + j \leq p$.

If $f^*(p^*) = a^* + b^*p^*$ and the constraint is binding, equation (25) can be rewritten as

$$\max_{p} \int (p - c) [(a + a^* + b^*j) - (b + b^*) p - x] e^{-rt} dt \tag{26}$$

$$\text{s. t. equations (3), (4), (6), and (7).}$$

Equation (26) is of exactly the same form as the original problem. The only difference is that both demand function parameters have been increased: "a" becomes $(a + a^* + b^*j)$ and "b" becomes $(b + b^*)$. An increase in "a" raises the original price, whereas an increase in "b" lowers it.[8] A discriminating monopolist who is forced to charge a single price tends to set that price between its original two prices. Thus, the price in the United States will rise initially. The long-run effect is extremely complex and can cause τ to change in either direction.

We now consider the intermediate case where the constraint is binding some of the time. Because the unregulated dominant firm will set a high price initially and then lower it along the upper part of this path, the constraint will not be binding while along the lower part it will be binding. The two parts of the path must meet; and p, $\dot{p}$, x, $\dot{x}$, y, and $\dot{y}$ must be continuous at that point. Call the time when they meet τ_0. If the roots of the original second-order differential equation are α_1 and α_2 and the roots of the constrained equation are α_1' and α_2', the following six conditions must be met.

First, along the upper part of the path at the initial time, $x(\alpha_1, \alpha_2, 0) = x_0$. Second, at the initial time, $p(\alpha_1, \alpha_2, 0)$ must be determined as above by the maximum principle and by x_0: $p(0) = [(a + bc - x_0)/2b]$. Third, where the constrained and unconstrained parts of the path join, $p_m^* + j = p(\alpha_1', \alpha_2', \tau_0) = p(\alpha_1, \alpha_2, \tau_0)$. Fourth and similarly, $x(\alpha_1', \alpha_2', \tau_0) = x(\alpha_1, \alpha_2, \tau_0)$. Fifth, the constrained part of the path must reach the corner, so $p(\alpha_1', \alpha_2', \tau) = \overline{p}$. Sixth, similarly, $x(\alpha_1', \alpha_2', \tau) = 0$. Thus, we have six equations which determine six unknowns (α_1, α_2, α_1', α_2', τ_0, and τ).

Therefore, while this policy affects the price path quantitatively (making consumers worse off, at least initially), it does not affect the qualitative results described above. That is, the path starts with a relatively high price; then the price falls until fringe firms are driven out of the market; and, finally, the dominant firm limit prices.

VII. Quota

An alternative policy that the U. S. government may use is to impose a quota, Q*, on the imports of the dominant firm, Q:

$$f(p) - x = a - bp - x = Q \leq Q^*. \tag{27}$$

Using the second equality in equation (27), we can express price as a function of imports and the production of the fringe firms:

$$p(Q, x) = \frac{a - x - Q}{b}. \tag{28}$$

The problem faced by the dominant firm is to

$$\max_{Q \leq Q^*} \int \{[p(Q, x) - x]\,[a - bp(Q, x) - x]\}\, e^{-rt}\, dt \tag{29}$$

$$\text{s. t. (3), (4), and (7).}$$

Since $Q \leq Q^*$, x is bounded away from zero, so this problem is different from the original one. It involves only a constraint on the control variable, Q, and not a constraint on the state variable, x.

The Hamiltonian for this problem is

$$H = R + zky + v(ry + \bar{p} - p) \tag{30}$$

where $R \equiv [(p - c)\,(a - bp - x)]\, e^{-rt}$. The maximum principle yields (where subscripts denote partial derivatives)

$$\begin{aligned} &R_p p_Q - v p_Q = 0 \text{ and } Q < Q^* \\ \text{or} \quad &R_p p_Q - v p_Q < 0 \text{ and } Q = Q^*. \end{aligned} \tag{31}$$

Since $p_Q = -1/b \neq 0$, in the interior the maximum principle gives the same expression (12) as before. The costate equations are

$$\dot{z} = -R_p p_x - R_x + v p_x \tag{32}$$

$$\dot{v} = -zk - rv. \tag{33}$$

These equations hold for all time. During those times for which Q is less than Q* [R_p = v from equation (31)], the equation for $\dot{z}$ (32) simplifies to give

$$\dot{z} = -R_x, \tag{32'}$$

which is the same equation as for an interior solution in the original problem which does not have a quota constraint.

There are two parts to the solution. First, in the interior solution (Q < Q*), the maximum principle, the costate equations, and the equations determining the price path are the same as those of an unconstrained interval in the original problem. Second, when the quota binds (Q = Q*), the constraint is

$$x = a - bp - Q. \tag{34}$$

Taking the time derivative and setting it equal to ky [using (4)], we obtain

$$\dot{x} = -b\dot{p} = ky. \tag{35}$$

Taking the time derivative of (35) and substituting $ry + \bar{p} - p$ for $\dot{y}$ using (7) and $-b\dot{p}/k$ for y using (35), we obtain the second-order ordinary differential equation,

$$\ddot{p} - r\dot{p} + \frac{k(\bar{p} - p)}{b} = 0. \tag{36}$$

Solving equation (36) for p gives

$$p(t) = \theta_1 e^{\mu_1 t} + \theta_2 e^{\mu_2 t} + \bar{p} \tag{37}$$

where

$$\mu_1 = \frac{1}{2}\left(r - \sqrt{r^2 - 4k/b}\right)$$

and

$$\mu_1 = \frac{1}{2}\left(r + \sqrt{r^2 - 4k/b}\right).$$

An optimal path may take one of several possible forms which we will describe using a set of lemmas. If the constraint is not binding at the beginning of a program, optimal paths begin with the short-run, profit-maximizing price (that is, on the $R_p e^{rt} = 0$ line) and continue as interior segments. These paths eventually reach the quota constraint and equilibrium at the cost of the fringe firms, $\bar{p}$, but the possible paths to this equilibrium are numerous.

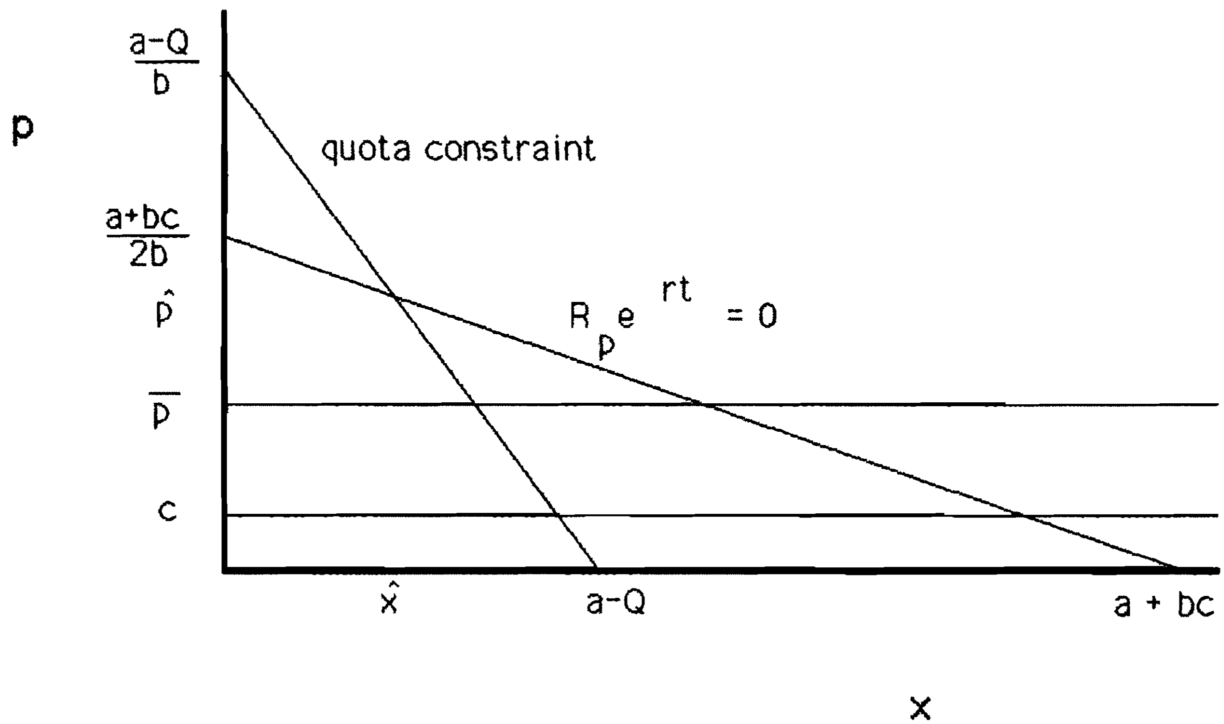
Figure 3 shows the relation of $\bar{p}$, $R_p e^{rt} = 0$, and the quota constraint in (x, p) space so that paths may be drawn in it as projections from (x, p, y) space into the (x, p) space. The figure includes four lines: (A) the quota constraint equation (34); (B) the cost to the domestic firm, $\bar{p}$; (C) $R_p e^{rt} = 0$, which is $x = a - 2bp + bc$; and (D) the cost to the foreign firm, c. The intersection of the constraint and $R_p = 0$ is labeled $(\hat{x}, \hat{p})$. Let $(x, p)^*$ denote the projection of an optimal path.

For small enough x, the optimal path follows the constraint:

LEMMA 1: If a path is optimal and $x(t) < \hat{x}$, then $[x(t), p(t)]^*$ lies on the share constraint.

PROOF: Points below the share constraint (see figure 3) are not feasible, so the optimal path must lie on or above the constraint. For $x(t) < \hat{x}$, points above the quota constraint are also above the $R_p e^{rt} = 0$ line.

Figure 3



Above that line, instantaneous profits fall as the price increases. As increases in price also encourage entry which decreases future profits, prices above the quota constraint lead to less instantaneous and future revenues and cannot be optimal.

By the same reasoning,

LEMMA 2: The interior portion of an optimal path does not lie above the $R_p e^{rt} = 0$ line at even a single point.

The next two lemmas, the corollaries, and theorems 1 and 2 describe the general direction that interior optimal paths may follow. They show that the direction of travel is away from the $R_p e^{rt} = 0$ line.

LEMMA 3: If along an optimal interior arc above c there is a time, t, at which $d(R_p e^{rt})/dt$ is negative, $d(R_p e^{rt})/dt$ remains negative as long as $(x, p)^*$ remains interior and above c.

PROOF: Since $(x, p)^*$ lies in the interior, from the maximum principle,

$$R_p = v. \tag{31'}$$

Multiplying both sides of equation (31') by e^{rt} and differentiating with respect to time gives

$$\frac{d}{dt}(R_p e^{rt}) = (\dot{v} + rv)\, e^{rt}. \tag{38}$$

The costate equation (33) requires $(\dot{v} + rv) = -zk$. Given the hypothesis of Lemma 3 that $d/dt(R_p e^{rt})$ is negative, equations (33) and (38) show that z is positive. The costate equation for z is $\dot{z} = (p - c)\, e^{-rt}$, so z is positive and increasing along $(x, p)^*$; hence, $dR_p e^{rt}/dt$ is negative and decreasing which establishes the lemma.

A geometric interpretation of Lemma 3 uses a vector, $N = (1, 2b)$, which is normal to $R_p e^{rt} = 0$. The N points in the direction of decreasing R_p. When $dR_p e^{rt}/dt$ is negative, the tangent vector to $(x, p)^*$, $(\dot{x}, \dot{p})^*$, points in the same half space as N, that is $|(\dot{x}, \dot{p})^*, N| > 0$.

A final lemma tells us the direction of motion in the x-plane:

LEMMA 4: When $p > \bar{p}$ and $\dot{x} < 0$, $\dot{x}$ remains negative as long as $(x, p)^*$ remains above $\bar{p}$. When $p < \bar{p}$ and $\dot{x} > 0$, $\dot{x}$ remains positive as long as $(x, p)^*$ remains below $\bar{p}$.

PROOF: Since $\dot{x} = ky$, if $\dot{x} < 0$, $y < 0$. Furthermore, because $\dot{y} = ry + \bar{p} - p$ if p is also greater than $\bar{p}$, $\dot{y}$ is negative. Thus, above $\bar{p}$, $\dot{x}$ must become more negative, which establishes the first half of the lemma. The second assertion is established by the same steps with the signs reversed.

COROLLARY: An optimal path that begins with $x < \hat{x}$ travels along the share constraint toward $(\hat{x}, \hat{p})$.

PROOF: Lemma 1 shows that the path is along the share constraint. Lemma 4 shows that, if the optimal path starts to move away from $\hat{x}$ toward lower x, $\dot{x}$ must remain negative forever and p must remain above $\bar{p}$ forever. Such a path is impossible since, if p is always above $\bar{p}$, y must be positive and, hence, $\dot{x}$ must be positive.

THEOREM 1: Let $(x, p)^*$ be an optimal path, and let $(\dot{x}, \dot{p})^*$ be its tangent. When p is above $\bar{p}$, $|(\dot{x}, \dot{p})^*, N| < 0$.

PROOF: The proof proceeds by contradiction: Assume that at some time $t > 0$, $p > \bar{p}$, and $|(\dot{x}, \dot{p})^*, N| > 0$. The contradiction is established in three steps. First, the optimal path may not remain above $\bar{p}$. Second, if the optimal path passes below $\bar{p}$, it will recross the $\bar{p}$ line; when it recrosses, the optimal path will be closer to the $R_p e^{rt} = 0$ line than when it first crossed. Third, since there is no limit point on the $R_p e^{rt} = 0$ line and optimal paths cannot cross $R_p e^{rt} = 0$, an optimal path cannot become ever closer to $R_p e^{rt} = 0$. This contradiction establishes the theorem.

It is impossible for an optimal interior path to remain within the triangle formed by the $R_p e^{rt} = 0$ line, the $p = \bar{p}$ line, and the constraint. Since Lemma 3 shows that once $(x, p)^*$ moves in the same half space as N (it must move in the same half space as N as long as it remains in the interior), the path must eventually come to the boundary of the triangle. By Lemma 2, an optimal path cannot lie above the $R_p e^{rt} = 0$ line, so the optimal path must eventually hit one of the other two boundaries.

The optimal path $(x, p)^*$ could join the constraint, but it would still have to move in the same direction as N which would mean that it either becomes interior again in the same direction as N or crosses $R_p e^{rt} = 0$, which is impossible. For $(x, p)^*$ to join the constraint while moving in the same half space as N, $\dot{x}$ must be negative. Using Lemma 4, when $\dot{x} < 0$ for p above $\bar{p}$, $\dot{x}$ cannot become positive. Since $\dot{x}$ cannot change signs, the direction of travel while on the constraint cannot reverse. When $\bar{x} < 0$ and $(\dot{x}, \dot{p})^*$ is on the constraint, $|(\dot{x}, \dot{p})^*, N| > 0$. By continuity, if $(x, p)^*$ again becomes interior, $|(\dot{x}, \dot{p})^*, N|$ will still be positive. Thus, paths that move in the same direction as N and are above $\bar{p}$ must move in that half space and may not exit the triangle through either the constraint or the $R_p e^{rt} = 0$ line.

The only remaining possibility is that the price on such a path falls below $\bar{p}$. The only way for a path to have $|(\dot{x}, \dot{p}), N| > 0$ and $|(\dot{x}, \dot{p}), (0, -1)| > 0$ (so that the path crosses the $p = \bar{p}$ line) is for $\dot{x} > 0$.

Since the path $(x, p)^*$ must dip below $\bar{p}$ with $\dot{x} > 0$, all that remains is to describe the behavior of the path below $\bar{p}$. Lemma 4 shows that $\dot{x}$ cannot change signs while the path remains below $\bar{p}$. Since $\dot{x} > 0$ and firms enter only when the present value of the profits is positive $(y > 0)$, there must be some time when instantaneous profits are again positive so that $(x, p)^*$ must again cross $\bar{p}$.

The value of p is obviously the same $(\bar{p})$ at both points where the optimal path crosses the $\bar{p}$ line, while the value of x is larger at the second point (because $\dot{x}$ was positive at all times in between), so, at the second point, $R_p e^{rt}$ must be smaller. Thus, $R_p e^{rt}$ must decrease, regardless of whether or not the path is above or below $\bar{p}$. Since an optimal path cannot cross the $R_p e^{rt} = 0$ line and there is no limit point on that line, it is not possible for $(x, p)^*$ to be constantly moving in the same half space as N, which establishes the theorem.

COROLLARY: When $(x, p)^*$ is optimal and $p < \bar{p}$, $\dot{x} < 0$.

PROOF: The proof proceeds by contradiction. Assume $\dot{x} > 0$. By the argument of the previous theorem, p must eventually exceed $\bar{p}$ and, at that instant, $\dot{p} > 0$. Since the direction of increasing x is the direction of decreasing p along the constraint, when p crosses $\bar{p}$, $(x, p)^*$ must be interior. An interior path that has increasing p and increasing x must have $|(\dot{x}, \dot{p})^*, N| > 0$, which is impossible by the theorem.

THEOREM 2: An optimal path does not cross $\bar{p}$ from below.

PROOF: The proof proceeds by contradiction: Assume that $(x, p)^*$ is optimal and crosses the $p = \bar{p}$ line from below. Lemma 4 and the corollary show that, along a path crossing $\bar{p}$ from below, $\dot{x} < 0$. Because $\dot{x} < 0$ at the time of crossing, there is some later time during which the fringe firms suffer losses, which implies that $(x, p)^*$ must recross $\bar{p}$. Since the optimal p is the sum of two real exponential functions with real coefficients, $\dot{p}$ can change signs, at most, once. There must be a change of sign for the path to recross $\bar{p}$, so $\dot{p} < 0$. Given decreasing p and x, $(x, p)^*$ must intersect the constraint. The direction of decreasing p on the constraint is, however, the direction of increasing x, so $(\dot{x}, \dot{p})^*$ could not be continuous at the point it meets the constraint which is a contradiction.

THEOREM 3: Once $(x, p)^*$ lies on the constraint below $\bar{p}$, it will continue on the constraint until it reaches $\bar{p}$ where it will stop.

PROOF: The proof proceeds by showing that the alternative--exiting to an interior arc--requires the tangent to the interior arc to point in a direction that is not "high" enough to leave the constraint. Along an interior arc, since $v = R_p$,

$$ve^{rt} = (a - 2bp + bc - x). \tag{39}$$

Differentiating (39) with respect to time gives

$$(\dot{v} + rv)\, e^{rt} = -2b\dot{p} - \dot{x}$$

which can be solved for $\dot{p}/\dot{x}$:

$$\left.\frac{dp}{dx}\right|_{\text{interior}} = \frac{\dot{p}}{\dot{x}} = \frac{\dfrac{-(\dot{v} + rv)\, e^{rt}}{\dot{x}} - 1}{2b}.$$

Along the quota constraint

$$\left.\frac{dp}{dx}\right|_{\text{constraint}} = \frac{-1}{b}.$$

For an optimal path to leave the constraint, it must climb above it:

$$\left.\frac{dp}{dx}\right|_{\text{interior}} < \left.\frac{dp}{dx}\right|_{\text{constraint}}$$

or

$$\frac{\dot{v} + rv}{e^{-rt}\,\dot{x}} > 1. \tag{40}$$

Because leaving the constraint with $\dot{x} < 0$ means moving in the same direction as N, $(\dot{v} + rv)$ must be negative. Therefore, the inequality asserts that one positive number is greater than another. For the path to return to the constraint, however, requires the inequality to be reversed, which cannot happen. Since $(\dot{v} + rv) = -zk$ and z is growing along the path, the left-hand-side numerator of equation (40) is growing in absolute value. The corresponding denominator, $\dot{x}e^{-rt}$, is shrinking in absolute value because

$$\frac{d}{dt}\,\dot{x}e^{-rt} = \frac{d}{dt}\left[k\int_t^{\infty} (p - \bar{p})\, e^{-rs}\, ds\right] = k(\bar{p} - p)\, e^{-rt} > 0$$

and $\dot{x}$ was initially negative. Since the numerator grows and the denominator shrinks (in absolute value) and neither of the parts changes sign, the inequality in equation (40) can never be reversed, so an interior path beginning on the constraint below $\bar{p}$ can never return to the constraint. Thus, as with all other potential interior paths traveling in the same half space as N, this path cannot exist. The conclusion is that, once an optimal path joins the constraint below $\bar{p}$, it continues along the constraint.

We are now in a position to describe the optimal path that begins in the interior (not on the constraint). The path begins on the $R_p\, e^{rt} = 0$ line. It travels in the same direction as -N; that is, the distance from $R_p e^{rt} = 0$ increases over time. It may or may not travel along the quota constraint while still above $\overline{p}$. An optimal path which starts with $p > \overline{p}$ either terminates at the intersection of $\overline{p}$ and the constraint or it passes below $\overline{p}$ in the interior. Should it drop below (or start below) $\overline{p}$, there will be a decreasing number of fringe firms. Eventually the path hits the constraint. The path then follows the constraint up to $\overline{p}$ where it stops.

VIII. The Tariff

A constant per unit tariff on the dominant firm has a different effect in a dynamic model than in the static model. In a static model (where the fringe firms enter instantaneousy), the dominant firm would price at p. The U. S. government, by setting a tariff equal to $\overline{p}$ - c, could capture all of the profits of the dominant firm, $(\overline{p} - c)\ [f(\overline{p}) - x]$, without creating any distortions. That is, the tariff redistributes income internationally and has no real effect.

In contrast, where entry is gradual, the tariff has real effects. The tariff is equivalent to an increase in c (which has ambiguous comparative static effects on τ, α_1, and α_2). At any moment in time, it will affect pricing by the dominant firm, the rate of entry of fringe firms, and output.

Presumably, the objective of the U. S. government is to maximize domestic welfare (e.g., consumer surplus plus domestic fringe profits plus tariff revenues). Given this objective, the optimal tariff in the dynamic model will lie between no tariff and the optimal static model tariff. The heuristic explanation is that consumers benefit when the dominant firm prices below $\overline{p}$ for a period to drive out the domestic firms. Were the government to set the tariff at the optimal static level, $\overline{p} - c$, the benefits of such low pricing would never be obtained.

Table 1 illustates how the optimal tariff varies with the initial number of firms and a constraint on the minimum number of U. S. fringe firms (section V). In the example in the table, U. S. welfare is higher in the case where there are a large initial number of fringe firms, $x_0 = 30$, than when there is initially only one firm. In the latter case, the dominant firm initially prices high and only after a while lowers the price below p. In contrast, when there are many firms to begin with, the dominant firm prices aggressively from the start. When x_0 is relatively large (at least in this example), the optimal tariff is also relatively large.

As all of the examples show, going from no tariff to the optimal tariff lowers the U. S. consumer surplus, raises the profits of the fringe firms, raises U. S. welfare, lowers the profits of the dominant firm, and lowers the total welare (U. S. welfare plus the profits of the dominant firm). Similarly, as the table shows, an increase in the minimum number of firms constraint, $\underline{x}$, lowers welfare. As this constraint increases, the optimal tariff rises. Where there is a constraint, adding the tariff lowers the surplus and total welfare less while raising the profits of the fringe firms and the U. S. welfare by more than when there is no constraint. Thus, at least in this example, some of the harmful effects of the tariff and the constraint are partially offsetting.

IX. Conclusions

A low-cost foreign firm will dump during certain time periods but eventually it will limit price. It is never in the best interests of this dominant firm to predate.

TABLE 1

Tariff Simulation Results

x_0:	1	1	30	30	30
x:	0	1	0	10	25
No tariff					
τ	30.189	29.757	36.105	33.209	27.615
Present values					
U. S. consumer surplus	34.795	34.747	37.128	36.739	36.028
Profits of fringe firms	1.040	1.026	1.335	1.241	1.133
U. S. welfare	35.835	35.774	38.462	37.979	37.161
Profits of dominant firm	24.850	24.782	21.828	21.142	20.056
Total welfare	60.685	60.556	60.290	59.121	57.218
Optimal tariff	4.023	4.025	3.796	3.875	4.025
τ	84.560	83.966	82.913	80.388	77.223
Present values (000's)					
U. S. consumer surplus	31.153	31.145	33.055	32.869	32.537
Tariff revenues	1.065	1.067	1.575	1.618	1.722
Profits of fringe firms	15.404	15.370	13.783	13.504	13.053
U. S. welfare	47.623	47.582	48.413	47.990	47.313
Profits of dominant firm	7.925	7.913	6.604	6.283	5.725
Total welfare	55.547	55.495	55.022	54.273	53.038
Percent change from no tariff to optimal tariff					
U. S. consumer surplus	-10.47	-10.37	-10.97	-10.53	- 9.69
Profits of fringe firms	2.40	4.00	17.98	30.38	51.99
U. S. welfare	32.90	33.01	25.87	26.36	27.32
Profits of dominant firm	-68.11	-68.07	-69.75	-70.28	-71.45
Total welfare	- 8.47	- 8.36	- 8.74	- 8.20	- 7.31

NOTE.--The parameters are: $p = 10$, $c = 5$, $k = 0.01$, $r = 0.03$, $a = 250$, and $b = 10$.

Were the government of the domestic country to impose such a stiff tariff that the dominant firm never prices below its domestic price, its pricing pattern would still be similar to the unconstrained case. That is, it initially charges a relatively high price, then lowers its price to drive out the domestic firms, and eventually limit prices. Further, under a policy that mandates a minimum number of domestic firms, pricing by the dominant firm follows the same pattern.

A quota, on the other hand, prevents the dominant firm from eliminating the domestic industry. Nevertheless, its price starts high, falls, and then rises. Thus, under all of these policies, a complex temporal price path will be observed rather than the constant or cyclical patterns described in earlier studies. As a result of this temporal price path, the optimal tariff in a dynamic model is lower than the optimal tariff in a static model. While such a tariff must lower international welfare, it can raise domestic welfare. In contrast, policies that prevent high-cost domestic firms from going out of business are likely to lower domestic welfare.

Footnotes

*We thank Larry Karp and the participants at workshops at Rice University, the University of California (Berkeley), the Hebrew University, Ben Gurion University, and North Carolina State University for valuable comments on our research.

[1]These figure are based on figures on completed U. S. Treasury Department and U. S. International Trade Commission cases from 1962 through 1977 in the Michigan Yearbook of International Legal Studies (Antidumping Law: Policy and Implementation, 1979).

[2]As Rodriquez (1979, p. 194) points out based on a U. S. Senate Report, the antidumping law was directed not at infrequent sales at less than cost but at "the practice of systematically selling at prices which will not permit recovery of all costs. . . ." The U. S. Treasury, however, in its informal guidelines has interpreted the terms to mean "within a business cycle." Rodriquez notes that it has been alleged that the Treasury has concluded that the first one or two months of a six-month investigative period are an extended period of time.

[3]See Flaherty (1980) for a defense of open-loop rather than closed-loop models in which firms incur adjustment costs and choose output rates.

[4]We assume that the Japanese do not allow U. S. firms to export to Japan. In any case, in most of the models discussed below it would not pay for a U. S. firm to export to Japan.

[5]Gaskins (1970) gives a justification for a constant rate, k, of adjustment: If costs of entry are quadratic, this linear entry specification will result.

[6]In this example, $x_0 = 1$, $\bar{p} = 10$, $c = 5$, $k = 0.01$, $r = 0.1$, $a = 10$, and $b = 250$. As a result, $\alpha_1 = 7.45$, $\alpha_2 = 1.46873 \times 10^{-14}$, $\tau = 298.2061$, $p(0) = 14.95$, and $y(0) = 43.2479$.

[7]Indeed, the U. S. International Trade Commission has often cited predatory intent as one factor that may lead to a finding of injury when it is combined with some other evidence of competitive harm (see Krauland, 1979, p. 168).

[8]$dp(0)/da = (1/2b) > 0$ and $dp(0)/db = (x_0 - a)/(2b^2) < 0$.

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