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Children's Estimation of Peripheral Information Drives Improvements in Approximate Number Sense

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Abstract

Children rely on their approximate number system (ANS) to guess quantities from a young age. Studies have shown that older children displayed better ANS performance. However, previous research did not provide an explanation for this ANS improvement. We show that children's development in ANS is primarily driven by improved attentional control and awareness of peripheral information. Children guess the number of dots on a computer screen while being eye-tracked in our experiment. The behavioral and eye-tracking results provide supporting evidence for our account. Our analysis shows that children estimate better under the longer display-time condition and more visual foveation, with the effect of visual foveation mediating that of time. It also shows that older children make fewer underestimations because they are better at directing their attention and gaze toward areas of interest, and they are also more aware of dots in their peripheral vision. Our finding suggests that the development of children's ANS is significantly impacted by the development of children's non-numerical cognitive abilities.

Keywords: approximate number system; eye-tracking

Introduction

The approximate number system (ANS) is the ability to represent large numerical magnitudes (Dehaene, 1997; Cantlon, Platt, & Brannon, 2009; Dehaene, 2009). This ability to guess numbers is present in infants (Xu & Spelke, 2000; Lipton & Spelke, 2003), human adults (Whalen, Gallistel, & Gelman, 1999), and also non-human animals (Brannon, Wusthoff, Gallistel, & Gibbon, 2001; Dehaene, Dehaene-Lambertz, & Cohen, 1998; Cantlon & Brannon, 2006). ANS is important in the sense that it is considered to be the basis for mathematical abilities (Dehaene, 2001), and the performance of ANS is related to the development of school mathematics in children (Halberda, Mazzocco, & Feigenson, 2008; Mazzocco, Feigenson, & Halberda, 2011; Libertus, Feigenson, & Halberda, 2011; Piazza et al., 2010).

Previous research has shown that the ANS is an imprecise, innate ability that stems from our evolutionary relatives (Cantlon, 2012; Cantlon et al., 2009; Dehaene, 2001; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004; Cantlon & Brannon, 2007). We start to develop this ability in infancy and continue developing until we reach adulthood (Piazza et al., 2010; Halberda & Feigenson, 2008). Studies have shown that animals have a basic number sense in a simpler form (Dehaene et al., 1998). This sense is useful to guide them through everyday tasks like foraging (Yang & Chiao, 2016). Other studies show that, in human children, ANS performances improve with development: Infants at 6 months of age can distinguish a different number of displayed elements with a ratio of 1:2 (Xu & Spelke, 2000; McCrink & Wynn, 2007), whereas infants at 10 months of age can do so with a ratio of 2:3 (Xu & Arriaga, 2007). It is also shown that the acuity of ANS in children from 3 to 6 years old improve and continues improving until early adolescence (Halberda & Feigenson, 2008).

Many studies that involve the psychophysical models of ANS either follow or derive the Weber's law (Cheyette & Piantadosi, 2019; Feigenson, Dehaene, & Spelke, 2004; Barth et al., 2006; Whalen et al., 1999; Halberda et al., 2008; Gallistel & Gelman, 2000). This ANS models assume that the mean estimate varies linearly as a function of the true numerosity. At the same time, the estimation error (standard deviation) also varies linearly as a function of the estimate. The Weber fraction, w, is the scalar factor that captures the relationship between the mean and the variability, with a lower value of w indicating better acuity. Data from previous studies show that while 6-month-old infants only have a Weber fraction of 1.0 (Xu & Arriaga, 2007), their acuity improves with age and can achieve a Weber fraction of 0.1 in adulthood (Halberda & Feigenson, 2008).

Some studies have shown that this psychophysical model of ANS is oversimplified: participants display different ANS acuity when performing different tasks with varying non-numerical stimulus features, such as the clustering of dots, presentation format, and total surface area (Price, Palmer, Battista, & Ansari, 2012; Im, Zhong, & Halberda, 2016; DeWind & Brannon, 2012; DeWind, Adams, Platt, & Brannon, 2015; Tokita & Ishiguchi, 2010). These results suggest that other domain-general cognitive or decision processes contribute to the performance of ANS (Price et al., 2012; Sophian & Chu, 2008).

Our goal for this paper is to identify factors in attention control and visual foveation that drive children's development in ANS. To explain the huge individual differences in ANS performances and the development of ANS in children, researchers have proposed that this development can be attributed to the development of some non-numerical abilities in children, including working memory capacity, spatial skills, and executive functioning that influence children's mathematical reasoning (Halberda & Feigenson, 2008; Fuhs

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& McNeil, 2013; Halberda et al., 2008; Gilmore et al., 2013; Susperreguy, Douglas, Xu, Molina-Rojas, & LeFevre, 2020; LeFevre et al., 2013). However, the relationship between ANS and other cognitive domain-general processes remains obscure.

We conducted a behavioral experiment with an eye-tracking technique to investigate cognitive processes involved in ANS. The eye-tracking technique has become increasingly popular in mathematical education and learning research (Strohmaier, MacKay, Obersteiner, & Reiss, 2020; Lai et al., 2013), because it provides data on eye movements that are important for understanding the underlying cognitive processes of learning that were hard to analyze otherwise (Hartmann, 2015). Eye movements are crucial to understanding the role of attention in children's learning process. Studies have shown that cognitive focus can be inferred from visual foveation (Just & Carpenter, 1980). In particular, overt attention is achieved by visually foveate on an area of interest and covert attention can arise from being attentive to peripheral information (Carrasco, 2011). For these reasons, the eye-tracking technique is a beneficial addition to the behavioral experiment, as it provides data on eye movements that are important for the quantitative modeling of children's ANS that investigate the relationship among the underlying cognitive processes.

There are two popular paradigms to measure ANS performances: the estimation paradigm in which participants estimate the numerosity of dot arrays (Izard & Dehaene, 2008; Cheyette & Piantadosi, 2019; Whalen et al., 1999; Cordes, Gelman, Gallistel, & Whalen, 2001; Le Corre & Carey, 2007) and the discrimination paradigm in which participants compare the numerical magnitude of two numerical stimuli (Xu & Spelke, 2000; Sophian & Chu, 2008; Price et al., 2012; Piazza et al., 2010). In this study, we used an estimation paradigm. Given that there is a fuzzy boundary between ANS and other numerical cognitive processes, namely counting, subitizing, or groupitizing (Ciccione & Dehaene, 2020; Burr, Turi, & Anobile, 2010; Schleifer & Landerl, 2011; Starkey & McCandliss, 2014), participants may have used systems other than or in addition to ANS when performing the estimation tasks. Since each of the stimuli was presented for a short period of time (2.5 seconds to 5 seconds), there is, not enough time for children to count all dots. Our results show that participants' error patterns follow Weber's law, which suggests that this experiment design mainly measured ANS performances, instead of counting.

We generated our findings from the results of behavioral experiments and model-driven analysis, and we show that children's development of ANS is attributed to their improvement in attentional control and awareness of peripheral information. Our results show that children's estimation improves with more time and more visual foveation. We also found that older children were better at estimation tasks. In addition, we show that children displayed no statistical difference in their estimation of foveated dots; however, individual differences are significant in the estimation of peripheral dots, with older children displaying an improvement in their estimation. Therefore, we suggest that older children perform better on ANS tasks because they take into account the dots that are in their peripheral vision when making an estimation. Our results provide a possible explanation for some previous findings that suggested that ANS performances are dependent on non-numerical factors (Im et al., 2016), and our result can shed some light on the relationship between ANS performances and other non-numerical cognitive processes.

Methods

Participants

71 children between the ages of 3 to 6 participated in the experiment either at the museum or in the lab. Data from 11 participants were removed because participants' ages were greater than 7. 5 participants were removed due to the incompletion of the experiment. 2 of them were further removed due to data incompletion.

Design

Throughout the experiment, we asked children to estimate a varied number of dots on a monitor (from 3 to 15). Each child participant did this 26 times, for a total of 26 trials, while they were eye tracked. We analyzed their eye-tracking data to determine whether the estimates were influenced by the number of dots they visually fixated. Figure 1 illustrates each trial. For each trial, the participant viewed a fixation cross for 1500ms and saw between 3 and 15 dots in randomly scattered positions for either 2500ms or 5000ms, depending on the condition. Then the participant saw an image of a toy for 1000ms. Finally, the researcher asked the child participant to guess the number of dots that appeared on the screen. The researcher gave positive feedback to the answer, regardless of the answer's accuracy.

Analysis

Data from a total of 1378 trials were collected from 53 participants who met the experimental requirement. 93 trials were removed because they contain an estimate of more or less than 3 standard deviations away from the mean. 368 trials were further removed because the eye tracker didn't capture the participant's eye movement or the participant failed to focus on the screen during the testing phase for more than 50% of the display time. Further data analysis and modeling were conducted on the remaining 917 trials.

Results

More time improves estimation mean and standard deviation

Figure 2 shows children's overall ANS performances. In Figure 2A, data from participants is shown with the best-fitted line from linear regression. The best-fit line has a positive slope, which indicates that children's mean estimations increase as the number of dots increases. The slope, however,

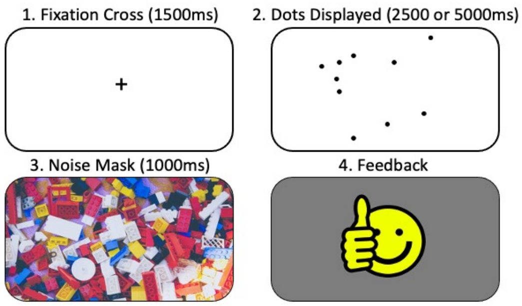

Figure 1: Each of the four images represents a stage of a trial. Stage 1: A fixation cross appears for 1,500 ms. Stage 2: The fixation cross is removed, and dots appear on the screen for either 2500ms or 5000ms, depending on the display-time condition. Stage 3: The display is masked by a picture of toys for 1000 ms. Stage 4: The researcher asks the participant to state the number of dots they thought they saw, then the researcher provides positive feedback to the answer.

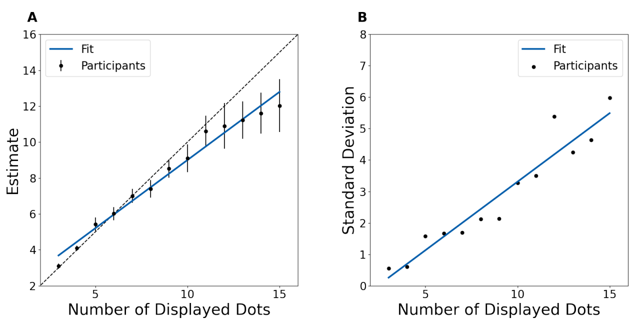

Figure 2: Estimates and standard deviations as a function of displayed dots shown.

is less than 1, indicating that children underestimate higher values. This result replicates findings in earlier studies that showed an underestimation bias in an estimation paradigm (Cheyette & Piantadosi, 2019; Izard & Dehaene, 2008). Figure 2B shows that the standard deviation of estimates also increases linearly with the number of dots, which follows Weber's law. This indicates that children made more estimation errors as the numerosity increases.

We performed a Bayesian hierarchical regression to test the effect of time on the mean estimate and the Weber fraction. The model includes regression terms that capture both the group-level effects and the individual-level effects. Group-level effects demonstrate the effect of time on the estimate on all participants, as a group. Individual-level effects capture the between-subject effects in this participant group. In this model, it is assumed that given a trial with n dots, the mean estimate of every participant is sampled from a normal distribution $\mathcal{N}(\mu, \sigma^2)$ with $\mu = \beta \cdot n$ and $\sigma = \omega \cdot \beta \cdot n$, where β (slope) and ω (Weber fraction) are fit parameters determined

Table 1: Group level regression weights and their 95% confidence intervals for the effect of time on slopes and Weber fractions.

Parameter	Value	2.5%	97.5%
β_0	-0.10	-0.16	-0.03
β_t	0.03	-0.01	0.06
w_0	-0.84	-1.10	-0.61
w_t	-0.39	-0.59	-0.20

by the model. The model also assumed a logarithmic effect of time (t) on the mean and the Weber fraction. We denote the group baseline slopes and Weber fractions as β_0 and w_0, and denote the effect of time on slopes as β_t and Weber fractions w_t. All subject effects are denoted with a prefix of $subj$. In this model, the slope and Weber fraction are calculated as:

$$\log(beta) = \beta_0 + subj\beta_0 + (\beta_t + subj\beta_t) \cdot \log(t)$$

$$\log(w) = w_0 + subjw_0 + (w_t + subjw_t) \cdot \log(t)$$

Table 1 shows the results from this model. As shown, the group-level regression weights demonstrate the effect of time on the slope and the Weber fraction. The result reveals that there is a small effect of time on the slope (β_t = 0.03; CI=[-0.01, 0.06]) in which more time leads to less underestimation. There is also a significant effect of time on the Weber fraction (w_t = –0.39; CI=[-0.59, -0.20]). The negative value of w_t means that, with more time, the standard deviation of participants' estimates decreases.

Figure 3 shows the behavioral data as well as our modeling results, with different colored dots and lines indicating data from two separate display-time conditions. As it shows, longer display time improves participants' mean estimates slightly but drives great improvements in the standard deviations of estimates. Firstly, Figure 3A shows the relationship between estimates (y-axis) and displayed numbers (x-axis). As it shows, the slope increased for the long display-time condition by 1% (0.92 - 0.93), indicating that children underestimate less slightly when given more time. Secondly, Figure 3B shows the relationship between the standard deviation of estimates (y-axis) and displayed number (x-axis). As it shows, the standard deviation decreases significantly for the long display-time condition, and the Weber fraction decreases for the long display-time condition by 20% (0.30 - 0.24). Therefore, we conclude that participants underestimate, with a reduced underestimation and less standard deviation as display-time increases. This result is consistent with previous finding that also shows that ANS is not a parallel process: it depends on a serial accumulation mechanism (Cheyette & Piantadosi, 2019).

Visual foveation, not time, has a greater effect on estimation

We used an eye tracker to record the participants' eye movements while they were performing number estimation tasks.

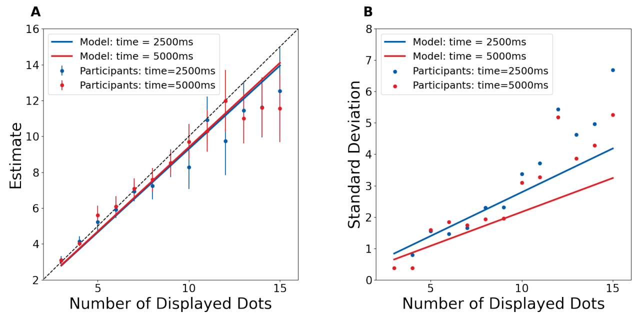

Figure 3: Results of Bayesian hierarchical regression with behavioral data. (A) Mean estimates as a function of the number of displayed dots, under two display-time conditions. (B) The standard deviation of participants' estimates as a function of the number of displayed dots, under two display-time conditions.

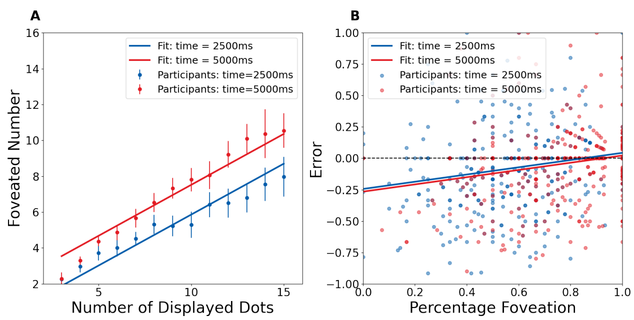

Figure 4: (A) The number of dots foveated as a function of the number of displayed dots. (B) Percentage estimation error as a function of percentage foveation.

Then, we analyze this eye-tracking data to determine the effect of visual foveation on estimates. We counted the number of dots that lie within the 5° of the center of the participants' gaze path as the number of dots that the participants foveated on. We denote the percentage of dots they foveated on as the percentage foveation.

Figure 4A shows the relationship between the number of foveated dots (y-axis) and the number of dots displayed (x-axis) for both display-time conditions. It shows that under the long display-time condition, participants were able to foveate on more dots. This result is consistent with a previous study that argues that longer display time leads to better ANS acuity in a dot comparison task (Inglis & Gilmore, 2013). This phenomenon could also explain the previous observation that participants made better estimates under the long display-time condition: it could be that in addition to or instead of display time, visual foveation has an impact on participants' estimates.

To test this hypothesis, we first look at participants' estimation performances as a function of their percentage foveation. Figure 4B shows the relationship between the error in estimation (y-axis) and the number of foveated dots (x-axis) for both display-time conditions, with a positive error denoting an overestimation and a negative error denoting an underestimation. As it shows, the mean of error differs significantly when percentage foveation increases from 0% to 100%: as percentage foveation increases, the percentage error increased from negative to almost 0. However, when controlling the percentage foveation, the mean of error (slope) for both time conditions doesn't exhibit a significant difference. As a result, the effect of time disappears when both time and foveation are considered.

Next, we aim to quantify the effect of visual foveation on estimates. We would also like to investigate whether visual foveation mediates the effect of time. We performed another Bayesian hierarchical regression, including both group-level effects and individual-level effects of visual foveation (s) and time (t) on the mean estimate and the Weber fraction. In this model, we denote the group-level effect of visual foveation on the slope as β_s and its effect on the Weber fraction as w_s, with their individual effects terms denoted with a prefix of $subj$. The slope and Weber fraction are calculated as:

$$\log(beta) = (\beta_0 + subj\beta_0) +$$
$$(\beta_t + subj\beta_t) \cdot \log(t) + (\beta_s + subj\beta_s) \cdot s$$
$$\log(w) = (w_0 + subjw_0) +$$
$$(w_t + subjw_t) \cdot \log(t) + (w_s + subjw_s) \cdot s$$

Figure 5 A, B show our modeling results. It shows that visual foveation, represented by the percentage of dots foveated by participants, has a significant effect on both the group mean slope and Weber fraction, as depicted by the black solid line and black dots. From 0% to 100% percentage foveation, the mean slope increased by 18% (0.82 - 0.97), and the Weber fraction decreased by 67% (0.55 - 0.18), suggesting that with more visual foveation, participants made less underestimation with less standard deviation.

Our model also demonstrates that visual foveation mediates the effect of time on both the mean estimate and the Weber fraction. Table 2 shows the regression weights for the effect of time and visual foveation. Comparing parameter values in Table 1 and Table 2, we show that as the model includes the visual foveation effect (β_s and w_s), the effect of time on the slope (β_t) gets closer to 0 (from 0.03 to -0.02), which means that the effect of time diminishes when taking into account visual foveation. It is also clear that the effect of time on the Weber fraction (w_t) reduced significantly (from -0.39 to -0.04) in this model, which is a result of visual foveation mediating the effect of time on the Weber fraction.

The result from this model supports our hypothesis that the ability to foveate on quantities influences ANS performances and contributes to the underestimation of displayed dots. It also supports our hypothesis that visual foveation is the primary contributing factor to ANS performances, and not time. Furthermore, it shows that the ANS performance of children depends on their ability to foveate on displayed dots, which

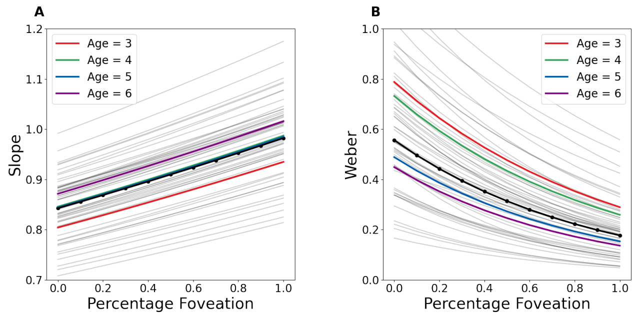

Figure 5: Results of Bayesian hierarchical regression. (A) The mean (bold black line with dots) and individual (transparent black lines) slope as a function of percentage foveation, with the mean from each age group (colored lines). (B) The mean (bold black line with dots) and individual (transparent black lines) Weber fraction as a function of percentage foveation, with the mean from each age group (colored lines).

Table 2: Group level regression weights and their 95% confidence intervals for the effect of time and visual foveation on slopes and Weber fractions.

Parameter	Value	2.5%	97.5%
β_0	-0.15	-0.22	-0.08
β_t	-0.02	-0.06	0.02
β_s	0.15	0.07	0.23
w_0	-0.52	-0.77	-0.26
w_t	-0.04	-0.24	0.16
w_s	-1.15	-1.48	-0.82

suggests that ANS is influenced by other nonnumerical processes.

Older children perform better on ANS tasks and have better attention and gaze control

We want to understand the difference in ANS performances in children from different age groups. As Figure 6 shows, older children made more accurate estimates: they performed the estimation tasks with less underestimation and smaller standard deviation. This is consistent with previous research that shows an improvement with age in ANS performances for children from 3 to 6 years old (Halberda & Feigenson, 2008).

To put the effect of age in a clearer picture, we analyzed previous modeling results by showing the mean of the slope and the Weber fraction grouped by participants' age. Effects of children's age on estimation is demonstrated in Figure 5, shown by bold colored lines to differentiate among each age group. From the age of 3 to the age of 6, older children's estimates show greater mean slopes and smaller Weber fractions across different percentage foveation (Figure 5 C, D).

Figure 7 also shows that older children are better at directing their attention to displayed dots, resulting in an increase of percentage foveation. The results are consistent with the

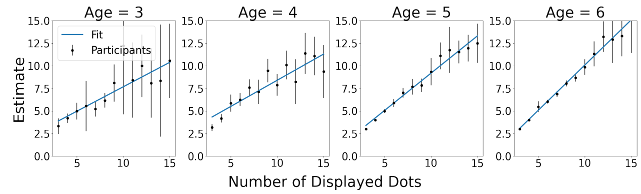

Figure 6: Estimated number of dots as a function of the displayed number of dots from each age group.

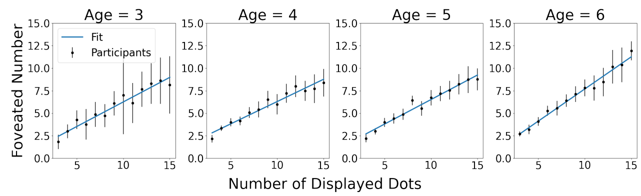

Figure 7: Foveated number of dots as a function of displayed number of dots from each age group.

theory that attention and gaze control contribute to the development of children's ANS.

Older children are better at estimating peripheral dots

To further investigate the mechanism behind children's improvement in estimation, we performed a third Bayesian hierarchical regression. We focused on the visual factors that contribute to children's development of ANS. This comprehensive model investigates the influence of different visual inputs on children's estimates. Components in this model are: the number of foveated dots (N_f), the number of non-foveated dots (dots in the peripheral visual field) (N_p), and the number of dots that participants foveated on initially and then re-foveated on after looking away (N_d). These components' corresponding regression parameters include terms that capture the group effects (β_f, β_p, and β_d) as well as terms that capture the individual effects ($subj\beta_f$, $subj\beta_p$, and $subj\beta_d$). The mean estimate of every participant is sampled from a normal distribution $\mathcal{N}(\mu, w^2)$ with μ and w expressed as:

$$\mu = (\beta_f + subj\beta_f) \cdot (N_f + (\beta_d + subj\beta_d) \cdot N_d) +$$
$$((\beta_p + subj\beta_p) \cdot N_p)$$
$$w = (\beta_n + subj\beta_n) \cdot \mu$$

We ran this model on all participants across age groups. Our modeling results show that the key factor influencing children's ANS performance in the different age groups is the amount of peripheral information they used to make an estimation. Table 3 shows that foveated dots almost always contribute to children's estimates (β_f = 0.96; CI=[0.91, 1.00]) while less percentage of peripheral dots were included in estimates (β_p = 0.84; CI=[0.76, 0.97]). It also shows that children don't double count dots that they have already foveated on (β_d = 0.04; CI=[0.00, 0.10]).

Table 3: Group level regression weights and their 95% confidence intervals for the effect of foveated (β_f), peripheral (β_p) and double counted dots (β_d).

Parameter	Value	2.5%	97.5%
β_f	0.96	0.91	1.00
β_d	0.04	0.00	0.10
β_p	0.87	0.76	0.97

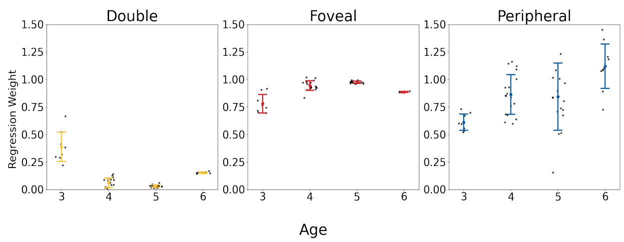

Figure 8: The double, foveal, and peripheral contribution to estimates.

We then ran this model four times on participants from each of the four age groups. Figure 8 shows our modeling results. As it shows, the mean peripheral regression weight for older children is significantly higher than that of younger children: The mean peripheral regression weight for 6-year-old children increases 84% (from 0.61 to 1.12) compared with that for 3-year-old children. This can explain the improvement in ANS acuity for older children. It also shows that the peripheral regression weights vary among individuals, which offers a hypothesis as to why there exist large individual differences in ANS performances. It shows that regression weights for foveal information are very close to 1, and don't have much difference among individuals and across age groups. This result indicates that children from 3 to 6 years old can almost always incorporate seen information into their estimates. The low and invariant values of double-counting regression weights show that children from 3 to 6 years old almost never double-count. Therefore, our model provides evidence that older children have more accurate estimations because they are more capable of incorporating information in their peripheral vision.

Conclusions and Discussion

Our experiment demonstrated that children's development in ANS can be attributed to their improvement in attention and gaze control, as well as their increased processing power for peripheral information. Firstly, We replicate previous findings which show that children develop their ANS and estimate more accurately as they age (Halberda & Feigenson, 2008). Additionally, our results also agree with previous studies that ANS is a serial accumulation process in which more time and visual foveation lead to better estimation, and that visual foveation mediates the effect of time (Cheyette & Piantadosi, 2019). Then, we analyzed the contribution of each type of visual input to children's ANS performances in an estimation. We show that while foveal information is almost always included in the estimation, the inclusion of peripheral information is significantly different across age groups. Older children incorporate much more peripheral information in their estimates. Combined with the finding that older children have more foveation on displayed dots, we conclude that as children develop their attention control and awareness of peripheral information, their ANS performances improve.

We provide evidence that participants used ANS during the experiment, and that other numerical processes, counting or groupitizing, had little effect. Our data shows properties of Weber's law, which is a result of estimation, instead of counting. The pattern of counting error is different from that of Weber's law (Odic, Im, Eisinger, Ly, & Halberda, 2016). Counting errors obey binomial statistics: the error should grow in proportion to the square root of the numerosity. However, in estimation, the error follows Weber's law that the error grows in proportion to the estimates (Cordes et al., 2001). Figure 2 shows the mean and standard deviation of estimates. As it shows, the error of estimation grows linearly as the numerosity, demonstrates an estimation behavior from the participants. If participants were counting, the relationship between error and numerosity should be sublinear. Other studies have also found that young children require a long time to perform a serial counting task: it is shown that 7-year-old children need approximately 5000ms to count 7 dots, but adults took only about 2000ms (Svenson & Sjöberg, 1983). The display-time conditions that were used in our experiment, although long enough for adults to count, were likely not long enough for children. Furthermore, studies that focused on the groupitizing process show that while adults and adolescents can use groupitizing strategy in their counting procedure to achieve a faster response time, 6-year-old children cannot effectively use such strategy to shorten their response time in counting tasks (Starkey & McCandliss, 2014; Ciccione & Dehaene, 2020). Since participants in the current study are between the ages of 3 to 6, they are not able to use groupitizing strategies in the experiment. Therefore, it is safe to conclude that participants mainly used ANS techniques.

The present experiment is limited in the degree to which it can disambiguate between different possible mechanisms driving participant differences in the amount of peripheral information that they take into account when they make estimations. The improvement in estimating peripheral information may be attributed to a development in the visual information-gathering system or in the way children process this visual information. This improvement could also be affected by the development of other factors in visual cognition as children mature. While our work provides a fuller picture of the relationship between ANS and other non-numerical cognitive functions, further research on the visual-cognitive processes is needed to advance the current understanding of ANS mechanisms.

References

Barth, H., La Mont, K., Lipton, J., Dehaene, S., Kanwisher, N., & Spelke, E. (2006). Non-symbolic arithmetic in adults and young children. *Cognition*, *98*(3), 199–222.

Brannon, E. M., Wusthoff, C. J., Gallistel, C., & Gibbon, J. (2001). Numerical Subtraction in the Pigeon: Evidence for a Linear Subjective Number Scale. *Psychological Science*, *12*(3), 238–243.

Burr, D. C., Turi, M., & Anobile, G. (2010). Subitizing but not estimation of numerosity requires attentional resources. *Journal of Vision*, *10*(6), 20–20.

Cantlon, J. F. (2012). Math, monkeys, and the developing brain. *Proceedings of the National Academy of Sciences*, *109*(supplement_1), 10725–10732.

Cantlon, J. F., & Brannon, E. M. (2006). Shared System for Ordering Small and Large Numbers in Monkeys and Humans. *Psychological Science*, *17*(5), 401–406.

Cantlon, J. F., & Brannon, E. M. (2007). Basic Math in Monkeys and College Students. *PLoS Biology*, *5*(12), e328.

Cantlon, J. F., Platt, M. L., & Brannon, E. M. (2009). Beyond the number domain. *Trends in Cognitive Sciences*, *13*(2), 83–91.

Carrasco, M. (2011). Visual attention: The past 25 years. *Vision Research*, *51*(13), 1484–1525.

Cheyette, S. J., & Piantadosi, S. T. (2019). A primarily serial, foveal accumulator underlies approximate numerical estimation. *Proceedings of the National Academy of Sciences*, *116*(36), 17729–17734.

Ciccione, L., & Dehaene, S. (2020). Grouping Mechanisms in Numerosity Perception. *Open Mind*, *4*, 102–118.

Cordes, S., Gelman, R., Gallistel, C. R., & Whalen, J. (2001). Variability signatures distinguish verbal from nonverbal counting for both large and small numbers. *Psychonomic Bulletin & Review*, *8*(4), 698–707.

Dehaene, S. (1997). *The number sense: how the mind creates mathematics*. New York: Oxford University Press.

Dehaene, S. (2001). Precis of The Number Sense. *Mind and Language*, *16*(1), 16–36.

Dehaene, S. (2009). Origins of Mathematical Intuitions: The Case of Arithmetic. *Annals of the New York Academy of Sciences*, *1156*(1), 232–259.

Dehaene, S., Dehaene-Lambertz, G., & Cohen, L. (1998). Abstract representations of numbers in the animal and human brain. *Trends in Neurosciences*, *21*(8), 355–361.

DeWind, N. K., Adams, G. K., Platt, M. L., & Brannon, E. M. (2015). Modeling the approximate number system to quantify the contribution of visual stimulus features. *Cognition*, *142*, 247–265.

DeWind, N. K., & Brannon, E. M. (2012). Malleability of the approximate number system: effects of feedback and training. *Frontiers in Human Neuroscience*, *6*.

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, *8*(7), 307–314.

Fuhs, M. W., & McNeil, N. M. (2013). ANS acuity and mathematics ability in preschoolers from low-income homes: contributions of inhibitory control. *Developmental Science*, *16*(1), 136–148.

Gallistel, C., & Gelman, R. (2000). Non-verbal numerical cognition: from reals to integers. *Trends in Cognitive Sciences*, *4*(2), 59–65.

Gilmore, C., Attridge, N., Clayton, S., Cragg, L., Johnson, S., Marlow, N., … Inglis, M. (2013). Individual Differences in Inhibitory Control, Not Non-Verbal Number Acuity, Correlate with Mathematics Achievement. *PLoS ONE*, *8*(6), e67374.

Halberda, J., & Feigenson, L. (2008). Developmental change in the acuity of the "number sense": The approximate number system in 3-, 4-, 5-, and 6-year-olds and adults. *Developmental Psychology*, *44*(5), 1457–1465.

Halberda, J., Mazzocco, M. M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature*, *455*(7213), 665–668.

Hartmann, M. (2015). Numbers in the eye of the beholder: What do eye movements reveal about numerical cognition? *Cognitive Processing*, *16*(S1), 245–248.

Im, H. Y., Zhong, S.-h., & Halberda, J. (2016). Grouping by proximity and the visual impression of approximate number in random dot arrays. *Vision Research*, *126*, 291–307.

Inglis, M., & Gilmore, C. (2013). Sampling from the mental number line: How are approximate number system representations formed? *Cognition*, *129*(1), 63–69.

Izard, V., & Dehaene, S. (2008). Calibrating the mental number line. *Cognition*, *106*(3), 1221–1247.

Just, M. A., & Carpenter, P. A. (1980). A theory of reading: From eye fixations to comprehension. *Psychological Review*, *87*(4), 329–354.

Lai, M.-L., Tsai, M.-J., Yang, F.-Y., Hsu, C.-Y., Liu, T.-C., Lee, S. W.-Y., … Tsai, C.-C. (2013). A review of using eye-tracking technology in exploring learning from 2000 to 2012. *Educational Research Review*, *10*, 90–115.

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition*, *105*(2), 395–438.

LeFevre, J.-A., Jimenez Lira, C., Sowinski, C., Cankaya, O., Kamawar, D., & Skwarchuk, S.-L. (2013). Charting the role of the number line in mathematical development. *Frontiers in Psychology*, *4*.

Libertus, M. E., Feigenson, L., & Halberda, J. (2011). Preschool acuity of the approximate number system correlates with school math ability: Approximate number system and math abilities. *Developmental Science*, *14*(6), 1292–1300.

Lipton, J. S., & Spelke, E. S. (2003). Origins of Number Sense: Large-Number Discrimination in Human Infants. *Psychological Science*, *14*(5), 396–401.

Mazzocco, M. M. M., Feigenson, L., & Halberda, J. (2011). Impaired Acuity of the Approximate Number System Underlies Mathematical Learning Disability (Dyscalculia): Impaired Numerical Acuity Contributes to MLD. *Child Development*, *82*(4), 1224–1237.

McCrink, K., & Wynn, K. (2007). Ratio Abstraction by 6-Month-Old Infants. *Psychological Science*, *18*(8), 740–745.

Odic, D., Im, H. Y., Eisinger, R., Ly, R., & Halberda, J. (2016). PsiMLE: A maximum-likelihood estimation approach to estimating psychophysical scaling and variability more reliably, efficiently, and flexibly. *Behavior Research Methods*, *48*(2), 445–462.

Piazza, M., Facoetti, A., Trussardi, A. N., Berteletti, I., Conte, S., Lucangeli, D., … Zorzi, M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition*, *116*(1), 33–41.

Piazza, M., Izard, V., Pinel, P., Le Bihan, D., & Dehaene, S. (2004). Tuning Curves for Approximate Numerosity in the Human Intraparietal Sulcus. *Neuron*, *44*(3), 547–555.

Price, G. R., Palmer, D., Battista, C., & Ansari, D. (2012). Nonsymbolic numerical magnitude comparison: Reliability and validity of different task variants and outcome measures, and their relationship to arithmetic achievement in adults. *Acta Psychologica*, *140*(1), 50–57.

Schleifer, P., & Landerl, K. (2011). Subitizing and counting in typical and atypical development. *Developmental Science*, *14*(2), 280–291. (_eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1467-7687.2010.00976.x)

Sophian, C., & Chu, Y. (2008). How do people apprehend large numerosities? *Cognition*, *107*(2), 460–478.

Starkey, G. S., & McCandliss, B. D. (2014). The emergence of “groupitizing” in children’s numerical cognition. *Journal of Experimental Child Psychology*, *126*, 120–137.

Strohmaier, A. R., MacKay, K. J., Obersteiner, A., & Reiss, K. M. (2020). Eye-tracking methodology in mathematics education research: A systematic literature review. *Educational Studies in Mathematics*, *104*(2), 147–200.

Susperreguy, M. I., Douglas, H., Xu, C., Molina-Rojas, N., & LeFevre, J.-A. (2020). Expanding the Home Numeracy Model to Chilean children: Relations among parental expectations, attitudes, activities, and children’s mathematical outcomes. *Early Childhood Research Quarterly*, *50*, 16–28.

Svenson, O., & Sjöberg, K. (1983). Speeds of Subitizing and Counting Processes in Different Age Groups. *The Journal of Genetic Psychology*, *142*(2), 203–211.

Tokita, M., & Ishiguchi, A. (2010). How might the discrepancy in the effects of perceptual variables on numerosity judgment be reconciled? *Attention, Perception & Psychophysics*, *72*(7), 1839–1853.

Whalen, J., Gallistel, C., & Gelman, R. (1999). Nonverbal Counting in Humans: The Psychophysics of Number Representation. *Psychological Science*, *10*(2), 130–137.

Xu, F., & Arriaga, R. I. (2007). Number discrimination in 10-month-old infants. *British Journal of Developmental Psychology*, *25*(1), 103–108.

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition*, *74*(1), B1–B11.

Yang, T.-I., & Chiao, C.-C. (2016). Number sense and state-dependent valuation in cuttlefish. *Proceedings of the Royal Society B: Biological Sciences*, *283*(1837), 20161379.