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What Goods Do Countries Trade? New Ricardian Predictions

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### WHAT GOODS DO COUNTRIES TRADE? NEW RICARDIAN PREDICTIONS

#### ARNAUD COSTINOT AND IVANA KOMUNJER

ABSTRACT. Though one of the pillars of the theory of international trade, the extreme predictions of the Ricardian model have made it unsuitable for empirical purposes. A seminal contribution of Eaton and Kortum (2002) is to demonstrate that stochastic productivity differences at the firm-level are sufficient to make the Ricardian model empirically relevant. While successful at explaining trade volumes, their model remains silent with regards to one important question: What goods do countries trade? Our main contribution is to generalize their approach and provide an empirically meaningful answer to this question.

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#### 1. Introduction

Though one of the pillars of the theory of international trade, the extreme predictions of the Ricardian model have made it unsuitable for empirical purposes. As Leamer and Levinsohn (1995) point out: "The Ricardian link between trade patterns and relative labor costs is much too sharp to be found in any real data set."

A seminal contribution of Eaton and Kortum (2002) is to demonstrate that stochastic productivity differences at the firm level are sufficient to make the Ricardian model empirically relevant. When drawn from an extreme value distribution, random productivity shocks imply a gravity-like equation in a Ricardian framework with a continuum of goods, transport costs, and more than two countries. While successful at explaining trade volumes, their model remains silent with regards to one important question: What goods do countries trade? Our main contribution is to generalize their approach and provide an empirically meaningful answer to this question.

Section 2 describes the model. We consider an economy with one factor of production, labor, and multiple products, each available in many varieties. There are constant returns to scale in the production of each variety. The key assumption of our model is that labor productivity may be separated into: a deterministic component, which is country and industry specific; and a stochastic component, randomly drawn across countries, industries, and varieties. The former captures factors such as climate, infrastructure, and institutions that affect the productivity of all producers in a given country and industry, whereas the latter reflects differences in technological know-how at the firm level.

Section 3 derives our predictions on the pattern of trade. Because of random productivity shocks, we can no longer predict trade flows in each variety. Yet, by assuming that each product comes in a large number of varieties, we generate sharp predictions at the industry level. In particular, we show that for any pair of exporters, the (first-order stochastic dominance) ranking of their relative labor productivity fully determines their relative export

<sup>&</sup>lt;sup>1</sup>Acemoglu, Antras, and Helpman (2006), Costinot (2005), Cuñat and Melitz (2006), Levchenko (2004), Matsuyama (2005), Nunn (2005), and Vogel (2004) explicitly model the impact of various institutional features—e.g. labor market flexibility, the quality of contract enforcement, or credit market imperfections—on labor productivity across countries and industries.

performance across industries. Compared to the standard Ricardian model—see e.g. Dornbusch, Fischer, and Samuelson (1977)—our predictions hold under fairly general assumptions on: preferences, transport costs, and number of industries and countries.<sup>2</sup> Moreover, they do not imply the full specialization of countries in a given set of industries.

Section 4 investigates how well our model squares with the empirical evidence. We consider linear regressions tightly connected to our theoretical framework. Using OECD trade and labor productivity data from 1988 to 2003, we find strong support for our new Ricardian predictions: countries do tend to export relatively more (towards any importing country) in sectors where they are relatively more productive.

Our paper contributes to the previous trade literature in two ways. First, it contributes to the theory of comparative advantage. Our model generates clear predictions on the pattern of trade in environments—with both multiple countries and industries—where the standard Ricardian model loses most of its intuitive content; see e.g. Jones (1961) and Wilson (1980). Our approach mirrors Deardorff (1980) who shows how the law of comparative advantage may remain valid, under standard assumptions, when stated in terms of correlations between vectors of trade and autarky prices. In this paper, we weaken the standard Ricardian assumptions—the "chain of comparative advantage" only holds in terms of first-order stochastic dominance—and derive a deterministic relationship between exports and labor productivity across industries.

Second, our paper contributes to the empirical literature on international specialization, including the previous "tests" of the Ricardian model; see e.g. MacDougall (1951), Stern (1962), Balassa (1963), and more recently Golub and Hsieh (2000). While empirically successful, these tests have long been criticized for their lack of theoretical foundations; see Bhagwati (1964). Our model provides such foundations. Since it does not predict full international specialization, we do not have to focus on ad-hoc measures of export performance. Instead, we may use the theory to pin down explicitly what the dependent variable in crossindustry regressions ought to be.

As we discuss in our concluding remarks, our model may also provide an alternative theoretical underpinning of cross-industry regressions when labor is not the only factor of

<sup>&</sup>lt;sup>2</sup>Deardorff (2005) reviews the failures of simple models of comparative advantage at predicting the pattern of trade in economies with more than two goods and two countries.

production. Traditionally, the validity of these regressions depends on strong assumptions on either demand—see e.g. Petri (1980), Romalis (2004), and the voluminous gravity literature based on Armington's preferences—or (the absence of) transport costs—see e.g. Harrigan (1997). Our paper suggests that many of these assumptions may be relaxed, as long as there are random productivity shocks at the firm level.

#### 2. The Model

We consider a world economy comprising i=1,...,I countries and one factor of production—labor. There are j=1,...,J products and constant returns to scale in the production of each product. Labor is perfectly mobile across industries and immobile across countries. The wage of workers in country i is denoted  $w_i$ . Up to this point, this is a standard Ricardian model. We generalize this model by introducing stochastic productivity differences at the firm level. Following Eaton and Kortum (2002), we assume that each product j may come in  $N_j$  varieties  $\omega = 1,...,N_j$ , and denote  $a_{ij}(\omega)$  the constant unit labor requirements for the production of the  $\omega$ th variety of product j in country i. Our first assumption is that:

**A1.** For all countries i, products j, and their varieties  $\omega$ 

(1) 
$$\ln a_{ij}(\omega) = \ln a_{ij} + u_{ij}(\omega),$$

where  $a_{ij} > 0$  and  $u_{ij}(\omega)$  is a random variable drawn independently for each triplet  $(i, j, \omega)$  from a continuous distribution  $F(\cdot)$  such that:  $E[u_{ij}(\omega)] = 0$ .

We interpret  $a_{ij}$  as a measure of the fundamental productivity of country i in sector j and  $u_{ij}(\omega)$  as a random productivity shock. The former, which can be estimated using aggregate data, captures cross-country and cross-industry heterogeneity. It reflects factors such as climate, infrastructure, and institutions that affect the productivity of all producers in a given country and industry. Random productivity shocks, on the other hand, capture intraindustry heterogeneity. They reflect differences in technological know-how at the firm level, which are assumed to be drawn independently from a unique distribution  $F(\cdot)$ . In our setup, cross-country and cross-industry variations in the distribution of productivity levels derive from variations in a single parameter:  $a_{ij}$ .

We assume that trade barriers take the form of "iceberg" transport costs:

**A2.** For every unit of commodity j shipped from country i to country n, only  $1/d_{ij}^n$  units arrive, where:

(2) 
$$\begin{cases} d_{ij}^n = d_i^n \cdot d_j^n \ge 1, & \text{if } i \ne n, \\ d_{ij}^n = 1, & \text{otherwise.} \end{cases}$$

The indices i and n refer to the exporting and importing countries, respectively. The first parameter  $d_i^n$  measures the trade barriers which are specific to countries i and n. It includes factors such as: physical distance, existence of colonial ties, use of a common language, or participation in a monetary union. The second parameter  $d_j^n$  measures the policy barriers imposed by country n on product j, such as import tariffs and standards. In line with "the most-favored-nation" clause of the World Trade Organization, these impediments may not vary by country of origin.

We assume that markets are perfectly competitive. Together with constant returns to scale in production, perfect competition implies:

**A3.** In any country n, the price  $p_j^n(\omega)$  paid by buyers of variety  $\omega$  of product j is

(3) 
$$p_j^n(\omega) = \min_{1 \le i \le I} \left[ c_{ij}^n(\omega) \right],$$

where  $c_{ij}^n(\omega) = d_{ij}^n \cdot w_i \cdot a_{ij}(\omega)$  is the cost of producing and delivering one unit of this variety from country i to country n.

For each variety  $\omega$  of product j, buyers in country n are "shopping around the world" for the best price available. Here, random productivity shocks lead to random costs of production  $c_{ij}^n(\omega)$  and in turn, to random prices  $p_j^n(\omega)$ . In what follows, we let  $c_{ij}^n = d_{ij}^n \cdot \mathbf{w}_i \cdot a_{ij} > 0$ .

On the demand side, we assume that consumers have a two-level utility function with CES preferences across varieties. This implies:

**A4(i).** In any country n, the total spending on variety  $\omega$  of product j is

(4) 
$$x_j^n(\omega) = \left[ p_j^n(\omega) / p_j^n \right]^{1 - \sigma_j^n} k_j^n,$$

where  $k_j^n > 0$ ,  $\sigma_j^n > 0$  and  $p_j^n = \left[\sum_{\omega'=1}^{N_j} p_j^n(\omega')^{1-\sigma_j^n}\right]^{1/(1-\sigma_j^n)}$ .

The above expenditure function is a standard feature of the "new trade" literature; see e.g. Helpman and Krugman (1985).  $\sigma_j^n$  is the elasticity of substitution between two varieties of product j in country n, and  $p_j^n$  is the CES price index.  $k_j^n$  is an endogenous variable that represents total spending on product j in country n. It depends on the upper tier utility function in this country and the equilibrium prices. It is worth emphasizing that we let demand conditions vary across countries and industries:  $k_j^n$  and  $\sigma_j^n$  are functions of both n and j.

Finally, we assume that:

**A4(ii).** In any country n, the elasticity of substitution  $\sigma_j^n$  between two varieties of product j is such that  $E\left[p_j^n(\omega)^{1-\sigma_j^n}\right] < \infty$ .

Assumption A4(ii) is a technical assumption that guarantees the existence of a well defined price index. Whether or not A4(ii) is satisfied ultimately depends on the shape of the distribution  $F(\cdot)$ .

In the rest of the paper, we let  $x_{ij}^n = \sum_{\omega=1}^{N_j} x_{ij}^n(\omega)$  denote the value of exports from country i to country n in sector j, where total spending on each variety  $x_{ij}^n(\omega)$  is given by:

(5) 
$$\begin{cases} x_{ij}^n(\omega) = x_j^n(\omega), & \text{if } c_{ij}^n(\omega) = \min_{1 \le i' \le I} c_{i'j}^n(\omega), \\ x_{ij}^n(\omega) = 0, & \text{otherwise.} \end{cases}$$

<sup>&</sup>lt;sup>3</sup>Suppose, for example, that  $u_{ij}(\omega)$ 's are drawn from a (negative) exponential distribution with mean zero:  $F(u) = \exp[\theta u - 1]$  for  $-\infty < u \le 1/\theta$  and  $\theta > 0$ . This corresponds to the case where labor productivity  $z_{ij}(\omega) \equiv 1/a_{ij}(\omega)$  is drawn from a Pareto distribution:  $G_{ij}(z) = 1 - (b_{ij}/z)^{\theta}$  for  $0 < b_{ij} \le z$  and  $b_{ij} \equiv a_{ij}^{-1} \exp(-\theta^{-1})$ , as assumed in various applications and extensions of Melitz's (2003) model; see e.g. Helpman, Melitz, and Yeaple (2004), Antras and Helpman (2004), Ghironi and Melitz (2005) and Bernard, Redding, and Schott (2006). Then, our assumption A4(ii) holds if the elasticity of substitution  $\sigma_j^n < 1 + I\theta$ . Alternatively, suppose that  $u_{ij}(\omega)$ 's are distributed as a (negative) Gumbel random variable with mean zero:  $F(u) = 1 - \exp[-\theta u - \gamma)$ ] for  $u \in \mathbb{R}$  and  $\theta > 1$ , where  $\gamma$  is Euler's constant  $\gamma \simeq 0.577$ . This corresponds to the case where labor productivity  $z_{ij}(\omega)$  is drawn from a Fréchet distribution:  $G_{ij}(z) = \exp(-b_{ij}z^{-\theta})$  for  $z \ge 0$  and  $b_{ij} \equiv a_{ij}^{-\theta} \exp(-\gamma)$ , as assumed, for example, in Eaton and Kortum (2002) and Bernard, Eaton, Jensen, and Kortum (2003). Then, like in the Pareto case, A4(ii) holds if  $\sigma_i^n < 1 + I\theta$ .

Similarly, we denote  $\pi_{ij}^n(\omega)$  the probability that country i exports a variety  $\omega$  of product j to country n:

(6) 
$$\pi_{ij}^n(\omega) = \Pr\left\{c_{ij}^n(\omega) = \min_{1 \le i' \le I} [c_{i'j}^n(\omega)]\right\}.$$

By Assumption A1, the probabilities  $\pi_{ij}^n(\omega)$  remain the same across all varieties  $\omega$  of product j, so we can let  $\pi_{ij}^n(\omega) = \pi_{ij}^n$  in Equation (6).

#### 3. The Pattern of Trade

We now describe the restrictions that Assumptions A1–A4 impose on the pattern of trade; and how they relate to those of the standard Ricardian model.

3.1. **Predictions.** In order to make predictions on the pattern of trade, we follow a two-step approach. First, we derive a log-linear relationship between total exports  $x_{ij}^n$  and the probability of being an exporter  $\pi_{ij}^n$ , using the law of large numbers. Second, we relate the probability  $\pi_{ij}^n$  to the fundamental productivity level  $a_{ij}$ , using a first-order Taylor series development. All proofs can be found in the Appendix.

Our first result can be stated as follows.

**Lemma 1.** Suppose that assumptions A1-A4 hold. Then, for any importer n, any exporter  $i \neq n$ , and any product j, with probability one,

(7) 
$$\ln x_{ij}^n \to \ln k_j^n + \ln \pi_{ij}^n, \text{ as } N_j \to \infty.$$

As mentioned above, Lemma 1 is an asymptotic result, which we derive using a law of large numbers. It states that if the number of varieties is large enough, then country n's total spending on product j coming from country i should be proportional to the fraction of varieties that this country exports.<sup>4</sup> Lemma 1 illustrates a simple idea: conditional on exporting a given variety to country n, the expected value of exports has to be identical across countries. Transport costs, wages and fundamental productivity levels only affect the extensive margin—how many varieties are being exported—not the intensive margin—how much of each variety is being exported. This idea was already present in Eaton and

<sup>&</sup>lt;sup>4</sup>The previous limit can be written equivalently:  $x_{ij}^n \xrightarrow{a.s.} k_j^n \cdot \pi_{ij}^n$ , as  $N_j \to \infty$ ; see proof of Lemma 1.

Kortum's (2002) model; our lemma simply shows that their original insight is robust to changes in the distribution of random productivity shocks and the number of industries.

The second step of our analysis, however, deviates substantially from Eaton and Kortum's (2002) approach. Instead of making strong assumptions on  $F(\cdot)$  in order to compute  $\ln \pi_{ij}^n$  explicitly, we start from a symmetric situation where costs are identical across exporters,  $(c_{1j}^n = \dots = c_{Ij}^n)$ , and compute the first-order Taylor series approximation of  $\ln \pi_{ij}^n$  around this point.

Our second and main result can be stated as follows.

**Theorem 2.** Suppose that assumptions A1-A4 hold. In addition, assume that the number of varieties  $N_j$  of any product j is large, and that technological differences across exporters are small:  $c_{1j}^n \simeq ... \simeq c_{Ij}^n$ . Then, for any exporter i, any importer  $n \neq i$ , and any product j,

(8) 
$$\ln x_{ij}^n \simeq \alpha_i^n + \beta_j^n + \gamma \ln a_{ij}.$$

where  $\gamma < 0$ .

The first term  $\alpha_i^n$  is importer and exporter specific; it reflects wages  $\mathbf{w}_i$  in the exporting country and trade barriers  $d_i^n$  between countries i and n. The second term  $\beta_j^n$  is importer and industry specific; it reflects the policy barriers  $d_j^n$  imposed by country n on product j and demand differences  $k_j^n$  across countries and industries. The main insight of Theorem 2 comes from the third term  $\gamma \ln a_{ij}$ , in which the parameter  $\gamma$  is constant across countries and industries. Since  $\gamma < 0$ , Theorem 2 predicts that  $\ln x_{ij}^n$  should be decreasing in  $\ln a_{ij}$ : ceteris paribus, countries should export less in sectors where their firms are, on average, less efficient.

It is worth emphasizing that Theorem 2 cannot be used for comparative static analysis. If the fundamental productivity level goes up in a given country and industry, this will affect wages and, in turn, exports in other countries and industries through general equilibrium effects. In other words, changes in  $a_{ij}$  also lead to changes in the country and industry fixed effects,  $\alpha_i^n$  and  $\beta_j^n$ . By contrast, Theorem 2 can be used to analyze the cross-sectional variations of bilateral exports, as we shall further explore in Section 4.

Though the assumptions of Theorem 2 may seem unreasonably strong—in practice, technological differences across *all* exporters are unlikely to be small—its predictions hold more

generally. Suppose that, for each product and each importing country, exporters can be separated into two groups: small exporters, whose costs are very large (formally, close to infinity), and large exporters, whose costs of production are small and of similar magnitude. Then, small exporters export with probability close to zero and the results of Theorem 2 still apply to the group of large exporters.<sup>5</sup>

If we impose more structure on the distribution of random productivity shocks, we can further weaken the assumptions of Theorem 2. Suppose that the distribution  $F(\cdot)$  of  $u_{ij}(\omega)$  is Gumbel as in Eaton and Kortum (2002). Then, it can be shown that the property in Equation (8) holds exactly for any  $(c_{1j}^n, ..., c_{Ij}^n)$ . In other words, if Eaton and Kortum's (2002) distributional assumption is satisfied, then our local results become global; they extend to environments where technological differences across all countries are large.

In order to prepare the comparison between our results and those of the standard Ricardian model, we conclude this section by offering a Corollary to Theorem 2. Consider an arbitrary pair of exporters  $i_1$  and  $i_2$ , an importer  $n \neq i_1, i_2$  and an arbitrary pair of goods  $j_1$  and  $j_2$ . Taking the differences-in-differences in Equation (8) we get that  $(\ln x_{i_1j_1}^n - \ln x_{i_1j_2}^n) - (\ln x_{i_2j_1}^n - \ln x_{i_2j_2}^n) \simeq \gamma \left[ (\ln a_{i_1j_1} - \ln a_{i_1j_2}) - (\ln a_{i_2j_1} - \ln a_{i_2j_2}) \right]$ , for  $N_{j_1}$  and  $N_{j_2}$  large enough. Since  $\gamma < 0$ , we then obtain that

(9) 
$$\frac{a_{i_1j_1}}{a_{i_2j_1}} > \frac{a_{i_1j_2}}{a_{i_2j_2}} \Rightarrow \frac{x_{i_1j_1}^n}{x_{i_2j_1}^n} < \frac{x_{i_1j_2}^n}{x_{i_2j_2}^n}.$$

Still considering the pair of exporters  $i_1$  and  $i_2$  and generalizing the above reasoning to all J products, we derive the following Corollary:

**Corollary 3.** Suppose that the assumptions of Theorem 2 hold. Then, the ranking of relative unit labor requirements determines the ranking of relative exports:

$$\left\{\frac{a_{i_11}}{a_{i_21}} > \dots > \frac{a_{i_1j}}{a_{i_2j}} > \dots > \frac{a_{i_1J}}{a_{i_2J}}\right\} \Rightarrow \left\{\frac{x_{i_11}^n}{x_{i_21}^n} < \dots < \frac{x_{i_1j}^n}{x_{i_2j}^n} < \dots < \frac{x_{i_1J}^n}{x_{i_2J}^n}\right\}.$$

3.2. Relation to the standard Ricardian model. Note that we can always index the J products so that:

(10) 
$$\frac{a_{i_11}}{a_{i_21}} > \dots > \frac{a_{i_1j}}{a_{i_2j}} > \dots > \frac{a_{i_1J}}{a_{i_2J}}.$$

<sup>&</sup>lt;sup>5</sup>In other words, our theory does not require Gambia and Japan to have similar costs of producing and delivering cars in the United States. It simply requires that Japan and Germany do.

Ranking (10) is at the heart of the standard Ricardian model; see e.g. Dornbusch, Fischer, and Samuelson (1977). When there are no random productivity shocks, Ranking (10) merely states that country  $i_1$  has a comparative advantage in (all varieties of) the high j products. If there only are two countries, the pattern of trade follows:  $i_1$  produces and exports the high j products, while  $i_2$  produces and exports the low j products. If there are more than two countries, however, the pattern of pairwise comparative advantage no longer determines the pattern of trade. In this case, the standard Ricardian model loses most of its intuitive content; see e.g. Jones (1961) and Wilson (1980).

When there are stochastic productivity differences at the industry level, Assumption A1 and Ranking (10) further imply:

(11) 
$$\frac{a_{i_11}(\omega)}{a_{i_21}(\omega)} \succ \dots \succ \frac{a_{i_1j}(\omega)}{a_{i_2j}(\omega)} \succ \dots \succ \frac{a_{i_1J}(\omega)}{a_{i_2J}(\omega)},$$

where  $\succ$  denotes the first-order stochastic dominance order among distributions.<sup>6</sup> In other words, Ranking (11) is just a stochastic—hence weaker—version of the ordering of fundamental costs  $a_{ij}$ , which is at the heart of the Ricardian theory. Like its deterministic counterpart in (10), Ranking (11) captures the idea that country  $i_1$  is relatively better at producing the high j products. But whatever j is, country  $i_2$  may still have lower costs of production on some of its varieties.

According to Corollary 3, Ranking (11) does not imply that country  $i_1$  should only produce and export the high j products, but instead that it should produce and export relatively more of these products. This is true irrespective of the number of countries in the economy. Unlike the standard Ricardian model, our stochastic theory of comparative advantage generates a clear and intuitive correspondence between labor productivity and exports. In our model, the pattern of comparative advantage for any pair of exporters fully determines their relative export performance across industries.

$$\Pr\left\{\frac{a_{i_1j}(\omega)}{a_{i_2j}(\omega)} \le A\right\} < \Pr\left\{\frac{a_{i_1j'}(\omega)}{a_{i_2j'}(\omega)} \le A\right\} \Leftrightarrow \frac{a_{i_1j}(\omega)}{a_{i_2j}(\omega)} \succ \frac{a_{i_1j'}(\omega)}{a_{i_2j'}(\omega)}$$

To see this, note that for any  $A \in \mathbb{R}^+$  we have  $\Pr\{a_{i_1j}(\omega)/a_{i_2j}(\omega) \leq A\} = \Pr\{u_{i_1j}(\omega) - u_{i_2j}(\omega) \leq \ln A - \ln a_{i_1j} + \ln a_{i_2j}\}$ . Since for any j < j',  $u_{i_1j}(\omega) - u_{i_2j}(\omega)$  and  $u_{i_1j'}(\omega) - u_{i_2j'}(\omega)$  are drawn from the same distribution by A1, Ranking (10) implies:

This may seem paradoxical. As we have just mentioned, Ranking (11) is a weaker version of the ordering at the heart of the standard theory. If so, how does our stochastic theory lead to finer predictions? The answer is simple: it does not. While the standard Ricardian model is concerned with trade flows in each variety of each product, we only are concerned with the total trade flows in each product. Unlike the standard model, we recognize that random shocks—whose origins remain outside the scope of our model—may affect the costs of production of any variety. Yet, by assuming that these shocks are identically distributed across a large number of varieties, we manage to generate sharp predictions at the industry level.

#### 4. Empirical Evidence

We now investigate whether the predictions of Theorem 2 are consistent with the data.

4.1. **Data description.** We use yearly data from the OECD Structural Analysis (STAN) Databases from 1988 to 2003. Our sample includes 25 exporters, all OECD countries, and 49 importers, both OECD and non-OECD countries. It covers 21 manufacturing sectors aggregated (roughly) at the 2-digit ISIC rev 3 level. To the best of our knowledge, it corresponds to the largest data set available with *both* bilateral trade data *and* comparable labor productivity data. See Table 1 for details.

The value of exports  $x_{ij}^n$  by exporting country i, importing country n, and industry j is directly available (in thousands of US dollars, at current prices) in the STAN Bilateral Trade Database. The unit labor requirement  $a_{ij}$  in country i and industry j is measured as total employment divided by value added (in millions or billions of national currency, at current prices), which can both be found in the STAN Industry Database.<sup>7</sup>

4.2. **Specification.** The main testable implication derived in Theorem 2 is that:

(12) 
$$\left(\partial \ln x_{ij}^n\right) / \left(\partial \ln a_{ij}\right) = \gamma < 0$$

<sup>&</sup>lt;sup>7</sup>Any difference in units of account across countries shall be treated as an exporter fixed effect in our regression (13). Hence, we do not need to convert our measures of  $a_{ij}$  into a common currency. Similarly, we do not correct for the number of hours worked per person and per year, which only is available for a very small fraction of our sample. But it should be clear that cross-country differences in hours worked will also be captured by our exporter fixed effect.

**Table 1: Data Set Description** 

Source: OECD Structural Analysis (STAN) Databases

Years 1988-2003

Exporters: Twenty-five OECD countries (Australia, Austria, Belgium, Canada, Czech Republic, Danemark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Japan, Korea, Luxembourg, Netherlands, Norway, Poland, Portugal, Slovak Republic, Spain, Sweden, United Kingdom, United States)

Importers: Exporters + Five OECD Countries (Iceland, Mexico, New Zealand, Switzerland, Turkey) + Others (Argentina, Brazil, Chile, China, Cyprus, Estonia, Hong Kong, India, Indonisia, Latvia, Lithuania, Malaysia, Malta, Phillipines, Russian Federation, Singapore, Slovenia, South Africa, Thailand)

*Product Classification System:* The industrial breakdown presented for the STAN indicators database is based upon the International Standard Industrial Classification (ISIC) Revision 3.

Industry:	ISIC Rev. 3
Food products, beverages and tobacco	15-16
Textiles, textile products, leather and footwear	17-19
Wood and products of wood and cork	20
Pulp, paper, paper products, printing and publishing	21-22
Coke, refined petroleum products and nuclear fuel	23
Pharmaceuticals	243
Rubber and plastics products	25
Other non-metallic mineral products	26
Iron and steel	271
Non-ferrous metals	272+2732
Fabricated metal products, except machinery and equipment	28
Machinery and equipment, n.e.c.	29
Office, accounting and computing machinery	30
Electrical machinery and apparatus, n.e.c.	31
Radio, television and communication equipment	32
Medical, precision and optical instruments, watches and clocks	33
Motor vehicles, trailers and semi-trailers	34
Building and repairing of ships and boats	351
Aircraft and spacecraft	353
Railroad equipment and transport equipment n.e.c.	352+359
Manufacturing n.e.c.	36-37

In other words, the elasticity of exports with respect to the average unit labor requirement should be negative (and constant across importers, exporters, and industries). Accordingly, we shall consider a linear regression model of the form

(13) 
$$\ln x_{ij}^n = \alpha_i^n + \beta_j^n + \gamma \ln a_{ij} + \varepsilon_{ij}^n,$$

where  $\alpha_i^n$  and  $\beta_j^n$  are treated as importer–exporter and importer–industry fixed effects, respectively, and  $\varepsilon_{ij}^n$  is an error term.

2002 2001 1997 1996 2003 2000 1999 1998 Variable In a -0.78-0.95 -0.73-0.67-0.91-0.80-0.71-0.89(-10.31)\*\*\*(-10.69)\*\*12.74)\*·16.47)\*\* (-15.91)\*\*\*(-12.87)\*\*(-13.72)\*-14.16)\* Yes Yes Exporter-Importer FE Yes Yes Yes Yes Yes Yes Industry-Importer FE Yes Yes Yes Yes Yes Yes Yes Yes Observations 8778 15051 18167 18597 18805 18187 18233 17780  $R^2$ 0.805 0.794 0.793 0.787 0.792 0.792 0.789 0.785 1995 1994 1993 1992 1991 1990 1989 1988 Variable In a -0.79-0.75-0.53-0.50-0.42-0.37-0.39-0.08

(-7.03)\*\*\*

Yes

Yes

12755

0.7936

(-5.60)\*\*\*

Yes

Yes

11827

0.783

(-5.05)\*\*\*

Yes

Yes

11655

0.780

-5.77)\*\*\*

Yes

Yes

11606

0.779

(-1.06)

Yes

Yes

10672 0.776

Table 2: Year-by-Year OLS Regressions (Dependent Variable: Inx)

Note: Absolute value of t-statistics in parentheses, calculated from heteroskedasticity-consistent (White) standard errors

.7.74)\*\*\*

Yes

Yes

13489

0.7936

-9.79)\*\*\*

Yes

Yes

14245

0.7877

(-12.22)\*\*\*

Yes

Yes

17423

0.7849

Exporter-Importer FE

Industry-Importer FE

Observations

There are (at least) two possible interpretations of the error term  $\varepsilon_{ij}^n$ . First, we can think of  $\varepsilon_{ij}^n$  as a measurement error in trade flows. This is the standard approach in the gravity literature; see e.g. Anderson and Wincoop (2003). Alternatively, we can think of  $\varepsilon_{ij}^n$  as representing the impact of unobserved trade barriers, not accounted for in Assumption A2. Indeed, we can generalize A2 as  $\ln d_{ij}^n = \ln d_i^n + \ln d_j^n + \varepsilon_{ij}^n$ , which—given the expressions of  $\alpha_i^n$  and  $\beta_j^n$  provided in the proof of Theorem 2—immediately leads to Equation (13). This is the approach followed by Eaton and Kortum (2002) and Helpman, Melitz, and Rubinstein (2005).

Under either interpretation, we shall assume that  $\varepsilon_{ij}^n$  is independent across countries i and n as well as across industries j, and that  $\varepsilon_{ij}^n$  is uncorrelated with  $\ln a_{ij}$ .<sup>8</sup> It is worth pointing out that we allow  $\varepsilon_{ij}^n$  to be heteroskedastic conditional on i, n and j.

4.3. Estimation Results. Table 2 reports the OLS estimates of the regression parameter  $\gamma$  obtained independently for each year 1988-2003. In line with our theory, we find that the regression parameter  $\gamma$  is nonpositive for every year in the sample. Further, it is significant at the 1% level for 15 out of 16 years, the only exception being 1988 (which also is the

<sup>\*</sup> Significant at 10% confidence level

<sup>\*\*</sup> Significant at 5% confidence level

<sup>\*\*\*</sup> Significant at 1% confidence level

<sup>&</sup>lt;sup>8</sup>Thus, we ignore any potential errors in the measurement of labor productivity at the industry level, which obviously is a very strong assumption.

only year for which we do not have US data). Overall, we view these results as strongly supportive of our new Ricardian predictions.

Is the impact of labor productivity on the pattern of international specialization economically significant as well? As mentioned in Section 3, we cannot use our estimate of  $\gamma$  to predict the changes in *levels* of exports associated with a given change in labor productivity. However, we can follow a difference-in-difference approach to predict relative changes in exports across countries and industries. Consider, for example, two exporters,  $i_1$  and  $i_2$ , and two industries,  $j_1$  and  $j_2$ , in 2003. If  $a_{i_1j_1}$  decreases by 10%, then our prediction is that:

$$\left(\Delta \ln x_{i_1 j_1}^n - \Delta \ln x_{i_1 j_2}^n\right) - \left(\Delta \ln x_{i_2 j_1}^n - \Delta \ln x_{i_2 j_2}^n\right) = -\widehat{\gamma}_{2003} \Delta \ln a_{i_1 j_1} \simeq 7.8\%.$$

This is consistent with a scenario where country  $i_1$ 's exports of good  $j_1$  (towards any importer) go up by 5% and those of  $j_2$  go down by 2.8%, while they remain unchanged in both sectors in country  $i_2$ .

#### 5. Concluding Remarks

The Ricardian model has long been perceived has a useful pedagogical tool with, ultimately, little empirical content. Building on the seminal work of Eaton and Kortum (2002), we introduce random productivity shocks at the firm level in a standard Ricardian model with multiple countries and industries. The predictions that we derive are both intuitive and empirically meaningful: countries should export relatively more (towards any importing country) in sectors where they are relatively more productive. Using OECD trade and labor productivity data from 1988 to 2003, we find strong support for our new Ricardian predictions.

We believe that the tight connection between the theory and the empirical analysis that our paper offers is a significant step beyond the existing literature. First, we do not have to rely on ad-hoc measures of export performance. The theory tells us exactly what the dependent variable in the cross-industry regressions ought to be: ln(exports), disaggregated by exporting and importing countries. This allows us to move away from the country-pair comparisons inspired by the two-country model, and in turn, to take advantage of a much richer data set. Second, our clear theoretical foundations make it possible to discuss the economic origins of the error terms—measurement errors in trade flows or unobserved trade barriers—and as a result, the reasonableness of our orthogonality conditions.

Another attractive feature of our theoretical approach is that it relies on fairly general assumptions on preferences, transport costs, and the number of industries and countries. Hence, we believe that it may be fruitfully applied to more general environments, where labor is not the only factor of production. The basic idea, already suggested by Bhagwati (1964), is to reinterpret differences in  $a_{ij}$  as differences in total factor productivity. With multiple factors of production, the probability of being an exporter, and in turn the volume of exports, would be a function of both technological differences, captured by  $a_{ij}$ , and differences in relative factor prices. The rest of our analysis would remain unchanged.

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#### APPENDIX

Proof of Lemma 1. Fix  $i \neq n$ ; by the definition of total exports  $x_{ij}^n$ , we have

$$\begin{split} x_{ij}^n &= \sum_{\omega=1}^{N_j} x_j^n(\omega) \cdot \mathbb{I}\left\{c_{ij}^n(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^n(\omega)\right\} \\ &= \frac{k_j^n}{\left(p_j^n\right)^{1-\sigma_j^n}} \sum_{\omega=1}^{N_j} p_j^n(\omega)^{1-\sigma_j^n} \cdot \mathbb{I}\left\{c_{ij}^n(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^n(\omega)\right\} \\ &= k_j^n \left[\frac{1}{N_j} \sum_{\omega'=1}^{N_j} p_j^n(\omega')^{1-\sigma_j^n}\right]^{-1} \left[\frac{1}{N_j} \sum_{\omega=1}^{N_j} p_j^n(\omega)^{1-\sigma_j^n} \cdot \mathbb{I}\left\{c_{ij}^n(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^n(\omega)\right\}\right], \end{split}$$

where the function  $\mathbb{I}\{\cdot\}$  is the standard indicator function, i.e. for any event A, we have  $\mathbb{I}\{A\} = 1$  if A true, and  $\mathbb{I}\{A\} = 0$  otherwise. By Assumption A1,  $u_{ij}(\omega)$  is independent and identically distributed (i.i.d.) across varieties so same holds for  $c_{ij}^n(\omega)$ . In addition,  $u_{ij}(\omega)$  is i.i.d. across countries so  $\mathbb{I}\{c_{ij}^n(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^n(\omega)\}$  is i.i.d. across varieties as well. This implies that  $p_j^n(\omega)^{1-\sigma_j^n}$  and  $p_j^n(\omega)^{1-\sigma_j^n} \cdot \mathbb{I}\{c_{ij}^n(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^n(\omega)\}$  are i.i.d. across varieties. Moreover, by Assumption A4(ii),  $E\left[p_j^n(\omega)^{1-\sigma_j^n}\right] < \infty$  so we can use the strong law of large numbers for i.i.d. random variables (e.g. Theorem 22.1 in Billingsley (1995)) to show that

(14) 
$$\frac{1}{N_j} \sum_{\omega'=1}^{N_j} p_j^n(\omega')^{1-\sigma_j^n} \stackrel{a.s.}{\to} E\left[p_j^n(\omega)^{1-\sigma_j^n}\right],$$

as  $N_j \to \infty$ . Note that  $a_{ij} > 0$ ,  $d_{ij}^n \ge 1$  ensure that  $c_{ij}^n > 0$  whenever  $w_i > 0$ ; hence  $E\left[p_i^n(\omega)^{1-\sigma_j^n}\right] > 0$ . Similarly, Assumption A4(ii) implies that

$$E\left[p_i^n(\omega)^{1-\sigma_j^n} \cdot \mathbb{I}\left\{c_{ij}^n(\omega) = \min_{1 \le i' \le I} c_{i'j}^n(\omega)\right\}\right] < \infty,$$

so we can again use the strong law of large numbers for i.i.d. random variables (e.g. Theorem 22.1 in Billingsley (1995)) to show that

(15) 
$$\frac{1}{N_{j}} \sum_{\omega=1}^{N_{j}} p_{j}^{n}(\omega)^{1-\sigma_{j}^{n}} \cdot \mathbb{I}\left\{c_{ij}^{n}(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^{n}(\omega)\right\}$$

$$\stackrel{a.s.}{\to} E\left[p_{j}^{n}(\omega)^{1-\sigma_{j}^{n}} \cdot \mathbb{I}\left\{c_{ij}^{n}(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^{n}(\omega)\right\}\right],$$

as  $N_j \to \infty$ . Combining Equations (15) and (14) together with the continuity of the inverse function  $x \mapsto x^{-1}$  away from 0, yields by continuous mapping theorem (e.g. Theorem 18.10

(i) in Davidson (1994))

(16) 
$$\left[\frac{1}{N_{j}} \sum_{\omega'=1}^{N_{j}} p_{j}^{n}(\omega')^{1-\sigma_{j}^{n}}\right]^{-1} \left[\frac{1}{N_{j}} \sum_{\omega=1}^{N_{j}} p_{j}^{n}(\omega)^{1-\sigma_{j}^{n}} \cdot \mathbb{I}\left\{c_{ij}^{n}(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^{n}(\omega)\right\}\right]$$

$$\stackrel{a.s.}{\to} \left\{E\left[p_{j}^{n}(\omega)^{1-\sigma_{j}^{n}}\right]\right\}^{-1} \left\{E\left[p_{j}^{n}(\omega)^{1-\sigma_{j}^{n}} \cdot \mathbb{I}\left\{c_{ij}^{n}(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^{n}(\omega)\right\}\right]\right\},$$

as  $N_j \to \infty$ . Now, by the law of iterated expectations, we have

$$E\left[p_{j}^{n}(\omega)^{1-\sigma_{j}^{n}} \cdot \mathbb{I}\left\{c_{ij}^{n}(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^{n}(\omega)\right\}\right]$$

$$= \pi_{ij}^{n} \cdot E\left[p_{j}^{n}(\omega)^{1-\sigma_{j}^{n}} \left| c_{ij}^{n}(\omega) = \min_{1 \leq i' \leq I} c_{i'j}^{n}(\omega)\right] + (1-\pi_{ij}^{n}) \cdot 0\right]$$

$$= \pi_{ij}^{n} \cdot E\left[\left(\min_{1 \leq i' \leq I} c_{i'j}^{n}(\omega)\right)^{1-\sigma_{j}^{n}}\right]$$

$$= \pi_{ij}^{n} \cdot E\left[p_{j}^{n}(\omega)^{1-\sigma_{j}^{n}}\right],$$

$$(17)$$

where the last equality comes from Assumption A3. The remainder of the proof is straightforward: Equation (17) combined with Equation (16) gives

(18) 
$$x_{ij}^n \stackrel{a.s.}{\to} k_j^n \cdot \pi_{ij}^n, \text{ as } N_j \to \infty.$$

Note that  $k_j^n \cdot \pi_{ij}^n > 0$ ; hence we can apply the continuous mapping theorem (e.g. Theorem 18.10 (i) in Davidson (1994)) to the logarithm of the quantities in Equation (18) to get, with probability one,

(19) 
$$\ln x_{ij}^n \to \ln k_i^n + \ln \pi_{ij}^n, \text{ as } N_j \to \infty.$$

This completes the proof of Lemma 1.

Proof of Theorem 2. Fix  $n \neq 1$  and j. We start by computing  $\ln \pi_{1j}^n$ . Assumption A1 and straightforward computations yield

(20) 
$$\pi_{1j}^n = \int_0^{+\infty} f(u) \prod_{i \neq 1} \left[ 1 - F(\ln c_{1j}^n - \ln c_{ij}^n + u) \right] du.$$

We now approximate  $\ln \pi_{1j}^n$  from Equation (20) by its first order Taylor series at  $\ln c_{1j}^n =$ ... =  $\ln c_{1j}^n = \ln c$  where c > 0. We have

(21) 
$$\ln \pi_{1j}^n \big|_{(\ln c, \dots, \ln c)} = -\ln I,$$

(22) 
$$\frac{\partial \ln \pi_{1j}^n}{\partial \ln c_{1j}^n} \bigg|_{(\ln c, \dots, \ln c)} = -(I-1) \cdot I \cdot \left[ \int_0^{+\infty} f^2(u) \left[ 1 - F(u) \right]^{I-2} du \right],$$

and, for  $i \neq 1$ ,

(23) 
$$\frac{\partial \ln \pi_{1j}^n}{\partial \ln c_{ij}^n} \Big|_{(\ln c, \dots, \ln c)} = I \cdot \left[ \int_0^{+\infty} f^2(u) \left[ 1 - F(u) \right]^{I-2} du \right],$$

where the last two equalities use the fact that  $\int_0^{+\infty} f(u) \left[1 - F(u)\right]^{I-1} du = I^{-1}$ . Combining Equations (21), (22), (23) we get

$$\ln \pi_{1j}^{n} = -\ln I - (\ln c_{1j}^{n} - \ln c) \cdot (I - 1) \cdot \delta + \sum_{i \neq 1} (\ln c_{ij}^{n} - \ln c) \cdot \delta + o\left(\left\|\ln c_{j}^{n}\right\|\right)$$

(24) 
$$= -\ln I - \delta \cdot I \cdot \ln c_{1j}^{n} + \delta \cdot \sum_{i=1}^{I} \ln c_{ij}^{n} + o\left(\left\|\ln c_{j}^{n}\right\|\right),$$

where  $\left\|\ln c_j^n\right\|^2 = \sum_{i=1}^I \left[\ln(c_{ij}^n/c)\right]^2$  denotes the usual  $L_2$ -norm and

$$\delta = I \cdot \left[ \int_{0}^{+\infty} f^{2}(u) \left[ 1 - F(u) \right]^{I-2} du \right],$$

is a positive constant  $\delta > 0$  which only depends on  $f(\cdot)$ ,  $F(\cdot)$  and I. Combining Equation (24) with the definition of  $c_{1j}^n = d_{1j}^n \cdot \mathbf{w}_1 \cdot a_{1j}$  and Assumption A2, then gives

$$\ln \pi_{1j}^n \simeq \alpha_1^n + b_j^n + \gamma \ln a_{1j},$$

where

$$\alpha_1^n \equiv -\ln I - \delta \cdot I \cdot \ln(d_1^n \cdot w_1), \ b_j^n \equiv -\delta \cdot I \cdot \ln d_j^n + \delta \cdot \sum_{i=1}^I \ln c_{ij}^n, \text{ and } \gamma \equiv -\delta \cdot I.$$

Note that  $\alpha_1^n$  does not depend on the product index j,  $b_j^n$  does not depend on the country index i = 1 and  $\gamma < 0$  is a negative constant which only depends on  $f(\cdot)$ ,  $F(\cdot)$  and I. Using the same reasoning as above for any  $i \neq n$  then yields

(25) 
$$\ln \pi_{ij}^n \simeq \alpha_i^n + b_j^n + \gamma \ln a_{ij}.$$

Combining Equations (19) and (25) then yields  $\ln x_{ij}^n \simeq \alpha_i^n + \beta_j^n + \gamma \ln a_{ij}$ , for  $N_j$  large, where we have let  $\beta_j^n \equiv \ln k_j^n + b_j^n$ . This completes the proof of Theorem 2.