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Publication Date

2005-10-01

Energy Policy and Economics 015

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October 2005

This paper is part of the University of California Energy Institute's (UCEI) Energy Policy and Economics Working Paper Series. UCEI is a multi-campus research unit of the University of California located on the Berkeley campus.

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The Natural Number of Forward Markets for Electricity

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ABSTRACT

Observers of restructured electricity markets emphasize: (1) Spot prices are extremely variable, because electricity is not storable; (2) long-dated forward markets rarely exist – those in California were a single day ahead. Actually, the first observation implies the second. With the aid of a simulation model, which replicates the seasonality, heteroskedasticity, and serial correlation in load, the precise constellations of forward prices can be deduced in a setting of perfect competition, risk neutrality, and best possible forecasting. Even at extreme conditions in the idealized spot market, the constellations of these forward prices converge to long-run seasonal means at the horizon of just a few days. Another reason long-dated forward markets for electricity are redundant is the futures market for natural gas on NYMEX, functioning at a horizon beyond two years, as demonstrated by analyses of forecasting power using the simulation data as well as the data from NYMEX and California over 1998-2000.

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1. INTRODUCTION

An often-noted consequence of restructured electricity markets has been price volatility. Some of the movements in spot wholesale prices are predictable, such as those resulting from diurnal or seasonal cycles in loads or input availability, and must have been present implicitly in the management of vertically integrated utilities. Some other volatility is due to truly unpredictable events. Other commodities experience "weather shocks," but, as is also often noted, the short-run constraints on generation, the fixing of retail prices, and the inability to store electricity combine to create large spot price movements out of weather shocks.

Economists instinctively believe that a full constellation of forward prices, stretching hour by hour to the distant horizon, would ameliorate the volatility in electricity spot prices. Distribution-oriented utilities could contract at these long-term prices to match their obligations to retail customers, while generators could use these forward prices as a signal for the timing of maintenance, investment, and so forth. Actual forward markets for electricity have nowhere near this horizon. Although some year-ahead contracts have been observed in England and Wales (Green, 1999), futures markets for electricity have failed even on the New York Mercantile Exchange (NYMEX), the most successful energy futures exchange (EIA, 2002). In California, the Power Exchange (CalPX) traded merely one day ahead while the distribution-oriented utilities signed few long-term contracts (Bushnell, 2004).

Because actual exchanges are costly to operate, economists' wish for a full constellation of forward prices may not be appropriate. Many of those forward prices could be redundant for all practical purposes. Is the forward price for electricity delivered during hour 18 on August 1 three years from now likely to differ from the corresponding price, as of today, for August 1 two years from now or from hour 18 on August 2 three years from now? Should a weather shock occur later today, are those three distant forward prices likely to change by tomorrow? If not, one of those three forward prices serves for the other two as an allocative signal or as a hedging mechanism. Furthermore, do any of these three forward prices add information beyond a general estimate of a normal "spark spread" and the futures price for natural gas in August two years ahead already traded on NYMEX?

The answers to these questions about redundant prices depend on the constellation of electricity forward prices themselves, specifically, how quickly they converge to some long-run averages and how closely their temporal movements relate to those in the futures market for a related commodity. But electricity forward prices do not exist sufficiently far ahead to demonstrate that even further ahead they would be redundant. This situation exposes a fundamental issue, one much more general than electricity: the endogeneity of whether prices can be observed. In this paper, we suggest three approaches to address this endogeneity issue.

A first approach to the lack of actual forward prices for electricity would be to find another commodity sharing many features with electricity yet having an actual forward market at a longer horizon. The best analogy is the lowly potato. Although storable for a few months after the fall harvest, potatoes are not carried over to the next cropyear. That crop, planted in the spring, is subject to considerable weather shocks, which result in prices differing by a factor of five to six from year to year (Paul et al., 1981). Before 1952, potato prices were controlled by many government programs; a "restructured" environment prevailed from 1952 through the mid 1970s; since the mid 1970s, the increasing importance of processed potatoes and the small number of processors have made market power the dominant issue. During the twenty-five years of relatively free trading, NYMEX offered a potato futures market (the demise of which encouraged the exchange to try energy futures). That futures market was controversial (Gray, 1964), for dubious reasons that any economist familiar with special interests in electricity would recognize, even though the futures prices were sensible by any *ex ante* standard (Tomek and Gray, 1970). Most interesting for an analogy with electricity, the spring-time futures price for November delivery was essentially constant from year to year, regardless of the spot price of potatoes at the same moment. As Gray and Tomek (1971) and Gray (1972) emphasized, the futures prices for November ought to have been constant at that horizon, because none of the previous weather conditions carried over to the new crop by way of potato inventories and no one could know in the spring whether growing conditions during the summer would be good or bad. Only as those weather conditions transpired did the harvest-time futures prices succeed in forecasting the ultimate harvest-time spot price. For corn, a commodity more storable than potatoes, Tomek and Gray (1970) presented evidence that futures prices more than one year ahead differ from year to year, which gave them an allocative role and a reason to exist. By this analogy, the constellation of forward prices for electricity, a commodity less storable than even the potato, should not go as far ahead as potato futures prices.

A second approach, one followed in this paper, is to construct an idealized electricity market, with known parameters in the stochastic processes as representing the weather and generators' cost functions. In the idealized setting, the constellation of forward prices can be calculated as far ahead as desired. Should the profile of forward prices in the idealized setting quickly converge to long-run averages, many prices are redundant because they could be inferred from others. The presumption would be that long-dated contracts would be redundant in actual electricity markets too. Similarly, the price of natural gas, one of the major inputs to electricity generation, can be represented by a known seasonal and stochastic process, albeit with patterns different from those for electricity loads. In these conditions, the constellations of forward prices for natural gas can also be determined as far ahead as desired. If prices of electricity and natural gas are related in a predictable way, as they are in actual settings as well as the idealized setting, a long-dated forward price of electricity can be duplicated from an estimate of this cross-commodity price relationship and existing forward prices of natural gas. If those "synthetic prices" forecast the subsequently realized spot price of electricity as accurately as equally distanced forward prices of electricity, which, in the idealized setting, is the best possible forecast, the electricity forward prices are redundant.

The ability of long-dated synthetic prices to forecast the subsequently realized spot price can also be compared to the forecasting ability of short-dated forward prices, such as the day-ahead market of the CalPX. The forecasting abilities of long-dated synthetic forward prices and the observed short-dated forward price of electricity place bounds on the forecasting ability of long-dated forward prices of electricity, had they been observed. Presumably, a long-dated forward price of electricity, had it existed in California, would have been more accurate than the synthetic price in predicting subsequent spot prices of electricity for it should have reflected information about factors other than the price of the primary input to generation, while it would have been less accurate than the short-dated forward price reflecting the information available at the period closer to dispatch. Should the long-dated synthetic forward price be only slightly less accurate as a forecast than the existing short-dated forward price of electricity, as will be shown in the final section here for the NYMEX natural gas and the CalPX market over 1998-2000, separate long-dated forward markets for electricity would not have added much to the existing forward prices of natural gas and, hence, would have been redundant. This bounding through forecasting ability is a third approach to the endogeneity of observed prices.

Of course, the idealized market in this paper is much less complicated than actual electricity markets, yet it is more complicated than the setting supposed in the typical model of hedging or oligopoly pressure. Previous studies of hedging pressure have not contemplated whether long positions in the futures market for the principal input commodity would be effective substitutes.[1] Similarly, studies of oligopolistic power have not considered how the opportunity to trade forward in input markets could have affected the exercise of oligopolistic power. Moreover, these studies typically consider a simple two-period model, in which the number and horizon of forward trading and the spot price volatility are exogenous. In this restricted setting, studies of oligopolistic pricing have suggested, both theoretically and empirically, that the creation of

[1] For example, see Bessembinder and Lemmon (2001) or Longstaff and Wong (2004).

forward markets mitigates the degree of oligopolistic pricing.[2] However, without specifying the term length of the forward contract, these studies imply that existing short-dated forward markets, such as the day-ahead market of the CalPX, should be as effective as long-dated forward markets in mitigating the degree of oligopolistic pricing. Similarly, studies of hedging pressure, although capable of examining the equilibrium relationship between the spot and a single forward price, cannot address the questions of the sensible number of forward markets and how far ahead they should cover. In contrast, the price variance is endogenous to the idealized market model as a function of the parameters characterizing the underlying stochastic factors, such as weather and fuel price, and is determined simultaneously with the natural term length of forward contracts.

2. DESCRIPTION OF THE IDEALIZED ELECTRICITY MARKET

2.1 A model of an idealized electricity market

The model of an idealized market needs to be constructed so that the resulting price series share features observed in actual settings such as California. In particular, the model should emphasize the seasonal and diurnal variations, heteroskedasticity, and serial correlation of the electricity load resulting from those of the underlying weather as well as the convexity of the electricity supply curve. The weather also affects the price dynamics of key inputs to electricity generation such as natural gas or hydro resources, yet the storability of these input commodities likely results in the distributions of their prices being different from those of electricity load. The simulation model seeks to accommodate these patterns with a manageable number of parameters so that the sensitivity of forward price profiles to these parameters is traceable.

The model constructed here is similar to a general equilibrium model of Bessembinder and Lemmon (2002) in their analysis of optimal hedging in an electricity forward market, except for their assumptions of risk-averse traders with an identical risk-aversion coefficient and of only one delivery period forward with trading of that forward contract on only one occasion. Some of key structures imposed on the idealized electricity market include:

- Generating and retailing firms trade wholesale electricity hour by hour for a full constellation of delivery hours, as far ahead into the future as can be imagined.

[2] See, for example, Powell (1993), Green (1999) and Wolak (2000) for mitigation of oligopolistic pricing through forward trading in the restructured electricity markets and Allaz (1990) in a more general case. In an empirical analysis of Australian (NEM1) market by Wolak (2000), the number and the term length of forward markets as well as forward position of each trader are exogenous to the model.

- Retailing firms meet exogenously determined hourly retail load at a fixed retail rate.[3] The complications of intertemporal substitution in demand are absent.
- Both retailers and generators are risk-neutral and price-taking.
- The aggregate cost function is characterized by, $TC(Q) = FC + \frac{bw}{c}Q^c$ where the first and second term are the fixed and variable cost, respectively, Q is the industry's aggregate supply, and w is the price of the primary input, fuel. The multi-period complications of ramping up or down are not present. This specification of the aggregate cost function allows a single parameter, c, to represent the degree of non-linearity (Wright, 1979).

Under these circumstances, the equilibrium spot price in any hour t is:

$$P_t^0 = bw_t \left(Q_t^D\right)^{c-1} \tag{1}$$

where P_t^0 is the spot price of wholesale electricity, Q_t^D is the aggregate retail demand, and the subscript t represents that the variables are in hour t.

In (1), only the fuel price and the load level vary over time, implying that the supply curve is constant over time aside from the temporal variations in the fuel price. Of course, in reality, electricity is generated from various resources whose availability varies across seasons.[4] To accommodate such seasonal variation, a stochastic component is added to (1):

$$P_t = P_t^0 (1 + \sigma_{AS}\, e_{1,t}) \qquad e_{1,t} = \rho_{AS}\, e_{1,t-1} + u_{1,t} \tag{2}$$

where $u_{1,t}$ is independently, identically distributed (iid) N(0, 1) and σ_{AS}, and ρ_{AS} are parameters. In (2), $P_t^0 = bw_t \left(Q_t^D\right)^{c-1}$ should be considered as representing the long-term average supply curve, any deviation from which is mean-reverting and has variance proportional to P_t^0.

In (1) and (2), the spot electricity price is endogenously determined, given two exogenous factors, Q_t^D and w_t. With the fixed retail price, the retail electricity demand is perfectly price

[3] A load exogenous to the model is consistent with the extreme price inelasticity of retail demand observed in California and many other regions where the retail price has been fixed even after the restructuring of the industry.

[4] For example, roughly 40% of electricity in the Northern California is generated from hydroelectric resources, supply of which is more abundant during the winter and early spring than in the rest of the year. Supply capacity of fossil fuel generation units also fluctuates due to required maintenance, either periodically or randomly. Such supply seasonality can be incorporated into (1) and (2) by allowing parameters to be time-variant. Such a modification will not alter the model's implication for the sensible number of forward markets.

inelastic and is largely determined by weather conditions such as temperature. We specify the seasonal and diurnal cycles of the electricity load by the following sinusoidal function, using a standard representation of weather (Hansen and Driscoll, 1977),[5]

$$Q_t^{DT} = Q_t^{DT0}\,(1 + \sigma_{QF}\, e_{2,t}) \qquad e_{2,t} = \rho_{QF}\, e_{2,t\text{-}1} + u_{2t} \tag{3}$$

$$Q_t^{DT0} = \alpha_0 + \alpha_1 \sin\left(\frac{2\pi h(t)}{8640}\right) + \alpha_2 \cos\left(\frac{2\pi h(t)}{8640}\right) + \alpha_3 \sin\left(\frac{2\pi h(t)}{24}\right) + \alpha_4 \cos\left(\frac{2\pi h(t)}{24}\right) + \alpha_5 WKEND_t$$

where $u_{2,t}$ ~ iid N(0, 1) and α's, σ_{QF}, and ρ_{QF} are parameters. In (3), Q_t^{DT0} creates the seasonal and diurnal cycles in retail demand as a function of two variables; $h(t)$, the hour count of observation t starting from January 1, hour 1 of the year, and, $WKEND_t$, a weekend dummy. Since these two variables will not replicate the complexity of electricity load dynamics in the real world, a stochastic term $Q_t^{DT0}\,\sigma_{QF}e_{2,t}$ is added to create the deterministic load, Q_t^{DT}, which is mean-reverting, with σ_{QF} determining the variance of Q_t^{DT} in ratio to Q_t^{DT0}. These properties of the load are known to the market to prior to t.[6]

The realized load deviates from the deterministic pattern due to factors unknown prior to t. This deviation is specified in a similar manner,

$$Q_t^{D} = Q_t^{DT}\,(1 + \sigma_{QA}\, e_{3,t}) \qquad e_{3,t} = \rho_{QA}\, e_{3,t-1} + u_{3,t} \tag{4}$$

where $u_{3,t}$ ~ iid N(0, 1) and σ_{QA} and ρ_{QA} are parameters.[7]

The fuel price, w_t, is modeled as,

$$w_{d(t)} = w_{0,d(t)} + \sigma_{w,d(t)}\, e_{4,d(t)} \qquad e_{4,d(t)} = \rho_w\, e_{4,d(t)-1} + u_{4,d(t)} \tag{5}$$

$$w_{0,d(t)} = \beta_0 + \beta_1 \sin\left(\frac{2\pi d(t)}{360}\right) + \beta_2 \cos\left(\frac{2\pi d(t)}{360}\right) + \beta_3 \sin\left(\frac{2\pi d(t)}{180}\right) + \beta_4 \cos\left(\frac{2\pi d(t)}{180}\right)$$

$$\sigma_{w,d(t)} = \sigma_w \frac{\beta_0 v_{d(t)}}{\gamma_0} \sqrt{1-\rho_w^2}$$

[5] The specification of the deterministic load in (3) is rather restrictive as it assumes the degree as well as the shape of diurnal variation to be identical across seasons. It can be replaced with more flexible functional form; yet, because this seasonality would be known to the market, this more complicated specification will not alter the model's implications for the temporal dynamics of electricity forward prices.

[6] Here, $e_{2,t}$ is stochastic in a statistical sense in that it is independent of Q_t^{DT0}, yet is deterministic as it is assumed observable by economic agents.

[7] A load variance proportional to the load level is evident in California, as shown in figure 1.

$$v_{d(t)} = \gamma_0 + \gamma_1 \sin\left(\frac{2\pi d(t)}{360}\right) + \gamma_2 \cos\left(\frac{2\pi d(t)}{360}\right) + \gamma_3 \sin\left(\frac{2\pi d(t)}{180}\right) + \gamma_4 \cos\left(\frac{2\pi d(t)}{180}\right)$$

where $w_{0,d(t)}$ and $v_{d(t)}$ are, respectively, seasonal variations in the long-run expected value and variance of the fuel price, $u_{4,d(t)}$ ~ iid N(0, 1), β's, γ's, σ_w, and ρ_w are parameters, and the subscript $d(t)$ represents that the variables are for day $d(t)$. The two sinusoidal functions acknowledge that the price of the primary input to electricity generation exhibits a seasonal trend, albeit in a pattern different from that of the electricity load. Specifically, the temporal price movements of a storable commodity with an efficient futures market, such as natural gas, should be characterized by an essentially linear increase at the rate of the storage cost (plus interest) during the period with positive inventory (Williams and Wright, 1991). (5) does not replicate such linearity completely; rather, it smoothes the seasonal variation of the equilibrium price without storage with a relatively large price drop at the end of the peak-demand period, after which inventory implicitly starts to build up. In addition, the price of a storable commodity is likely less volatile when inventory is ample, for shocks can be absorbed by releasing inventory. Separate sinusoidal functions in (5) seek to accommodate these features. The stochastic component, $\sigma_{w,d(t)}\ e_{4,d(t)}$, creates the *realized* fuel price, which is mean-reverting and is heteroskedastic with variance determined by the function $v_{d(t)}$.

2.2 Forward price as the best, unbiased forecast

In this idealized setting, suppose that forward trading takes place k periods prior to the actual dispatch, where k is any non-negative integer. Under the risk-neutrality and price-taking assumptions, an efficient forward price should form the best possible forecast of the spot price based on the information available at the time of forward trading. That is, the k-period-ahead forward price of the electricity for delivery at t is,

$$F_{t,t-k} = \mathrm{E}_{t-k}[P_t] \tag{6}$$

where $F_{t,t-k}$ is the equilibrium forward price with the first and second subscripts representing the delivery and trading period, respectively, and $\mathrm{E}_{t-k}[\cdot]$ denotes that the expectation is conditional on information available at $t-k$. To clarify the notation, we emphasize that k is a set of non-negative integers rather than a single number and, for each delivery hour t, there are as many forward prices as the number of values in this set. That is, there are J forward prices with $F_{t,t-k_j}$ where k_j is one value in the sequence $\{k_1, \ldots, k_J\}$. The spot price at t is merely one in the constellation of prices observed at t, the price with $k = 0$.[8] We further assume that the set of k is identical for all t whereas it often differs from one delivery hourly to another in real markets.[9] The spot price at t is thus also the culmination of a time-series first recorded k_J periods previously.

With the aggregate supply function as specified in (1) and (2), the conditional expectation of the spot price, for any combination of t and k, is,

$$E_{t-k}[P_t] = b\, E_{t-k}[w_{d(t)}]\, E_{t-k}[\, QA_t^{c-1}\,]\, (1 + \sigma_{AS}\, \rho_{AS}^{k}\, e_{1,t-k}) \tag{7}$$

where the independence of $u_{1,t}$, $u_{3,t}$ and $u_{4,t}$ and $E[u_{3,t-s}] = 0$, $\forall\ s = 0, \ldots, k-1$, are used. In (7), the conditional expectation of the fuel price is,

$$E_{t-k}[w_{d(t)}] = w_{0,d(t)} + \sigma_{w,d(t)}\ \rho_w^{\delta(t,k)}\ e_{4,d(t-k)} \tag{8}$$

where $\delta(t, k) = d(t) - d(t-k)$. The error in forecasting the fuel price is,

$$w_{d(t)} - E_{t-k}[w_d] = \sigma_{w,d(t)} \sum_{j=0}^{\delta(t,k)-1} \rho_w^{j} u_{4,d(t)-j} \tag{9}$$

Since $u_{4,d(t)-i} \sim$ iid $N(0, 1)\ \forall\ i = 0, \ldots, \delta(t, k) - 1$, the forecast error has its conditional distribution,

$$w_{d(t)} - E_{t-k}[w_{d(t)}]\ |\ \mathfrak{I}_{t-k} \sim N\left(0, \sigma_{w,d(t)}^2 \frac{1-\rho_w^{2\delta(t,k)}}{1-\rho_w^2}\right) \tag{10}$$

where $\mathfrak{I}_{t-k}$ is the information set available at $t - k$.

An analytic expression for $E_{t-k}[\, QA_t^{c-1}\,]$ in (7) is difficult to obtain due to the non-linearity within the expectation operator. Hence, for each t and k, it is calculated numerically by generating 1,000 values of QA_t from its conditional distribution,

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[8] The definition of a spot price could be extended to, say, a month-long block of hours.

[9] For example, in the CalPX day-ahead market, the bids for all 24 hourly blocks were closed at 7AM on the day previous to the actual dispatch. The CalPX also operated an "hour-ahead" market where the bids for 24 hourly blocks were closed on a rolling basis three hours before dispatch. In January 1999, that market was reconfigured and called the "day-of" market, where bids were closed three times a day: 4PM of previous day for delivery at hours 1-10, 6AM for delivery at hours 11-16, and noon for delivery at hours 17-24). In this trading sequence, the value of k corresponding to the day-ahead market ranges between 17 and 41 for hour 1 though hour 24 and that corresponding to the day-of market ranges between 5 and 18 hours depending on the delivery hour. There was no trading during the time between the two auctions.

$$QA_t \mid \mathfrak{I}_{t-k} \sim \mathrm{N}\left(QF_{t,t-k}, \left(\sigma_{QA} Q_t^{DT}\right)^2 \frac{1-\rho_{QA}^{2k}}{1-\rho_{QA}^2} \right) \tag{11}$$

and averaging the associated 1,000 values of QA_t^{c-1}. In (11), $QF_{t,t-k}$ is the *k*-period-ahead forecast of period *t* load with its expression,

$$QF_{t,t-k} = \mathrm{E}_{t-k}[QA_t] = Q_t^{DT}\,(1 + \sigma_{QA}\, \rho_{QA}^{k}\, e_{3,t-k}) \tag{12}$$

The model as specified in (1) through (5) has more state variables and parameters than models of electricity price dynamics suggested in previous studies, in which a common approach has been to model directly the stochastic dynamics of the electricity spot price with a few parameters, without including the companion stochastic processes labeled here the "fuel price" and "electricity load" (for example, Burger et al., 2004; Escribano et al., 2002; Etheir and Mount, 1998; Knittel and Roberts, 2005; Lucia and Schwartz, 2002; and Villaplana, 2003).[10] Implicit in this approach is the assumption that the stochastic dynamics of the spot electricity price can be expressed analytically in reduced-form. In contrast, in the idealized market model, the dynamics of the electricity spot price are endogenous, being determined by non-linear interactions of the three underlying factors whose stochastic dynamics are specified with twenty-six distinct parameters.[11] However, these differences among models are in degree rather than kind. All these models imply constellations of forward prices, the variability within which speaks to the sensible number of forward markets.

2.3 Descriptive analyses of the simulated electricity data

We constructed time series of the simulated electricity prices with different sets of the parameter values as shown in table 1. These time series are examined for sensitivity to: (i) the convexity of aggregate supply, (ii) the persistence of deviations from the deterministic load, fuel price, and long-run average supply curve, and (iii) the variance of the deviations from the deterministic load and fuel price. The intercept in supply function (1) is set as $b = 10^{-2c+1}$ in all scenarios so that the price according to the long-run average supply remains ten times the fuel price when Q_t^D = 100. All variance parameters (σ_i) are multiplied by $\sqrt{1-\rho_i^2}$, i = QF, QA, w, AS, so that the

[10] Pirrong and Jamakyan (2001) and Burger et al. (2004) follow an approach similar to ours by modeling the stochastic dynamics of underlying factors and thus electricity spot prices as a (non-linear) function of these underlying factors.

[11] Due to this non-linearity, forward price formula in the idealized market cannot be expressed in a closed form, implying that the studies following a reduced-form approach are inherently subject to errors in approximating true stochastic dynamics of electricity spot prices.

parameters before multiplying by this term represent the variances of the three stochastic components in proportion to the corresponding deterministic values. For each set of parameter values, 8,640 hourly observations were generated in one replication.[12]

Figure 1 and table 2 present the key distributional features of the simulation data and compare them with those of California data.[13] In figure 1, the idealized market model with the base parameter set creates seasonal and diurnal variation of load and load variance similar to those observed in California. Descriptive statistics of the simulation data in table 2 illustrate that the parameter values in table 1 create plausible seasonal cycles in the mean and variance of the simulated fuel price. The average price is the lowest in April and gradually increases until it reaches the annual high in the following March. This gradual increase of the average price and rapid drop at the end of the high demand season is appropriate for a storable commodity such as natural gas. The shape of the seasonal heteroskedasticity is also plausible for a storable commodity: Price variance is proportional to the price level while it is inversely related to the level of inventory as implied by the seasonal price cycle.

These seasonal and diurnal variations are amplified into the electricity spot price through the supply function, quadratic in the base parameter case ($c = 3$) according to table 2. The simulated electricity spot prices are highly volatile with the sample standard deviation ranging from 47% to above 60% of the mean spot price in the base parameter case, depending on hour of the day and on the season. Forward prices as well as spot price are positively skewed, with skewness increasing in the values of the cost function and variance parameters.[14]

The load and fuel price forecasts in (7) and (12) and the electricity forward price obtained by the method of numerical integration are, by construction, the best possible forecasts of their corresponding values in the spot market.[15] Nevertheless, large differences between the spot and previous forward prices for the same delivery period are not infrequent. The standard deviation of the price forecast error ranges from 8.387 to 12.810 or 75.1% to 85.5% of the mean of 24-hours-ahead forward price ($k = 24$) in base parameter case, depending on the hour of the day and the season. In other words, the forecasting performance of the forward prices is poor, even at such a short horizon, the result of the very properties causing the highly volatile spot prices.

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[12] In the simulations, we assumed 30 days per month for all twelve months of a year, resulting in 8,640 hourly observations per year.

[13] The California Independent System Operator (CAISO)'s website (www.caiso.com), includes, for Hour 1 of April 1, 1998 through Hour 24 of November 30, 2000, the actual and day-ahead forecast load for CAISO's control area (covering roughly 70% of the state), hourly zonal electricity prices from the CAISO's real-time imbalance energy (spot) market, and the day-ahead forward market operated by the CalPX for NP15 and SP15, the two major market regions within the CAISO control area.

[14] The positively skewed distribution of electricity prices is emphasized in Knittel and Roberts (2005).

[15] We verified this numerically: For each set of the underlying parameter values, the average, over 1,000 replications, of the sample means of the forecast errors is not significantly different from zero for the three variables by a *t*-test using their Monte Carlo standard errors.

3. TEMPORAL MOVEMENTS OF THE SIMULATED ELECTRICITY FORWARD PRICES

If two prices are closely related, one price can be deduced from the other. Thus, the sensible number of forward markets and how far ahead they should cover from the perspective of their price signaling function depend on the predictability of the price relationships among the forward contracts. Specifically, for these two interrelated questions, we seek answers in the temporal movements of the following three price relationships: (1) prices of electricity forward contracts for the same delivery traded in distinct periods some time ahead, (2) prices of electricity forward contracts for distinct delivery periods traded in the same period, and (3) the relationship between the forward prices of electricity and those of the major input to generation.

3.1 Movements of electricity forward price and spread

For the first two of these three price relationships, if forward prices of electricity for delivery at a particular period are closely related between two distinct trading periods or if forward prices for two distinct delivery periods are closely related at a particular trading period, one price is redundant, as it can be deduced from the other. Thus, the number of explicit forward markets necessary for price signaling is reduced to the extent that possible forward prices are closely related.

Figure 2 plots one particular realization, out of many possible sequences, of the simulated forward prices for delivery at August 1, hour 18 traded during a week-long period before dispatch, contrasting it with the movement of spot prices hour-by-hour during the 168 hours of the same week.[16] The figure illustrates how the price of a particular forward contract moves over time in response to the underlying shocks, which are manifest in the contemporaneous movement of spot prices. In the figure, the forward price, in the base parameter case, stays virtually constant until close to dispatch (panel b), even though the market experiences large shocks around two and five days before August 1, hour 18 (panel a). Changes in the forward price are visible only within 48 hours of delivery and diminish with the time to maturity. The figure can be extended to cover a longer horizon, yet, we can safely presume no noticeable movements before the week shown in the figure.

The sensitivity of the forward price movements to the values of the parameters in the data generating process (DGP) in figure 2 is intuitive. The changes in the forward prices between two

[16] This particular period is chosen for illustration since the spot electricity price as well as forward prices for August 1, hour 18 is inherently the most volatile according to the data generating process of the simulation data. However, since the spot and forward prices shown in figure 2 are standardized to their long-run averages, the implication for the sensible number of forward markets would not differ were other delivery periods used.

trading hours are larger for the series generated with large variance parameters (σ's = 0.20) or cost function parameter (c = 5) while they become visible earlier with higher AR(1) parameters (ρ's). The forward prices start fluctuating earlier when ρ's = 0.95, yet no earlier than 4 days before delivery.

From the temporal movements of the forward price shown in figure 2, it can be easily inferred, for the second of the three price relationships, that the spread between forward prices for two distinct delivery periods does not exhibit any noticeable movements until a few days before delivery for the first of the two contracts. Clearly, many long-dated forward prices, even those for merely three or four days to maturity, are redundant, because from one price can be deduced others.

These implications from the temporal movements of the forward price are not an artifact of the realization shown in figure 2. Figure 3 plots the standard deviation, over 1,000 realizations, of the forward price standardized to the deterministic price at each trading hour for the same 168 hours preceding hour 18 of August 1. It illustrates how the forward prices for this particular delivery hour vary at each trading hour over the 1,000 realizations and how this variation changes over the 168 hours. In the base parameter case, the standard deviation stays close to zero until around 36 hours prior to delivery and increases non-linearly after this point as the trading hour approaches delivery. In other words, for all 1,000 realizations, 36-hours-ahead and longer-dated forward prices for delivery on August 1 hour 18 do not deviate from the deterministic price level.[17] As for the sensitivity to the parameters in the DGP, the forward price variation across 1,000 replications is higher when the σ's = 0.20 or c = 5, yet, in either of these two cases, the variation is essentially zero 36 hours before delivery and earlier. When the ρ's = 0.95, the standard deviation starts rising earlier, but no earlier than 48 hours before delivery.

Temporal movements of forward prices necessarily depend on the DGP of the simulation data, especially the persistence of the three underlying stochastic factors. Depending on the degree of the persistence, current deviations from the long-run averages of fuel price, load, and aggregate supply curve deliver information about their future states. Specifically, the current forecasts of the future states of these three factors are given as their long-run expected levels multiplied by the current deviations from the long-run expectations discounted at the rates of the autoregressive coefficients. Because the discount factor converges to zero at an exponential rate with time to maturity of the contract, the current deviations become less important with the

[17] The standard deviation jumps up at 18 hours prior to delivery due to the particular trading sequence assumed in the idealized setting: a daily series of the spot price of fuel input is observed at hour 1 every day, 18 hours prior to the realization of the spot electricity price for hour 18 delivery. Consequently, electricity forward prices for hour 18 delivery traded within 18 hours of delivery representing the variation in the spot price of fuel exhibit large variations across replications.

forecast horizon.[18] Figures 2 and 3 illustrate that the current deviations affect the price of long-dated forward contracts only near the delivery period, even when the deviations from the long-run expected levels of the three stochastic factors are highly persistent.

What if the time series of the underlying factors are so persistent as to be characterized by a random walk? If so, the long-dated forward prices fluctuate every period exactly by the amount of the current deviations from the deterministic states of the three underlying factors. Nevertheless, in this extreme example, the spread between the prices of forward contracts for two distinct delivery periods will not fluctuate to any significant extent even until the delivery of the first of the two contracts. That is, to say, the two forward prices will be nearly perfectly correlated. Hence, forward contracts for a small number of delivery periods would suffice.

Although the idealized market model presupposes risk-neutral behavior, the same conclusion can be derived from a perspective of hedging motivated by risk aversion. In figure 4, the standard deviation of the spot electricity price conditional on the information available at each trading period stays constant until near the delivery period.[19] In other words, information arriving more than a few days before delivery does not reduce the price risk. Given the simulation results on the equilibrium spot-forward price relationships of Bessimbinder and Lemmon (2002), the figure implies that the equilibrium forward price deduced under the alternative assumption of risk-averse agents would not fluctuate until near delivery.

These examples may seem unrealistic, yet are the implications of the idealized market. The frequency of forward trading necessary for perfect price signaling increases with the horizon at which the information arrives regarding the future market conditions, or more precisely, regarding future deviations from the long-run expected state, whereas the number of contracts with distinct delivery periods in each trading period decreases with the persistence of stochastic deviations from the long-run expected state. Figures 2 and 3 provide one example, in which the current realizations persist only briefly. Consequently, the forward markets further away from the delivery period are unnecessary as the realizations during these periods express little additional information about the future.

3.2 Relationship between forward electricity prices and input prices

Should the price of the forward contract for a particular delivery period or the spread between two prices with distinct delivery periods be stable over multiple trading periods, only one price is

[18] Models directly representing the stochastic dynamics of electricity spot prices should imply a similar result because all have a mean-reverting process. Even models with stochastic jumps (for example, Villaplana, 2003) imply similar movements of forward prices because the stochastic jumps are unpredictable.
[19] The conditional variance of the electricity spot price was obtained by the numerical method used to determine risk-neutral forward price.

necessary, as it can be used to deduce the prices of contracts for related trading or delivery periods. The same logic applies to cross-commodity price relationships, which have long been studied in the agricultural economics under the term "cross hedging" (e.g, Anderson and Danthine, 1981). Sorghum is distinct from corn in use and production, but not very much, and not very far ahead. By the logic concerning redundant prices, the corn futures market suffices for both except at the short end of the constellation of prices (Williams, 1986). For electricity, of particular relevance is natural gas, which serves among the primary inputs to electricity generation and has been actively traded on NYMEX, with futures contracts covering a horizon beyond two years. To the extent that the relationship between the prices of these two commodities is predictable, long-dated forward contracts for electricity can be duplicated from existing natural gas futures contracts. Separate markets are necessary only at the horizon that forecasts of this price relationship diverge from the long-run expectation.

This cross-commodity price relationship is not necessarily constant across seasons. Indeed, the efficiency of transferring natural gas and other generation inputs into electricity (namely, the physical heat rate) varies both at the market level due to the heterogeneity across generation units and at the individual unit level since the efficiency of transforming fuel into electricity decreases as the unit's rated capacity is approached. Given the extremely price inelastic demand and the seasonality in load, the market-clearing price of wholesale electricity is determined by the heat rate of the least efficient unit operating and the price of the input used at this marginal unit. Such seasonal variation in the heat rate should be reflected in the natural gas-electricity price relationship, or in what is often termed the "market-implied heat rate," and anticipated in the forward markets of the two commodities. To the extent that this temporal variation in the market-implied heat rate is predictable at a particular horizon, forward markets of only one commodity that time ahead should provide essentially the same information as a forward market for the other commodity.

The idea of a cross-commodity relationship is closely related to the "spark spread," often defined as the implicit positions from taking explicit long and short positions, respectively, in natural gas and electricity futures markets. Emery and Liu (2002), for example, have examined, with the following regression, how the spark spread in a constant spread ratio or the market-implied heat rate varies over time,

$$F_{t,t-k} = a_0 + a_1 \, w_{t,t-k} + e_{t,k} \tag{13}$$

where $w_{t,t-k}$ is the k-period ahead futures price of natural gas for delivery at t, $e_{t,k}$ is the disturbance term with zero unconditional mean, and the two parameters, a_0 and a_1, can be time-variant to allow for seasonal variation in the price relationship.[20]

The specification (13), when estimated with the forward prices of the two commodities, should capture seasonal variation in the price relationship. Here, the existence of separate long-dated forward markets for electricity depends not only on the estimated forward price relationship but also on its accuracy in forecasting the same relationship in the respective spot markets.[21] More directly, we are interested in how accurately the futures price of the fuel input together with the estimated market implied heat rate forecasts the spot electricity price relative to the forward price of electricity. Such relative forecasting ability can be measured by the difference between the R-squared from the regression (13) with the spot electricity price as a regressand and that from the regression of the spot electricity price on the futures price of electricity. Because the electricity-fuel price relationship, in both simulation and empirical settings, often depends on the retail demand due to the non-linear aggregate supply curve, with the inclusion of the forecasted load into the right-hand side of (13), we have,

$$p_t = b_0 + b_1\ \omega_{t,t-k} + b_2\ qf_{t,t-k} + e_t \qquad (14)$$

where p_t is the log of spot electricity price at period t, $\omega_{t,t-k}$ and $qf_{t,t-k}$ are, respectively, log of the k-period-ahead futures price of the input for period t delivery and k-period-ahead forecast of the period t load.[22] The R-squared from (14) is compared with that from the alternative,

$$p_t = \beta_0 + \beta_1 f_{t,t\text{-}k} + e_t \qquad (15)$$

where $f_{t,t-k}$ is the log of the k-period-ahead electricity forward price for time t delivery. In the idealized setting, it is the best, unbiased forecast of p_t at $t - k$. Thus, the R-squared from (15) is the

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[20] For example, Emery and Liu (2003) estimated (13), using data from the NYMEX, by including monthly dummy variables and a time trend in the right-hand side of equation, which allows the electricity-natural gas futures price relationship to differ across the 12 delivery months per year.

[21] Our argument here suggests that data used in previous studies of the cross-commodity price relationship are endogenous to the price relationship itself. That is, price data for commodities for which price relationship is predictable would be precluded from an empirical analysis. Consequently, an empirical analysis of cross-commodity price relationship is limited to markets in which any significant price relationships are inherently difficult to depict or markets that are inactive for any rational traders should not trade through the exchange to avoid transaction costs. Inactive trading in the NYMEX electricity futures market that Emery and Liu (2003) studied provides one such example.

[22] The specification in (14) is the true DGP of the simulation data aside from the non-normal distribution of the error term. In empirical analyses, the true DGP is unknown, yet, since the non-linearity in the supply function is widely acknowledged, the log-linear specification will be a normal choice. Besides, the model, including only one regressor in addition to the fuel price, is more restrictive than Emery and Liu (2003) whose model includes eleven monthly dummies and a linear trend.

highest attainable given the information available at $t - k$. The difference in the R-squared between the two regressions represents the information contained in the explicit forward price of electricity beyond that provided by the "synthetic" forward price, a forecast of subsequently realized spot electricity price, based on the forecast of the market implied heat rate as well as the futures price of the input and the load forecast.[23] An R-squared considerably lower for (14) than (15) would indicate that a large share of the variation in the spot electricity price originates in factors other than the input price and load level and, hence, a separate forward market for electricity would be necessary, at least at the forecast horizon compared.

We estimated (14) and (15) with the simulation data for six different forecast horizons: k = 3, 12, 24, 48, 72, and 168 hours ahead of the delivery period. For each of these k's, the two regression models were estimated with 8,640 hourly observations and the process was repeated 1,000 times. Figure 5 plots the averages of these 1,000 R-squared values from the OLS estimates of (14) and (15). In panel a, the two variables in the right-hand side of (14) account for approximately 80 percent of the variation in the spot electricity price in the base parameter case, even when forecast variables from one week prior to the realization are used in estimation. The mean R-squared increases to only slightly above 0.808 when the model is estimated with 24-hour-ahead forecasts. An improvement in the R-squared is noticeable only with forecasts within 24 hours of delivery (0.835 and 0.905, respectively, for forecasts of 12 and 3 hours ahead). Panel b shows essentially the same picture for the regressions with the electricity forward price as the regressor. The electricity forward price explains above 80 percent of the variation in the spot electricity price and more so for the regressions with the forward price closer to the actual dispatch as a regressor (R-squared of 0.808, 0.837, and 0.921, respectively, for forecasts of 24, 12, and 3 hours ahead).

For both regressions (14) and (15), the R-squared increases with the persistence of the deviation from the deterministic load (ρ_Q) while it is inversely related to the values of the variance parameters for all three stochastic factors. This sensitivity to the DGP parameters is not surprising, because demand and, hence, the electricity-fuel price relationship is more accurately forecasted when the deviation from the deterministic load is highly persistent. In contrast, higher volatility of the three factors simply causes large stochastic variation relative to deterministic variation within the electricity spot price, which results in lower forecast power. Of the three factors, the persistence and variance of the deviation from the deterministic load have the largest impact on the R-squared, largely because of the non-linearity in the supply function.

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[23] For the simulation data, such additional information is the prediction of the time t deviation from the long-run average supply curve based on the current deviation at $t - k$.

The difference in the R-squared from the two regressions (14) and (15) is very small, for all sets of parameter values and at all horizons considered (panel c). In the base parameter case, the 3-hour-ahead forward price of electricity accounts for the spot price variation less than one percentage point more than the forward input price and the forecasted load, which increment is negligible given that 92% of spot price variation is accounted for by either regression. Even that small disadvantage disappears when the forecasts further away from the delivery period are used in estimation. As for the sensitivity to the DGP parameters, the difference in the R-squared increases with both the volatility and persistence of the deviation from the long-run average supply curve, the results as expected because the current supply condition delivers more information regarding its future state when they are more persistent or volatile. The electricity forward price, incorporating this information, predicts the spot price more accurately than the synthetic forward price. Nonetheless, the sensitivity to the two parameters is marginal, given the large share of the spot price variation explained by either regression.

While the high volatility of electricity prices has been emphasized in the literature, it is often unstated that this price volatility is partly predictable, just by knowing the hour, day, and month of the delivery period. The current deviation from this long-run expected price further improves the accuracy of price forecasts when it is persistent, yet only marginally so, given that the deterministic price variation accounts for a large share of the total.[24] In such a case, the forward price stays close to the long-run expected level for that hour, day, and month even until near the delivery period. In other words, a large share of the total price variation is accounted for by variation in the long-run expected price level, the rest in the last days of forward contracts' lives, and almost nothing in the medium term.

4. NYMEX NATURAL GAS FUTURES AS FORECASTS OF THE CAISO SPOT ELECTRICITY PRICE

In this section, we construct a synthetic forward price based on the NYMEX natural gas futures prices and load forecast in California and examine the ability of this synthetic forward price to predict the California spot electricity price over 1998-2000, relative to the predictive ability of the

[24] The relative magnitudes of stochastic and deterministic price variation can be inferred by comparing the unconditional variances of spot and forward prices. In the base parameter case, the standard deviation of the forward prices over all seasons and hours ranges from 16.634 to 18.079 ($/MWh) with numbers smaller for forward prices of longer maturity. These numbers correspond to about 64% to 95% of the standard deviations of the spot price, which represent the shares of the total price variation predicted at the time of forward trading. The forward prices converge to the long-run expected level as the trading takes place further away from the delivery period and, accordingly, the share of spot price variation accounted for by the forward price decreases with the time to maturity at a decreasing rate.

CalPX day-ahead forward prices, the only active forward market in California.[25] This long-dated synthetic forward price should predict the spot electricity price less accurately than the CalPX day-ahead electricity forward price, which, established on the day previous to actual dispatch, should reflect more updated information on the natural gas price and load level as well as other market conditions such as unscheduled outages of generation units. The forecasting ability of longer-dated forward prices of electricity, had they existed, would have fallen within these two boundaries.

Unlike in the previous section, the relative forecasting ability of the long-dated synthetic price and the CalPX forward price is measured by an out-of-sample test. We divide the available observations into two periods: April 3, 1998-March 31, 2000 and April 1, 2000-November 30, 2000, the latter of which starts shortly before the period identified as the California Energy Crisis. We estimate from the first observation period the aggregate supply function or market-implied heat rate, using the realized natural gas and electricity spot price as well as the realized load in the right-hand side of (14),

$$p_t = b_0 + b_1\,\omega_t + b_2\,q_t + e_t \tag{16}$$

where ω_t and q_t are the spot natural gas price and the realized load level. The estimate of (16) is then used to construct a time series of synthetic forward prices for the Energy Crisis period,

$$f_{t,t-k} = b_0 + b_1\,\omega_{t,t-k} + b_2\,qf_{t,t-k} + e_t \tag{17}$$

where $f_{t,t-k}$ is the k-period-ahead synthetic forward price of electricity for deliver at hour t, $\omega_{t,t-k}$ is the k-period-ahead futures price natural gas, and $qf_{t,t-k}$ is the k-period-ahead load forecast.

Unfortunately, due to the limited availability of California load data as well as the specific trading sequence of the NYMEX natural gas market, estimation of (16) requires several refinements. First, the data on the realized load in the right-had side of (16) is available only at the California Independent System Operator (CAISO)'s system-wide level, which comprises five congestion zones, whereas the electricity prices in the CAISO market is established for each zone.[26] Moreover, the load forecast in the right-hand side of (17) is available only for the CAISO's day-ahead and hour-ahead schedules. Because of the absence of the zonal-level realized load data, we estimated (16) using the CAISO system-wide realized load while using the CAISO real-

[25] Data used in this section include, in addition to those used in section 2, the NYMEX natural gas futures price and the spot natural gas price at the PG&E Malin delivery point located at the California-Oregon border for the period between April 1, 1998 and November 30, 2000.

time imbalance energy (spot) price from NP15, one of the five zones, the one covering a majority of the Northern California, as a regressand. We constructed a time series of CAISO system-wide load forecasts based on the following load seasonality model estimated with the CAISO system-wide realized load during the first observation period,[27]

$$QA_t = \alpha_0 t + \sum_{s=1}^{24} i(h_t = s)\left[\sum_{j=1}^{7}\alpha_{sj} i(wkday_t = j) + \sum_{i=1}^{8}\left\{\alpha_{1si}\sin\left(\frac{2\pi i h_t}{8760}\right) + \alpha_{2si}\cos\left(\frac{2\pi i h_t}{8760}\right)\right\}\right] + \varepsilon_t$$

$$\varepsilon_t = \rho\varepsilon_{t-1} + u_t \qquad (18)$$

where $i(\cdot)$ is an indicator variable with value one if its argument is true and zero otherwise, h_t is the hour count from the beginning of the calendar year, $wkday_t$ is the day of the week (1 = Sunday, 2 = Monday, and so on), and u_t is assumed iid N(0, σ^2). A time series of the k-period-ahead forecasts is constructed using the estimate of (18) as,

$$\hat{Q}F_{t,t-k} = \hat{\alpha}_0 t + \sum_{s=1}^{24} i(h_t = s)\left[\sum_{j=1}^{7}\hat{\alpha}_{sj} i(wkday_t = j) + \sum_{i=1}^{8}\left\{\hat{\alpha}_{1si}\sin\left(\frac{2\pi i h_t}{8760}\right) + \hat{\alpha}_{2si}\cos\left(\frac{2\pi i h_t}{8760}\right)\right\}\right] + \hat{\rho}^k e_{t-k} \qquad (19)$$

where e_{t-k} is the regression residual at $t-k$.

Second, specification (16), including only the natural gas price and the load forecast on the right-hand side, implicitly assumes that natural gas generation units, each with a different efficiency level, are always the marginal unit, setting the market-clearing price, regardless of the load level and the amount of electricity supplied by generation units of other types. Unfortunately, the absence of data on operations of individual generation units and price/ availability of these non-natural-gas generation resources allows us neither to verify this assumption nor to model explicitly the resulting temporal variations in the electricity-natural gas price relationship.[28] As an alternative, we consider the following specification for the three parameters in (16),

$$b_i = b_{i0} + b_{i1}\sin\left(\frac{2\pi h_t}{8760}\right) + b_{i2}\cos\left(\frac{2\pi h_t}{8760}\right) + b_{i3}\sin\left(\frac{2\pi h_t}{24}\right) + b_{i4}\cos\left(\frac{2\pi h_t}{24}\right) \qquad (16')$$

[26] Prices differ among the five zones whenever the transmission lines connecting them congest.
[27] Specification (16), which allows the seasonal cycle as well as the day-of-the-week effect to vary across the 24 hourly blocks a day, is more flexible than (3), for the purpose here is to obtain a function that fits well the observed load cycle.
[28] In California, electricity is supplied by various generation units, such as nuclear and hydroelectric plants whose availability varies across seasons. Of particular relevance are the hydroelectric units, which serve 40% of total electricity generation in the Northern California while their availability exhibits strong seasonal pattern with its supply lower during the summer than in the winter.

where i = 0, 1, and 2. The specification (16′), while it still assumes that a natural gas generation unit is the marginal unit at all times, seeks to capture seasonal variation in the link between natural gas-electricity price relationship and load level resulting from that of the availability of other generation resources.[29] If anything, the inevitable imprecision leading to (16′) widens the boundaries for the comparison of forecasting abilities.

Third, because of the price cap imposed on the CAISO real-time imbalance energy market, the observed prices in this spot market reflect the true generation cost only below the price cap.[30] A standard method to compensate for such censoring of the dependent variable is the tobit model, which is easy to estimate in the case where the censoring point is known. With the assumption of normality for e_t in the right-hand side of (16), the likelihood function becomes,[31]

$$\ln L(\mathbf{b}, \sigma | P_t, P_t^{cap}, \hat{Q}F_{t,t-k}, w_{t,t-k}) = i(P_t = P_t^{cap})\, \Phi(P_t^{cap} - P_t^*) + (1 - i(P_t = P_t^{cap}))\, \phi\left(\frac{P_t - P_t^*}{\sigma}\right) \qquad (20)$$

where $\Phi(\cdot)$ and $\phi(\cdot)$ are, respectively, the distribution and density function of the standard normal, P_t^{cap} is a price cap at t, $P^*_t = b_0 + b_1\, \omega_t + b_2\, q_t$, and σ is the variance of e_t.

Finally, the differences in the frequency of the natural gas and electricity price data and the specific delivery terms of a NYMEX contract complicate the construction of the synthetic price in (17). The NYMEX natural gas futures contracts are defined in monthly blocks and traded daily whereas electricity is traded in hourly blocks in both the CAISO and CalPX market. Hence, we constructed the hourly series of natural gas futures prices by using, for each combination of t and k, the closing price on trading day, $d(t - k)$, of the NYMEX futures contract for delivery in month, $m(t)$, where $d(\cdot)$ and $m(\cdot)$ converts observation in hour into day and month, respectively. A related issue is that because the NYMEX natural gas futures market trades only on weekdays and trading ends on three business days prior to the first calendar day of the delivery month, futures prices do not exist for some combinations of t and k with $d(t - k)$ corresponding to these non-trading days. For weekends and holidays, we utilized the futures price from the nearest previous

[29] The validity of this assumption relies on the operational inflexibility as well as ability to arbitrage temporally non-natural-gas-powered generation units. For example, units with low operational flexibility, such as nuclear plants due to high start-up cost, cannot adjust their generation schedules in response to the market price of electricity and hence submit their supply bids lower than the expected market-clearing price. On the other hand, hydroelectric units, with their relative operational flexibility due to substantially lower start-up cost and relative storability of their generation inputs before transforming to electricity, should operate when the spot price is expected to be high.

[30] The price cap was initially set at \$250/MWh. It was raised to \$750 on September 1, 1999, then reduced to \$500 on July first and further to \$250 on August 7, 2000. The price cap was hit repeatedly during the summer 2000 and after.

trading day. Because $d(t-k)$ and the nearest previous trading day can take large values for observations toward the end of the delivery month, we constructed the synthetic forward price in two ways: (a) k ranging from 1 to 30 days (k = 24 to 720) in every one-day step and 35 to 364 days (k = 816 to 8736) in every seven-day step, using the k-period lagged daily spot natural gas price, and (b) for k from 35 to 364 days in every seven-day step, using the NYMEX natural gas futures price.[32] That is,

$$f_{t,t-k} = \hat{b}_0 + \hat{b}_1 \ln(w^{PGE}_{d(t-k)}) + \hat{b}_2 \ln(\hat{Q}F_{t,-k}) \qquad \text{for all } k \text{ considered} \qquad (17a)$$

$$f_{t,t-k} = \hat{b}_0 + \hat{b}_1 \ln(w^{NYMEX}_{m(t),d(t-k)}) + \hat{b}_2 \ln(\hat{Q}F_{t,-k}) \qquad \text{for } k \text{ above 34 days} \qquad (17b)$$

where $f_{t,t-k}$ is the k-period-ahead synthetic forward price for period t delivery at NP15, $w^{NYMEX}_{m(t),d(t-k)}$ is the closing price on $d(t-k)$ of the NYMEX natural gas futures contract for delivery in $m(t)$, $w^{PGE}_{d(t-k)}$ is the spot natural gas price observed at the PG&E Malin delivery point located on California-Oregon border on $d(t-k)$, $\hat{Q}F_{t,t-k}$ is the estimate of k-period-ahead load forecast from (19), and $\hat{b}_i$, i = 0, 1, 2 are the parameters in (16′) estimated with the first observation period.

To summarize, we estimated the parameters, b's, ρ, and σ in (16) and (18), using the realized spot prices of electricity and natural gas as well as the realized load for the period between April 3, 1998 through March 31, 2000. Using these estimated functions and the NYMEX natural gas futures price or the lagged spot price of natural gas, we constructed a time series of synthetic forward prices for the period between April 1 and November 30, 2000.[33] The root-mean-squared error (RMSE) of the synthetic forward price as a forecast of the realized CAISO NP15 spot price is compared with the RMSE of the CalPX day-ahead forward price.

In figure 6, the RMSE of the synthetic forward price constructed with the lagged natural gas spot price gradually increases with k, yet stays around \$120/MWh for k as large as 10 weeks, only 20% above the RMSE of the CalPX day-ahead forward price. The RMSE of the synthetic forward price then increases at a relatively rapid rate, reaching 150 at k = 7 months, after which it stays at this level for the range of k considered. The RMSE of the synthetic forward price constructed with the NYMEX natural gas futures price exhibits the similar movements over the range of k.

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[31] Clearly, the normality assumption does not hold given the heteroskedasticity and serial correlation of the electricity price series. Nonetheless, because we are only interested in the forecasting ability rather than the coefficient estimates in (16′) and their distributions, we estimated (20) by the standard tobit method.

[32] For example, for any t from the 30th day of the month, the nearest previous trading day is more than 33 days prior to t. Clearly, using the nearest previous closing price in place of the missing prices for these observations reduces the informational content of the natural gas futures prices in the right-hand side of (17) by a large extent.

Figure 7 illustrates the accuracy of the two forecast variables in the right-hand side of (17). In panel a of the figure, the RMSE of the load forecast by (19) as a forecast of the CAISO system-wide realized load in the same period is initially low at k = 1 day, only 20% higher than the RMSE of the CAISO day-ahead load forecast. The RMSE of the load forecast by (19) increases rapidly to slightly above 2000MWh and stays at this level for k greater than 6 days. Even though this is almost twice the magnitude of the RMSE of the CAISO day-ahead load forecast, it is only of marginal importance relative to the amount of load variation explained by either forecast.[34] In contrast, the RMSE of the lagged natural gas spot price as a forecast of the spot natural gas price rapidly increases with k for k less than 21 days, after which it gradually increases and stays around $2.5/MBTu for k as large as 10 weeks (panel b). The RMSE then increases rapidly after this point until k reaches 7 months, after which it gradually increases and stays around $3.5/MBTu for k up to 12 months. The RMSE of the NYMEX natural gas futures price exhibits similar movements as that of the lagged spot price, except that it is slightly lower for the former than for the latter over the range of k considered. Apparently, the poor forecasting ability of natural gas price forecasts contribute to that of the synthetic forward price.

What is less apparent in figure 7 is that the RMSE of the synthetic electricity forward prices in forecasting the electricity spot prices exhibits only a mild increase whereas the RMSE of either of the two natural gas price forecast increases rather rapidly over the range of k considered. Although not presented in the figure, a close examination of the forecasting errors by the synthetic electricity forward price and the natural gas price forecasts provides two plausible explanations for the inconsistency of their sensitivity to the forecasting horizon (k). First, the lagged spot natural gas price and the NYMEX natural gas futures price, especially for those with longer forecast horizon, under-forecasted the subsequently realized spot natural gas price by large magnitude during October and November 2000. During these two months, the spot natural gas price increased dramatically from $5.33/MBTu on October 1, 2000 to $16.35/Mbtu on December 1, 2000 whereas the NYMEX natural gas futures price increased only moderately from $5.31/Mbtu for the October 2000 contract traded on September 27, 2000 to $6.02/Mbtu for the December 2000 contract traded on November 28, 2000.[35] These large forecasting errors in the natural gas spot price in California are expected to follow similarly large forecasting errors of the

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[33] Whenever the predicted price by (17a) and (17b) exceeded the price cap, we replaced them with the price cap. This replacement was necessary only for a few hours while the CAISO spot price for NP15 hit the price cap frequently during the summer 2000 and after.

[34] The comparison of the predicting accuracy of (16) relative to the CAISO day-ahead load forecast as measured by Theil's U statistics, $U = \sqrt{n^{-1}\sum_i (Q_i - \hat{Q}_i)^2 \Big/ n^{-1}\sum_i Q_i^2}$ where Q_i and $\hat{Q}_i$ are the realized and forecasted load, respectively, and n is the number of periods being forecasted, is 0.04 and 0.07 for the two forecasts, respectively, for k = 4 days and higher.

[35] This large increase in the price of natural gas within California relative to other areas of the U.S. has itself been controversial.

synthetic forward prices for electricity. Nonetheless, the forecasting errors of the synthetic forward price of electricity during the same two-month period of late 2000 are rather moderate even for large *k*, mainly due to the low price caps set in the CAISO spot electricity market.[36]

Second, even though a small forecasting error by the lagged spot natural gas price at a short horizon, say 2-days, is expected to follow accordingly a small forecasting error by the synthetic electricity forward price during this two-month period, the forecasting error by even 2-day-ahead synthetic forward price exceeds that of the CalPX day-ahead forward price by a considerable extent. By implication, that under-forecasting of the natural gas spot price is not only the source of the under-forecasting by the synthetic forwards. Rather, the high RMSE must trace to the estimated aggregate supply functions, (17a) and (17b), forecasting the market implied heat rate well below the realized heat rate during these two odd months. This under-forecasting is consistent with the structural changes of the California market discussed in Bushnell (2004). Among these changes, a dramatic increase in the cost of emission permits and a rapid increase in outages of generation units resulted in upward shifts of aggregate supply curve, which appears insufficiently captured by the specification (16′).[37]

In short, the results from out-of-sample forecast tests indicate that spot electricity prices in the California market were highly volatile such that even the existing CalPX day-ahead forward price anticipated less than half of that variation during the period of its operation. The synthetic forward price, constructed from a simple load seasonality model of (16) and either of the NYMEX futures price or the lagged spot price of natural gas, is less accurate than the CalPX day-ahead electricity forward price in predicting the subsequent period spot price. Yet, what is most important for present purposes is that the forecasting is only slightly less accurate for the price constructed based on even three-months-ahead forecast of these two variables. Forward markets of electricity longer dated than the CalPX day-ahead market, had they existed, would presumably have fallen between these bounds of forecasting ability. In other words, they would have added little information beyond that contained in the synthetic forward price. In particular, it is unlikely that they would have predicted the dramatic price increases in late 2000.

[36] Aside from these structural changes, inaccurate forecasting of the synthetic forward price in the latter half of 2000 is also attributable to a dramatic increase in natural gas price. Nonetheless, the price cap lowered well ahead of such dramatic increase in natural gas price limited the forecasting errors by the synthetic forward price in the two-month period.

[37] Bushnell argues that the reported outages increased in fall 2000 due to a potential suspension of payments from the incumbent utilities after wholesale price repeatedly have exceeded the fixed retail rate during summer 2000.

5. CONCLUSION

Having examined the temporal movements of representative forward price series generated from a model of an idealized electricity market, we find that forward prices fluctuate only near actual dispatch and that the constellation converges to the long-run expected level in a matter of days. By implication, prices of long-dated forward contracts, were the market to exist, would not fluctuate and, hence, the price from one representative period could be used to deduce those in other periods, just as positions in that representative period could serve as hedges for the others. By the same logic, because the relationship between electricity and natural gas prices can be predicted fairly accurately, long-term forward contracts for electricity can be duplicated using the existing forward contracts for the principal input and the seasonal heat rate.

Although the results from our simulation rely on assumed data generating processes, including that for natural gas, the implications for actual electricity markets should not be much different, as suggested by our application to three years of data from California. The relationship between electricity and natural gas forward prices in the real world is determined by factors other than the retail demand level, such as the availability of hydro resources, prices of emission quota, or even market power. Our analysis implies that even when forward markets for these factors are absent, the forecasts of the future states of these factors, including the exercise of market power, could be incorporated into the predictions of the relationship between natural gas and electricity prices. Indeed, the NYMEX natural gas futures prices, incorporated into a model of seasonal and diurnal cycles of load, predicted well the variation in the spot electricity price in California, only slightly less accurately than the CalPX day-ahead forward price, even in the extreme conditions experienced during the summer and fall of 2000.

These extreme, not to say notorious, conditions within California in the summer and fall of 2000 can be used to restate the main conclusions of this paper. As part of the restructuring that commenced in 1998, the two main distribution-oriented utilities had agreed to satisfy retail demand at the equivalent of \$60/MWh (Borenstein, 2002). When the wholesale price of electricity exceeded that \$60/MWh for long stretches of 2000, the two utilities' financial condition deteriorated, further discouraging those selling wholesale electricity. At least with the benefit of hindsight, it seems obvious that before 2000, if not in 1996 when the fixed retail price became part of the restructuring law, the utilities should have contracted for electricity to the horizon of their obligation. Given that restructuring within California amounted to a giant bet on natural gas prices, the distribution-oriented utilities could have offset that position directly in the NYMEX natural gas futures market. Consider a simple hedging strategy: From April 1996, month by month they could have gone "long" in the NYMEX delivery month two years in proportion to their forecasted seasonal loads,[38] and later could have liquidated that position for Henry Hub natural gas as it matured.[39] Given the rise in natural gas prices over April 1998 through December 2000, this hedging strategy would have resulted *ex post* in some $3.7 billion,[40] comparable to the *ex post* oligopoly profits extracted by generators (as estimated by Borenstein, et al., 2002). The "California Energy Crisis" happened for other reasons than because a long-term forward market for electricity had not been initiated as part of the original restructuring. Close substitutes did exist.

Because organized exchanges are themselves costly and because liquidity is desirable, relentless pressure concentrates trading into a few benchmark forward markets. The Henry Hub in Louisiana has the longest-dated forward market for natural gas, by virtue of being the delivery location for NYMEX futures. It may be that another location for distant forward prices would serve better but only one such benchmark location is necessary. Given that an active futures market exists for natural gas, to expect an active futures market for electricity in addition is rather like expecting a futures market for french fries in addition to the futures market for potatoes.

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[38] Alternatively, the utilities could have "stacked" positions in the distant delivery months to reach the horizon of their commitment to the fixed retail price. Each month through 1998-2000, they could have rolled the stack into the newly traded delivery months, the spreads among those distant prices likely staying constant. For corn, soybeans, and cotton, Gardner (1989) concluded that farmers could stack six years of output by placing six times annual output in the farthest available futures contract, typically 18 months ahead, and rolling the remainder each year into newly traded contracts as they become available.

[39] A more sophisticated hedging strategy would transfer the NYMEX positions upon liquidation into one-month contracts for California natural gas. This more sophisticated strategy, or even a less sophisticated one in which procurement would have been a constant amount per month, would have been resulted in much the same *ex post* hedging result, because the general rise in natural gas prices from early 2000 dominates the calculations. Similarly, the revenue from this simple strategy at different hedging horizons (say 12 months ahead rather than 24 months ahead) would have accomplished much the same *ex post* result over 1998-2000, provided the horizon was at least six months. Of course, even this simple strategy would have had complications such as the regulatory treatment of the financial results, the illiquidity in the distant NYMEX delivery months, and the basis risk between nearby California and Henry Hub natural gas prices at the time of liquidation of the long positions.

[40] Revenue from cross-hedging with the NYMEX natural gas futures market is calculated under the scenario where all distribution-oriented utilities in the CAISO control area fully hedge their procurement needs including those from their self-owned generation units, two years before that gas would be normally used. Under this scenario, the distribution-oriented utilities include what would be the equivalent in terms of natural gas of their inputs from hydro or nuclear. The price of natural gas measures the marginal value of these other fuels so they should be included to the extent that the utilities owned or had long-term contracts for those inputs, these hedging positions are overstated.) The *ex post* revenue from the hedge is calculated, for each delivery hour, $TR_t = E_{t-k}[P_t/w_t]\ E_{t-k}[Q_t](w^{NYMEX}_{m(t)} - w^{NYMEX}_{m(t),d(t-k)})$, and summed over the hours between April 1, 1998 through December 31, 2000. In this formula, $E_{t-k}[P_t/w_t] = \hat{P}_t / w^{NYMEX}_{d(t-k)}$ represents the forecast of the market-implied heat rate. That is, the number of the NYMEX natural gas contracts to be purchased at $t - k$ incorporates the extent that in the peak summer hours more natural gas is needed for the same amount of electricity, regardless of whether that greater need is caused by an inefficient generator at the margin or by an exercise of market power. The expected spot price, $\hat{P}_t$, is obtained as the predicted electricity spot price from the estimate of (14). The second term, $E_{t-k}[Q_t]$, is the load forecast according to the estimate of (16), which, multiplied by $E_{t-k}[P_t/w_t]$, yields the number of the NYMEX natural gas futures contracts required to full hedge. The last term in parentheses is the sum of changes in the price of the natural gas futures for delivery in month $m(t)$ from $d(t - k)$ to the last trading day of the contract (3 business days prior to the first calendar day of the delivery month). That is, natural gas for April, 1998 was supposedly purchased in April 1996 while gas for December 2000 in December 1998.

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Table 1. Parameter values used in the data generation process

Parameters and their values are used in the following stochastic processes defining the idealized market model:

Price-load curve: $P^0_t = bw_t\left(Q_t^D\right)^{c-1}\ (1 + \sigma_{AS}\, e_{1,t}),\qquad e_{1,t} = \rho_{AS}\, e_{1,t-1} + u_{1,t}$

Fuel price equation: $w_d = w_{0,d} + \sigma_{w,d}\, e_{4,d}\ ,\qquad e_{4,d} = \rho_w\, e_{4,d\text{-}1} + u_{4,d}$

$$w_{0,d} = \beta_0 + \beta_1 \sin\left(\tfrac{2\pi d}{360}\right) + \beta_2 \cos\left(\tfrac{2\pi d}{360}\right) + \beta_3 \sin\left(\tfrac{2\pi d}{180}\right) + \beta_4 \cos\left(\tfrac{2\pi d}{180}\right)$$

$$\sigma_{w,d} = \sigma_{w,0}\ \tfrac{\beta_0 v_d}{\gamma_0}\sqrt{1-\rho_w^2}$$

$$v_d = \gamma_0 + \gamma_1 \sin\left(\tfrac{2\pi d}{360}\right) + \gamma_2 \cos\left(\tfrac{2\pi d}{360}\right) + \gamma_3 \sin\left(\tfrac{2\pi d}{180}\right) + \gamma_4 \cos\left(\tfrac{2\pi d}{180}\right)$$

Load Equation: $Q^D_t = Q^{DT}_t\ (1 + \sigma_{QA}\, e_{3,t})\ ,\qquad e_{3,t} = \rho_{QA}\, e_{3,t-1} + u_{3,t},$

$Q^{DT}_t = Q^{DT0}_t\ (1 + \sigma_{QF}\, e_{2,t})\qquad e_{2,t} = \rho_{QF}\, e_{2,t-1} + u_{2t}$

$$Q_t^{DT0} = \alpha_0 + \alpha_1 \sin\left(\tfrac{2\pi h(t)}{8640}\right) + \alpha_2 \cos\left(\tfrac{2\pi h(t)}{8640}\right) + \alpha_3 \sin\left(\tfrac{2\pi h(t)}{24}\right) + \alpha_4 \cos\left(\tfrac{2\pi h(t)}{24}\right) + \alpha_5 WKEND_t$$

Parameters	Base Case	Convexity	Volatility	Persistence
Seasonal and diurnal variation in load				
α_0	110			
α_1	-15			
α_2	-15			
α_3	-20			
α_4	-10			
α_5	-20			
ρ_{QF}	0.9			0.95
σ_{QF}	0.1		0.2	
Seasonal variation in fuel price				
β_0	3			
β_1	-0.075			
β_2	0.3			
β_3	0.15			
β_4	0.075			
Seasonal variation in fuel price variance				
γ_0	1			
γ_1	1/7			
γ_2	3/7			
γ_3	2/7			
γ_4	-2/7			
Aggregate supply function				
c	3	5		
ρ_{AS}	0.9			0.95
σ_{AS}	0.1		0.2	
Load deviation				
ρ_{QA}	0.9			0.95
σ_{QA}	0.1		0.2	
Fuel price deviation				
ρ_w	0.9			0.95
σ_{w0}	0.1		0.2	

Table 2. Descriptive statistics of the simulated series for electricity price, load, and fuel price

Statistics are calculated for each of 1,000 sets of 8,640 hourly observations. Reported values are means over the 1,000 replications.

	Parameters			**Mean**			**Standard Deviation**						**Skewness**					
	Curvature in supply function (c)	Variance (σ)	AR(1) (ρ)	Fuel price (w_t)	Realized load (QA_t)	Spot price (P_t)	Fuel price (w_t)	Realized load (QA_t)	Spot price (P_t)	Price forecast error (k-hours ahead, $P_t - F_{t,t-k}$)			Fuel price (w_t)	Realized load (QA_t)	Spot price (P_t)	Forward price (k-hours ahead, $F_{t,t-k}$)		
										$k = 3$	24	168				$k = 3$	24	168
Base parameter set																		
By month																		
1	3	0.1	0.90	3.372	86.54	26.94	0.254	22.01	14.03	6.060	8.705	8.716	-0.017	0.271	0.962	0.830	0.623	0.589
2	3	0.1	0.90	3.255	83.60	24.29	0.343	21.72	13.03	5.869	8.472	8.484	-0.027	0.267	1.000	0.864	0.661	0.603
3	3	0.1	0.90	2.985	85.94	23.50	0.353	22.12	12.53	6.042	8.652	8.662	0.026	0.269	0.999	0.870	0.675	0.609
4	3	0.1	0.90	2.727	93.42	25.21	0.255	22.80	12.70	6.531	9.426	9.438	0.092	0.291	0.974	0.839	0.640	0.592
5	3	0.1	0.90	2.633	104.53	30.23	0.137	23.52	14.03	7.287	10.475	10.481	-0.003	0.301	0.932	0.795	0.591	0.578
6	3	0.1	0.90	2.755	115.11	38.25	0.098	24.72	17.03	8.007	11.535	11.551	0.007	0.294	0.904	0.777	0.580	0.571
7	3	0.1	0.90	2.937	121.45	45.23	0.137	25.32	19.66	8.450	12.150	12.160	0.107	0.319	0.930	0.806	0.594	0.581
8	3	0.1	0.90	3.031	125.71	49.85	0.214	25.29	21.15	8.712	12.525	12.536	0.019	0.304	0.927	0.800	0.590	0.555
9	3	0.1	0.90	3.011	122.83	47.36	0.256	25.06	20.44	8.506	12.257	12.270	0.015	0.308	0.932	0.812	0.610	0.570
10	3	0.1	0.90	2.989	114.34	40.91	0.222	24.52	18.39	7.964	11.523	11.539	-0.006	0.316	0.956	0.822	0.618	0.587
11	3	0.1	0.90	3.085	103.95	35.08	0.178	23.75	16.45	7.242	10.426	10.437	-0.026	0.292	0.926	0.797	0.602	0.578
12	3	0.1	0.90	3.258	94.09	30.49	0.191	22.69	15.01	6.590	9.500	9.519	0.024	0.282	0.936	0.798	0.595	0.571
By hour																		
1	3	0.1	0.90	-	89.45	25.24	-	21.74	12.55	6.256	9.119	9.144	-	0.292	1.039	0.903	0.697	0.664
2	3	0.1	0.90	-	85.64	23.20	-	21.41	11.82	6.007	8.744	8.768	-	0.283	1.038	0.901	0.699	0.666
3	3	0.1	0.90	-	83.08	21.89	-	21.21	11.34	5.831	8.493	8.516	-	0.280	1.040	0.905	0.705	0.672
4	3	0.1	0.90	-	81.98	21.34	-	21.14	11.16	5.768	8.387	8.410	-	0.280	1.052	0.915	0.709	0.674
5	3	0.1	0.90	-	82.40	21.55	-	21.18	11.24	5.796	8.428	8.452	-	0.280	1.051	0.911	0.706	0.671
6	3	0.1	0.90	-	84.31	22.52	-	21.33	11.58	5.934	8.623	8.648	-	0.286	1.048	0.910	0.705	0.671
7	3	0.1	0.90	-	87.57	24.22	-	21.58	12.19	6.145	8.937	8.965	-	0.291	1.045	0.908	0.700	0.668
8	3	0.1	0.90	-	91.97	26.64	-	21.97	13.05	6.437	9.380	9.411	-	0.298	1.043	0.907	0.697	0.665
9	3	0.1	0.90	-	97.23	29.67	-	22.42	14.11	6.795	9.900	9.931	-	0.308	1.038	0.903	0.693	0.663
10	3	0.1	0.90	-	102.96	33.17	-	22.91	15.30	7.186	10.459	10.491	-	0.314	1.034	0.897	0.689	0.658
11	3	0.1	0.90	-	108.78	36.94	-	23.44	16.58	7.577	11.022	11.057	-	0.318	1.025	0.889	0.681	0.651
12	3	0.1	0.90	-	114.29	40.68	-	23.96	17.81	7.939	11.562	11.597	-	0.319	1.003	0.880	0.670	0.644
13	3	0.1	0.90	-	119.11	44.11	-	24.42	18.95	8.273	12.050	12.085	-	0.317	0.994	0.869	0.659	0.634
14	3	0.1	0.90	-	122.96	46.97	-	24.81	19.89	8.558	12.442	12.478	-	0.318	0.990	0.865	0.655	0.629
15	3	0.1	0.90	-	125.50	48.89	-	25.06	20.53	8.717	12.687	12.724	-	0.319	0.991	0.857	0.652	0.627
16	3	0.1	0.90	-	126.61	49.75	-	25.19	20.83	8.794	12.810	12.848	-	0.320	0.995	0.864	0.651	0.625
17	3	0.1	0.90	-	126.20	49.42	-	25.12	20.70	8.768	12.778	12.815	-	0.321	0.991	0.866	0.653	0.626
18	3	0.1	0.90	-	124.29	47.97	-	24.95	20.24	8.639	12.599	12.636	-	0.321	0.990	0.870	0.656	0.629
19	3	0.1	0.90	-	121.02	45.52	-	24.62	19.43	8.402	12.259	12.294	-	0.317	0.992	0.869	0.656	0.629
20	3	0.1	0.90	-	116.61	42.31	-	24.18	18.36	8.099	11.815	11.850	-	0.317	1.003	0.871	0.661	0.634
21	3	0.1	0.90	-	111.35	38.65	-	23.66	17.13	7.733	11.286	11.318	-	0.316	1.014	0.878	0.669	0.641
22	3	0.1	0.90	-	105.62	34.85	-	23.12	15.84	7.345	10.710	10.741	-	0.311	1.015	0.883	0.676	0.647
23	3	0.1	0.90	-	99.80	31.21	-	22.60	14.60	6.958	10.136	10.165	-	0.304	1.024	0.895	0.685	0.653
24	3	0.1	0.90	-	94.28	27.94	-	22.13	13.48	6.592	9.605	9.630	-	0.296	1.028	0.901	0.690	0.659
All hours and months																		
	3	0.1	0.90	3.003	104.29	34.78	0.377	28.01	19.03	7.356	10.704	10.737	0.268	0.319	1.160	1.024	0.824	0.791
Sensitivity to DGP parameters																		
All hours and months																		
	3	0.1	0.95	3.004	104.27	34.78	0.366	27.99	18.98	5.530	10.263	10.717	0.236	0.319	1.149	1.075	0.850	0.797
	3	0.2	0.90	3.006	104.25	36.89	0.653	38.54	30.48	14.923	21.719	21.784	0.217	0.694	2.179	1.877	1.365	1.278
	5	0.1	0.90	3.003	104.29	51.76	0.377	28.01	58.04	7.356	10.704	10.737	0.268	0.319	2.890	2.524	2.056	2.026

Figure 1. **Seasonal and diurnal cycles of the simulated load series and the coefficient of variation compared to CAISO realized load (April 1, 1998 - December 31, 2000)**

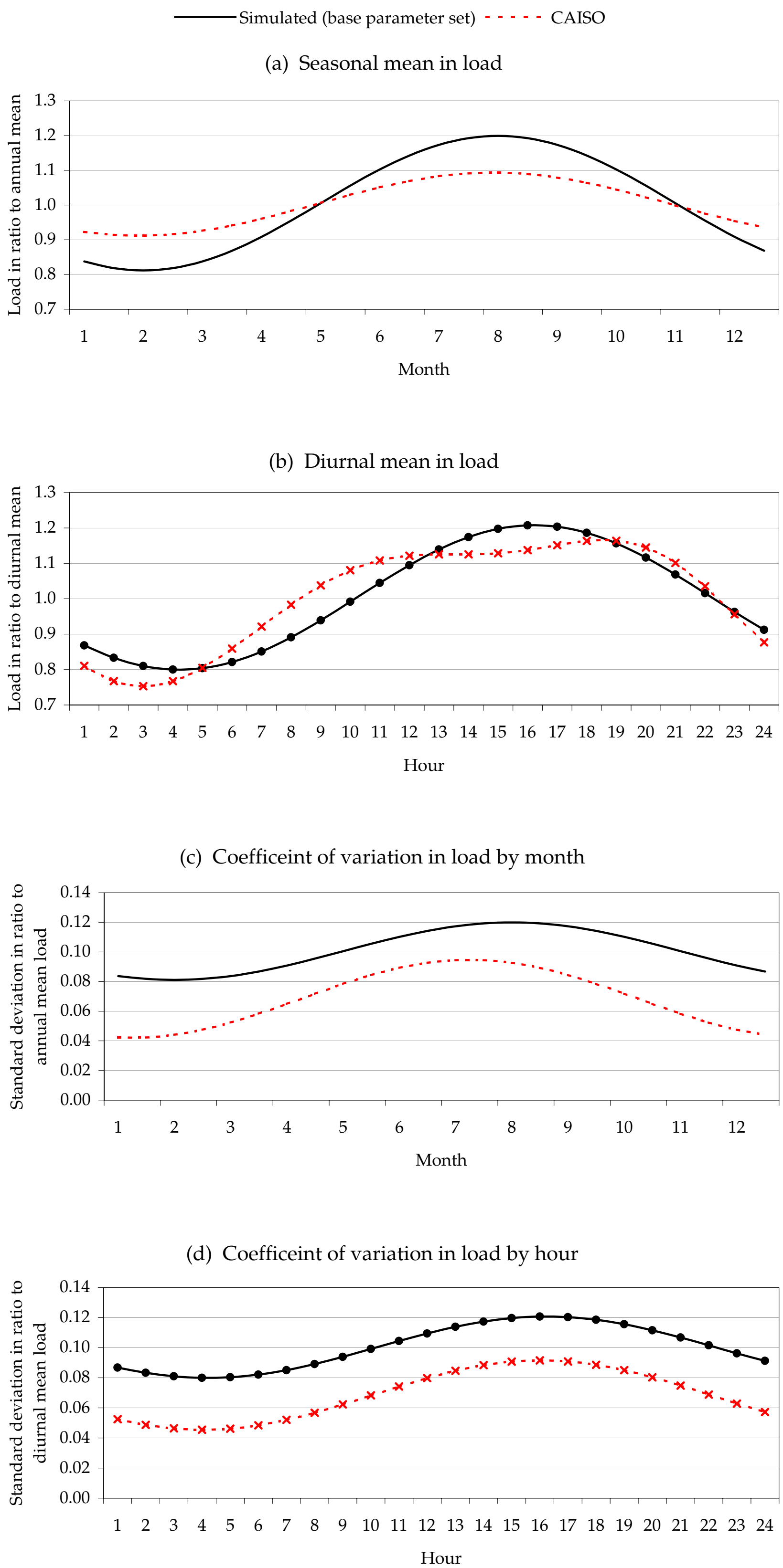
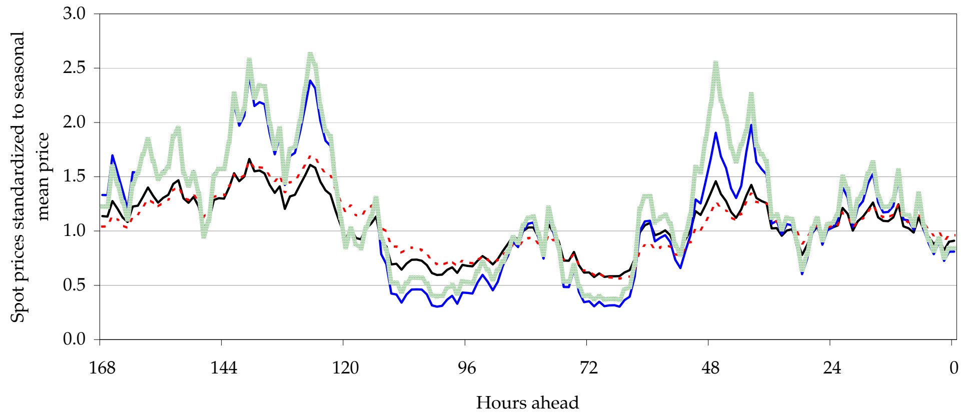


Figure 2. A representative path of spot prices and the corresponding movement of a forward price originally one week ahead

One sequence of the random processes for the week ending in August 1, Hour 18

base parameter set
σ = 0.20 (instead of σ = 0.10 in base parameter set)
ρ = 0.95 (instead of ρ = 0.90 in base parameter set)
c = 5 (instead of c = 3 in base parameter set)

(a) Realizations of the electricity spot prices standardized to the long-run expected value



(b) Movement of the forward price for August 1, Hour 18 standardized to the long-run expected value

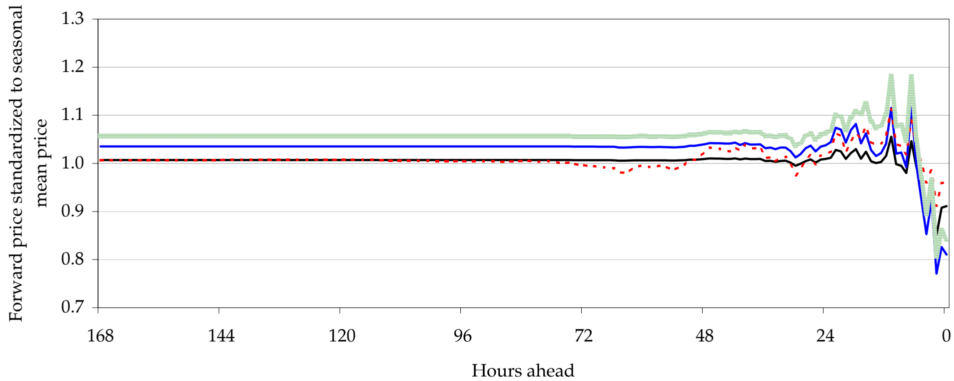


Figure 3. Standard deviation of forward prices as a function of the time ahead

Monte Carlo standard deviation, over 1,000 realizations, of the forward price for August 1, hour 18 delivery, standardized to the long-run expected value, obtained for each trading hour within 168 hours of the delivery period.

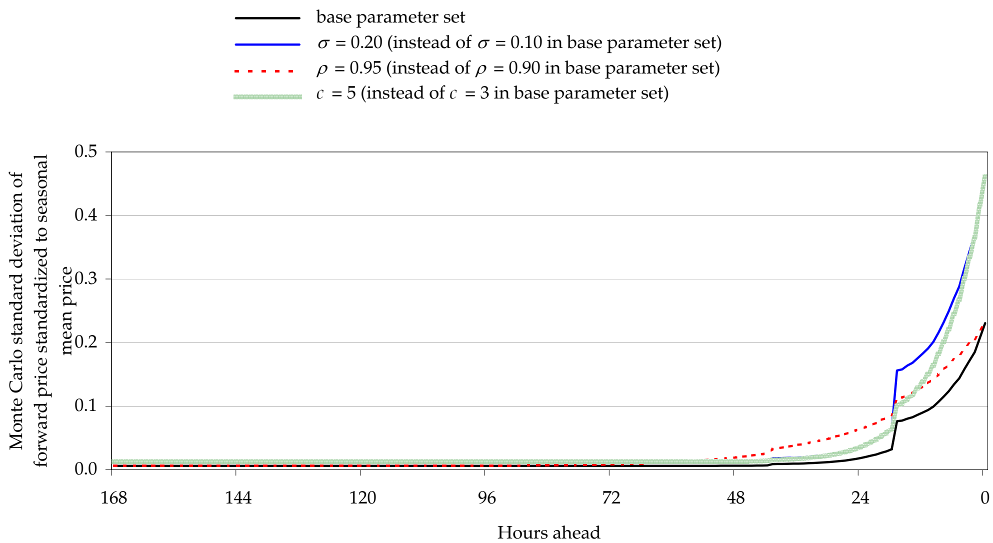


Figure 4. A representative path of standard deviation of the price forecast error as a function of the time ahead

Standard deviation of the spot electricity price for August 1, hour 18 delivery, conditional on the information available at each trading period, is obtained for 168 hours within the delivery period.

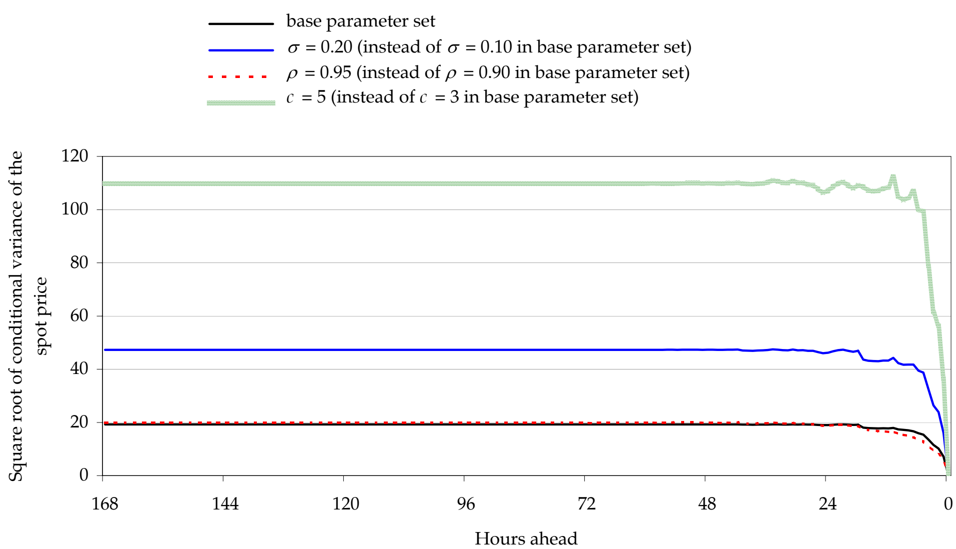
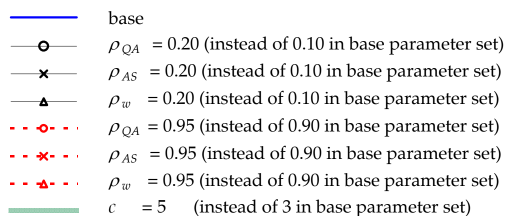
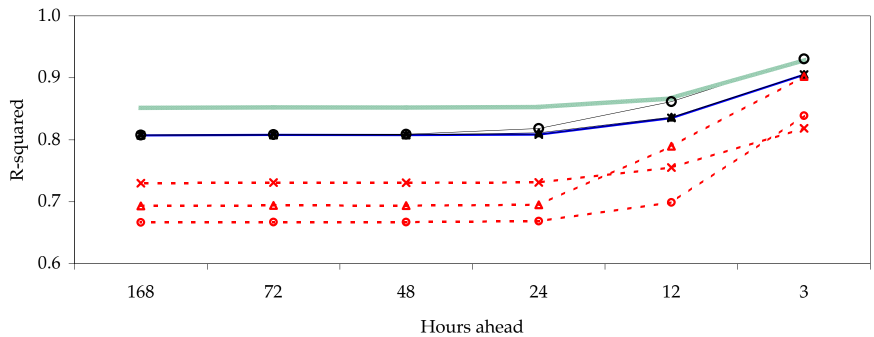


Figure 5. Relative forecasting ability of explicit and implicit forward prices

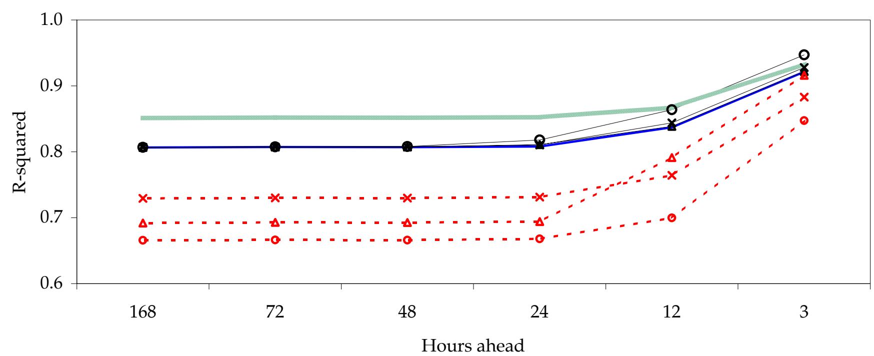
Eqns. (14) and (15) were estimated for six different forecast horizons (3, 12, 24, 48, 72, and 168 hours ahead). Equations were estimated with one year of hourly observations for each of 1,000 replications. Averages over these 1,000 replications are plotted.



(a) R-squared from the regressions of spot electricity price on forecasted load and futures price of fuel input (Eqn. 14)



(b) R-squared from the regressions of spot electricity price on forward electricity price (Eqn. 15)



(c) Difference in the R-squared from Eqns. (14) and (15)

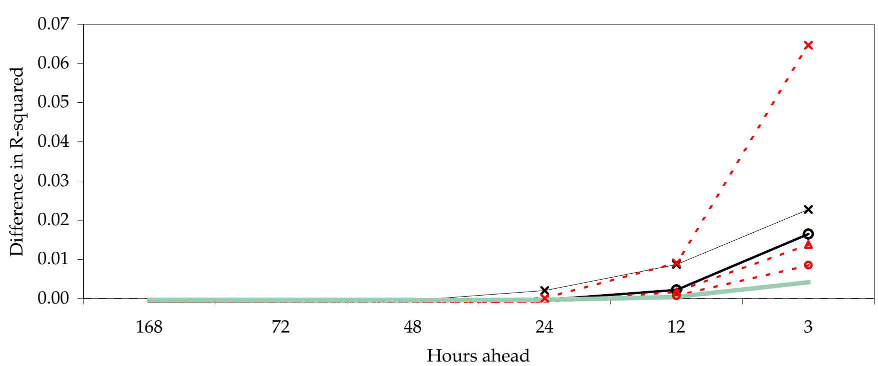


Figure 6. RMSE from the out-of-sample forecast test for the estimated synthetic electricity price

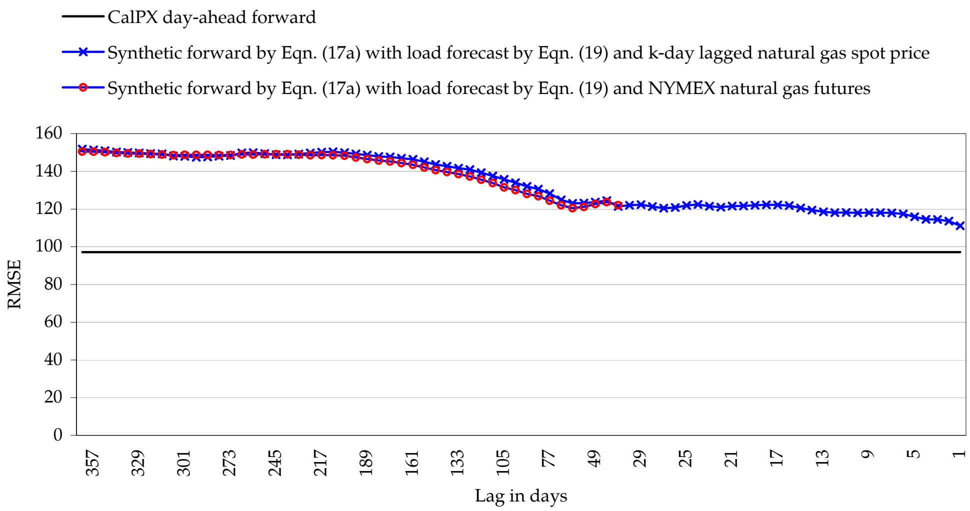
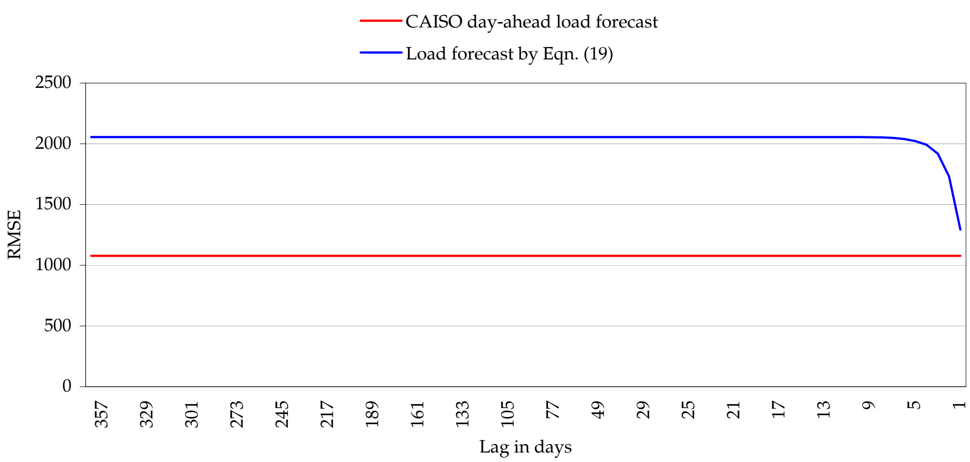


Figure 7. RMSE from the out-of-sample forecast test for the natural gas price and load forecast

(a) Load forecast



(b) Natural gas price forecast