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The Relation Between Competition and Investment - Towards a Synthesis

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The relation between competition on investment – Towards a synthesis

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Abstract: Using a general two-stage framework, this paper gives sufficient conditions for increasing competition to have negative or positive effects on R&D-investment, respectively. Both possibilities arise in plausible situations, even if one uses relatively narrow definitions of increasing competition. The paper also shows that competition is more likely to increase the investments of leaders than those of laggards. When R&D-spillovers are strong, competition is less likely to increase investments. The paper also identifies conditions under which low initial levels of competition make positive effects of competition on investment more likely. Extending the basic framework, the paper shows that separation of ownership and control, endogenous entry and cumulative investments make positive effects of competition on investment more likely. Imperfect upstream competition weakens the effects of competition on investment.

Keywords: competition, investment, cost reduction

JEL: L13, L20, L22

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1 Introduction

Even though economists have been trying to understand the effects of the intensity of competition on R&D-investment for decades, the issue remains unsettled. While some authors argue that competitive pressure is essential to induce R&D-investments, others emphasize the Schumpeterian idea that some monopoly power is necessary for innovation. As both arguments have some merit, it is unsurprising that the theoretical analysis of the subject has been inconclusive. Depending on the definition of "competitive intensity" and the underlying oligopoly framework, investments can be increasing or decreasing functions of competitive intensity.¹

Understanding the driving forces behind these different predictions is extremely difficult, because most models rely on specific functional forms. In the following, I will therefore provide a general framework that allows searching for robust predictions, because it captures many different notions of increasing intensity of competition and different types of oligopolistic interaction. To reveal the economic intuition in the most transparent fashion, I opted for simplicity in other respects: The most basic version of the game has two stages, with cost-reducing investment followed by product market competition. This simplification is not entirely innocuous, because it rules out situations where the investments are not observable by competitors and therefore have no strategic effect in the product market.

In most of the paper, I will consider a duopoly.² One firm (the leader) may be exogenously more efficient than the other one (the laggard), that is, it may have lower marginal costs. The initial efficiency levels and the cost-reducing investments determine the efficiency Y_i in the product market stage. Together with a competition parameter θ , the efficiency levels determine the output $Q^i(Y_i, Y_j; \theta)$ and the profit margin $M^i(Y_i, Y_j; \theta)$ of each firm in the second-stage product market equilibrium, and hence the profit $\Pi^i = Q^i \cdot M^i$ (gross of investment costs). By assumption, and in line with many examples,

¹For elementary models on this topic, see Motta (2004, ch.2); Vives (2008) provides a more sophisticated analysis. Similar issues are discussed in a macroeconomic context (Aghion et. al. 1997, 2001)

²Generalizations of most results to more than two firms are possible at the cost of additional notation.

higher own efficiency increases both components of a firm's profit: Lower marginal costs lead to higher outputs and profit margins.

The framework covers many familiar cases. In particular, the competition parameter can be interpreted quite broadly. It does not necessarily refer to a competition policy parameter, but more generally to some parameter of the market environment capturing the intensity of competition. The framework applies, for instance, to a homogeneous linear Cournot model where θ is the negative of market size; a Hotelling model where θ is the inverse of transportation costs; differentiated linear Cournot or Bertrand models where θ corresponds inversely to the extent of horizontal product differentiation, as captured for instance by the demand functions of Shubik and Levitan (1980) or Singh and Vives (1984). θ may also capture a shift from Cournot to Bertrand competition or an increase in the number of firms for an otherwise given environment. The parameter shift can also be interpreted as a change in cartel policy or intellectual property rights protection (see Schmutzler 2009).

Our defining assumptions on the competition parameter θ are inspired by two common properties of these examples (and many others). First, the profit margin M^i of each firm in the product market equilibrium decreases with θ ; competition thus has a negative margin effect.³ Second, the output sensitivity effect is non-negative: The positive effect of greater efficiency on equilibrium output $(Q_i^i \equiv \frac{\partial Q^i}{\partial Y_i})$ weakly increases with competition θ .⁴

In this framework, I give sufficent conditions for the effects of competition on investment to be positive and negative, respectively. I also provide conditions under which competition increases the investments of some firms (e.g., leaders) and decreases those of others (e.g., laggards). The analysis shows that there are very natural situations in which each possibility arises. Thus, searching for a general relation between competition and investment is in vain.

However, the conditions derived help to uncover the circumstances under

³Boone (2008) provides a reasonable example where this property of a competition parameter is *not* satisfied. The ideas of the following analysis could still be applied, but at the cost of having to distinguish more cases.

⁴Throughout the paper, we use subscripts to denote partial derivatives, with indices i referring to Y_i , y_i , etc.

which competition is more likely to have a positive or negative effect on a firm. The following testable predictions emerge from the basic model. First, quite generally, competition is more likely to have a positive effect on the investments of leaders than on those of laggards, and the effect on laggards is quite robustly negative.⁵ Second, when investments have higher spillovers, increasing competition is more likely to reduce investments. Third, an inverse U-shaped relation between competition and investment is not necessarily more likely than a U-shaped relation.

A possible objection to the conclusion that competition has ambiguous effects on investment is that the approach presented here is simply too general, and that natural restrictions on the class of parameterizations might lead to more conclusive results. I show that this is not the case for two plausible candidates. First, if one identifies "increasing competition" quite narrowly with decreasing product differentiation, the possibility of negative and positive effects still arises, even for symmetric firms. Second, one might want to add a further requirement to the definition of a competition parameter, namely that competition has an unambiguously positive effect on equilibrium output. This condition often holds because competition reduces prices. It clearly works towards a positive effect on investment, ⁶ but it is not sufficient to guarantee that competition increases investments. A somewhat more definite result can be obtained if one moves beyond the duopoly framework and identifies increasing competition with an increase in the number of firms. Then, there are strong forces suggesting a negative effect on per-firm investment.

The most closely related paper is Vives (forthcoming) who also considers the effects of competition on cost-reducing investments in general two-stage games.⁷ Vives arrives at more definite conclusions, suggesting that com-

⁵This is related to, but not identical, to the concept of weak increasing dominance, which requires that leaders invest more than laggards (Cabral and Riordan 1994, Athey and Schmutzler 2001, Cabral 2002, 2008): I am arguing that increasing competition works in favor of increasing difference.

⁶Inutitively, if competition leads to higher demand per firm, it becomes more attractive to increase markups by becoming more efficient.

⁷In a broader sense, the paper is related to Fudenberg and Tirole (1984) and Bulow et al. (1985). These papers also consider classes of two-stage investment games, and they identify general properties of the strategic interaction guaranteeing that strategic

petition quite generally has positive effects on investment. Several reasons explain these different findings. First, Vives does not consider initial asymmetries, so that the robust negative effect of competition on laggards does not show up. Second, Vives confines himself to product differentiation parameters. Third, even when increasing competition refers to lower product differentiation, there is at least one example not considered by Vives where increasing competition has a negative effect on investment in non-degenerate parameter regions even for symmetric firms.

The basic model captures the investment decisions under the following assumptions:

- (i) There is no separation of ownership and control;
- (ii) Investment decisions are one-shot;
- (iii) The number of firms is exogenously fixed;
- (iv) Firms provide R&D-inputs inhouse or from a competitive market rather than from an imperfectly competitive upstream market.

We will show that relaxing each of these restrictions has a clear-cut effect on the relation between competition and investment. The effects of competition on investment tend to be more positive with separation of ownership and control, with cumulative investments and with endogenously determined entry decisions. When firms buy R&D inputs from an upstream market, the effects of competition and investment tend to be reduced in absolute values, no matter whether they are positive or negative.

I will also sketch how the approach can help to understand the effects of downstream competition on the innovation incentives of a vertically integrated upstream monopolist who supplies downstream competitors and his own downstream subsidiary. This introduces a number of additional complications, which result from two sources. First, upstream investments also tend to benefit the downstream competitors, which influences the profits of the

considerations have a positive or negative effect on investment.

integrated firm on the downstream market. Second, by changing the downstream costs of both firms, these investments also affect the access revenue (the upstream profits) that a firm can obtain.

The paper is organized as follows. Section 2 introduces the basic model. Section 3 provides comparative statics results. Section 4 applies these results to familiar examples. Section 5 uses the general results and the examples to clarify under which circumstances a positive effect of competition is likely in the basic model. Section 6 re-examines this question in richer setting, allowing for separation of ownership and control, cumulative investments, endogenous entry and imperfectly competitive upstream suppliers. Section 7 concludes.

2 Set-up

I shall consider the following class of two-stage games. In period 1, firms i=1,2 can carry out a cost-reducing investment. In period 2, they engage in product-market competition. Initially, firm i has marginal cost $c_i = \bar{c} - Y_i^0$ for some exogenous reference level \bar{c} of marginal costs.⁸ In the first stage, given (Y_1^0, Y_2^0) , each firm chooses its investment y_i . In the second stage, firm i has marginal costs $c_i = \bar{c} - Y_i$, where $Y_i = Y_i^0 + y_i + \lambda y_j$ is the efficiency level after the investment stage and $\lambda \in [0,1]$ is a spillover parameter. We introduce a parameter θ from some partially ordered set to parameterize the intensity of competition; the defining properties of which will be introduced below. The product-market game is assumed to have a unique Nash equilibrium for arbitrary θ and $\mathbf{Y} = (Y_1, Y_2)$, corresponding to prices $p^i(Y_i, Y_i; \theta)$. The demand function for firm i is $q^i(p^i, p^j; \theta)$, where p^i and p^j are the prices of firm i and firm j, respectively. We allow for the case where competition does not enter demand directly, so that q^i is only a function of p^i and p^j . This will be reasonable when θ reflects stricter competition policy or a shift from Cournot to Bertrand competition, but not when θ stands for an increase in

⁸The choice of \bar{c} is arbitrary; to simplify calculations, I usually choose $\bar{c} = 0$ or $\bar{c} = a$, where a is the maximal willingness to pay for any unit of the good.

⁹For price competition, $p_i(Y_i, Y_j; \theta)$ is the equilibrium price; for quantity competition, it denotes the market clearing price for equilibrium outputs.

the degree of substitutability between goods.

The following notation will be used:

- 1. Equilibrium profit margins $M^{i}(Y_{i}, Y_{j}; \theta) \equiv p^{i}(Y_{i}, Y_{j}; \theta) \overline{c} + Y_{i}$
- 2. Equilibrium outputs $Q^{i}(Y_{i}, Y_{j}; \theta) \equiv q^{i}(p^{i}(Y_{i}, Y_{j}; \theta), p^{j}(Y_{i}, Y_{j}; \theta); \theta)$
- 3. Gross equilibrium profits $\Pi^{i}(Y_{i}, Y_{j}; \theta) = M^{i}(Y_{i}, Y_{j}; \theta) \cdot Q^{i}(Y_{i}, Y_{j}; \theta)$

I will maintain the following assumptions throughout, all of which hold in the examples to be discussed in Section 3 below.

- (A1) $q^{i}(p^{i}, p^{j}; \theta)$ is weakly decreasing in p^{i} and weakly increasing in p^{j} , $j \neq i$. Thus, the firms produce (potentially imperfect) substitutes.
- (A2) $p^{i}(Y_{i}, Y_{j}; \theta)$ is weakly decreasing in Y_{i} and $Y_{j}, j \neq i$.
- (A2) holds in most oligopoly models. Because the product market game has a unique equilibrium, the investment game reduces to a one stage game with payoff functions

$$\pi^{i}(y_{i}, y_{j}; \theta) = \Pi^{i}(Y_{i}^{0} + y_{i} + \lambda y_{j}, Y_{j}^{0} + y_{j} + \lambda y_{i}; \theta) - K(y_{i}).$$
(1)

(A3) $Q^{i}(Y_{i}, Y_{j}; \theta)$ is weakly increasing in Y_{i} and weakly decreasing in Y_{j} , $j \neq i$.

This assumption is related to (A1) and (A2). To see this, define

$$\eta^{o} \equiv \frac{\partial q^{i}}{\partial p^{i}} \left(p^{i} \left(Y_{i}, Y_{j}; \theta \right), p^{j} \left(Y_{i}, Y_{j}; \theta \right) \right) \cdot \frac{\partial p^{i}}{\partial Y_{i}} \left(Y_{i}, Y_{j}; \theta \right);$$

$$\eta^{c} \equiv \frac{\partial q^{i}}{\partial p^{j}} \left(p^{i} \left(Y_{i}, Y_{j}; \theta \right), p^{j} \left(Y_{i}, Y_{j}; \theta \right) \right) \cdot \frac{\partial p^{j}}{\partial Y_{i}} \left(Y_{i}, Y_{j}; \theta \right).$$

 η^o reflects the own-price effect of efficiency on output: By (A2), lower costs of firm i reduce its equilibrium price p^i which, by (A1) works towards higher equilibrium output Q^i . η^c reflects the competitor-price effect: As c_i falls, the competitor's price falls by (A2), which reduces firm i's output Q^i . As $Q^i_i \equiv \frac{\partial Q^i}{\partial Y_i} = \eta^o + \eta^c$, (A3) says that the own price effect dominates over the competitor price effect. Indeed, this is true in all our examples. The next assumption is slightly more problematic.

(A4) $M^{i}(Y_{i}, Y_{j}; \theta)$ is weakly increasing in Y_{i} and weakly decreasing in Y_{j} , $j \neq i$.

As $M^i(Y_i, Y_j; \theta) = p^i(Y_i, Y_j; \theta) - \overline{c} + Y_i$ and $\frac{\partial M^i}{\partial Y_i} = \frac{\partial p^i}{\partial Y_i} + 1$, the first part of the assumption states that the cost reductions are larger than the induced price reductions. This holds in many, but not all, oligopoly models.¹⁰ Finally, I introduce two defining properties of the competition parameter.

(C1) $M^{i}(Y_{i}, Y_{j}; \theta)$ is weakly decreasing in θ .

The notion that competition reduces margins (and prices) is standard. However, the relation between θ and output is less clear. To see why, assume for simplicity that all relevant functions are differentiable in the competition parameter.¹¹ Then

$$\frac{dQ^i}{d\theta} = \frac{\partial q^i}{\partial p^i} \frac{\partial p^i}{\partial \theta} + \frac{\partial q^i}{\partial p^j} \frac{\partial p^j}{\partial \theta} + \frac{\partial q^i}{\partial \theta}.$$
 (2)

If the own price effect dominates over the competitor price effect, the sum of the first two terms are positive. However, the direct effect $q_{\theta}^{i} \equiv \frac{\partial q^{i}}{\partial \theta}$ can be negative, potentially compensating the price-induced effects. Thus, equilibrium output may rise or fall as competition increases. Moreover, as we will see below, competition may have differential impacts on the output of leaders and laggards.

The next assumption concerns the effect of competition on $(\eta^o + \eta^c)$. Clearly, $|\eta^c|$, the output effect of higher own efficiency resulting from the induced lower competitor prices, is small for soft competition, suggesting a negative effect of θ on η^c . Indeed, the examples below confirm this. However, η^o is more likely to increase in θ : Part of the effect of higher efficiency on own output that is induced by lower own prices comes from a business-stealing effect that is absent with weak competition. In all examples where a change in θ refers to an increase in the intensity of competition for a given number

¹⁰For instance, it does not hold globally in a Cournot duopoly with demand generated from CES utility functions.

¹¹We shall maintain this assumption in the rest of the paper, even though nothing of substance depends on it.

¹²See the example in 4.1, where competition corresponds to a reduction in market size.

of firms, the own price effect dominates over the competitor price effect.¹³ This motivates the following assumption.

(C2)
$$Q_{i\theta}^i \geqslant 0$$
.

We are now ready to define a competition parameter.

Definition 1 In a duopoly model given by $Q^{i}(Y_{i}, Y_{j}; \theta)$ and $M^{i}(Y_{i}, Y_{j}; \theta)$, θ is a **competition parameter** if (C1) and (C2) hold.

We shall illustrate the definition with specific examples in Section 4.

For some results, it is useful to work with an alternative definition of increasing competition. To this end, I shall sometimes invoke two further properties.

(C3) $\frac{\partial \Pi^i}{\partial Y_i}$ is weakly decreasing in θ .

This condition seems plausible: As competition increases, the adverse effect of a more efficient competitor on own profits implied by (A3) and (A4) becomes larger in absolute value.¹⁴

(C4) $\frac{\partial y_i}{\partial y_j}$ is weakly decreasing in θ .

To understand this property, first note that, in a large class of investment games without strong spillovers, actions are strategic substitutes (see e.g. Bagwell and Staiger 1994, Athey and Schmutzler 2001). To understand why, note that

$$\Pi^i_{ij} = Q^i_i \cdot M^i_j + M^i_i \cdot Q^i_j + M^i \cdot Q^i_{ij} + Q^i \cdot M^i_{ij}.$$

In linear examples, the last two terms disappear. The first two terms are typically negative, because of (A3): If competitors invest a lot, own margins and outputs fall. This reduces the benefits from increasing own outputs and markups by becoming more efficient. Intuitively, as the competitor invests more, a firm's output and profit margin both fall, which reduces the benefits from increasing the own profit margin and output by investing more.

¹³If θ reflects the number of firms, this is no longer true (see Section 5.5).

¹⁴See the cautionary remarks in Section 5.2, however.

(C4) thus corresponds to the following intuitive notion: If the negative effect of the competitor's investments on own output and margin is more pronounced when competition is intense, then the reason to reduce own investments as a response becomes more pronounced.

3 General comparative statics results

I will now provide general results about the effects of competition on investment. These results are essentially well-known from other contexts, but I state them as a foundation for the following analysis. Assumptions (A1)-(A4) and (C1)-(C4) are not necessary to derive the results, but they are essential for the interpretation. I will suppose for simplicity that investments are chosen from some compact subset of the reals, and $\Pi^i(Y_i, Y_j; \theta)$ and $\pi^i(y_i, y_j; \theta)$ are twice continuously differentiable, even though much of the following easily generalizes to discrete choice sets and more general objective functions. Also, I assume existence and uniqueness of the equilibrium in the investment game. The following result shows that the properties of $\pi^i_{i\theta} \equiv \frac{\partial^2 \pi^i}{\partial y_i \partial \theta}$ are essential for comparative statics. When $\pi^i_{i\theta} > 0$, θ shifts out player i's reaction curve. This does not guarantee that competition increases player i's investment, but there are several sets of additional conditions that lead to this outcome.

Proposition 1 $y_i(\theta)$ is weakly increasing in θ for i = 1, 2 if, for i = 1, 2 and $j \neq i$, one of the following conditions (i)-(iii) holds:

(i) $\pi_{i\theta}^i \ge 0$ and $\pi_{ij}^i \equiv \frac{\partial^2 \pi^i}{\partial y_i \partial y_j} \ge 0$.

(ii) $\pi_{i\theta}^{i} \geq 0$, $\pi^{i}(y_{i}, y_{j}; \theta)$ is symmetric and concave in y_{i} ; $y_{i}(\theta) = y_{j}(\theta)$ for all θ considered, and the Hahn stability condition $\pi_{ii}^{i}\pi_{jj}^{j} \geq \pi_{ij}^{i}\pi_{ji}^{j}$ holds.

(iii) $\pi^i(y_i, y_j; \theta)$ is concave in y_i . Near the equilibrium, $\pi^i_{i\theta} \geq \frac{\pi^i_{ij}}{\pi^j_{jj}} \pi^j_{j\theta}$, and the Hahn-stability condition holds.

Proof. See Appendix 1. ■

¹⁵This follows from a well-known comparative statics result of Topkis (1978) for the maximizer of a supermodular function, as positivity of the relevant mixed partials for differentiable functions guarantees supermodularity.

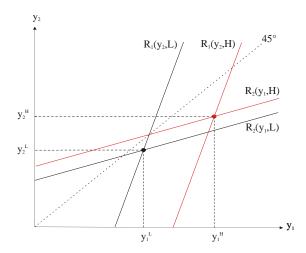


Figure 1: Strategic Complements

By switching the signs in the inequalities $\pi^i_{i\theta} \geq 0$ and $\pi^i_{i\theta} \geq \frac{\pi^i_{ij}}{\pi^j_{j\theta}} \pi^j_{j\theta}$ in (i) - (iii), one arrives at sufficient conditions for competition to have negative effects on investment. Also, for the benchmark case without spillovers $(\lambda = 0)$, $\pi^i_{i\theta} = \Pi^i_{i\theta} \equiv \frac{\partial^2 \Pi^i}{\partial Y_i \partial \theta}$, whereas, with positive spillovers $\pi^i_{i\theta} = \Pi^i_{i\theta} + \lambda \Pi^i_{j\theta}$. Either way, the conditions of the theorem reflect properties of the gross profit function Π^i that are independent of the precise form of the investment cost functions, because, by assumption, these functions and, in particular, marginal costs do not depend on θ . 16

To understand (i), consider Figure 1. Here, and in the following $\theta = L$ refers to the situation before a parameter increase, $\theta = H$ to the situation after the increase. Recall that $\pi^i_{i\theta} \geq 0$ implies that reaction functions shift out as θ increases. The supermodularity condition in (i), $\pi^i_{ij} = \Pi^i_{ij} \geq 0$, implies increasing reaction functions, so that the indirect effects of competition reinforce the direct effects. Thus, competition increases both players' investments.

However, as argued at the end of Section 2, unless spillovers are sufficiently large, investments are typically strategic substitutes, so that the

¹⁶In Section 6.2 I will give reasons why costs may sometimes depend on competition, and I will discuss the implications.

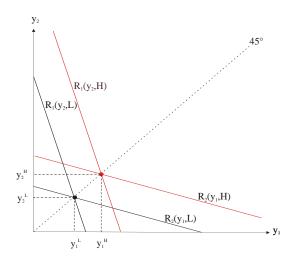


Figure 2: Strategic Substitutes (Symmetric Case)

direct and indirect effects have opposite signs. Even then, part (ii) shows that, if $\pi_{i\theta}^i \geq 0$ for both firms (so that both reaction functions are shifted outwards) competition still increases both players' investments as long as the functions π^i are symmetric (see Figure 2).

The case of asymmetric firms is more complex with strategic substitutes. Figure 3 shows that it is possible that only one firm increases its investments, even though both reaction functions are shifted outwards as competition increases. The intuition is straightforward. If the shift is more pronounced for, say, firm 1 than for firm 2, and the reaction function of firm 2 is sufficiently steep, then the direct positive effect of competition on investment for firm 2 (outward shift of own reaction functions) is outweighed by the negative effect that firm 1 increases investments, to which firm 2 reacts by reducing investments. However, as Figure 4 shows, even in the asymmetric case with strategic substitutes, an outward shift of both reaction function guarantees a positive effect on both equilibrium investments as long as reactions to changes in the other player's investment are not too strong. This requirement is captured by the condition in (iii): $\frac{\pi_{12}^2}{\pi_{22}^2}$ is the slope of the reaction function of firm 2.

The following proposition is useful to identify such situations where competition increases the investments of one firm and decreases those of the other

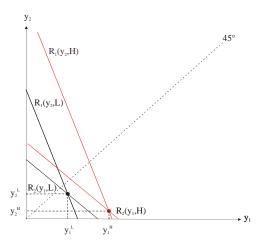


Figure 3: Strategic Substitutes: Counterexample

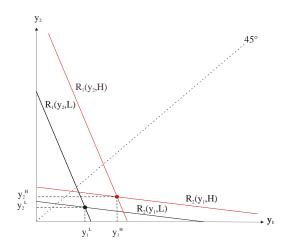


Figure 4: Strategic Substitutes: Asymmetric Case

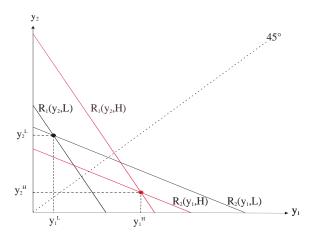


Figure 5: Asymmetric Effects on Investment Incentives

one, which will be shown to arise naturally when one firm is the leader and the other firm is the laggard.

Proposition 2 Suppose for some $i \in \{1, 2\}$ and $j \neq i$, the following conditions hold: (a) $\pi^i_{i\theta} \geq 0$; (b) $\pi^j_{j\theta} \leq 0$; (c) $\pi^i_{ij} \leq 0$ and (d) $\pi^j_{ji} \leq 0$. Then y_i is weakly increasing in θ and y_j is weakly decreasing.

Proof. Conditions (a)-(d) imply $\pi_{i\theta}^i \geq 0$; $\pi_{j\theta}^j \leq 0$; $\pi_{ij}^i \leq 0$ and $\pi_{ji}^j \leq 0$. The result therefore follows from Theorem 5 in Milgrom and Roberts (1990) by reversing the order on the strategy space of one firm.

The intuition is captured in Figure 5: By (a) and (b), θ has the direct effect of shifting out firm i's reaction curve and shifting the reaction curve of firm j inwards. By (c) and (d), these direct effects are mutually reinforcing: As both reaction functions are decreasing, an increase of firm i's investment reduces firm j's investment incentives and vice versa.

As $\Pi^i = Q^i \cdot M^i$, Proposition 1 implies the following loosely stated result:

Corollary 1 Suppose for i = 1, ..., I,

$$\Pi_{i\theta}^{i} = Q_{i}^{i} \cdot M_{\theta}^{i} + M_{i}^{i} \cdot Q_{\theta}^{i} + Q^{i} \cdot M_{i\theta}^{i} + M^{i} \cdot Q_{i\theta}^{i}$$

$$\tag{3}$$

is sufficiently large (small). Then $y_i(\theta)$ is weakly increasing (weakly decreasing) in θ for i = 1, ..., I.

Here, "sufficiently large" reduces to "positive" for symmetric firms and for games with strategic complementarities (Parts (i) and (ii)). For other games, "sufficiently large" means that expression (3) must be greater than $\frac{\pi_{ij}^i}{\pi_{jj}^j}\pi_{j\theta}^j$, which is positive (Part (iii)).

Expression (3) captures the total effects of competition on investment incentives, $\Pi_{i\theta}^i$. Each of the four terms corresponds to one intuitive transmission channel by which competition affects investment incentives. The first term in (3), $Q_i^i \cdot M_{\theta}^i$, reflects the margin effect of competition: By (A3), investment has a positive effect on output $(Q_i^i > 0)$. Also, by (C1), M_θ^i is negative. Thus, as competition increases, margins decrease, so that the positive effect of expanding output on profits becomes smaller. The second term, $M_i^i \cdot Q_\theta^i$, reflects the *output effect* of competition: By (C1), investment increases margins, M_i^i . If $Q_\theta^i > 0$ the output effect of competition on marginal investment incentives is positive; if $Q_{\theta}^{i} < 0$, it is negative. The third term, $Q^i \cdot M_{i\theta}^i$, reflects the cost-pass-through effect of competition. Because $M_{i\theta}^i = p_{i\theta}^i$, the sign of the cost-pass-through effect is positive if and only if $p_{i\theta}^i \equiv \frac{\partial}{\partial \theta} \left(\frac{\partial p^i}{\partial Y_i} \right) \geq 0$, that is, competition reduces the sensitivity of equilibrium prices to costs. The examples below will show that the cost-pass-through effect is ambiguous.¹⁷ The fourth term, $M^i \cdot Q^i_{i\theta}$ reflects the output-sensitivity effect of competition. Under (C2), the output-sensitivity effect is positive: As θ increases, output reacts more strongly to efficiency, which enhances the incentive to invest.

Summing up, the analysis in this section suggests why more intense competition does not have clear-cut effects on investment. The effect of competition on investment incentives, $\Pi^i_{i\theta}$, consists of the four transmission channels just discussed. The margin effect is negative, whereas the output-sensitivity effect is positive. The output effect and the cost-pass-through effect can be positive or negative.

¹⁷For instance, when competition corresponds to increasing substitutability, the sign depends on whether firms compete à la Bertrand or à la Cournot.

4 Examples

The following examples show how (3) helps to understand under which circumstances competition has positive or negative effects on investments. Several of these examples are well-known, but they nevertheless are useful to identify the four transmission channels. Whenever I calculate equilibrium investment levels explicitly, the investment cost function is $K(y_i) = y_i^2$; importantly, however, the comparative statics also hold for more general cost functions.

4.1 Inverse market size

The first example is perhaps the least convincing case of "increasing competition", but it is a useful illustration. Suppose firms are Cournot competitors, with homogeneous goods and market demand Q(p) = a - p for some a > 0, and constant marginal costs c_i . Define $\theta = -a$. Hence, more intense competition corresponds to a smaller market.¹⁸ Defining $Y_i = -c_i$,

$$Q^{i}(Y_{i}, Y_{j}; \theta) = M^{i}(Y_{i}, Y_{j}; \theta) = (2Y_{i} - Y_{j} - \theta)/3.$$

Equilibrium investments can easily be calculated as

$$y_i = \frac{1}{7} \left(-2\theta + 8Y_i^0 - 6Y_j^0 \right).$$

The effect of increasing competition on investments is thus negative. To see the economic logic behind this, note that $Q_i^i = M_i^i = \frac{2}{3}$; $Q_\theta^i = M_\theta^i = -\frac{1}{3}$. Thus, in line with (C1), the margin effect is negative. The output effect happens to be identical to the margin effect and thus negative. Finally, as $Q_{i\theta}^i = M_{i\theta}^i = 0$, the effect of competition on investment incentives is fully determined by the negative output and margin effects: $\Pi_{i\theta}^i < 0$, so that the effect of competition on investment incentives is negative.

¹⁸Boone (2008) also treats inverse market size as a competition parameter.

4.2 Substitutability (Shubik-Levitan)

In a market with differentiated goods, let inverse demands be

$$p^{i}(q_{i}, q_{j}) = 1 - q_{i} - bq_{j}, \tag{4}$$

where $0 \le b \le 1$ (Shubik and Levitan 1980). The corresponding demand functions $q^i(p^i, p^j)$ satisfy $\frac{\partial q^i}{\partial p^j} > 0$ for b > 0; thus the goods are substitutes. For b = 0, firms are monopolists; b = 1 corresponds to homogeneous goods. Higher b corresponds to better substitutability. Thus, define $\theta = b$.

4.2.1 Quantity competition

The middle line in Figure 6 plots investments as a function of the competition parameter for $c_1^0 = c_2^0 = 0.5$. The line is U-shaped: Starting from a monopoly, an increase in competition first reduces investment; beyond $\theta = 2/3$ further increases lead to higher investments.

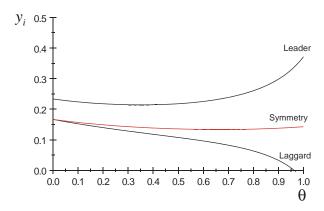


Figure 6: Differentiated Cournot competition

With small heterogeneities between firms, the qualitative pattern is similar: Competition has a U-shaped effects on leaders and laggards.²⁰ For firms

¹⁹The results for the Cournot case are taken from Sacco and Schmutzler (forthcoming), which also contains experimental evidence for the U-shape.

²⁰However, the level of competition from which on competition has a positive effect on investment is lower for leaders than for laggards.

that lag far behind, however, the effects of competition on investment are negative. For instance, the respective lines in Figure 6 plot the relation between competition and investments for $c_1^0 = 0.3$; $c_2^0 = 0.7$ for leaders (laggards). To understand this pattern, note that $Q_i^i = M_i^i > 0$ (see Appendix 9). $Q_\theta^i = M_\theta^i$ is negative unless firm i has a very strong lead; $\frac{Y_i}{Y_j} > \frac{4+\theta^2}{4\theta} (> 1.25)$. Thus, quite generally, output and margin effects are negative.²¹ As $Q_{i\theta}^i = M_{i\theta}^i > 0$, the remaining effects are positive. Hence, the U-shaped relation between competition and investment for all firms except strong laggards reflects the interplay between the negative output and margin effects and the positive cost-pass-through and output-sensitivity effects: Starting from low competition, greater competition, by reducing output and margins, reduces incentives to increase efficiency. Beyond a certain threshold, the effect of competition on investment is positive, reflecting the positive output-sensitivity and margin effects. The unambiguously negative effect for firms that are lagging far behind results because their margin and hence the positive output-sensitivity effect $M^iQ^i_{i\theta}$ is small.²² Therefore, the negative output and margin effects dominate.

4.2.2 Price competition

Figure 7 plots investments for price competition, with the same initial costs as in Figure 6, assuming $\theta \in [0, 1)$.²³ Investments decrease with competition when firms are neck-to-neck or laggards, but for the leader they increase as competition becomes very intense.

Hence, even though the fundamentals (demand and technology) are the same as for quantity competition, competition has a strictly negative effect except for strong leaders, for whom the relation is U-shaped. The economic logic for the negative effect differs from Section 4.1. There decreasing market size had negative output and margin effects, and the remaining effects were zero. Here substitutability has a negative effect on investments in spite of countervailing underlying effects. To see this, note that that $Q_i^i > 0$; $M_i^i > 0$;

²¹When firms are very asymmetric, (A5) no longer applies.

²²By the same token, they have low demand, so that the positive cost-pass-through effect $D^i M^i_{i\theta}$ is small.

²³I will comment on the homogeneous Bertrand case $\theta = 1$ below.

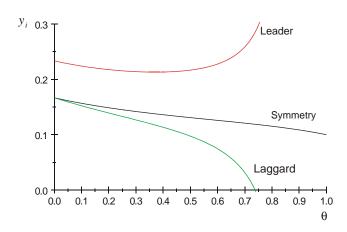


Figure 7: Differentiated Bertrand Competition

 $M_{\theta}^{i} < 0$; $Q_{i\theta}^{i} > 0$; $M_{i\theta}^{i} < 0$. Further, under symmetry $Q_{\theta}^{i} > 0$ if $\theta > 0.5$ (see Appendix 9).²⁴ Thus, while the margin effect and the cost-pass-through effect are both negative, the output-sensitivity effect is always positive and the output effect is positive for intense competition ($\theta > 0.5$). The U-shaped rather than decreasing investment function for leaders reflects the fact that the output effect is more likely to be positive for leaders.

To understand why reducing product differentiation has a more positive effect in the Cournot case than in the Bertrand case, note that $M^i_{i\theta} > 0$ for Cournot competition, whereas $M^i_{i\theta} < 0$ for Bertrand competition. To see why, compare situations where products are essentially monopolists, with situations with relatively close substitutes. In the latter case, for Cournot competition, higher efficiency of a firm induces an output reduction of the competitive interaction, this output reduction dampens the price-reducing effect $|p^i_i|$, so that the cost-pass-through effect should be positive. Under price competition, however, greater efficiency induces lower prices of both firms, enhancing the price-reducing effect of greater efficiency. Thus, compared to the case with little product differentiation where such considerations play no

²⁴More generally, $D_{\theta}^{i} > 0$ if and only if $\frac{Y_{i}}{Y_{j}}$ is above a critical level that is a suitable function of θ .

role, cost reductions induce more substantial price reductions, so that $|p_i^i|$ should increase. Summing up, the cost-pass-through effect works towards a positive relation between competition and investment under Cournot competition, and conversely under Bertrand competition.

A final comment concerns the case $\theta = 1$. Clearly with homogeneous Bertrand competition, there can no longer be a symmetric pure-strategic equilibrium:²⁵ Clearly, if both firms invest a positive amount, at least one is earning a negative amount. If it invests a small positive amount y_i , it earns gross profits of $y_i (1 - c_j^0)$. The incentive to invest is thus

$$\frac{\partial \Pi_i}{\partial y_i} = \left(1 - c_j^0\right) = D(c_j^0):$$

The demand discontinuity at 0 translates into a positive investment incentive for b = 1. If we consider investment incentives for the differentiated model as b approaches 1, however, investment incentives approach 0. This is interesting because if we compare the case b = 0 and the case b = 1, we obtain the famous result that investment incentives in the homogeneous Bertrand case are higher than those in a monopoly (Arrow 1962).²⁶ The observation that investment approaches 0 as b approaches 1 implies that this result is not robust to a small amount of product differentiation!

4.3 Substitutability (Singh-Vives)

In the examples of Section 4.2, an increase in $\theta = b$ not only increases substitutability; in addition, θ shifts both demand functions inwards, so that it mixes two sources of increasing competition. An inverse demand function without this property was analyzed by Singh and Vives (1984), namely

$$p^{i}(q_{i}, q_{j}; \theta) = 1 - \frac{1}{1+\theta}q_{i} - \frac{\theta}{1+\theta}q_{j}.$$
 (5)

It can be shown that, in both the Bertrand and the Cournot case, invest-

²⁵The game has multiple asymmetric pure-strategic equilibria as well as mixed-strategy equilibria (Sacco and Schmutzler 2007).

For b = 0, $\frac{\partial \Pi_i}{\partial Y_i} = 0.5 \left(1 - c_0^i\right)$ in a symmetric situation.

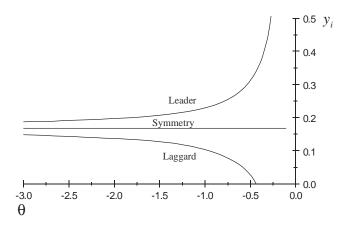


Figure 8: The effects of increasing transportation costs

ment depends positively on the substitution parameter θ for this demand function, except for firms that are lagging far behind; in which case the relation may become negative.²⁷ The main reason behind this more positive effect of competition on investment than in the Shubik-Levitan case is that the output effect is now unambiguously positive (See Appendix 9).

4.4 Transportation costs

Next, consider a Hotelling duopoly. Consumers buy at most one unit of a homogeneous good, and are uniformly distributed on [0,1]. Firms are located at $q_1 = 0$ and $q_2 = 1$. Consumers incur transportation costs t per unit distance in addition to the price p^i . Competition affects the leader's investments positively and the laggard's negatively, as depicted in Figure 8.²⁸ This figure is drawn for $c_1^0 = c_2^0 = 0.5$ (symmetric case), $c_1^0 = 0.3$ (leader) and $c_2^0 = 0.7$ (laggard).

Simple calculations show that $M_{\theta}^{i} < 0$; $Q_{i}^{i} > 0$; $M_{i}^{i} > 0$; $Q_{i\theta}^{i} > 0$; $M_{i\theta}^{i} = 0$ (See Appendix 9). Crucially, $Q_{\theta}^{i} > 0$ if and only if i is a leader; hence the

 $^{^{27}\}mathrm{Again},$ in the Bertrand case, a restriction on b~(b<0.85) is necessary for symmetric investment equilibria to exist.

²⁸We assume that transportation costs are in an intermediate range where second-order conditions hold, both firms are active and all consumers buy one unit.

same is true of the output effect.²⁹ As a result, the sign of $\Pi_{i\theta}^i$ is determined by whether a firm is leader or laggard. Also, it is straightforward to show that $\Pi_{ij}^i < 0$, so that Proposition 2 can explain the differential impact of competition on the investments of the two firms: Intuitively, because competition has a positive output effect for leaders and a negative output effect for laggards, increasing θ has the direct effect that it raises the leader's investment incentives and reduces those of the laggard. As investments are strategic substitutes, both effects are mutually reinforcing.

4.5 Cournot vs. Bertrand

Our framework can be adapted to understand how switching from Cournot competition to Bertrand competition affects investments. To this end, reconsider the differentiated goods examples of Section 4.2.1. Let $\theta \in \{0,1\}$, where $\theta = 0$ for Cournot and $\theta = 1$ for Bertrand. Even though θ does not affect demand functions $q^i(p^i, p^j)$, it affects equilibrium outputs, margins and profits. Therefore the terms $Q^i(Y_i, Y_j; \theta)$, $M^i(Y_i, Y_j; \theta)$, $\Pi^i(Y_i, Y_j; \theta)$ still make sense. Figure 9 plots the investments displayed in Figures 6 and 7 in one diagram for $c_1^0 = c_2^0 = 0.5$. Investments are thus always higher for soft (Cournot) competition, though the difference approaches zero as b does.³⁰

What lies behind this clear negative effect of competitive intensity (in the sense of moving from Cournot to Bertrand competition) on investments? To understand this, we compare $\Pi_i^i = Q^i M_i^i + M^i Q_i^i$ for $\theta = 0$ and $\theta = 1$. In Figure 10, the middle line describes equilibrium output and margin as a function of b in the Cournot case. The upper line describes equilibrium output in the Bertrand case.³¹ The lower line describes equilibrium margin in the Bertrand case. The figure thus shows that the margin effect is negative, that is, M^i is greater for $\theta = 0$ than for $\theta = 1$, and the output effect is positive, that is, Q^i is smaller for $\theta = 0$ than for $\theta = 1$. Similarly, the cost-

²⁹The remaining two non-zero effects, the positive demand-sensitivity effect and the negative markup effect, happen to sum up to a positive effect for leaders, a negative effect for laggards, and they cancel out in the symmetric case.

 $^{^{30}}$ For the Bertrand case, the figure is drawn for the parameter region where the second-order condition holds (b < 0.933).

³¹Recall that a symmetric equilibrium only exists for b < 0.923.

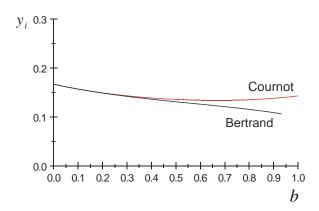


Figure 9: Cournot vs. Bertrand competition

pass-through (output-sensitivity) effects can be obtained by comparing M_i^i (Q_i^i) in the Bertrand and the Cournot case.

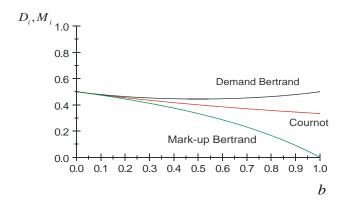


Figure 10: Cournot vs. Bertrand: Absolute demand and markup effects

Figure 11 shows that the output-sensitivity effect is positive, whereas the cost-pass-through effect is negative.

Summing up, increasing competition by moving from Cournot to Bertrand competition has a negative effect on investments for two reasons. First, it reduces the margin, which reduces the incentive to increase output. Second, it reduces the positive reaction of margins to reducing own marginal costs. However, under Bertrand competition, equilibrium output is higher, making

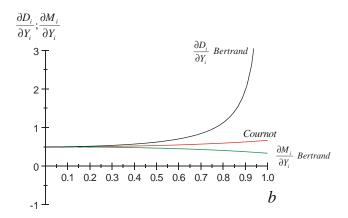


Figure 11: Cournot vs. Bertrand: Cost-pass-through and demand-sensitivity

margin increases through investments more attractive. Also, the sensitivity of equilibrium output to efficiency is higher. Nevertheless, the negative effects dominate.

4.6 Towards a taxonomy

Table 12 summarizes the examples.³²

For simplicity, it only contains the symmetric cases. In line with (C1) and (C2), the margin effect is always non-positive, and the output sensitivity effect is always non-negative, suggesting countervailing effects. The output effect and the cost-pass through effect are ambiguous, which complicates matters further. Table 13 shows which combinations of absolute output effects and cost-pass through effects arise in the different cases. In each case, the sign after the colon shows whether the marginal investment incentive is negative, positive, zero or U-shaped.³³ Note that there is no example where both the output effect and the cost-pass through effect are negative.³⁴

³²In the differentiated Bertrand and Cournot examples the number in brackets refers to the number of the underlying demand function.

³³Again, the numbers in brackets refer to the number of the underlying demand function.

³⁴When asymmetries are allowed, some modifications are necessary. For instance, in the differentiated Bertrand example from Section 4.2, both the cost-pass-through and the demand effect are negative for laggards.

	Absolute Demand	Absolute Markup	Demand Sensitivity	Cost-Pass- Through	Total Effect
Linear Cournot	-	=	0	0	-
Differentiated Cournot (3)	-	-	+	+	U
Differentiated Bertrand (3)	+	-	+	1	-
Differentiated Cournot (4)	+	-	+	+	+
Differentiated Bertrand (4)	+	-	+	-	+
Hotelling	0	-	+	0	0
Bertrand vs. Cournot	+	-	+	-	-

Figure 12: Summary of examples (Symmetric Case)

Cost-Pass-Through \rightarrow	negative	zero	positive
Absolute Demand \downarrow			
negative		Hom. Cournot: -	Diff. Cournot (3): -
zero		Hotelling: 0	
positive	Diff. Bertrand (3): - Bertrand vs. Cournot: - Diff. Bertrand (4): +		Diff. Cournot (4): +

Figure 13: Towards a taxonomy of examples

Otherwise, however, arbitrary combinations of the two effects arise.

5 When does competition raise investments (Basic Model)?

The examples show that, depending on the oligopoly model and the notion of competition, the effect of competition on investment may be positive or negative. I now use the general approach of Section 3 and the examples to identify which factors work towards a positive or negative effect of competition.

5.1 Leaders vs. laggards

In the Hotelling case, competition increases the investments of leaders and decreases those of laggards. In the Cournot example with differentiated goods (Shubik-Levitan 1980), competition has a negative effect on strong laggards, but a U-shaped effect for leaders, symmetric firms and firms that are not lagging behind too far. With price competition, the effect is U-shaped for strong leaders; it is negative for all other firms. With Singh-Vives demand, the effects are positive except for strong laggards. Based on the examples, we therefore obtain the following results:

Observation 1: Investment tends to have a more positive effect for leaders than for laggards; and they are robustly negative for laggards.

There are two reasons why increasing competition is more likely to have a positive investment effect for leaders than for laggards, and why the effect is robustly negative for laggards. Both relate to (C2). First, the positive output sensitivity effect $M^iQ^i_{i\theta}$ implied by (C2) is substantial only when margins are large – but when firms are lagging far behind, their margins are low. Second, because of (C2), Q^i_{θ} and hence the output effect $M^i_iQ^i_{\theta}$ is more likely to be positive when a firm is efficient. Reflecting this intuition, $\Pi^i_{i\theta}$ is increasing in Y_i and decreasing in Y_j in all the examples. Thus, increasing the efficiency of a laggard and decreasing the efficiency of the competitor until the roles of both parties are changed will typically increase the laggard's investment incentives.

5.2 Spillovers

Though Section 2 applies to cases with spillovers ($\lambda > 0$), we have not treated this case in the examples. The following result suggests a tendency for spillovers to make a negative effect of competition on investments more likely.

Proposition 3 Suppose (C3) holds and (i) $\frac{\partial^3 \Pi^i}{\partial Y_i \partial Y_j \partial \theta} = \frac{\partial^3 \Pi^i}{(\partial Y_i)^2 \partial \theta} = \frac{\partial^3 \Pi^i}{(\partial Y_j)^2 \partial \theta} = 0$ for $i = 1.2, j \neq i$ or (ii) investment costs are sufficiently large. As spillovers (λ) increase, $\pi^i_{i\theta}$ falls.

Proof. See 8. ■

To repeat, condition (C3) that $\frac{\partial^2 \Pi^i}{\partial Y_j \partial \theta} < 0$ appears plausible: It requires that, as competition increases, the adverse effect of a more efficient competitor on own profits becomes larger in absolute value. Clearly, this must be true for a move from no competition (two monopolists) to some degree of competition, because in the former case $\frac{\partial \Pi^i}{\partial Y_j} = 0$, whereas in the latter case $\frac{\partial \Pi^i}{\partial Y_j} = 0$. However, closer scrutiny suggests that moving from low to more intense competition does not necessarily lead to a decline in $\frac{\partial \Pi^i}{\partial Y_j}$. Proceeding as in 3,

$$\Pi^i_{j\theta} = Q^i_j \cdot M^i_\theta + M^i \cdot Q^i_{j\theta} + Q^i \cdot M^i_{j\theta} + M^i_j \cdot Q^i_\theta.$$

For instance, the first term, $Q_j^i \cdot M_{\theta}^i$, is positive: As competition reduces margins, it reduces the negative effect of the output reduction following a competitor's increase in efficiency. Nevertheless, in all our examples, at least for sufficently symmetric firms, the remaining effects dominate, so that $\Pi_{j\theta}^i < 0.35$

We are left with the following, slightly tentative, conclusion.

Observation 2: If investments have higher spillovers, marginal investment incentives are more likely to be negatively affected by competition.

5.3 The effects of pre-existing competition

There is a quite common rough intuition that, while some competition is good for investments, "excessive competition" may have negative effects, suggesting an inverted-U relation between competition and investment. In other words, low initial levels of competition would appear to make it more likely that further increases of competition increase investments. The above examples already show that such a general statement cannot be supported in our partial equilibrium framework.³⁶ In fact, the only non-monotone examples feature a U-shape. Even so, (C2) suggests two reasons why increasing competition is indeed more likely to have positive effects when the initial level of competition is low. First, with low competition, margins and hence

³⁵ The second term introduces an effect that cannot be strictly positive, because $Q_j = 0$ for $\theta = 0$, whereas $Q_j < 0$ for $\theta > 0$.

³⁶Aghion et al. (1997, 2001) derive an inverse U-shape from general equilibrium considerations.

the output sensitivity effect $(M_iQ_{i\theta}^i)$ should be high. Second, by (C2), Q_i^i is higher when competition is intense, suggesting that the negative margin effect $Q_i^iM_{\theta}^i$ is more pronounced when competition is intense.

The fact that the effect of competition on investment can be U-shaped even so comes from a simple source: While M_{θ}^{i} has a negative effect on margins, this effect is typically convex: When competition has already reduced margins substantially, further competition does not reduce them much more. In the differentiated Cournot example of 4.2, this effect dominates, resulting in the U-shaped relation observed there. We summarize the discussion as follows:

Observation 3: It is not necessarily more likely that competition has a positive effect on investment incentives when the initial level of competition is low than when it is high.

5.4 Positive output effects

Using (2), one might argue that it is "natural" for competition to have a positive effect on output $(Q_{\theta} > 0)$: If the demand-enhancing effect of lower own price $(\frac{\partial q^i}{\partial p^i} \frac{\partial p^i}{\partial \theta})$ dominates over the demand-reducing effect of lower competitor prices $(\frac{\partial q^i}{\partial p^j} \frac{\partial p^j}{\partial \theta})$, output can only fall if $\frac{\partial q^i}{\partial \theta} < 0.37$ Even $Q_{\theta} > 0$ does not necessarily make for less ambiguity: There are several examples where the output effect is positive, but competition nevertheless reduces investments, even in the symmetric case. For instance, this is true for the substitution parameter in the differentiated Bertrand model of Shubik and Levitan,³⁸ and it also holds when one moves from Cournot to Bertrand competition in the Shubik-Levitan case. Intuitively, while competition increases output (and also by (C2), the sensitivity of output to investment), it also reduces margins, which reduces investment incentives. Hence:

Observation 4: Even when competition increases output, a positive effect of competition on investment does not follow.

³⁷The negative direct effect dominates, for instance, in the homogeneous Cournot example with decreasing market size.

³⁸In the Singh-Vives case, the effect is negative for sufficiently large initial levels of competition.

5.5 The effects of the number of firms

Rather than changes in the intensity of competition for a given number of firms, consider now increases in the number of firms for an otherwise unchanged environment. First, I shall provide an analogous result to Proposition 1 that gives conditions under which the sign of the effect of the change of the number of firms on investment is given exactly by the sign of the effect on marginal investment incentives.

Suppose there are $n \geq 2$ firms. Replace the parameter θ by n and write

$$\Pi^{i}\left(Y_{i},\mathbf{Y}_{-i};n\right)=M^{i}\left(Y_{i},\mathbf{Y}_{-i};n\right)\cdot Q^{i}\left(Y_{i},\mathbf{Y}_{-i};n\right).$$

Apart from that, proceed as in Section 2. Write net profits as $\pi^i(y^i, \mathbf{y}_{-i}; n)$. For any investment level y, let \mathbf{y}_n be the n-1-dimensional vector consisting of identical entries y. Finally, introduce the following weak strategic substitutes condition.

Definition 2 The investment game satisfies **strategic substitutes at the diagonal** (SSD) if $\frac{\partial \pi^i}{\partial y_i}(y_i, \mathbf{y}_n; n)$ is weakly decreasing in \mathbf{y}_n for all y_i and y.

Thus, (SSD) requires player i's investment incentives to fall as the other players' investments increase symmetrically along the diagonal. The condition is motivated by the observation that strategic substitutes typically hold in duopoly investment games with no spillovers.³⁹ The following result holds.

Proposition 4 Consider a symmetric investment game with objective functions $\pi^i(y_i, \mathbf{y}_{-i}; n)$ that are concave in y_i and satisfy (SSD). Suppose for $n^L < n^H$ the game has symmetric equilibria $\mathbf{y}_L \equiv \mathbf{y}(n^L) = (y_L, ..., y_L)$ and $\mathbf{y}_H \equiv \mathbf{y}(n^H) = (y_H, ..., y_H)$. Suppose for all $i \in \{1, 2, ..., n\}$ and $n^L < n^H$,

$$\frac{\partial \pi^i}{\partial u_i} \left(y, \mathbf{y}_L; n^L \right) > \frac{\partial \pi^i}{\partial u_i} \left(y, \mathbf{y}_H; n^H \right). \tag{6}$$

Then $\mathbf{y}_L > \mathbf{y}_H$.

³⁹See the discussion in Section 4.4.

Proof. See Appendix 8.

Under the conditions of Proposition 4, if an increase in the number of firms reduces marginal investment incentives of each firm, as required by (6), it also reduces investments in the symmetric equilibrium. Similar to (3), we obtain

$$\Pi_{in}^{i} = Q_{i}^{i} \cdot M_{n}^{i} + M_{i}^{i} \cdot Q_{n}^{i} + Q^{i} \cdot M_{in}^{i} + M^{i} \cdot Q_{in}^{i}. \tag{7}$$

Thus, as in Section 3, we can identify four transmission channels by which the number of firms affects marginal incentives. However, a higher number of firms quite robustly reduces both margins and output, so that both the margin effect $Q_i^i \cdot M_n^i$ and the output effect $M_i^i \cdot Q_n^i$ are negative. This suggests a clearer negative effect of increasing competition on investments, unless M_{in}^i and Q_{in}^i are positive and very large, though positive signs of Q_{in}^i and M_{in}^i could work in the opposite direction in principle. However, most examples tend to confirm the following:

Observation 5: For symmetric firms, an increase in the number of firms tends to reduce investments per firm.

To illustrate the asymmetric case, return to the example of Section 4.2.1, and compare the investments of both firms in duopoly with the investments that each firm would have in monopoly with the same demand functions. It turns out that, if firm i is the monopolist, it will invest less in monopoly than in the duopoly if and only if $\frac{17}{21}Y_0^i - \frac{6}{7}Y_0^j - \frac{2}{7} > 0$. Thus, interestingly, while introducing competition by a second firm always reduces investments of the former monopolist when the entrant is at least as efficient as the incumbent, entry of a less efficient firm can increase the incumbent's investments. This is a variant of the theme that competition is more likely to have a positive effect on the investments of relatively efficient firms than on those of relatively inefficent firms (Section 5.1).

⁴⁰This condition is consistent with both firms producing positive outputs.

6 When does competition raise investments (Beyond the basic model)?

The previous analysis has exposed several channels by which competition affects investment, suggesting that there is little hope of expecting a robust and non-ambiguous relationship. It also identified some factors that are conducive to positive effects of competition on the investments of a firm. To make further progress in this direction, I will extend the framework in several directions.

6.1 Cumulative Investments

So far, we have considered a model that is entirely static. This may be appropriate in some contexts, but it clearly is an incomplete characterization when cumulative investments are concerned. When firms are faced with the opportunity to improve their technology repeatedly and when they can observe each other's investments, additional strategic considerations are necessary. These considerations affect the relation between competition and investment in a clear-cut way.

For simplicity, suppose the game is played twice (periods t = 1, 2). Let Y_{t-1}^i be the efficiency level of the firm at the beginning of period t. Similarly, y_t^i is the investment in period t. Then we obtain the following result.

Proposition 5 Suppose conditions (C3) and (C4) hold. If competition has a non-negative effect on investment incentives in the static game, then it also has a non-negative effect in each period of the two-stage game.

The intuition is clearest when investments are strategic substitutes. Then investments in the two-stage game have the additional benefit of lowering future investments of the competitor. By (C3), competition increases the negative effect of own investments on the future investments of the competitor. Furthermore, by (C4), competition makes the negative effect of first-period investments on the future investments of the competitor more desirable.

Observation 6: If the effects of competition on investment are positive in the static game, they are also positive in the game with cumulative

investments.

6.2 Competition-dependent investment costs

So far, innovation costs were assumed to be independent of the competition parameter. Though this may appear to be innocuous at first sight, there are at least two natural reasons why competition might affect (marginal) investment costs.

6.2.1 Imperfect upstream competition

So far, we have summarized the R&D process in the cost function without specifying the source of the costs. We now assume that R&D requires inputs that are produced by an upstream supplier. Suppose further that there is an industry-specific component to R&D. Even without an explicit model of the interaction between the supplier and the downstream firms, it is plausible that the intensity of downstream competition has an impact on investment costs. According to our previous considerations, competition affects the willingness of downstream firms to pay for cost reductions. Whenever competition would increase investment incentives with marginal investment costs that are independent of θ , then increasing competition drives up the demand for the upstream input. With this in mind, marginal investment costs should be increasing in competition in this case, and conversely when competition would decrease marginal investment costs. These upstream cost effects should therefore dampen the original effects of competition and investment: When R&D inputs are bought from an imperfectly competitive upstream supplier, the effects of competition would appear to be less pronounced than when the inputs are supplied competitively (or inhouse).

Observation 7: Imperfect upstream competition tends to reduce the strength of the relation between competition and investment.

6.2.2 Agency models

When firms are controlled by managers rather than owners, competition can have the effect of decreasing the costs of investment. To see this, I adjust the model of Schmidt (1996) to the present oligopolistic framework.⁴¹

Suppose that, in both firms, marginal costs can take values L or H > L. Suppose further, that each firm employs a risk-neutral agent who can affect the probability of low marginal costs by exerting effort costs $G(p^i)$, where G is increasing, convex and differentiable. For effort choices of p^i and p^j , we obtain expected profits

$$\widetilde{\Pi}^{i}(p_{i}, p_{j}; \theta) = p^{i}(1 - p^{j}) \Pi^{i}(Y_{i}(L), Y_{j}(H); \theta) + p^{i}p^{j}\Pi^{i}(Y_{i}(L), Y_{j}(L); \theta) + (1 - p^{i}) (1 - p^{j}) \Pi^{i}(Y_{i}(H), Y_{j}(H); \theta) + (1 - p^{i}) p^{j}\Pi^{i}(Y_{i}(H), Y_{j}(L); \theta)$$

Next suppose there is a probability $l(p^i, p^j; \theta)$ that the agent loses his job, where l is differentiable, decreasing in p^i , increasing in p^j and $\frac{\partial l}{\partial p^i}$ is decreasing in θ .⁴² Intuitively, own effort reduces the risk of losing the job, and this becomes more pronounced as competition increases. Assume that losing the job involves costs of $\lambda > 0$.

In the original model of Schmidt (1996), the principal in firm i chooses wages (w_i^L, w_i^H) so as to maximize expected profits subject to the incentive, participation and wealth constraints of the agent. For simplicity of exposition, I confine myself to incentive constraints, assuming that the optimal contract involves $w_i^H = 0$ in line with wealth constraints. The incentive constraint

$$\max_{p^i} w_i^L p^i - G(p^i) - l(p^i, p^j; \theta) \lambda$$

leads to the first-order condition

$$w_i^L = G'(p^i) + \frac{\partial l}{\partial p^i} \lambda.$$

The agent must be compensated for the net cost of effort, which consists of the actual effort cost minus the expected gain from reducing the lay-off

⁴¹Schmidt (1996) assumes that competition corresponds to a parameter change that reduces a firm's profits for given effort levels. He does not model a competitor explicitly.

⁴²A simple specification with this property is $l(p^i, p^j; \theta) = \theta (1 - p^i) p^j$: Layoffs can only arise in the worst state that an unsuccessful firm is facing a successful competitor, and the intensity of competition determines the fractions of cases in which this happens.

probability. Thus, the principal effectively maximizes

$$\widetilde{\Pi}^{i}\left(p_{i},p_{j};\theta\right)-C\left(p,\theta\right),$$

where

$$C(p, \theta) = p^{i} \left(G'(p^{i}) + \frac{\partial l}{\partial p^{i}} \lambda \right).$$

The incentive to induce marginally higher effort is thus

$$(1 - p^{j}) \left(\Pi^{i} (Y_{i}(L), Y_{j}(H); \theta) - \Pi^{i} (Y_{i}(H), Y_{j}(H); \theta) \right)$$

+
$$p^{j} \left(\Pi^{i} (Y_{i}(L), Y_{j}(L); \theta) - \Pi^{i} (Y_{i}(H), Y_{j}(L); \theta) \right)$$

-
$$G' \left(p^{i} \right) - p^{i}G'' \left(p^{i} \right) - \frac{\partial l}{\partial p^{i}} \lambda.$$

The first two rows summarize the positive effects of investment on (expected) gross profits, and the effect of competition on these terms is as before. The third row describes the marginal cost effect. Competition reduces marginal costs: By increasing the effect $\frac{\partial l}{\partial p^i}$ of effort on the lay-off probability, it increases the agent's own interest in exerting effort to avoid layoff. Because competition reduces the marginal costs of investment ($\frac{\partial^2 C^i}{\partial p^i \partial \theta} \leq 0$), there is an additional force in firms with separation of ownership and control that works towards a positive effect of competition on investment.

Observation 8: If the effects of competition on investment are positive in a model with owner-managed firm, the same is true in a model with separation of ownership and control.

6.3 Endogenous market participation

So far, we have assumed that a change in the level of competition leaves the number of firms unaffected. Clearly, however, when there are fixed costs of market participation, the number of firms should be adversely affected by the intensity of competition. Taking into account that a lower number of firms increases investment incentives, ignoring the effects of competition on market participation biases the effects of competition on innovation downwards.

For a simple formalization of the idea, suppose firms i = 1, 2 have to

decide whether to enter the market at a fixed cost F, before the investment game is played. Denote the profits of a monopolist firm i as $\Pi^{i}(Y_{i};\theta)$. Then the equilibrium structure is described as follows.

- 1. If $\Pi^{i}(Y_{i};\theta) < F$, then no firm enters.
- 2. If $\Pi^i(Y_i^*, Y_j^*; \theta) \geq F$ for the SPE choices Y_i^* and Y_j^* of the investment game with two firms, then both firms enter.
- 3. In all other cases, only one firm enters in SPE.

As long as the the number of firms entering the market is independent of θ , the previous analysis applies. If an increase in competition reduces the number of firms, then the analysis of Section 5.5 kicks in. This analysis suggests that a reduction in the number of firms is likely to increase investment incentives of the remaining firm(s).

A slightly different approach would have firms deciding on investments before entering the markets. This problem is more complex, because multiple equilibria will typically arise in the second stage rather than in the first stage, and investment decisions have to be made before the equilibrium is selected. Intuitively, however, this introduces another positive effect of competition on investment, an intimidation effect: By investing more, a firm should increase the chances that the competitor exits. As competition intensifies, inducing such exit becomes more desirable.

Observation 9: If the effects of competition on investment are positive in a model with an exogenous number of firms, the same is true in a model with endogenous market participation.

6.4 Upstream investments

Recent literature has dealt with the investment incentives in vertical structures, e.g. network industries.⁴³ I will briefly sketch how the above approach can be modified to provide a framework for the analysis of upstream investments in such structures.

 $^{^{43}}$ See Bühler and Schmutzler (2005, 2008) for downstream investments, Chen and Sappington (2009) for upstream investments.

Downstream competition is modeled exactly as before, with duopolists obtaining profits

$$\Pi_D^i(Y_i, Y_j; \theta) = Q_D^i(Y_i, Y_j; \theta) \cdot M_D^i(Y_i, Y_j; \theta)$$

for each vector (Y_1,Y_2) of efficiency levels and a competition parameter θ . To carry out production, downstream firms require the input of an upstream monopolist supplier U; for simplicity, suppose that the technology is one-to one, so that one input unit is required for each output unit. The upstream firm has initial constant marginal costs of \overline{u} and can carry out upstream cost-reductions u at costs K(u). Suppose the upstream monopolist is integrated with the downstream firm i=1, whereas it supplies the downstream firm 2 at an access price $a(u;\theta)$, with $\frac{\partial a}{\partial u} < 0$. The functional form of $a(u;\theta)$ could either result from optimization of the upstream firm, a negotiation process or from regulation. It is natural to assume that lower upstream costs not only translate into lower access prices, but also into lower costs of the integrated firm. Thus, we assume that $Y_1 = Y_1(u)$, $Y_2 = Y_2(a(u;\theta))$, where $Y_1'(u) > 0$, $Y_2'(a) < 0$, $\frac{dY_1}{du} = 1$ and $\frac{dY_2}{da} = -1$.

The upstream monopolist obtains revenues from downstream activities of its own subsidiary (firm 1) and from access revenues from firm 2. Write downstream output of firm 2 as

$$\widetilde{Q}_D^2(u;\theta) \equiv Q_D^2(Y_2(a(u)), Y_1(u);\theta)$$
.

Denoting the upstream margin as

$$\widetilde{M}_U^2(u) \equiv a(u) - (\overline{u} - u),$$

total upstream profits thus become

$$\Pi^{T}(u;\theta) = \Pi^{1}(Y_{1}(u), Y_{2}(a(u));\theta) + \widetilde{M}_{U}^{2}(u) \cdot \widetilde{Q}_{D}^{2}(u;\theta).$$

Incentives to invest are thus

$$\frac{\partial \Pi^T}{\partial u} = \frac{\partial \Pi^1}{\partial Y_1} - \frac{\partial \Pi^1}{\partial Y_2} \frac{\partial a}{\partial u} + \frac{\partial \widetilde{M}_U^2}{\partial u} \widetilde{Q}_D^2 + \frac{\partial \widetilde{Q}_D^2}{\partial u} \widetilde{M}_U^2. \tag{8}$$

The first two terms reflect the effects of upstream investments on the integrated firm's downstream profits: $\frac{\partial \Pi^1}{\partial Y_1}$ is the incentive to reduce own (downstream) costs, as in the model without vertical structure. The analysis of competition in a horizontal setting (Section 3) thus applies verbatim to this term: Competition affects investments via the four transmission channels identified there.

The term $-\frac{\partial \Pi^1}{\partial Y_2}\frac{\partial a}{\partial u}$ captures a disincentive to invest which is related to well-known effects in models with spillovers: Investment reduces access costs and hence production costs of the downstream competitor, which is undesirable because it reduces own profits. Intuitively, this effect should be stronger with intense competition. The arguments are similar to those advanced in Section 5.2

The remaining two terms in (8) introduce concerns for wholesale profits from the sale of access to the intermediate input. The term $\frac{\partial \widetilde{M}_U^2}{\partial u}\widetilde{Q}_D^2 = \left(1+\frac{\partial a}{\partial u}\right)\widetilde{Q}_D^2\left(u;\theta\right)$ reflects the effects of own investments on the profit margin from supplying the competitor: Both the costs and the price for each unit of access fall as upstream investments increase. As long as the direct cost reduction effect dominates over the induced price effect $\left(1+\frac{\partial a}{\partial u}>0\right)$, the term is positive. The sign of $\frac{\partial \widetilde{Q}_D^2}{\partial u}\widetilde{M}_U^2 = \frac{\partial \widetilde{Q}_D^2}{\partial u}\left(a(u)-\left(\overline{u}-u\right)\right)$ reflects the net effect of upsteam cost reductions on the output of the competitor who benefits from lower access costs, but suffers from lower costs of the integrated firm. This term may well be negative: If $\left|\frac{\partial a}{\partial u}\right|$ is sufficiently small, the separated firm suffers from lower costs of the competitor, but does not have much lower costs itself. Hence, $\frac{\partial \widetilde{Q}_D^2}{\partial u} < 0$ is conceivable.

The structural similarity between $\frac{\partial \widetilde{\Pi}_D^2}{\partial u}$ and $\frac{\partial \Pi^i}{\partial Y^i}$ might suggests that the effects of competition on investment incentives that come from access revenue considerations are similar to those discussed earlier. However, the discussion of the sign of $\frac{\partial \widetilde{Q}_D^2}{\partial u}\widetilde{M}_U^2$ already shows that there are important differences. While there are four transmission channels by which competition affects

$$\frac{\partial \widetilde{\Pi}_D^2}{\partial u} = \frac{\partial \widetilde{M}_U^2}{\partial u} \widetilde{Q}_D^2 + \frac{\partial \widetilde{Q}_D^2}{\partial u} \widetilde{M}_U^2, \tag{9}$$

the interpretation of the terms differs from the corresponding terms in $\frac{\partial \Pi^i}{\partial Y^i}$ in

several ways. The crucial difference reflects two facts. First, \widetilde{M}_U^2 is not the margin of the downstream competitors, but of the upstream firm supplying them. Second, contrary to the investments of firms in own cost reduction in a standard horizontal duopoly, investments of the integrated firm also reduce the costs of the competitor. As far as access revenue is concerned, it is this aspect of cost reduction that makes investments desirable for the upstream firms (whereas the reduction of own costs is undesirable).

We now sketch the effects of competition on each term in (9). First, it is quite conceivable that the effect of competition on margins \widetilde{M}_U^2 is positive. For instance, if $a(u,\theta)$ results from negotiations between downstream firms and U, greater competition may involve better outside options of the upstream firm, so that greater downstream competition should increase the upstream margin.

For the effect of competition on downstream output Q_D^2 , there are no substantial difference to the earlier considerations. Thus the somewhat ambiguous effects highlighted there carry over. However, one important aspect must be taken into consideration: In many relevant applications, there is a clear asymmetry between integrated and separated firms. The integrated firm is often an established incumbent, whereas the entrants may be less experienced. Depending on the precise context, these differences may show up in cost differences, in which case the considerations from the leader-laggard model apply. Specifically, if the separated firm has cost disadvantages, the effect of greater competition on \widetilde{Q}_D^2 will tend to be negative.

To understand the effects of competition on $\frac{\partial \tilde{Q}_D^2}{\partial u}$, first suppose the two firms are monopolists. Then $\frac{\partial \tilde{Q}_D^2}{\partial u}$ must be positive, because the separated firm faces lower access costs, whereas the lower downstream costs of the integrated firm have no adverse effect on the output of the separated firm. As competition increases, this adverse effect kicks in. Thus, it appears plausible that the downstream output-sensitivity effect is negative.

Finally, consider the effect of competition on $\frac{\partial M_U^2}{\partial u}$, or equivalently, the effect on $\frac{\partial a}{\partial u}$. Without specifying the details of the model, it is hard to come up with any definite result for the sign of the effect of competition on this expression. If access prices are determined by regulation one could easily imagine that this regulation is insensitive to the details of downstream

competition, so that there might well be no effect.

Thus, perhaps unsurprisingly, increasing competition leads to additional positive and negative effects on upstream investment incentives. Future research will explore under which circumstances the positive effects dominate over the negative ones.

7 Conclusion

The paper has identified several channels by which competition affects investment. In the main part, increasing competition referred not to an increase in the number of firms, but to a more competitive strategic interaction for a given number of firms. By assumption and consistent with many examples, competition reduces margins, and increases the sensitivity of equilibrium output with respect to efficiency. Adding to these ambiguities, competition can have positive or negative effects on equilibrium output and on the sensitivity of prices with respect to marginal costs. Unless one opts for very narrow notions of increasing competitions, the ambiguities do not disappear. Further, a positive effect of competition is more likely for leaders than for laggards, and it is less likely when spillovers are strong. Next, no general case can be made that an inverse relation between competition and investment is more likely than a U-shaped relation. With the alternative interpretation of increasing competition as an increase in the number of firms, however, competition has a clear negative effect.

Extensions of the basic model helped to identify various factors that influence the effects of competition on investment. A positive effect is likely to be fostered when investments are cumulative, when there is separation of ownership and control and when market participation is determined endogenously. Imperfect upstream markets reduce the effects of competition on investment, no matter whether they are positive or negative. The analysis also helps to obtain some intuition for the effects of downstream competition on upstream investments.

Though the approach presented here allows to incorporate a large number of issues concerning the relation between competition and investment in one framework, there are obvious limitations. First, I have not treated product innovations. It is very likely that a treatment along the lines sketched here would help to understand how the effects of competition on product innovation differ from those on process innovation.⁴⁴ In principle a decomposition of investment incentives analogous to (3) could shed light on the relation between competition and (vertical or horizontal) product innovation. However, one important distinction is that, with product innovations, an innovating firm may want to continue to use the old product (Greenstein and Ramey 1998, Chen and Schwartz 2008). Second, I have assumed that R&D investments are observable to competitors before they take their product market decisions. Taken literally, this is certainly a strong assumption. Most of the arguments appear to rely, however, on the weaker notion that in the product-market stage firms should be aware of their relative competitive position as determined by previous investments to some extent.

8 Appendix 1: Proofs

8.1 Proof of Proposition 1

- (i) follows from Theorem 5 in Milgrom and Roberts (1990).⁴⁵
- (ii) By (i), it suffices to consider $\pi^i_{ij} < 0$. Total differentiation of the system of first order conditions shows that a negative effect of θ on investment would require $\pi^j_{j\theta}\pi^i_{ij} < \pi^i_{i\theta}\pi^j_{jj}$, and therefore, using symmetry $\pi^i_{ij} < \pi^j_{jj}$. For $\pi^i_{ij} < 0$ and symmetry, this condition is incompatible with stability.
- (iii) follows from total differentiation of the system of first order conditions.

8.2 Proof of Proposition 3

First note that

$$\frac{\partial^{2} \pi^{i} (y_{i}, y_{j}; \theta)}{\partial y_{i} \partial \theta} = \frac{\partial^{2} \Pi^{i} (Y_{i}^{0} + y_{i} + \lambda y_{j}, Y_{j}^{0} + y_{j} + \lambda y_{i}; \theta)}{\partial Y_{i} \partial \theta} + \lambda \frac{\partial^{2} \Pi^{i} (Y_{i}^{0} + y_{i} + \lambda y_{j}, Y_{j}^{0} + y_{j} + \lambda y_{i}; \theta)}{\partial Y_{j} \partial \theta}.$$

⁴⁴Gilbert (2006) summarizes some arguments pertaining to this discussion.

⁴⁵This theorem is a comparative-static result for supermodular games.

Therefore,

$$\begin{split} &\frac{\partial^{3}\pi^{i}\left(y_{i},y_{j};\theta\right)}{\partial y_{i}\partial\theta\partial\lambda} = \\ &\frac{\partial^{2}\Pi^{i}}{\partial Y_{j}\partial\theta} + y_{j}\left(\frac{\partial^{3}\Pi^{i}}{\left(\partial Y_{i}\right)^{2}\partial\theta} + \frac{\partial^{3}\Pi^{i}}{\partial Y_{i}\partial Y_{j}\partial\theta}\right) + y_{i}\left(\frac{\partial^{3}\Pi^{i}}{\partial Y_{i}\partial Y_{j}\partial\theta} + \frac{\partial^{3}\Pi^{i}}{\left(\partial Y_{j}\right)^{2}\partial\theta}\right). \end{split}$$

Thus, if either y_i and y_j or the terms in brackets are sufficently small, $\frac{\partial^3 \pi^i(y_i, y_j; \theta)}{\partial y_i \partial \theta \partial \lambda} < 0$. If (i) or (ii) holds, the statement thus holds.

8.3 Proof of Proposition 4

As $\frac{\partial \pi^i}{\partial y^i} \left(y^H, \mathbf{y}_{n^H}^H; n^H \right) = 0$, (6) implies $\frac{\partial \pi^i}{\partial y^i} \left(y^H, \mathbf{y}_{n^L}^H; n^L \right) > 0$. By concavity, $\frac{\partial \pi^i}{\partial y_i} \left(y_i, \mathbf{y}_{n^L}^H; n^L \right) > 0$ for any $y_i < y^H$. Finally, (SSD) implies $\frac{\partial \pi^i}{\partial y^i} \left(y_i, \mathbf{y}_{n^L}^i; n^L \right) > 0$. Therefore, $\mathbf{y}_L < \mathbf{y}_H$ is impossible.

8.4 Proof of Proposition 5

The game in period 2 corresponds to the static game. Hence, we only need to consider period 1. Denote the equilibrium investment of player i in the second stage game for each vector $\mathbf{Y}_1 = (Y_1^1, Y_1^2)$ of interim states as $y_2^i(\mathbf{Y}_1)$. Then, if players invest y_1^i in period 1, their total payoffs can be written as functions of first-period investments:

$$\Pi_{T}^{i}\left(y_{1}^{i}, y_{1}^{j}; \theta\right) = \Pi^{i}\left(Y_{0}^{i} + y_{1}^{i}, Y_{0}^{j} + y_{1}^{j}; \theta\right) +
\Pi^{i}\left(Y_{0}^{i} + y_{1}^{i} + y_{2}^{i}\left((\mathbf{Y}_{0} + \mathbf{y}_{1})\right), Y_{0}^{j} + y_{1}^{j} + y_{2}^{j}\left((\mathbf{Y}_{0} + \mathbf{y}_{1})\right); \theta\right) - K(y_{1}^{i}) - K(y_{2}^{i}(\mathbf{Y}_{0} + \mathbf{y}_{1})).$$

Because y_2^i will be chosen so as to satisfy

$$\frac{\partial \Pi^i}{\partial y_2^i} = \frac{\partial K^i}{\partial y_2^i},$$

investment incentives in period 1 are

$$\frac{\partial \Pi_T^i}{\partial y_1^i} = 2 \frac{\partial \Pi^i}{\partial Y_1^i} + \frac{\partial \Pi^i}{\partial Y_2^j} \frac{\partial y_2^j}{\partial y_1^i}$$

Conditions (C3) and (C4) imply that $\frac{\partial \Pi^i}{\partial Y_2^j} \frac{\partial y_2^j}{\partial y_1^i}$ is increasing in θ .

9 Appendix 2: The Examples

9.1 Substitutability (Shubik-Levitan)

9.1.1 Quantity competition

Define $Y_i = 1 - c_i$, that is, $\bar{c} = 1$. For $2Y_i \ge \theta Y_j$; $2Y_j \ge \theta Y_i$, ⁴⁶

$$Q^{i}\left(Y_{i}, Y_{j}; \theta\right) = M^{i}\left(Y_{i}, Y_{j}; \theta\right) = \frac{2Y_{i} - \theta Y_{j}}{4 - \theta^{2}}.$$

9.1.2 Price competition

With price competition,

$$Q^{i}(Y_{i}, Y_{j}; \theta) = \frac{(2 - \theta^{2}) Y_{i} - \theta Y_{j}}{(4 - \theta^{2}) (1 - \theta^{2})}; M^{i}(Y_{i}, Y_{j}; \theta) = \frac{(2 - \theta^{2}) Y_{i} - \theta Y_{j}}{4 - \theta^{2}}.$$

9.2 Substitutability (Singh-Vives)

With quantity competition,

$$Q^{i}(Y_{i}, Y_{j}; \theta) = \frac{(1+\theta)(2Y_{i}-\theta Y_{j})}{(4-\theta^{2})}; M^{i}(Y_{i}, Y_{j}; \theta) = \frac{2Y_{i}-\theta Y_{j}}{4-\theta^{2}}.$$

With price competition,

$$Q^{i}(Y_{i}, Y_{j}; \theta) = \frac{(2 - \theta^{2}) Y_{i} - \theta Y_{j}}{(4 - \theta^{2}) (1 - \theta)}; M^{i}(Y_{i}, Y_{j}; \theta) = \frac{(2 - \theta^{2}) Y_{i} - \theta Y_{j}}{4 - \theta^{2}}.$$

9.3 Hotelling

In the Hotelling model, demand functions are given by

$$q^{1}(p^{1}, p^{2}; \theta) = (p^{1} - p^{2} + \theta)/2\theta \text{ and } q^{2}(p^{2}, p^{1}; \theta) = (p^{2} - p^{1} + \theta)/2\theta.$$

⁴⁶The following results are taken from Sacco and Schmutzler (forthcoming).

Defining $Y_i = -c_i$, it is straightforward to show that

$$Q^{i}(Y_{i}, Y_{j}; \theta) = (Y_{j} - Y_{i} + 3\theta) / 6\theta; M^{i}(Y_{i}, Y_{j}; \theta) = (Y_{i} - Y_{j} - 3\theta) / 3.$$

Thus,

$$Q_{\theta}^{i} = (Y_{i} - Y_{j})/6\theta^{2}; M_{\theta}^{i} = -1; Q_{i}^{i} = -1/6\theta; M_{i}^{i} = 1/3; Q_{i\theta}^{i} = 1/6\theta^{2}; M_{i\theta}^{i} = 0$$

Simple but tedious calculations show that equilibrium investments are

$$y_i = \frac{1}{6} + \frac{Y_j^0 - Y_i^0}{2(9\theta + 1)}.$$

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