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IDENTIFICATION OF SUPPLY MODELS OF RETAILER AND MANUFACTURER OLIGOPOLY PRICING*

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ABSTRACT

This note outlines conditions under which we can identify a vertical supply model of multiple retailers' and manufacturers' oligopoly-pricing behavior. This is an important question particularly when the researcher believes, contrary to the traditional assumption followed in the empirical literature, that retailers may not be neutral pass-through intermediaries. We show that a data-set of an industry's product prices, quantities, and input prices over time is sufficient to identify the vertical model of retailers' and manufacturers' oligopoly-pricing behavior given nonlinear demand, for homogeneous-products industries, and given multi-product firms, for differentiated-products industries.

JEL Classifications: L13, L22. Keywords: Identification, Vertical relationships, Oligopoly models of multiple manufacturers and retailers.

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1. INTRODUCTION

A number of recent studies have introduced retailers into structural econometric models of sequential vertical-pricing games (Goldberg and Verboven 2001, Mortimer 2002, Villas-Boas and Zhao 2004 and Villas-Boas 2004), relaxing the conventional assumption that manufacturers set prices and that retailers act as neutral pass-through intermediaries (as in Bresnahan 1982 and Nevo 1998, for example). In traditional structural econometric models, the model of firms' pricing behavior (and the degree of market power) is identified via the estimation of a conduct parameter that measures manufacturers' deviations from price-taking behavior for homogeneous-products industries and from Bertrand-pricing behavior for differentiated-products industries.

This note provides conditions under which we can identify a vertical model of multiple retailers' and manufacturers' oligopoly-pricing behavior. Modeling firms' behavior along a vertical channel has important implications for the analysis of price dynamics in the economy as a whole (Chevalier, Kashyap and Rossi 2003), for manufacturer merger analysis (Manuszak 2001), and for the pass-through effects of foreign-trade policy (Feenstra 1989 and Hellerstein 2004).

Suppose that a researcher observes a time series of retail price-quantity pairs which he believes to be market-equilibrium outcomes of demand and supply conditions. When modeling a vertical relationship, the researcher typically does not have access to wholesale-price data, that is, to the prices retailers pay to manufacturers. In many industries, however, the researcher can get data on retailers' and manufacturers' input prices. The general identification problem is, thus, to infer the distributions of consumers' and firms' decision rules, which are *not* observable, from the decisions themselves, which *are* observable, as price-quantity pairs. But without additional information various combinations of demand and supply models may appear to produce the same observable decisions, that is, the same observable price-quantity pairs over time (Working 1926 and Bresnahan 1982).

The econometric problem we face is, thus, a standard simultaneous-equation model in which a demand and a supply pricing equation, both derived from behavioral assumptions, are to be estimated. The demand equation relates quantity purchased to price, product characteristics, and unobserved determinants of demand. The supply equation relates retail and wholesale prices to a markup and to observed and unobserved determinants of cost. Our main goal in this paper is to establish when data on an industry's product prices, quantities, and input prices over time are sufficient to identify the vertical model of manufacturers' and retailers' oligopoly-pricing behavior given that demand and supply relations are not known *a priori*.

The rest of the note proceeds as follows. In the next section we introduce the model and set

out the identification problem for a homogeneous-products model. We show that the supply model of manufacturer and retailer oligopoly pricing is identified in a homogeneous-products model if demand is nonlinear. The third section examines the identification problem for a differentiated-products model. In this case, the supply model of manufacturer and retailer oligopoly-pricing is identified under very general conditions, even with linear demand, except in special cases where an industry has exclusive dealerships with single product firms.

2. THE HOMOGENEOUS-PRODUCTS MODEL

In this section we set out a general demand model and describe the identification problem for the homogeneous-products model. Given identification of demand, one can identify the oligopoly solution as well as the parameters that measure deviations from a benchmark model of retailer and manufacturer pricing.

Let the inverse demand for a particular product be given by

$$p = h(Q, Y, \alpha) + \epsilon, \tag{1}$$

where p is the retail price, Q is quantity, Y contains exogenous variables that affect demand, α contains demand parameters to be estimated, and ϵ is the random error term.

On the supply side let us assume the standard linear pricing model that leads to double-marginalization in which manufacturers set wholesale prices p^w first and retailers follow setting retail prices p . Let retailers have constant marginal costs given by: $c^r = \beta_r + \gamma_r W$ where W are exogenous variables that affect cost and β_r and γ_r are parameters to be estimated. Let manufacturers have constant marginal costs given by: $c^w = \beta_w + \gamma_w W$, where β_w and γ_w are manufacturer cost parameters to be estimated.

2.1. Retailers

Looking at retailers first, if they behave as price-takers, one can write $p = p^w + c^r$. Otherwise, they set their perceived marginal revenue, and not their price, equal to marginal cost. To formalize, let us define a parameter λ_r to be estimated that is interpreted as measuring retail firms' deviations from price-taking behavior (when it is significantly different from zero). Given that marginal revenue is given by $p + h'(Q, \cdot)Q$, retailers' perceived marginal revenue is given by $p + \lambda_r h'(Q, \cdot)Q$. This means

that the supply relation of the retailers is:

$$p = p^w - \lambda_r h'(Q)Q + \beta_r + \gamma_r W + \eta_r, \quad (2)$$

where η_r is the retail supply random term that contains unobserved components of retail costs.

2.2. Manufacturers

Given that retailers behave according to (2), the inverse derived demand faced by manufacturers is:

$$p^w = h(Q) + \lambda_r h'(Q)Q - \beta_r - \gamma_r W. \quad (3)$$

Manufacturers' marginal revenue is then: $p^w + h'(Q)Q + \lambda_r h''(Q)Q^2 + \lambda_r h'(Q)Q$, their perceived marginal revenue is: $p^w + \lambda_w [h'(Q)Q + \lambda_r h''(Q)Q^2 + \lambda_r h'(Q)Q]$, and their supply relation is:

$$p^w = -\lambda_w \lambda_r [h''(Q)Q^2 + h'(Q)Q] - \lambda_w h'(Q)Q + \beta_w + \gamma_w W + \eta_w, \quad (4)$$

where in η_w is the manufacturer supply random term that contains unobserved components of manufacturer costs.

If the researcher has wholesale-price data as well as retail and manufacturer cost data, he can estimate (1), (2), and (4) simultaneously, treating price and quantity as endogenous variables. In most cases, however, neither wholesale-price data nor information on what part of marginal cost is attributable to retailers and what to manufacturers are available. In this more common case, the pricing equation to be estimated is obtained by substituting (4) into (2), which gives:

$$p = -(\lambda_r + \lambda_w)h'(Q)Q - \lambda_r \lambda_w [h''(Q)Q^2 + h'(Q)Q] + \beta_{r+w} + \gamma_{r+w}W + \eta. \quad (5)$$

2.3. Identification in the Homogeneous-Products Model

This section describes the conditions under which the parameters λ_r and λ_w , the demand parameters α , and the cost parameters β and γ are identified. First, given constant marginal costs, the exogenous cost variables W must differ from the exogenous demand variables Y and the dimensions of W must be such that the demand parameters α are identified.¹

¹The identifying assumption is that changes in input prices are uncorrelated with unobserved determinants of demand that are in ϵ .

Second, the parameters λ_r and λ_w are identified if demand is non-linear. In the special case of linear demand the parameters λ_r and λ_w cannot be separately identified. A linear demand function, e.g., $Q = \alpha_0 + \alpha_1 p + \alpha_2 Y$ yields $h = \frac{-\alpha_0}{\alpha_1} + \frac{1}{\alpha_1} Q - \frac{\alpha_2}{\alpha_1} Y$ and $h'(Q) = \frac{1}{\alpha_1}$ and finally $h''(Q) = 0$. Therefore (5) becomes

$$p = - \underbrace{[\lambda_r + \lambda_w + \lambda_r \lambda_w]}_{\omega} \frac{1}{\alpha_1} Q + \beta_{r+w} + \gamma_{r+w} W + \eta. \quad (6)$$

One can estimate $\omega = -[\lambda_r + \lambda_w + \lambda_r \lambda_w] \frac{1}{\alpha_1}$ and, though we can treat α_1 as known since demand can be estimated, we cannot identify λ_r and λ_w separately. Note also that only β_{r+w} , and γ_{r+w} can be identified and not the retailers' β_r and γ_r or the manufacturers' β_w and γ_w separately.

3. THE DIFFERENTIATED-PRODUCTS MODEL

Let there be N differentiated products and let q_n be the demand for a certain product n given by:

$$q_n = q(p_1, \dots, p_N, Y, \alpha) + \epsilon_n, \quad (7)$$

where p_1, \dots, p_N are the retail prices of all products, Y contains exogenous variables that affect demand, α contains demand parameters to be estimated, and ϵ_n is the random error term.

On the supply side let us assume the standard linear pricing model that leads to double-marginalization in which M manufacturers set wholesale prices p^w and R retailers follow setting retail prices p . Let retailers' marginal costs be constant and given by: $c^r = \beta_r + \gamma_r W$ as above, and let manufacturers' marginal cost be constant and given by: $c^w = \beta_w + \gamma_w W$.

3.1. Retailers

Each retailer maximizes his profit function:

$$\pi_r = \sum_{j \in S_r} [p_j - p_j^w - c_j^r] q_j(p) \quad \text{for } r = 1, \dots, R. \quad (8)$$

where S_r is the set of products sold by retailer r . The first-order conditions, assuming a pure-strategy Nash equilibrium in retail prices, are:

$$q_j + \sum_{m \in S_r} T_r(m, j) [p_m - p_m^w - c_m^r] \frac{\partial q_m}{\partial p_j} = 0 \quad \text{for } j = 1, \dots, N. \quad (9)$$

Switching to matrix notation, let us define $[A * B]$ as the element-by-element multiplication of two matrices of the same dimensions A and B . Let us also define a matrix T_r with general element $T_r(i, j) = 1$ if the retailer sells both products i and j and equal to zero otherwise. Finally, let Δ_r be a matrix with general element $\Delta_r(i, j) = \frac{\partial q_i}{\partial p_i}$. Solving (9) for the price-cost margins for all products in vector notation gives the price-cost margins m_r for the retailers under Nash-Bertrand pricing:

$$\underbrace{p - p^w - c^r}_{m^r} = -[T_r * \Delta_r]^{-1}q(p), \quad (10)$$

which is a system of N implicit functions that expresses the N retail prices as functions of the wholesale prices. If retailers behave as Nash-Bertrand players then equation (10) describes their supply relation. Let us define an N -by-1 vector of parameters, Λ_r , that measures the deviation from the underlying retail-pricing model for each product and let us construct a matrix M_r with diagonal elements given by the vector m^r :

$$M_r = \begin{bmatrix} m_1^r & 0 & \cdots & 0 \\ 0 & m_2^r & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & m_N^r \end{bmatrix} \quad (11)$$

Then the supply relation becomes

$$p = p^w + M_r \Lambda_r + \beta_r + \gamma_r W + \eta_r, \quad (12)$$

where η_r is the retail supply random error term which contains unobserved components of retail costs.

3.2. Manufacturers

Manufacturers choose wholesale prices p^w to maximize their profits knowing that retailers behave according to (12). Solving for the first-order conditions from the manufacturers' profit-maximization problem, assuming again a pure-strategy Nash equilibrium in wholesale prices and using matrix notation, yields:

$$\underbrace{(p^w - c^w)}_{m^w} = -[T_w * \Delta_w]^{-1}q(p), \quad (13)$$

where T_w is a matrix with general element $T_w(i, j) = 1$ if the manufacturer sells both products i and j and equal to zero otherwise, Δ_w is a matrix with general element $\Delta_w(i, j) = \frac{\partial q_i}{\partial p_i^w}$, and $*$ represents

the element-by-element multiplication of both matrices.

To obtain Δ_w , first note that $\Delta_w = \Delta_p' \Delta_r$, where Δ_p is a matrix of derivatives of all retail prices with respect to all wholesale prices. To get the expression for Δ_p , let us start by totally differentiating for a given j equation (9) with respect to all retail prices ($dp_k, k = 1, \dots, N$) and with respect to a single wholesale price p_f^w , with variation dp_f^w :

$$\sum_{k=1}^N \underbrace{\left[\frac{\partial q_j}{\partial p_k} + \sum_{i=1}^N (T_r(i, j) \frac{\partial^2 q_i}{\partial p_j \partial p_k} (p_i - p_i^w - c_i)) + T_r(k, j) \frac{\partial q_k}{\partial p_j} \right]}_{g(j, k)} dp_k - \underbrace{T_r(f, j) \frac{\partial q_j}{\partial p_j}}_{S(j, f)} dp_f^w = 0. \quad (14)$$

Putting all $j = 1, \dots, N$ products together, let G be a matrix with general element $g(j, k)$ and let S_f be an N -dimensional vector with general element $S(j, f)$. Then $G dp - S_f dp_f^w = 0$. Solving for the derivatives of all retail prices with respect to the wholesale price p_f^w , the f -th column of Δ_p is obtained:

$$\frac{dp}{dp_f^w} = G^{-1} S_f. \quad (15)$$

Stacking all N columns together, $\Delta_p = G^{-1} S$, which has the derivatives of all retail prices with respect to all wholesale prices. The general element of Δ_p is $(i, j) = \frac{\partial p_j}{\partial p_i^w}$.

If manufacturers behave as Nash-Bertrand players then equation (13) describes their supply relation. If one associates an N -by-1 vector of parameters Λ_w , one for each product, that measures deviations from the underlying model of manufacturer-pricing behavior, then the supply relation for the manufacturers becomes:

$$p^w = M_w(\Lambda_r, M_r) \Lambda_w + \beta_w + \gamma_w W + \eta_w, \quad (16)$$

where η_w is a supply random term which contains unobserved components of manufacturer costs and $M_w(\Lambda_r, M_r)$ is a matrix containing the m^w defined by (13) on its diagonal elements, and is in general a function of the retail-pricing ‘‘behavior’’ represented by Λ_r and of the retail margins M_r .

If there are no wholesale-price data then the supply equation to be estimated is obtained by substituting (16) into (12) which yields:

$$p = M_r \Lambda_r + M_w(\Lambda_r, M_r) \Lambda_w + \beta_{r+w} + \gamma_{r+w} W + \eta. \quad (17)$$

3.3. Identification of Retail and Manufacturer Pricing in the Differentiated-Products Model

Let us assume that W in (17) and Y in (7) are exogenous variables that differ from one another and that the dimension of W is such that the parameters of demand α are identified.²

For each product j , equation (17) is given by:

$$p_j = \lambda_j^w \underbrace{[-[T_w * \Delta'_p \Delta_r]^{-1}]_{j\text{-line}} q(p)}_{m_j^w(T_w, T_r)} + \lambda_j^r \underbrace{[-[T_r * \Delta_r]^{-1}]_{j\text{-line}} q(p)}_{m_j^r(T_r)} + \beta_{r+w} + \gamma_{r+w} W + \eta. \quad (18)$$

If for a certain product j we have $m_j^w(T_w, T_r) = K m_j^r(T_r)$ where K is a non-zero constant (that is, where manufacturer margins are proportional to retail margins) then the retail and manufacturer models are not identified: for a certain j we could estimate $\lambda_r + K\lambda_w$ but not λ_r and λ_w separately. This case is unlikely to occur in practice, however, regardless of the demand model. Unlike in the homogeneous-products model, identification in the differentiated-products model is possible with linear demand. In the next subsection, we show formally that with linear demand the parameters in Λ_r and Λ_w can be identified separately.

3.3.1. Simple Model

Let us consider without loss of generality a simple model of two manufacturers selling two products to two retailers, where the goal is to examine equation (18). In this simplified model, two manufacturers a and b produce one good each, which they sell to two retailers 1 and 2. Without loss of generality it is assumed that the good produced by manufacturer b is a private label of retailer 2. This implies that there are three retail-level products in this model: let product 1 be produced by manufacturer a and sold to retailer 1; product 2 be produced by manufacturer a and sold to retailer 2; and product 3 be produced by manufacturer b and sold to retailer 2. This implies that the retailer's (T_r) and the manufacturer's (T_w) product matrices are given by:

$$T_r = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \text{ and } T_w = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (19)$$

²The identifying assumption is that changes in input prices W are uncorrelated with unobserved determinants of demand ϵ such as advertising and changes in consumer preferences.

Let $d_{ij} = \frac{\partial q_i}{\partial p_j}$. Solving (10) for each product yields

$$\begin{bmatrix} m_1^r \\ m_2^r \\ m_3^r \end{bmatrix} = \frac{1}{d_{11}(d_{22}d_{33} - d_{23}d_{32})} \begin{bmatrix} d_{22}d_{33} - d_{23}d_{32} & 0 & 0 \\ 0 & d_{11}d_{33} & -d_{11}d_{32} \\ 0 & -d_{11}d_{23} & d_{11}d_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \quad (20)$$

To compute manufacturers' price-cost margins, one needs to compute Δ_w , a matrix with a very complicated expression even for this simple model. Note that the matrix $\Delta_w = \Delta_p' \Delta_r$, where Δ_p is a matrix of derivatives of each retail price with respect to each wholesale price. In particular, $\Delta_p = G^{-1}S$. Let $h_{ijk} = \frac{\partial q_i^2}{\partial p_j \partial p_k}$. Note that in this case:

$$G = \begin{bmatrix} 2d_{11} + h_{111}m_1^r & d_{12} + h_{112}m_1^r & d_{13} + h_{113}m_1^r \\ d_{21} + h_{221}m_2^r + h_{321}m_3^r & 2d_{22} + h_{222}m_2^r + h_{322}m_3^r & d_{23} + h_{223}m_2^r + h_{323}m_3^r + d_{32} \\ d_{31} + h_{231}m_2^r + h_{331}m_3^r & d_{32} + h_{232}m_2^r + h_{332}m_3^r + d_{23} & 2d_{33} + h_{233}m_2^r + h_{333}m_3^r \end{bmatrix} \quad (21)$$

and

$$S = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & d_{32} \\ 0 & d_{23} & d_{33} \end{bmatrix}. \quad (22)$$

Solving for the manufacturers' mark-ups in (13) for each product gives us:

$$\begin{bmatrix} m_1^w \\ m_2^w \\ m_3^w \end{bmatrix} = \frac{1}{\frac{\det(T_w * \Delta_w)}{\det G}} \begin{bmatrix} \mu d_{11}^2 + \varepsilon + \varsigma & -((\eta + \theta)d_{11} + (\iota + \kappa)d_{21} + \varepsilon) & 0 \\ -(\mu d_{11}d_{12} + \nu d_{11}d_{22} + \varsigma d_{11}d_{32}) & (\eta + \theta)d_{12} + (\iota + \kappa)d_{22} + (\rho + \varpi)d_{32} & 0 \\ 0 & 0 & (\pi + \theta)d_{13} + (\xi + \kappa)d_{23} + (\sigma + \varpi)d_{33} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \quad (23)$$

where the expressions for the Greek letters from ε through ς in the above equation are in the appendix.

Finally we need to substitute the relevant lines of m_j^w from (23) and m_j^r from (20) into equation (18) for this simple model and to look at cases where m_j^w is directly proportional to m_j^r , which leave us unable to identify manufacturer and retailer pricing separately. In the next subsection, we show that even for the linear case m_j^w is generally an affine transformation of m_j^r and not simply a proportional change. In the general non-linear case it is clear that m_j^w is not directly proportional to m_j^r .

3.3.2. Illustration of Identification - Simple Model, Linear-Demand Case

Let demand for each product be given by

$$q_n = \alpha_{n0} + \alpha_{n1}p_1 + \alpha_{n2}p_2 + \alpha_{n3}p_3 + Y\alpha_{n4} + \epsilon_n, \text{ for } n = 1, 2, 3. \quad (24)$$

Let us look without loss of generality at product 1. The retail margin from (20) for the linear case is:

$$m_1^r = \frac{q_1}{\alpha_{11}}, \quad (25)$$

and the manufacturer margin is:

$$m_1^w = K \frac{q_1}{\alpha_{11}} - K_2 q_2, \quad (26)$$

where

$$K = T_w(1, 1) \frac{\det G}{\det(T_w * \Delta_w)} \alpha_{11} [2\alpha_{11}^3 - \alpha_{12}\alpha_{11}\alpha_{21} + \alpha_{13}\alpha_{11}\alpha_{31}], \quad (27)$$

$$K_2 = T_w(1, 2) \frac{K}{\alpha_{11}} \underbrace{[2\alpha_{22}^2\alpha_{21} + \alpha_{31}\alpha_{33}\alpha_{11} - \alpha_{32}\alpha_{33}\alpha_{21} - \alpha_{23}\alpha_{33}\alpha_{21} - \alpha_{21}\alpha_{22}\alpha_{11} - \alpha_{12}\alpha_{11}\alpha_{21}]}_{K_3}, \quad (28)$$

and $T_w(1, 1) = T_w(1, 2) = 1$. Given that the manufacturer margin in (26) generally is not proportional to the retail margin in (25), the models of firms' pricing behavior are identified. Only if $K_2 = 0$ does identification fail. This happens if $T_w(1, 2) = 0$ (which implies by definition that $T_w(2, 1) = 0$), that is, if the matrix T_w is diagonal in its upper-left 2-by-2 minor. In terms of the structure of the market, this corresponds to single-product manufacturers each working with single-product retailers that each act as an exclusive dealer of one product in the upper-left minor. In this extraordinary case, given linear demand, the manufacturer margin is proportional to the retail margin and we do not have identification.³ This case is unlikely to occur in practice, however. That is, we rarely observe single-product retailers as the exclusive outlets for single-product manufacturers.

4. CONCLUSION

This note outlines conditions under which we can estimate and identify a model of multiple retailers' and manufacturers' oligopoly-pricing behavior. This is an important question particularly when the researcher believes, contrary to the traditional assumption followed in the empirical literature, that retailers may not be neutral pass-through intermediaries.

³For the particular cases of the demand parameters satisfying $K_3 = 0$ we would still have no identification for this retail and manufacturer multi-product simple case.

The modeling approach has two steps. First, one must estimate demand parameters consistently. Second, given the demand parameters, one computes the implied price-cost margins that retailers and manufacturers choose in maximizing profits in a particular vertical supply model with well-defined retailer and manufacturer oligopoly behavior. The oligopoly model has two identifying assumptions. First, it is assumed that product choice, the portfolio of products produced by manufacturers and sold by retailers, is exogenous. This means that firms, both retailers and manufacturers, don't change the selection of products they offer following an input-price change. Second, the model assumes that marginal costs are constant.

Given these two assumptions, we show that models of manufacturer and retailer oligopoly-pricing behavior are identified in homogeneous-products models given nonlinear demand. In the case of differentiated products, we show that vertical models are identified in general for multi-product retailers and manufacturers, even with linear demand. Lack of identification may arise in special cases where an industry has exclusive dealerships and single product firms.

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APPENDIX

$$[T_w * \Delta_w]^{-1} = \frac{\det(T_w * \Delta_w)}{\det G} \begin{bmatrix} \mu d_{11}^2 + \varepsilon + \zeta & -((\eta + \theta) d_{11} + (\iota + \kappa) d_{21} + \varepsilon) & 0 \\ -(\mu d_{11} d_{12} + \nu d_{11} d_{22} + \varsigma d_{11} d_{32}) & (\eta + \theta) d_{12} + (\iota + \kappa) d_{22} + (\rho + \varpi) d_{32} & 0 \\ 0 & 0 & (\pi + \theta) d_{13} + (\xi + \kappa) d_{23} + (\sigma + \varpi) d_{33} \end{bmatrix}$$

where $d_{ij} = \frac{\partial q_i}{\partial p_j}$, $h_{ijk} = \frac{\partial q_i^2}{\partial p_j \partial p_k}$ and where:

$$\varepsilon = \nu d_{11} d_{21}$$

$$\zeta = \varsigma d_{11} d_{31}$$

$$\eta = (-d_{21} - h_{221} m_2^r - h_{321} m_3^r) d_{22}$$

$$\theta = (d_{31} + h_{231} m_2^r + h_{331} m_3^r) d_{33}$$

$$\iota = (2d_{22} + h_{222} m_2^r + h_{322} m_3^r) d_{22}$$

$$\kappa = (-d_{32} - h_{232} m_2^r - h_{332} m_3^r - d_{23}) d_{33}$$

$$\mu = (2d_{11} + h_{111} m_1^r)$$

$$\nu = (-d_{12} - h_{112} m_1^r)$$

$$\xi = (2d_{22} + h_{222} m_2^r + h_{322} m_3^r) d_{32}$$

$$\pi = (-d_{21} - h_{221} m_2^r - h_{321} m_3^r) d_{32}$$

$$\varpi = (2d_{33} + h_{233} m_2^r + h_{333} m_3^r) d_{33}$$

$$\rho = (-d_{23} - h_{223} m_2^r - h_{323} m_3^r - d_{32}) d_{22}$$

$$\sigma = (-d_{23} - h_{223} m_2^r - h_{323} m_3^r - d_{32}) d_{32}$$

$$\varsigma = (d_{13} + h_{113} m_1^r)$$