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Abstract

We study a dynamic regulation model where firms' actions contribute to a stock externality. The regulator and firms have asymmetric information about serially correlated abatement costs. With price-based policies such as taxes, or if firms trade quotas efficiently, the regulator learns about the evolution of both stock and costs. This ability to learn about costs is important in determining the ranking of taxes and quotas, and in determining the value of a feedback rather than an open-loop policy.

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1 Introduction

The possibility of global warming has revived interest in comparing taxes and quotas when the regulator and firms have asymmetric information about abatement costs. Weitzman (1974) showed that there is a simple criterion for ranking the policies when abatement costs and environmental damages are quadratic functions of the *flow* of pollution, uncertainty enters additively (i.e., it affects the level but not the slope of the firm's marginal costs), and the optimal quantity restriction is binding with probability one.[1] When the externality is caused by a *stock* rather than a flow the regulatory problem is dynamic, and the comparison of policies is more complicated. When stocks decay slowly, as do greenhouse gasses, current emissions may cause future environmental damages. Regulatory policies should balance current abatement costs and the stream of future environmental damages. In formulating these policies, the regulator should also consider the possibility of learning about the firms' abatement costs, thereby reducing the future asymmetry of information.

We show how the regulator's ability to update information about serially correlated abatement costs affects both the problem of regulating a stock pollutant, and the comparison of taxes and quotas. In a dynamic model with serially correlated private information about abatement costs, past observations can provide information about current costs.[2] In order to take advantage of this information, the regulator needs to use a feed-back policy.

Staring (1995) considers the simplest dynamic model where the regulator uses an open-loop policy, i.e. he announces the entire policy trajectory at the initial time. Weitzman's basic result, and the intuition for it, still holds: a steeper marginal damage function or a flatter marginal abatement cost function favor the use of quotas. A lower discount rate or a lower stock-decay rate – both features that increase the importance of future damages resulting from current pol-

[1] There is a large literature that examines other aspects of the problem of choosing policies under asymmetric information. Important contributions to this literature include Dasgupta, Hammond, and Maskin (1980),Kwerel (1977),Malcomson (1978), Roberts and Spence (1976), Stavins (1996) and Watson and Ridker (1984). These papers are concerned with the problem of flow rather than stock externalities.

Non-linear policies can achieve higher payoffs than linear taxes or quotas. Roberts and Spence (1976) points out that a non-linear tax, with the marginal tax rate equals the marginal damage, achieves the first-best outcome in regulating a single firm and is superior to a quantity policy.

[2] In addition to learning about abatement costs, the regulator may learn about the relation between stocks and environmental damages. This second kind of learning leads to a different problem, addressed in Karp and Zhang (2002b)

lution – favor the use of quotas.

Hoel and Karp (2002) show that both the ability to change the policy frequently and the use of a feedback rather than an open-loop policy favors the use of taxes.[3] They assume that cost are uncorrelated, so past observations provide no information about the current cost shock. The effect of all parameters on the policy ranking is qualitatively the same in the open-loop and feedback settings. Newell and Pizer (in press) extend the open-loop model by allowing costs to be serially correlated. They show that a more positive degree of autocorrelation favors the use of quotas. They study only the open-loop case, where the regulator learns nothing about either the evolution of stocks or abatement costs.

We consider the stock-regulation problem with correlated costs when the regulator uses a feedback policy. Correlated costs increase the value of feedback rather than the open-loop decision rules, because the regulator has the opportunity to learn about both the evolution of stocks and costs. Not surprisingly, most of the intuition developed in the earlier papers survives in this more general setting. With a feedback policy, the distinction between tradeable and non-tradeable quotas is important, because of the two types of policies provide the regulator with different amounts of information. This paper thus extends our intuition for the stock regulation problem and confirms that previous results hold in a more general setting.

Although these theoretical insights are valuable, the implications for empirical work are probably more important. For some pollution problems (such as global warming) where we would like to compare taxes and quotas, we have only rough estimates (or guesses) of the slopes of marginal damages and abatement costs, and estimates of other parameters such as decay and discount rates and cost correlations. An empirical challenge is to use the existing data to rank policies. This challenge cannot be met merely by knowing the *qualitative* characteristics of a problem. For example, Hoel and Karp (2002) show that despite the lack of a qualitative difference, there is a large quantitative difference in the criterion for ranking taxes and quotas, depending on whether the regulator uses open-loop or feedback policies; however, for all plausible parameter estimates, taxes dominate quotas for the control of greenhouse gasses under either open-loop or feedback policies. Newell and Pizer (in press) reach a similar conclusion with respect to changes in the cost correlation parameter, given that the regulator uses an open-loop policy. The formulae that we derive enable us to rank taxes and quotas in a more realistic

[3]The effect of the length of a period is the same in the more general setting discussed in this paper (where costs are serially correlated). Details are available upon request.

and general setting (e.g., with or without cost correlation, under open-loop or feedback policies, with or without trade in quotas).

The literature to which this paper contributes compares efficient quotas and efficient taxes. To be consistent with this literature we assume that when there is no trade in quotas, firms have the same marginal abatement costs. In this case, there would be no efficiency gain from trade. When firms have different cost shocks, we assume that there is trade in quotas, so the potential efficiency gain is realized. That is, in both of these cases the quota is efficient. The difference between the two cases is that with trade the equilibrium quota price contains the same information about the aggregate cost shock as does the equilibrium response to a tax; without trade, the regulator learns nothing about the cost shock when he uses a quota. Thus, we identify the role of trade in providing information (via the quota price). One way to interpret this model is that trade in quotas is always permitted. In equilibrium trade actually occurs if and only if firms are heterogenous.

A third possibility, that we do not study (but mention in footnote 9), is that firms are heterogenous and there is no trade under quotas. In this case the quota is inefficient. This model would illustrate the combined effects of the greater information and the greater allocative efficiency provided by taxes or by quotas with trade.

Section 2 describes the model and Section 3 explains the intuition for our results. Section 4 generalizes Newell and Pizer's policy ranking under the open-loop assumption and shows that more positively correlated cost shocks always favors the use of quotas. Section 5 shows that under the feedback policy without quota trading, taxes tend to dominate quotas when the cost shocks are highly positively or negatively correlated. Section 6 shows that efficient quota trading eliminates the informational advantage of taxes; as in the open-loop setting, higher autocorrelation of cost shocks then favors the use of quotas. Subsequent sections assess the likelihood that the policy ranking is different in the three scenarios (open-loop, feedback with and without quota trading), and provide an empirical illustration.

2 The model

We begin the analysis with the assumption that all firms are identical, so there is no incentive for firms to trade quotas. Firms behave non-strategically towards the regulator. We fix units of time equal to years and assume that a period lasts for one year. All time-dependent variables are constant within a period. At time t the stock of pollutant is S_t and the flow of new pollution is x_t. The fraction $1-\Delta$ of the stock decays within a period:

$$S_{t+1} = \Delta S_t + x_t. \tag{1}$$

In period t the flow of environmental damages is $D(S_t)$:

$$D\left(S_t\right) = cS_t + \frac{g}{2}S_t^2, \qquad g > 0.$$

The representative firm's business-as-usual (BAU) level of emissions in period t is $x_t^b = \overline{x} + \tilde{\theta}_t$ where $\tilde{\theta}_t$ is a random variable. With an actual emission level $x_t < x_t^b$, the firm's abatement cost is a quadratic function of abatement $A\left(x_t\right) = \frac{b}{2}\left(x_t^b - x_t\right)^2$ with $b > 0$. The firm's benefit from higher emission equals the abatement costs that it avoids having to pay. Defining the cost shock $\theta_t \equiv b\tilde{\theta}_t$, we write the benefit as a linear-quadratic function, concave in the emission with an additive cost shock. The benefit function for a representative firm is defined as the flow of cost saving due to more pollution (less abatement):[4]

$$B\left(x_t, \theta_t\right) = f + \left(a + \theta_t\right)x_t - \frac{b}{2}x_t^2, \qquad b > 0. \tag{2}$$

At time t only the firm observes θ_t ; there is persistent asymmetric information. The regulator knows the parameters of the $AR\left(1\right)$ process that determines the evolution of θ:

$$\theta_t = \rho\theta_{t-1} + \mu_t; \qquad \mu_t \sim iid\left(0, \gamma^2\right) \tag{3}$$

for $t \geq 1$. The regulator's subjective distribution for θ_0 is

$$\theta_0 \sim \left(\bar{\theta}_0, \sigma_0^2\right).$$

The random variable θ_0 has (subjective) mean $\bar{\theta}_0$ and variance σ_0^2. θ_0 and μ_t are independent and the correlation coefficient ρ satisfies $-1 < \rho < 1$.

If the regulator uses quotas, we assume that these are always binding.[5] Depending on the choice of parameter values and the initial value of S, the probability that the quota is binding

[4] The parameters satisfy $f = -\frac{b}{2}\overline{x}^2$ and $a = b\overline{x}$. We ignore the effect of $\tilde{\theta}$ on f since f has no effect on the regulator's control.

[5] Costello and Karp (2002) study a dynamic model with flow pollution, in which the possibility that the quota is not binding enables the regulator to learn about the firm's cost. Brozovic, Sunding, and Zilberman (2002) point out that even in the simplest static problem, the regulator's payoff might not be globally concave. In this case, a binding quota might be a local but not a global maximum.

for an arbitrarily large but finite number of periods can be made arbitrarily close to one. Since the regulator discounts the future, this fact means that the loss from ignoring the possibility that the quota is slack is very small. Therefore, we view the assumption that the quota is always binding as an approximation.

If the regulator uses a tax p_t, firms in each period maximize the difference between the savings in abatement cost and the tax payment:

$$\underset{x_t}{\text{Max}} \quad \Pi_t = B(x_t, \theta_t) - p_t x_t \;=\; \left[f + (a + \theta_t) x_t - \frac{b}{2} x_t^2 \right] - p_t x_t.$$

The firms' first order condition implies[6]

$$x_t^* = \frac{a - p_t}{b} + \frac{\theta_t}{b}. \tag{4}$$

When the regulator uses a tax, the flow of emissions and the evolution of the pollutant stock S_t are stochastic. At time t the regulator knows the actual level of emissions and the tax at time $t-1$, and (using equation (4)) he is able to infer the value of θ_{t-1}. The tax-setting regulator does better using a feedback rather than an open-loop policy because he learns about the cost variable and the pollution stock, and he conditions his policy on this information.

Define z_t^i as the regulator's expected emission given the tax p_t, for $i = OL$ (open loop) or $i = FB$ (feedback). Under the open-loop tax policy,

$$z_t^{OL} \;=\; E_0 x_t^* \;=\; \frac{a - p_t}{b} + \frac{1}{b} E_0 \theta_t \;=\; \frac{a - p_t}{b} + \frac{1}{b} \rho^t \bar{\theta}_0.$$

Under the feedback tax policy,

$$z_t^{FB} \;=\; E_t x_t^* \;=\; \frac{a - p_t}{b} + \frac{1}{b} E_t \theta_t.$$

The regulator's expectation of the cost variable (under the feedback policy) is $E_t\theta_t = \bar{\theta}_0$ when $t = 0$ and $E_t\theta_t = \rho\theta_{t-1}$ when $t \geq 1$. Choosing a tax p_t is equivalent to choosing expected emissions z_t^i. The firm's actual level of emissions is

$$x_t^*(z_t,\ \theta_t) = \begin{cases} z_t^{OL} + \frac{1}{b}\left(\theta_t - \rho^t \bar{\theta}_0\right) & \text{(open-loop)} \\ z_t^{FB} + \frac{1}{b}\left(\theta_t - E_t\theta_t\right) & \text{(feedback).} \end{cases} \tag{5}$$

Hereafter we model the tax-setting regulator as choosing z_t^i; we drop the superscript i ($i = OL$ or $i = FB$) where the meaning is clear.

[6]Non-strategic firms solve a succession of static optimization problems. If firms made investment decisions which affect their abatement costs, as in Karp and Zhang (2002a), firms would solve dynamic problems. This problem requires the solution of a dynamic game.

3 The intuition for policy ranking

The policy ranking obtained in subsequent sections depends on four considerations. The first two, which we refer to as the flexibility effect and the stochasticity effect, form the basis for policy ranking in Weitzman (1974)'s static model, and in all of dynamic models previously cited. Under taxes, emissions and marginal abatement costs are positively correlated. This flexibility increases expected cost saving, favoring taxes. However, taxes result in a stochastic stock of pollution. Stochastic stocks increase expected damages because damages are convex in stocks. The stochasticity effect favors quotas.

The third and fourth considerations are related to the autocorrelation parameter ρ. The change in stocks approximately equals the sum of flows. (A positive decay rate means that the two are not exactly the same.) Other things equal, the variation in the stock is smaller when the flows are negatively autocorrelated – as occurs under taxes when costs are negatively correlated. Thus, negative autocorrelation of costs reduces the characteristic (stochasticity of stocks) that tends to make taxes unattractive. Similarly, positive autocorrelation of costs increases the characteristic that tends to make taxes unattractive. This relation – "the stock correlation effect" – explains why the preference for quotas is monotonically increasing in ρ under an open-loop policy.

The stock correlation effect exists but is less important under the feedback tax policy, because the regulator is able to adjust the policy in every period to accommodate the previous shock. In choosing the current tax he need only consider next-period stock variability.

The fourth consideration is the learning effect. A higher *absolute value* of ρ means that knowledge of the previous cost provides more information about the current cost. By observing the level of lagged emissions and taxes, the tax-setting regulator learns the previous cost variable under a feedback policy. Under the feedback quota the regulator learns the previous cost variable only if quotas are traded.

When quotas are traded, the learning effect is the same under taxes and quotas. This symmetry means that the only difference (related to ρ) between taxes and quotas is the stock correlation effect. Consequently, a higher value of ρ favors (tradable) quotas under feedback policies.

When quotas are not traded, the learning effect and the stock correlation reinforce each other for $\rho < 0$, so a smaller value of ρ favors taxes; when $\rho > 0$ the two effects tend to counteract each other. The interplay of the two effects explains why the ranking of feedback taxes and non-tradable feedback quotas may be non-monotonic in ρ.

4 Open-loop policy

With an open-loop policy, the regulator chooses an infinite sequence of policy levels, $\{x_t\}_{t=0}^{\infty}$ under quotas and $\{z_t\}_{t=0}^{\infty}$ under taxes, based on the information he has at time 0. This decision depends on the variance and covariance of the costs shocks, conditional on the information at time 0.

Conditional on information at time 0, $\lim_{t\to\infty} var\left(\theta_t\right) = \frac{\gamma^2}{1-\rho^2}$. If the initial conditional variance, σ_0^2, also equals $\frac{\gamma^2}{1-\rho^2}$ (as in Newell and Pizer (in press)) the regulator's open-loop problem is stationary. For any other value of σ_0^2, the problem is non-stationary. We consider the general case (an arbitrary value of σ_0^2) in order to be able to compare the policy ranking in the open-loop and feedback settings. In the feedback setting the regulator would have the prior $\sigma_0^2 = \frac{\gamma^2}{1-\rho^2}$ after the first period only if he begins with these beliefs and uses a non-tradable quota. This prior is therefore not useful for comparing the open-loop and feedback settings.[7]

The regulator wants to maximize the expectation of the present value of the difference between abatement cost saving and pollution damages

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ B\left(x_t, \theta_t\right) - D\left(S_t\right) \right\}.$$

β is a constant discount factor. The expectation is taken over the sequence of random variables $\{\theta_t\}_{t=0}^{\infty}$ with respect to the information available at $t = 0$. Define $T^{OL}(S_0)$ as the maximized expected value of the regulator's open-loop (OL) program when he uses taxes and the initial stock of pollutant is S_0. Define $Q(S_0)$ as the maximized expected value of the regulator's program when he uses a quota. (Since – as we explain in Section 5 – this value is the same under open-loop and feedback quotas without trading, we do not use a superscript on the function $Q(S)$.)

The expectation of the trajectories of the flow and the stock of pollution are the same for every scenario that we consider: open-loop and feedback, taxes and quotas, with and without quota trading. This fact is a consequence of the Principle of Certainty Equivalence of the linear-quadratic model with additive uncertainty.[8] Consequently, the policy ranking depends on the second moment of the random variable.

[7] An earlier version of this paper explains a second reason for considering general priors. The variance of costs changes in a predictable manner, raising the possibility that the choice of policy instruments (as distinct from the choice of policy levels) might be time-inconsistent. Our earlier paper shows that this type of time-inconsistency does not occur.

[8] This Principle states that in the linear-quadratic control problem with additive random variables, the optimal

The payoff difference under taxes and quotas, given the open-loop policy, is

$$\Psi_0^{OL} \equiv T^{OL}(S_0) - Q(S_0) = \frac{1}{2b(1-\rho^2\beta)}\left(\sigma_0^2 + \frac{\gamma^2\beta}{1-\beta}\right)\left\{1 - \frac{g}{b}\frac{\beta}{1-\beta\Delta^2}\frac{1+\rho\beta\Delta}{1-\rho\beta\Delta}\right\}. \tag{6}$$

(Details of this and other derivations, discussions of tangential issues, and some technical proofs, are contained in an appendix that is available upon request.) Since the term outside the curly brackets is always positive, the policy ranking is independent of the regulator's priors $(\bar{\theta}_0,\ \sigma_0^2)$; that is, Newell and Pizer (in press)'s criterion for ranking policies is correct for all priors. We restate this result as:

Remark 1 *The preference for quotas under the open-loop policy is monotonically increasing in g, β, Δ, ρ, and monotonically decreasing in b.*

Since the possibility of learning about cost shocks depends on the value of ρ, we emphasize the role of this parameter.

Proposition 1 *Under an open-loop policy, the preference for quotas is monotonically increasing in ρ. Taxes dominates quotas iff*

$$\rho \leq \frac{1-\beta\Delta^2-\beta\frac{g}{b}}{\beta\Delta\left(1-\beta\Delta^2+\beta\frac{g}{b}\right)}.$$

5 Feedback policy without quota trading

The assumption that the optimal quota is always binding means that the quota-setting regulator learns nothing about the previous cost shock and also means that the evolution of the stock of pollution is nonstochastic. Since no new information becomes available, the open-loop and feedback quota policies (and payoffs) are identical when quotas are not traded. When the regulator uses taxes, the evolution of S_t is stochastic. By observing the firms' response to the tax, the regulator learns the value of the random cost. With taxes the regulator obtains

control rule does not depend on second moments of the random variable. Hoel and Karp (2001) examine the case where the slope rather than the intercept of firms' marginal abatement cost is uncertain. In that case uncertainty is multiplicative, and the Principle of Certainty Equivalance does not hold.

information over time, so a feedback tax policy results in a higher payoff than does an open-loop policy.

We compare the payoffs under the two policies by comparing their respective value functions. With correlated costs, there are two state variables, the stock of pollution S_t and the current expected value of the cost variable $y_t \equiv E_t\theta_t$. When necessary to avoid confusion, we use superscripts to distinguish state variables under tax or quota policies, e.g. y_t^{tax} and y_t^{quota}.

The value functions under both taxes and quotas are quadratic in the state, i.e. they both have the form

$$V_{0,\,t}^i + \underbrace{(v_1\ v_2)}_{V_1}\begin{pmatrix} S_t \\ y_t \end{pmatrix} + \frac{1}{2}(S_t\ y_t)\underbrace{\begin{pmatrix} V_{11} & V_{12} \\ V_{12} & V_{22} \end{pmatrix}}_{V_2}\begin{pmatrix} S_t \\ y_t \end{pmatrix}. \tag{7}$$

The first term of the value function $V_{0,t}^i$, $i = tax, quota$, depends on t because $\text{Var}_t(\theta_t)$ changes exogenously in the first period.

The appendix contains explicit expressions for the parameters of the value functions. The matrices V_1 and V_2 are the same under taxes and quotas. *For a given state*, the optimal control, z_t^* under taxes and x_t^* under quotas are equal and are given by

$$\frac{a(1-\beta\Delta)-\beta c}{(b-\beta V_{11})-b\beta\Delta} + \frac{1-\rho\beta\Delta}{b(1-\rho\beta\Delta)-\beta V_{11}}y_t + \frac{\beta\Delta V_{11}}{b-\beta V_{11}}S_t, \tag{8}$$

with

$$V_{11} \;=\; \frac{-\left(b\beta\Delta^2+\beta g-b\right)-\sqrt{\left(b\beta\Delta^2+\beta g-b\right)^2+4\beta gb}}{2\beta} \;<\; 0. \tag{9}$$

The function V_{11} is independent of ρ; the correlation parameter affects only the slope of the control rule with respect to the state y_t. Equation (8) implies that an increase in current expected costs, y_t, increases the current (expected) flow of pollution, as in the static model.

The values of z_t^* and x_t^* differ over time because the actual trajectories of the state vector differ under taxes and quotas. However, as we remarked above, the first moments of pollution flows and stocks are the same in all of our scenarios, because of the Principle of Certainty Equivalence. Taking expectations at time 0, we have $E_0 y_t^{tax} = E_0(\rho\,\theta_{t-1}) = \rho^t\bar{\theta}_0 = y_t^{quota}$. This relation and equation (8) imply

$$E_0 z_t^* = x_t^*. \tag{10}$$

The expected values of the programs are different because of the second moments of the cost shocks. It makes sense to compare the values of these programs only under the same information set, e.g. at time $t = 0$. The same comparison holds at an arbitrary time, provided that the regulator evaluates the two policy instruments using the same information set.

In the initial period, $y_0^{tax} = y_0^{quota} = \bar{\theta}_0$. The payoff difference under taxes and quotas, given the feedback policy, is due to the difference in the term $V_{0,t}^i$ (evaluated at $t = 0$) in equation (7) :

$$\Psi_0^{FB} \equiv T^{FB}\left(S_0, \bar{\theta}_0\right) - Q\left(S_0, \bar{\theta}_0\right) = \frac{1}{2b(1-\rho^2\beta)}\left(\sigma_0^2 + \frac{\gamma^2\beta}{1-\beta}\right)\frac{\left(1-\frac{\beta}{b}V_{11}\right)\left\{(1-\rho\beta\Delta)^2 - \frac{\beta^2}{b^2}V_{11}^2\left(1-\rho^2\beta\right)\right\}}{\left(1-\rho\beta\Delta-\frac{\beta}{b}V_{11}\right)^2}. \tag{11}$$

The ranking of feedback tax and quota policies depends only on the sign of the term in curly brackets. Hoel and Karp (2002) show that for $\rho = 0$ and $\sigma_0^2 = \gamma^2$, the parameters b, g, β and Δ have qualitatively the same effect on the policy ranking under both open-loop and feedback policies. The comparative statics of the ranking with respect to b, g, and Δ are unchanged when $\rho \neq 0$:

Remark 2 *The preference for quotas under the feedback policy is monotonically increasing in g, Δ, and monotonically decreasing in b.*

However, the effect of ρ and β is different in the open-loop and feedback setting. Equation (11) implies

Proposition 2 *Feedback taxes dominate non-tradable feedback quotas iff*

$$f(\rho) \equiv \rho^2\beta^2\left(\Delta^2 + \frac{\beta}{b^2}V_{11}^2\right) - 2\rho\beta\Delta + 1 - \frac{\beta^2}{b^2}V_{11}^2 \geq 0. \tag{12}$$

The function $f(\rho)$ is convex in ρ and for some parameter values is nonmonotonic in ρ over $\rho \in (-1,\ 1)$. For such parameter values, the preference for quotas is non-monotonic in ρ.

The function $f(\rho)$ reaches a minimum at $\rho \geq 0$; the minimum occurs at $\rho = 0$ if and only if $\Delta = 0$, i.e. for a flow externality. We define ρ_1 and ρ_2 as the smaller and the larger roots of $f(\rho) = 0$, provided that these roots are real. If both of these roots are in the interval $(-1, 1)$, then for low values of ρ an increase in ρ makes quotas more attractive, and the opposite holds for high values of ρ. The following are sufficient conditions for either taxes or quotas to dominate:

Corollary 1 *A sufficient condition for taxes to dominate quotas is*

$$\frac{1-\beta\Delta^2}{\beta} > \frac{g}{b}.$$

A sufficient condition for quotas to dominate taxes is

$$\rho_1 < -1 \ \textit{and} \ \rho_2 > 1.$$

Under an open-loop policy, a larger value of β favors the use of quotas, because a larger β increases the importance of the future stock variability arising from the current flow variability. Under feedback policies, the comparative statics of β is ambiguous. We have:

Proposition 3 *Under a feedback policy, a higher discount factor favors quotas (i.e., it decreases Ψ_0^{FB}) iff*

$$\frac{-\left(\Delta^2 + \frac{g}{b}\right)\frac{\beta}{b}V_{11} + \frac{g}{b}}{\sqrt{\left(\beta\Delta^2 + \beta\frac{g}{b} - 1\right)^2 + 4\beta\frac{g}{b}}} \geq \frac{\rho\left(\rho^2\beta\Delta - 2\Delta + \rho\right)}{2\left(1 - \rho^2\beta\right)^{\frac{3}{2}}}. \tag{13}$$

A sufficient condition for inequality (13) is $\beta \leq \frac{2\Delta - \rho}{\rho^2\Delta}$ and $\rho \geq 0$. These two inequalities are satisfied if $0 \leq \rho \leq \Delta$.

A higher discount factor favors the use of quotas if the gain from the informational advantage under taxes is not great enough to offset the higher expected damage from future stock variability. The sufficient condition $0 \leq \rho \leq \Delta$ means that equation (13) is very likely to hold for stock pollutants that decay slowly. For example, when a period is one year a half-life of 15 years corresponds to $\Delta = 0.9548$; greenhouse gasses have a half-life of over 80 years, for $\Delta = 0.99$ (see Section 8).

In the limiting case where $\Delta = 0$ the externality is a flow pollutant. Define $\tilde{g} = \beta g$, the present value of the slope of marginal damages and set $\sigma_0^2 = \gamma$, the variance of the innovation to costs. The difference between payoffs under taxes and quotas for a flow pollutant is

$$\Psi_0^{FB} = \frac{\sigma_0^2}{2b\left(1-\beta\right)}\left[\frac{\beta\rho^2}{\left(1-\beta\rho^2\right)\left(1+\frac{\tilde{g}}{b}\right)} + \left(1 - \frac{\tilde{g}}{b}\right)\right].$$

This expression shows how the learning that is made possible by a nonzero value of ρ favors the use of taxes for the case of a flow pollutant. When $\rho = 0$ we obtain Weitzman (1974)'s criterion.

6 Ranking with quota trading

Previous sections assume that all firms are identical, so firms have no incentive to trade emissions permits. When firms are heterogeneous, emissions trade increases efficiency and also reveals industry-wide costs with a one-period lag. The informational advantage of taxes disappears in this case.

Suppose there are n firms, where n is large. Let $x_{i,t}$ be firm i's emissions at time t. The benefit for firm i of emitting x_{it} is

$$B_i\left(x_{i,t},\,\theta_t,\epsilon_{i,t}\right)=\frac{f}{n}+\left(a+\theta_t+\epsilon_{i,t}\right)x_{i,t}-\frac{bn}{2}x_{i,t}^2, \tag{14}$$

where $\epsilon_{i,t}$ is the firm-specific deviation from the industry-wide cost θ_t. These firm-specific deviations are $i.i.d.$ over time with mean 0 and constant variance σ_ϵ^2. They are uncorrelated with each other and are independent of the industry-wide average θ_t, which follows the $AR(1)$ process defined in equation (3).

We use p_t (p_t^{tax} or p_t^{quota}) to denote either the tax level or the quota price from trading. The first order condition to firm i's problem gives its emission response as

$$x_{i,t}^*=\frac{a-p_t}{bn}+\frac{\theta_t+\epsilon_{i,t}}{bn}. \tag{15}$$

Summing over $x_{i,t}^*$ gives the aggregate industry level emission

$$x_t=\sum_{i=1}^{n}x_{i,t}^*=\frac{1}{b}\left((a-p_t)+\theta_t+\frac{\sum\epsilon_{it}}{n}\right). \tag{16}$$

The last term in equation (16) is $iid\left(0,\frac{\sigma_\epsilon^2}{n}\right)$; since n is large, we replace $\frac{\sigma_\epsilon^2}{n}$ with 0. Thus, once the regulator knows p_t and x_t, he knows θ_t. Under quotas the regulator chooses x_t, and p_t is endogenous; under taxes the regulator chooses p_t, and x_t is endogenous.

Quotas. Under quotas, in each period the aggregate emissions, and consequently the pollutant stock, are deterministic. Equation (16) implies the equilibrium quota price $p_t^{quota}=a+\theta_t-bx_t$. By observing p_t^{quota}, the regulator learns θ_t. Substituting this price into equation (15) gives firm i's emission level as $\frac{x_t}{n}+\frac{\epsilon_{it}}{bn}$. Substituting this expression into equation (14), summing over i, and taking expectations gives the aggregate expected cost saving under quotas[9]:

$$E_t \sum_{i=1}^{n} B_i \left(x_{i,t}^{quota}, \theta_t, \epsilon_{i,t}\right) = f + (a + E_t\theta_t)\, x_t - \frac{b}{2}x_t^2 + \frac{1}{2b}\sigma_\epsilon^2. \tag{17}$$

Taxes. A tax policy results in a stochastic level of aggregate emissions. Choosing a tax is equivalent to choosing the expected aggregate emissions $z_t \equiv E_t x_t = \frac{1}{b}\left(a - p_t^{tax}\right) + \frac{1}{b}E_t\theta_t$. The actual level of aggregate emissions is $x_t = z_t + \frac{1}{b}\left(\theta_t - E_t\theta_t\right)$. By observing x_t the regulator learns θ_t. Firm i's emission level is $\frac{1}{n}z_t + \frac{1}{bn}\left(\theta_t - E_t\theta_t\right) + \frac{1}{bn}\epsilon_{i,t}$. Substituting this expression into equation (14), summing over i, and taking expectations gives the aggregate expected cost saving under taxes:

$$E_t \sum_{i=1}^{n} B_i \left(x_{i,t}^{tax}, \theta_t, \epsilon_{i,t}\right) = f + (a + E_t\theta_t)\, z_t - \frac{b}{2}z_t^2 + \frac{1}{2b}\sigma_\epsilon^2 + \frac{1}{2b}Var_t\left(\theta_t\right). \tag{18}$$

Taxes vs. Quotas. The firms' individual deviations have the same effect on the aggregate expected cost saving under taxes and quotas. These firm-specific deviations do not affect the policy ranking.

As before, the regulator maximizes the expectation of the difference between firms' aggregate cost saving from polluting and environmental damage:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \sum_{i=1}^{n} B_i \left(x_{i,t}, \theta_t, \epsilon_{i,t}\right) - D\left(S_t\right) \right\}.$$

Under quotas, the control variable is x_t and the expected aggregate benefit is (17). Under taxes, the control variable is z_t and the expected aggregate benefit is (18). The expectation is taken over sequences of random variables $\{\theta_t\}_{t=0}^{\infty}$ and $\{\epsilon_{i,t}\}_{t=0}^{\infty}$ with respect to the information available at the current time, $t = 0$.

With the open-loop policy, the regulator decides his future policy trajectory at the initial period $t = 0$ and commits to it. There is no learning. All of the conclusions in Section 4 also hold when quotas are traded. The payoff difference under taxes and quotas, $\Psi_0^{OL+Trade}$, is the same as in equation (6).

With the feedback policy, the regulator adjusts the instrument level as he learns about cost shocks. Under both taxes and quotas, the regulator's posterior is

$$\theta_t \sim \left(\rho\theta_{t-1}, \gamma^2\right), \quad \forall t \geq 1.$$

[9]If firms are heterogeneous and each firm receives the same allocation of quotas, but there is no trade in quotas, the expected single period benefit of emissions is given by $f + (a + E_0\theta_t)\, x_t - \frac{b}{2}x_t^2$. The difference between this expression with the right side of equation (17) identifies the informational advantage of trade (the fact that with trade we have $E_t\theta_t$ rather than $E_0\theta_t$) and the allocative efficiency (the presence of the term $\frac{1}{2b}\sigma_\epsilon^2$).

The corresponding law of motion for $y_t \equiv E_t\theta_t$ under both quotas and taxes is $y_{t+1} = \rho y_t + \rho\mu_t$. (Without quota trading, $y_{t+1} = \rho y_t$; see the appendix.) This difference identifies the informational advantage of tradeable quotas (relative to non-traded quotas).

The optimal control, z_t under taxes and x_t under quotas, obeys the linear control rule (8). The regulator has the same amount of information about costs under taxes and quotas, although in general the realization of S is different under the two policies. Consequently, the optimal controls are identical, for a given value of S:

$$z_t^* = x_t^*. \tag{19}$$

Note the qualitative difference between equations (10) ($E_0 z_t^* = x_t^*$) and (19) ($z_t^* = x_t^*$). By observing the price of emissions permits, the regulator has the same information about costs under taxes and quotas in every period, not just at time $t = 0$.

Although – as previously noted – the first moment of stocks and flows are the same under taxes and quotas, their second moments differ, leading to different expected payoffs:

$$\begin{array}{c} \Psi_0^{FB+Trade} = J_0^T\left(S_0, \bar{\theta}_0\right) - J_0^Q\left(S_0, \bar{\theta}_0\right) = \\ \frac{1}{2b}\left(\sigma_0^2 + \frac{\gamma^2\beta}{1-\beta}\right)\frac{\left(1-\frac{\beta}{b}V_{11}\right)\left(1-\rho\beta\Delta+\frac{\beta}{b}V_{11}\right)}{1-\rho\beta\Delta-\frac{\beta}{b}V_{11}}. \end{array} \tag{20}$$

$V_{11} < 0$ is given in equation (9).

The payoff under taxes is higher than under quotas if and only if

$$1 - \rho\beta\Delta + \frac{\beta}{b}V_{11} \geq 0.$$

This inequality implies

Remark 3 *With efficient quota trading, changes in parameters b, g, β, Δ, and ρ have qualitatively the same effect on the policy ranking under open-loop and feedback policies.*

The qualitative differences in policy ranking between open-loop and feedback policies depend only on the informational advantage of taxes under feedback policies. Emissions trading eliminates this informational advantage. Again, we emphasize the effect of cost autocorrelation:

Proposition 4 *With quota trading, more positively autocorrelated cost shocks (higher ρ) favors the use of quotas under both open-loop and feedback strategies. Feedback taxes dominate tradeable feedback quotas iff*

$$\rho \leq \frac{1}{\beta\Delta}\left(1 + \frac{\beta}{b}V_{11}\right).$$

7 Sensitivity of the ranking

This section identifies the region of parameter space where the open-loop and feedback assumptions lead to different policy rankings We define the critical ratio of $\frac{g}{b}$ as the value of the ratio that makes the regulator indifferent between taxes and quotas. The critical ratio is obtained by setting the differences in payoffs equal to 0 and solving for $\frac{g}{b}$. The preference for quotas is monotonically increasing in $\frac{g}{b}$ under both open-loop and feedback policies, so the quota is the right instrument if and only if the actual value of the ratio of slopes exceeds the critical value.

The critical values in the open-loop and feedback cases are respectively

$$\left(\frac{g}{b}\right)^{*OL} = \left(\frac{g}{b}\right)^{*OL+Trade} = \left(\frac{1-\rho\beta\Delta}{1+\rho\beta\Delta}\right)\left(\frac{1}{\beta}-\Delta^2\right),$$

$$\left(\frac{g}{b}\right)^{*FB} = \frac{(1-\rho\beta\Delta)\left\{(1-\rho\beta\Delta)+(1-\beta\Delta^2)\sqrt{1-\rho^2\beta}\right\}}{\beta\left\{(1-\rho^2\beta)+(1-\rho\beta\Delta)\sqrt{1-\rho^2\beta}\right\}},$$

$$\left(\frac{g}{b}\right)^{*FB+Trade} = (1-\rho\beta\Delta)\left(\frac{1}{\beta}-\frac{\Delta^2}{2-\rho\beta\Delta}\right).$$

In the static model, the critical ratio of $\frac{g}{b}$ is 1. When both $\rho = 0$ and $\Delta = 0$, the critical ratio under both open-loop and feedback policies is β^{-1} rather than 1, since (by assumption) the current flow of pollution causes damages in the next period. The following Propositions describe the relation between the critical ratio of $\frac{g}{b}$ and ρ.

Proposition 5 *The critical ratio of $\frac{g}{b}$ is monotonically decreasing in ρ under both an open-loop policy and a feedback policy with tradable quotas. Under a feedback policy without tradable quotas, the critical ratio of $\frac{g}{b}$ is nonmonotonic in ρ:*

$$\frac{\partial\left(\frac{g}{b}\right)^{*FB}}{\partial\rho}\begin{cases} <0, & \text{if } \rho < \Delta \\ =0, & \text{if } \rho = \Delta \\ >0, & \text{if } \rho > \Delta. \end{cases}$$

Proposition 6 *Without efficient quota trading, the critical ratio of $\frac{g}{b}$ under feedback policies is always greater than or equal to the open-loop level:*

$$\left(\frac{g}{b}\right)^{*FB} \geq \left(\frac{g}{b}\right)^{*OL}, \qquad \forall\, \rho. \tag{21}$$

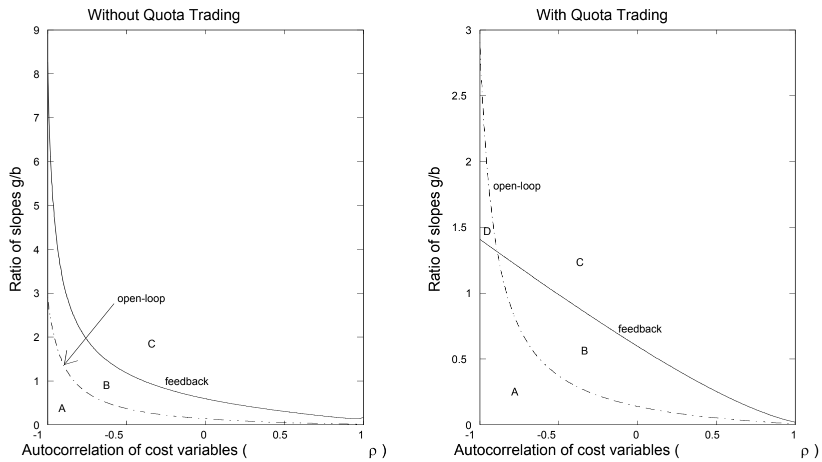

Figure 1:

With efficient quota trading, the critical ratio of $\frac{g}{b}$ under feedback policies is greater than the open-loop level only for a subset of parameters:

$$\left(\frac{g}{b}\right)^{*FB+Trade} \geq \left(\frac{g}{b}\right)^{*OL+Trade}, \qquad \textit{iff} \;\; \rho \geq \tilde{\rho}; \tag{22}$$

where $\tilde{\rho}$ is a function (of only β and Δ) that satisfies $-1 \leq \tilde{\rho} < 0$.

The equality in (21) holds only if $\rho = \Delta = 0$, i.e. with a flow externality and uncorrelated cost shocks. The function $\tilde{\rho}\,(\beta, \Delta)$ used in equation (22) equals -1 only if $\beta = \Delta = 1$, i.e. where both discount rate and stock decay rate are zero. The proposition implies

Corollary 2 *Taxes will necessarily be the right instrument choice in the feedback setting if taxes dominate quotas in the open-loop setting, provided that*

- *there is no quota trading; or*
- *there is quota trading, and cost shocks are non-negatively autocorrelated.*

With negatively correlated shocks and quota trading, there exist parameter values such that it is optimal to choose taxes under the open-loop policy but quotas under the feedback policy – or vice versa.

Figure 1 plots the critical ratios of $\frac{g}{b}$ against ρ, holding $\beta = 0.9512$ and $\Delta = 0.9548$. Since a period is one year, these values imply a continuous discount rate of 0.05 and a half-life of 15 years. The left panel shows the case without quota trading, and the right panel shows the case with quota trading. In both panels, the solid curve graphs the critical ratio under the feedback policy and the dotted curve graphs the ratio under the open-loop policy. The tax is better than the quota if and only if the actual ratio of $\frac{g}{b}$ lies below the critical ratio. The left panel of Figure

optimal instrument choice					
without quota trading			with quota trading		
region	open-loop	feedback	region	open-loop	feedback
A	tax	tax	A	tax	tax
B	quota	tax	B	quota	tax
C	quota	quota	C	quota	quota
			D	tax	quota

Table 1: Feedback vs. open-loop optimal instrument choice.

1 illustrates equation (21) and the right panel illustrates equation (22). Table 1 summarizes the different ranking possibilities.

8 An Application to Global Warming

There is little agreement about the likely magnitude of abatement costs and environmental damages related to greenhouse gasses. Using a linear-quadratic formulation, we can construct a simple model that incorporates – in a transparent manner – a particular belief about these magnitudes. If we had a good knowledge of the physics and economics of global warming, it would be worth constructing complex models. At this stage we know little more than that there is a probable connection between greenhouse gasses and global warming, and that the economic consequences of this relation may be important. The extent of the uncertainty and disagreement about these magnitudes makes the simplicity of the model very important. It is easy to see how policy conclusions depend on beliefs about the unknown parameters. For example, we can determine whether a particular conclusion would change if we increase our estimate of environmental damages by a factor of 10 or 100. Of course, since these experiments maintain the assumption of the linear-quadratic structure, they tell us nothing about whether the policy conclusions are sensitive to functional form.

A number of papers use the linear-quadratic structure, together with existing estimates of costs and benefits of greenhouse gas abatement, to examine particular policy issues. We briefly review these papers. Under the assumption of zero autocorrelation of cost shocks, Hoel and Karp (2002) show that taxes dominate quotas even if environmental damages associated with greenhouse gasses are much more severe than is commonly believed. This conclusion holds under open-loop and feedback policies, with a period of commitment of anywhere from one to ten years. Newell and Pizer (in press) find that taxes dominate quotas under open loop policies with positive autocorrelation. Hoel and Karp (2001) compare feedback taxes and quotas with multiplicative (rather than additive) and serially uncorrelated cost shocks. The multiplicative structure means that the Principle of Certainty Equivalence does not hold, and thus raises issues that are not present in the previous papers. Subsequent papers examine more complicated (feedback) models, where the regulator learns about environmental damages (Karp and Zhang 2002b) or where there is endogenous investment that reduces abatement costs (Karp and Zhang 2002a). Those papers review previous integrated assessment models that have been used to study greenhouse gas abatement.

We use a time period of one year. Many economic studies (Kolstad 1996), (Nordhaus 1994) use an annual decay rate of 0.0083 (a half-life of 83 years) for atmospheric concentrations of the primary greenhouse gas, CO_2, implying $\Delta = 0.9917$. We set $\beta = 0.9704$ (a continuous discount rate of 0.03). Using these values, Table 2 shows the critical values of the ratio $\frac{g}{b}$ for five values of ρ, under open-loop and feedback policies with either homogenous or heterogenous firms.[10]

This table shows that for $\mid \rho \mid \approx 1$ the ranking depends primarily on whether taxes have an informational advantage over quotas – as they do only if firms are homogenous so that there is no trade in quotas. The critical ratios under open-loop and under feedback with heterogenous firms and tradable quotas are similar; these ratios are quite different than the critical ratio under feedback without tradable quotas. That is, when costs are highly (positively or negatively) autocorrelated, the ranking of policies is not particularly sensitive to the open-loop versus feedback distinction provided that quotas are traded; if quotas are not traded, this distinction is important for ranking policies. For moderate values of ρ (i.e. $\mid \rho \mid \leq .5$), the ranking depends

[10]Recall that in the absence of trade there is no allocative inefficiency because (by assumption) firms are homogenous; the quota is always efficient. The reader interested in comparing taxes and inefficient quotas can carry out the calculations using the single period benefit function described in footnote 9. This calculation requires an additional parameter that measures the extent of firm heterogeneity, given by σ_ϵ^2.

	open-loop	feedback & homogenous firms (no trade)	feedback & heterogenous firms (with trade)
$\rho = -0.99$	1.9427	8.2154	1.3619
$\rho = -0.5$	0.1343	1.1344	0.9393
$\rho = 0$	0.0470	0.5388	0.5388
$\rho = 0.5$	0.0165	0.2470	0.1987
$\rho = 0.99$	0.0011	0.0471	0.0043

Table 2: The critical ratio g/b

primarily on whether the regulator uses a feedback or an open-loop policy; allowing trade in quotas causes a relatively small change in the critical ratio. That is, when costs are not highly autocorrelated, the open-loop versus feedback distinction is important in ranking the policies; current information on cost shocks is not particularly important, so the possible informational advantage of taxes

has little effect on the ranking.

The survey in Hoel and Karp (2002) suggests a point estimate of $\frac{g}{b} = 1.406\,2 \times 10^{-5}$ (based on the estimate that doubling carbon stocks causes a 5% reduction in Gross World Product (GWP) and that a 50% reduction in emissions causes a 1% reduction in GWP).[11] The estimated value of $\frac{g}{b}$ is linearly related to the estimate of environmental damages. (A ten-fold increase in the estimated reduction in GWP associated with a doubling stocks leads to a ten-fold increase in the estimate of $\frac{g}{b}$.) This evidence suggests that in the case of greenhouse gasses, the serial correlation of cost shocks does not overturn the preference for taxes. This conclusion does not depend on whether the regulator uses feedback policies and learns about the cost shock.

9 Summary

This paper provides a criteria for ranking taxes and quotas for the control of a stock pollutant in a linear-quadratic model. We extended previous results by including serially correlated abate-

[11] We cannot assess the plausibility of these estimates. A reader who thinks that the damage estimate understates actual damages by, for example, a factor of 10, should magnify the point estimate of $\frac{g}{b}$ by a factor of 10. The lack of "reliable" estimates of costs and damages is, as we have emphasized, one of the main attractions of using a simple linear-quadratic model.

ment costs under either open-loop or feedback policies, with or without trading in emissions quotas.

Under a tax policy, or under a quota policy with efficient quota trading among polluting firms, the regulator learns about the industry's abatement cost schedule by observing the aggregate emission response. The feedback strategy, unlike the open-loop strategy, enables the regulator to use new information to adjust his policy level, leading to higher welfare.

With feedback policies, the ranking of taxes and non-traded quotas may be nonmonotonic with respect to both the autocorrelation parameter and the discount factor. In contrast, the effects of the other parameters (the relative slopes of marginal costs and damages, and the decay rate) on the ranking are monotonic and are qualitatively the same under the open-loop and feedback assumptions. A large absolute value of autocorrelation increases the potential for learning, thus increasing the advantage of feedback taxes relative to both the non-traded quota and to open-loop taxes. When quota trading occurs, the feedback tax loses its informational advantage over feedback quotas, and more autocorrelated cost shocks favor the use of quotas.

Without quota trading, taxes will certainly be the right instrument choice in the feedback setting if taxes dominate quotas in the open-loop setting. However, this conclusion does not hold if firms can trade quotas. With tradeable permits, endogenous learning occurs under quotas. In this case, a regulator who is required to use an open-loop policy might want to use one instrument, where a regulator who is able to use a feedback policy would use the other instrument. Using estimates of greenhouse gas-related damages and abatement costs, we provide evidence that taxes dominate quotas for the control of greenhouse gasses regardless of the opportunities for learning.

References

BROZOVIC, N., D. SUNDING, AND D. ZILBERMAN (2002): "Prices vs. Quantities Reconsidered," *Giannini Foundation Working paper*.

COSTELLO, C., AND L. KARP (2002): "Dynamic taxes and quotas with learning," Working Paper, Giannini Foundation of Agricultural Economics, University of California, Berkeley.

DASGUPTA, P., P. HAMMOND, AND E. MASKIN (1980): "On Imperfect Information and Optimal Pollution Control," *Review of Economic Studies*, 47(5), 857–60.

HOEL, M., AND L. KARP (2001): "Taxes and Quotas for a Stock Pollutant with Multiplicative Uncertainty," *Journal of Public Economics*, 82, 91–114.

HOEL, M., AND L. KARP (2002): "Taxes Versus Quotas for a Stock Pollutant," *Resource and Energy Economics*, 24, 367 – 384.

KARP, L., AND J. ZHANG (2002a): "Controlling a Stock Pollutant with Endogenous Abatement Capital and Asymmetric Information," Giannini Foundation of Agricultural Economics, Working Paper No. 928, University of California, Berkeley; http://are.Berkeley.EDU/ karp/.

——— (2002b): "Regulation with Anticipated Learning about Environmental Damage," Giannini Foundation of Agricultural Economics, Working Paper No. 926, University of California, Berkeley; http://are.Berkeley.EDU/ karp/.

KOLSTAD, C. D. (1996): "Learning and Stock Effects in Environmental Regulation: The Case of Greenhouse Gas Emissions," *Journal of Environmental Economics and Management*, 31(1), 1–18.

KWEREL, E. (1977): "To tell the truth: imperfect information and optimal pollution control," *Review of economic studies*, 44(3), 595–601.

MALCOMSON, J. (1978): "Prices vs. Quantities: A Critical Note on the Use of Approximations," *Review of Economic Studies*, 45(1), 203–07.

NEWELL, R., AND W. PIZER (in press): "Stock Externality Regulation Under Uncertainty," Journal of Environmental Economics and Management.

NORDHAUS, W. D. (1994): "Expert Opinion on Climate Change," *American Scientist*, pp. 45–51.

ROBERTS, M., AND A. SPENCE (1976): "Effluent charges and licenses under uncertainty," *Journal of Public Economics*, 5(3-4), 193–208.

STARING, K. (1995): "Regulering av forurensing under usikkerhet - en dynamisk modell - Regulation of pollution under uncertainty - a dynamic model," Master degree thesis, Department of Economics, University of Oslo.

STAVINS, R. (1996): "Correlated uncertainty and policy instrument choice," *Journal of Environmental Economics and Management*, 30(2), 218–232.

WATSON, W., AND R. RIDKER (1984): "Losses from effluent taxes and quotas under uncertainty," *Journal of Environmental Economics and Management*, 11(4), 310–26.

WEITZMAN, M. (1974): "Prices versus quantities," *Review of Economic Studies*, 41(4), 477–91.

A Appendix

A.1 General Solution for a Linear Quadratic Dynamic Programming Problem

We set up the dynamic programming equations in general matrix notation as:

$$
\begin{aligned}
J_t(X_t) = \underset{Y_t}{\text{Max}} \Big\{ & U_0 + \frac{1}{2}X_t'QX_t + Y_t'WX_t + M'X_t + U_1'Y_t - \frac{1}{2}Y_t'U_2Y_t \\
& + U_3 Var_t(\theta_t) + \beta E_t J_{t+1}(X_{t+1}) \} , \qquad (23) \\
\text{s.t.} \qquad & X_{t+1} = AX_t + BY_t + C\mu_t + D.
\end{aligned}
$$

The subscript t in J_t denotes the change in $Var_t(\theta_t)$: $Var_t(\theta_t) = \sigma_0^2$ when $t = 0$; and $Var_t(\theta_t) = \sigma_\mu^2$ when $t > 0$.

X_t is a $n \times 1$ vector of state variables; Y_t is a $m \times 1$ vector of control variables; μ_t is a white noise. Dimensions for those coefficient matrices are: Q is $n \times n$ symmetric; W is $m \times n$; M is $n \times 1$; U_0 is 1×1; U_1 is $m \times 1$; U_2 is $m \times m$; U_3 is 1×1; A is $n \times n$; B is $n \times m$; C is $n \times 1$; D is $n \times 1$.

Given the quadratic value function $J_t(X_t) = V_{0,t} + V_1'X_t + \frac{1}{2}X_t'V_2X_t$, the first order condition with respect to Y_t is

$$
WX_t + U_1 - U_2Y_t + \beta E_t \left[B'V_1 + B'V_2 (AX_t + BY_t + C\mu_t + D) \right] = 0.
$$

The optimal feedback control rule is

$$
Y_t^* = \left(U_2 - \beta B'V_2B\right)^{-1} \left[U_1 + \beta B'(V_1 + V_2D) + \left(W + \beta B'V_2A\right) X_t \right], \qquad (24)
$$

a linear function of state variables. Substituting Y_t^* back into the dynamic programming equation and equating coefficients gives the algebraic Riccati matrix equation for V_2:

$$
V_2 = Q + \beta A'V_2A + \left(W' + \beta A'V_2B\right)\left(U_2 - \beta B'V_2B\right)^{-1}\left(W + \beta B'V_2A\right). \qquad (25)
$$

V_2 is a $n \times n$ symmetric negative-semidefinite matrix. After obtaining V_2, we can solve for the $n \times 1$ coefficient matrix V_1:

$$
\begin{aligned}
V_1 = & \left[I - \beta A' - \beta (W' + \beta A'V_2B)(U_2 - \beta B'V_2B)^{-1} B' \right]^{-1} \\
& \left[M + \beta A'V_2D + (W' + \beta A'V_2B)(U_2 - \beta B'V_2B)^{-1}(U_1 + \beta B'V_2D) \right], \qquad (26)
\end{aligned}
$$

and the constant term $V_{0,t}$

$$
\begin{aligned}
V_{0,t} = & \eta_t \left(U_3 + \frac{\beta}{2}C'V_2C \right) + \frac{1}{1-\beta} \Big\{ U_0 + \beta V_1'D + \frac{\beta}{2}D'V_2D \\
& + \frac{1}{2}\left[U_1 + B'(V_1 + V_2D)\right]'(U_2 - \beta B'V_2B)^{-1}\left[U_1 + B'(V_1 + V_2D)\right] \Big\} \qquad (27)
\end{aligned}
$$

where η_t depends on the second moment of cost shocks: $\eta_t = \sigma_0^2 + \frac{\beta}{1-\beta}\sigma_\mu^2$ when $t = 0$, and $\eta_t = \frac{1}{1-\beta}\sigma_\mu^2$ when $t > 0$. The first moment of the cost shock affects both the value function and the optimal control, but the second moment affects only the constant term of the value function.

A.2 Feedback Policy

First we write the law of motion for $y_t \equiv E_t(\theta_t)$. For the feedback quota policy, no new information becomes available over time, and

$$y_{t+1} = E_{t+1}\theta_{t+1} = E_0\left(E_{t+1}\theta_{t+1}\right) = E_0\theta_{t+1} = \rho^{t+1}\bar{\theta}_0 = \rho y_t.$$

Under the feedback tax policy, the regulator infers θ_t by observing the firm's emissions, and

$$y_{t+1} = E_{t+1}\theta_{t+1} = E_{t+1}\left(\rho\theta_t + \mu_{t+1}\right) = \rho\theta_t = \begin{cases} \rho\theta_0 = \rho y_0 + \rho\mu_0, & t = 0 \\ \rho\left(\rho\theta_{t-1} + \mu_t\right) = \rho y_t + \rho\mu_t, & t \geq 1 \end{cases} \tag{28}$$

with $\mu_0 \equiv \theta_0 - \bar{\theta}_0$. The distribution of cost shocks in the initial and subsequent periods have different variances:

$$\mathrm{Var}_t\left(\theta_t\right) = \begin{cases} \sigma_0^2, & t = 0 \\ \mathrm{Var}_t\left(\mu_t\right) = \gamma^2, & t \geq 1. \end{cases} \tag{29}$$

Under both feedback taxes and quotas, the equation of motion for the state variable $y_t \equiv E_t\theta_t$ is independent of the regulator's actions. However, there is a qualitative difference in y_t under taxes and quotas because of endogenous learning.

Under feedback tax policies, using equation (5), the expected benefit in period t, is

$$E_t\left\{B\left[x_t^*\left(z_t, \theta_t\right),\ \theta_t\right]\right\} \ = \ f + \left(a + E_t\theta_t\right) z_t - \frac{b}{2} z_t^2 + \frac{1}{2b} Var_t\left(\theta_t\right).$$

The regulator's value function under taxes, $T^{FB}\left(S_t,\ y_t\right)$, solves the dynamic programming equation

$$\begin{aligned} T^{FB}\left(S_t,\ y_t\right) &= \underset{z_t}{\mathrm{Max}}\left\{E_t B\left[x_t^*\left(z_t, \theta_t\right), \theta_t\right] - D\left(S_t\right) + \beta E_t T^{FB}\left(S_{t+1},\ y_{t+1}\right)\right\} \\ &= \underset{z_t}{\mathrm{Max}}\left\{f + \left(a + y_t\right) z_t - \frac{b}{2} z_t^2 + \frac{1}{2b} Var_t\left(\theta_t\right)\right. \\ &\qquad \left. - \left(cS_t + \frac{g}{2} S_t^2\right) + \beta E_t T^{FB}\left(S_{t+1},\ y_{t+1}\right)\right\} \\ \text{s.t.} \qquad & S_{t+1} = \Delta S_t + z_t + \frac{1}{b}\mu_t \\ & y_{t+1} = \rho y_t + \rho\mu_t. \end{aligned}$$

Under quotas, the regulator's value function, $Q(S_t,\ y_t)$, solves the dynamic programming equation

$$\begin{aligned} Q(S_t,\ y_t) &= \underset{x_t}{\text{Max}}\left\{E_t B(x_t,\theta_t) - D(S_t) + \beta E_t Q(S_{t+1},\ y_{t+1})\right\} \\ &= \underset{x_t}{\text{Max}}\left\{f + (a+y_t)x_t - \frac{b}{2}x_t^2 - \left(cS_t + \frac{g}{2}S_t^2\right) + \beta Q(S_{t+1},\ y_{t+1})\right\} \\ \text{s.t.} \qquad & S_{t+1} = \Delta S_t + x_t \\ & y_{t+1} = \rho y_t. \end{aligned}$$

Solving the Dynamic Programming Equations. Using a two-dimensional state vector $X_t = (S_t,\ y_t)'$, the dynamic programming equations (DPE) can be written in general matrix notations as in the previous subsection. The control variable is z_t under taxes, and x_t under quotas. Those coefficients are

$$Q = \begin{pmatrix} -g & 0 \\ 0 & 0 \end{pmatrix}, \qquad W = (0\ \ 1), \qquad M = \begin{pmatrix} -c \\ 0 \end{pmatrix},$$

$$A = \begin{pmatrix} \Delta & 0 \\ 0 & \rho \end{pmatrix}, \qquad B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad C = \begin{cases} C_{tax} = \left(\frac{1}{b},\ \rho\right)' & \text{(tax)} \\ C_{quota} = (0,\ 0)' & \text{(quota)} \end{cases}, \qquad D = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$$U_0 = f, \qquad U_1 = a, \qquad U_2 = b, \qquad U_3 = \begin{cases} \frac{1}{2b} & \text{(tax)} \\ 0 & \text{(quota)} \end{cases}.$$

We see the DPE for taxes and quotas only differ in U_3 and C. Thus at any time with the same state vector X_t, the payoff difference between feedback taxes and quotas is

$$\begin{aligned} T^{FD}(X_t) - Q(X_t) = V_{0,t}^{tax} - V_0^{quota} &= \eta_t \left(U_3 + \frac{\beta}{2}C_{tax}' V_2 C_{tax}\right) \\ &= \begin{cases} \frac{1}{2b}\left(\sigma_0^2 + \frac{\gamma^2\beta}{1-\beta}\right)\left(1 + b\beta C_{tax}' V_2 C_{tax}\right), & t = 0; \\ \frac{\gamma^2}{2b(1-\beta)}\left(1 + b\beta C_{tax}' V_2 C_{tax}\right), & t \geq 1. \end{cases} \end{aligned}$$

With $C_{tax} = \left(\frac{1}{b},\ \rho\right)'$, expanding $C_{tax}' V_2 C_{tax}$ leads to

$$T^{FD}(X_0) - Q(X_0) \ = \ \frac{1}{2b}\left(\sigma_0^2 + \frac{\gamma^2\beta}{1-\beta}\right)\left(1 + \frac{\beta}{b}V_{11} + 2\beta\rho V_{12} + \beta b\rho^2 V_{22}\right). \qquad (30)$$

where $V_{11},\ V_{12},\ V_{22}$ are elements of V_2 which can be solved for from equation (25). Substituting $Q,\ A,\ B,\ W$ into (25) and equating elements of matrices at different side of the "=" sign, we get the following system of equations for $V_{11},\ V_{12},\ V_{22}$.

$$V_{11} = -g + \beta\Delta^2 V_{11} + (b - \beta V_{11})^{-1} \beta^2 \Delta^2 V_{11}^2; \tag{31}$$

$$V_{12} = \rho\beta\Delta V_{12} + (b - \beta V_{11})^{-1} \beta\Delta (1 + \beta\rho V_{12}) V_{11}; \tag{32}$$

$$V_{22} = \rho^2 \beta V_{22} + (b - \beta V_{11})^{-1} (1 + \beta\rho V_{12})^2 . \tag{33}$$

Given that the value function is quadratic in S_t and bounded above, $V_{11} < 0$. $V_{22} < 0$ is not required since the equation of motion for y_t is not affected by the regulator's actions. Obtaining the negative root of equation (31) and then solving the linear equations (32) and (33) recursively lead to the expression for $V_{11},\ V_{12}$, and V_{22}:

$$V_{11} = \frac{-\left(b\beta\Delta^2 + \beta g - b\right) - \sqrt{\left(b\beta\Delta^2 + \beta g - b\right)^2 + 4\beta g b}}{2\beta} \ < \ 0;$$

$$V_{12} = \frac{\beta\Delta V_{11}}{b(1 - \rho\beta\Delta) - \beta V_{11}} \ < \ 0;$$

$$V_{22} = \frac{\left(1 - \frac{\beta}{b} V_{11}\right)(1 - \rho\beta\Delta)^2}{b(1 - \rho^2\beta)\left(1 - \rho\beta\Delta - \frac{\beta}{b} V_{11}\right)^2} \ > \ 0.$$

Similarly we obtain elements $(v_1,\ v_2)$ of matrix V_1 from equation (26) by substituting in $V_2,\ M,\ A,\ B,\ W$:

$$v_1 = \frac{-c + (b - \beta V_{11})^{-1} a\beta\Delta V_{11}}{1 - \beta\Delta - (b - \beta V_{11})^{-1} \beta^2 \Delta V_{11}} \ < \ 0;$$

$$v_2 = \frac{1}{1 - \rho\beta} \frac{a(1 - \beta\Delta) - \beta c}{(b - \beta V_{11}) - b\beta\Delta} \frac{(1 - \rho\beta\Delta)(b - \beta V_{11})}{b - \beta V_{11} - b\rho\beta\Delta}.$$

After obtaining V_2 and V_1, we can get the constant term $V_{0,t}$. The constant term under quotas is

$$V_0^{quota} = \frac{1}{1 - \beta}\left\{ f + \frac{1}{2} \frac{[a(1 - \beta\Delta) - \beta c]^2}{(b - \beta V_{11}) - 2b\beta\Delta + (b - \beta V_{11})^{-1} b^2 \beta^2 \Delta^2} \right\}.$$

We obtain the optimal control rule (equation (8) in the text) by substituting matrices $V_1,\ V_2,\ A,\ B,\ W$ into equation (24); and the payoff difference (equation (11) in the text) by substituting $V_{11},\ V_{12},\ V_{22}$ into equation (30).

The procedure for solving the DPE with quota trading is similar. The only necessary change for the coefficient matrices is to replace C_{quota} by $C_{q+Trade} = (0, \rho)'$.

Proof. (Remark 2) Feedback emission taxes are preferred to quotas if and only if

$$(1 - \rho\beta\Delta)^2 - \frac{\beta^2}{b^2} V_{11}^2 \left(1 - \rho^2\beta\right) \ \geq \ 0. \tag{34}$$

Given $V_{11} < 0$, equation (34) is equivalent to

$$\begin{aligned} &\frac{V_{11}}{b} \geq -\frac{1-\rho\beta\Delta}{\beta\sqrt{1-\rho^2\beta}} \\ \iff &\frac{\left(\beta\Delta^2+\beta\frac{g}{b}-1\right)+\sqrt{\left(\beta\Delta^2+\beta\frac{g}{b}-1\right)^2+4\beta\frac{g}{b}}}{2} \leq \frac{1-\rho\beta\Delta}{\sqrt{1-\rho^2\beta}}. \end{aligned} \tag{35}$$

It is easy to see that the left-hand-side of the above inequality is monotonically increasing in g, Δ, β, and monotonically decreasing in b. The right-hand-side of the above inequality is independent of g and b, and monotonically decreasing in Δ. Hence, higher g and Δ and lower b all make quotas more attractive, relative to taxes. ■

Proof. (Proposition 2) Equation (34) can be transformed to

$$f(\rho) \equiv \rho^2\beta^2\left(\Delta^2+\frac{\beta}{b^2}V_{11}^2\right)-2\rho\beta\Delta+1-\frac{\beta^2}{b^2}V_{11}^2 \geq 0. \tag{36}$$

$f(\rho)$ is a convex quadratic function in ρ, and symmetrical with respect to

$$\rho_0 = \frac{\Delta}{\beta\left(\Delta^2+\frac{\beta}{b^2}V_{11}^2\right)} \geq 0.$$

■

Proof. (Corollary 1) The inequality in (36) will always hold if

$$\begin{aligned} &(2\beta\Delta)^2-4\beta^2\left(\Delta^2+\frac{\beta}{b^2}V_{11}^2\right)\left(1-\frac{\beta^2}{b^2}V_{11}^2\right) < 0 \\ \iff &4\beta^3\frac{V_{11}^2}{b^2}\left(-1+\beta\Delta^2+\beta^2\frac{V_{11}^2}{b^2}\right) < 0 \\ \iff &4\beta^3\frac{V_{11}^2}{b^2}\left(1-\frac{\beta}{b}V_{11}\right)\left(\beta\Delta^2+\beta\frac{g}{b}-1\right) < 0 \\ \iff &\frac{1-\beta\Delta^2}{\beta} > \frac{g}{b}. \end{aligned}$$

Otherwise, there will be two real roots

$$\rho_{1,2} = \frac{\Delta \pm \sqrt{\frac{\beta}{b^2}V_{11}^2\left(1-\frac{\beta}{b}V_{11}\right)\left(\beta\Delta^2+\beta\frac{g}{b}-1\right)}}{\beta\left(\Delta^2+\frac{\beta}{b^2}V_{11}^2\right)}$$

satisfying the equality, and the turning point

$$0 \leq \rho_0 = \frac{\Delta}{\beta\left(\Delta^2+\frac{\beta}{b^2}V_{11}^2\right)} \leq \Delta.$$

■

Proof. (Proposition 3) The effect of β on the right hand side of (35) is given by

$$\frac{\partial\left(\frac{1-\rho\beta\Delta}{\sqrt{1-\rho^2\beta}}\right)}{\partial\beta}=\frac{\rho}{2}\frac{\rho^2\beta\Delta-2\Delta+\rho}{(1-\rho^2\beta)^{\frac{3}{2}}}$$

which is non-positive if $\beta\le\frac{2\Delta-\rho}{\rho^2\Delta}$ when $\rho\ge 0$ or $\beta>\frac{2\Delta-\rho}{\rho^2\Delta}$ when $\rho<0$. Otherwise, higher β will increase both sides of the inequality (35) and make the effect of higher discount factor on the ranking more complicated. We obtain the left hand side of equation (13) in the text by taking partial derivative of the left hand side of equation (35) with respect to β. The condition $\beta\le\frac{2\Delta-\rho}{\rho^2\Delta}$ is satisfied if

$$0\le\rho\le\Delta\Longrightarrow 2\Delta\ge 2\rho\ge\rho+\rho^2\Delta\Longrightarrow\frac{2\Delta-\rho}{\rho^2\Delta}\ge 1\ge\beta.$$

■

Proof. (Remark 3) With quota trading, feedback taxes are preferred to quotas if and only if

$$1-\rho\beta\Delta+\frac{\beta}{b}V_{11}\ \ge\ 0 \tag{37}$$

$$\Longleftrightarrow\quad 1-\rho\beta\Delta-\frac{\left(\beta\Delta^2+\beta\frac{g}{b}-1\right)+\sqrt{\left(\beta\Delta^2+\beta\frac{g}{b}-1\right)^2+4\beta\frac{g}{b}}}{2}\ \ge\ 0.$$

It is easy to see that the left-hand-side of the above inequality is monotonically decreasing in g, Δ, β, and monotonically increasing in b. Hence, higher g, β and Δ and lower b all make quotas more attractive relative to taxes, qualitatively the same as under open-loop policies. ■

Proof. (Proposition 4) It is straightforward from equation (37). ■

A.3 Comparison of Ranking

Proof. (Proposition 5)

$$\frac{\partial\left(\frac{g}{b}\right)^{*OL}}{\partial\rho}=\frac{-2\beta\Delta}{(1+\rho\beta\Delta)^2}\left(\frac{1}{\beta}-\Delta^2\right)<0.$$

$$\frac{\partial\left(\frac{g}{b}\right)^{*FB}}{\partial\rho}=\frac{2\left(\rho-\Delta\right)}{(1-\rho^2\beta)^{\frac{3}{2}}}\frac{\sqrt{\left(\beta\Delta^2+\beta\frac{g}{b}-1\right)^2+4\beta\frac{g}{b}}}{\left(\beta\Delta^2+\beta\frac{g}{b}+1\right)+\sqrt{\left(\beta\Delta^2+\beta\frac{g}{b}-1\right)^2+4\beta\frac{g}{b}}}$$

with the sign depending on the sign of $\rho-\Delta$.

$$\frac{\partial\left(\frac{g}{b}\right)^{*FB+Trade}}{\partial\rho}=\frac{\Delta\left[\beta\Delta-(2-\rho\beta\Delta)^2\right]}{(2-\rho\beta\Delta)^2}<0$$

since in general $0<\beta\Delta<1<2-\rho\beta\Delta$. ■

Proof. (Proposition 6)

$$
\begin{aligned}
& \left(\frac{g}{b}\right)^{*FB} \geq \left(\frac{g}{b}\right)^{*OL} \\
\Longleftrightarrow\ & \frac{(1-\rho\beta\Delta) + \left(1-\beta\Delta^2\right)\sqrt{1-\rho^2\beta}}{(1-\rho^2\beta) + (1-\rho\beta\Delta)\sqrt{1-\rho^2\beta}} \geq \frac{1-\beta\Delta^2}{1+\rho\beta\Delta} \\
\Longleftrightarrow\ & \left(1-\rho^2\beta^2\Delta^2\right) - \left(1-\rho^2\beta\right)\left(1-\beta\Delta^2\right) \geq -2\rho\beta\Delta\left(1-\beta\Delta^2\right)\sqrt{1-\rho^2\beta} \\
\Longleftrightarrow\ & \rho^2\beta\left(1-\beta\Delta^2\right) + \beta\Delta^2\left(1-\rho^2\beta\right) \geq -2\rho\beta\Delta\left(1-\beta\Delta^2\right)\sqrt{1-\rho^2\beta}
\end{aligned}
$$

which holds in general.

$$
\begin{aligned}
& \left(\frac{g}{b}\right)^{*FB+Trade} \geq \left(\frac{g}{b}\right)^{*OL+Trade} \\
\Longleftrightarrow\ & \rho^2\beta\Delta - 2\left(1-\beta\Delta^2\right)\rho - \Delta \leq 0 \\
\Longleftrightarrow\ & \tilde{\rho} \leq \rho \leq \bar{\rho}
\end{aligned}
$$

with

$$
-1 \leq \tilde{\rho} \equiv \frac{1-\beta\Delta^2 - \sqrt{\left(1-\beta\Delta^2\right)^2 + \beta\Delta^2}}{\beta\Delta} < 0,
$$

$$
\bar{\rho} \equiv \frac{1-\beta\Delta^2 + \sqrt{\left(1-\beta\Delta^2\right)^2 + \beta\Delta^2}}{\beta\Delta} \geq 1.
$$

■