INTEGRITY OF INTERCONNECTED WATER SYSTEMS

BY

Roland Schinzinger,

Lemba Davy Nyirenda, Magdy Saeb

and

Ali Peiravi

School of Engineering
University of California, Irvine

Office of the Director

CALIFORNIA WATER RESOURCES CENTER

University of California   Davis, California 95616

The research leading to this report was supported in part by the United States Department of the Interior, under the Annual Cooperative Program of Public Law 95-467, Project No. A-086-CAL, and by the University of California Water Resources Center, Project UCAL-WRC-W-614. Contents of this publication do not necessarily reflect the views and policies of the U. S. Department of the Interior, nor does mention of trade names or commercial products constitute their endorsement or recommendation for use by the U. S. Government.

TECHNICAL COMPLETION REPORT

October 1983

TABLE OF CONTENTS

ABSTRACT

Various theoretical and applied network theory approaches for assessing the reliability of water delivery systems are examined including signal flow methodology and connectivity as well as structural integrity in a utility-type power system network. Problems of topological and flow reliability and associated sensitivity analyses are discussed. Various criteria are applied to identify critical and structurally important components, including a method of calculating frequency of system failure. Various concepts are illustrated by simple examples.

1. INTRODUCTION

The problem of evaluating the performance of water delivery systems in case of emergencies can be approached in many different ways. We present a technique that is based on network reliability computations and associated sensitivity analyses. Flow as well as topological reliability, as they relate to the delivery of water, are discussed. Since in all but very simple cases the calculation of system reliability can become cumbersome, selected algorithms suitable for large systems are presented.

A theoretical framework for ultimate consideration of the possibility of water network system failures was first established by review and evaluation of various measures of structural integrity useful in the overall analysis and synthesis of utility-type networks. The vulnerability of a utility system to interruption and the possibility of damage to the user is of utmost concern. Because improvements in a utility network system are costly, any design procedure attempting to improve system performance must include cost as a constraint. But in the absence of accurate reliability figures for many of the components of utility systems, other measures of network integrity must be identified that provide insight into design of affordable and reliable utility networks. It was shown that a number of measures of structural integrity that have been useful in public utilities can be applied to water delivery systems as well as power transmission and distribution systems, communication networks, computer networks and transportation networks. [13]

As necessary background to the final evaluation of system failure, various approaches to problems of connectivity were reviewed. The conclusions of this phase of the study, beginning with the definition of global node-connectivity of a directed graph, led to an alternative approach. Signal flow methodology was applied to derive a convenient expression for the paths between "source(s)" and "sink(s)" in a utility-type network. Reachability and connectivity properties of a network were obtained directly using the minors of a modified adjacency matrix, which can be applied to utility networks with any number of sources and sinks. [11, 12, 13]

A component or cut-set's contribution to system failure is called its importance.[3, 4, 5, 6, 8] Importance analysis is compatible with sensitivity analysis and is used for identifying a system's critical, structurally important or vulnerable elements. In this type of analysis, components are ordered based on their relative criticality in the system. The knowledge of this component's ranking provides a rational basis for both system design and operation. Applying such measures, the problem of upgrading system performance subject to cost constraints can be handled more effectively. Inspection and maintenance of components can be performed based on their order of criticality. One of the three importance measures presented in this analysis can also be used to estimate the frequency of system failure. These concepts are illustrated by means of numerical examples.

2. RELIABILITY MODELS

The functional behavior of water delivery systems can be analyzed by means of a network diagram or graph whose branches denote the components of the system and whose nodes represent the functional relationship between these components. The reliability of a system is defined as the probability that a system performs adequately within a specified period of time. Based on this definition, we consider two categories of reliability models- flow and topological reliability-depending on whether pressure and flow rate are of major concern or whether the mere existance of a connection represents adequate service.

2.1 Flow reliability [1, 7, 9, 10,]

The flow reliability model assumes that the system components (represented by branches in a graph) are of finite capacity; the flow in any branch cannot exceed its capacity. The system is considered performing adequately if and only if it allows a certain amount of flow to be transmitted from source to sink(s). Note that we have just now chosen to mention "flow" rather than flow rate. This is not precise but helps us in visualizing the system as a transmitter of a vital commodity.

Proceeding, we distinguish between single and multiple sink reliabilities.

i) Source-to-terminal flow reliability R1(f) is the probability that a certain amount of flow "f" can be transmitted from a specified node to another specified node (sink or terminal) during a specified time period.

ii) Source-to-k-terminal flow reliability Rk(f) is the probability that a certain amount of flow can be transmitted from a certain node (source) to k-specified nodes (sinks or terminals) during a specified time period.

It should be noted that R1(f) is a special case of Rk(f) with k=1. On other hand, an R1(f) evaluation can be used to find Rk(f) with a simple manipulation of the network to be analyzed. This is shown in Figure (1.b). An example will be presented to illustrate means of evaluating R1(f) and Rk(f).

2.2 Topological reliability [2, 7, 14]

The topological reliability analysis assumes that the system is represented by a probabilistic graph and the system is considered performing adequately if and only if there exists a path from the source node to the terminal node. The analysis is mainly considered with the enumeration of paths in the graph. This model assumes the mere fact of remaining connected to the system to be adequate for successful operation.

It should be noticed that the topological reliability of a system can be obtained from its flow reliability by either assuming that the capacities of all components are infinite, or equivalently, that the flow demand is very small. In the following analysis we concentrate on the flow reliability model because of its more general nature.

Example 1:

Given the network shown in Figure 1, we wish to find the flow reliability R1(f) between nodes 1,4. Flow "f" can assume any of the values: 5, 10, 15, 20. Component reliabilities and capacities are shown in Table 1.

TABLE 1

Data for Example 1

component #	reliability(p)	capacity
1	.9	15
2	.9	5
3	.8	5
4	.7	1Ø
5	.8	15

Solution:

To find R1(f) with, say f=10, we have to find a procedure that finds the "f-cuts", i.e. the branches whose removal causes the network to be unable to transmit f-units of flow between source and sink. For such a simple system these cuts can be found by inspection. For other values of flow, we can proceed in a similar manner.

Table 2

The f-cuts for Example 1

f	first order cuts	second order cuts
1Ø	(1)	(4,5), (3,4), (2,4)

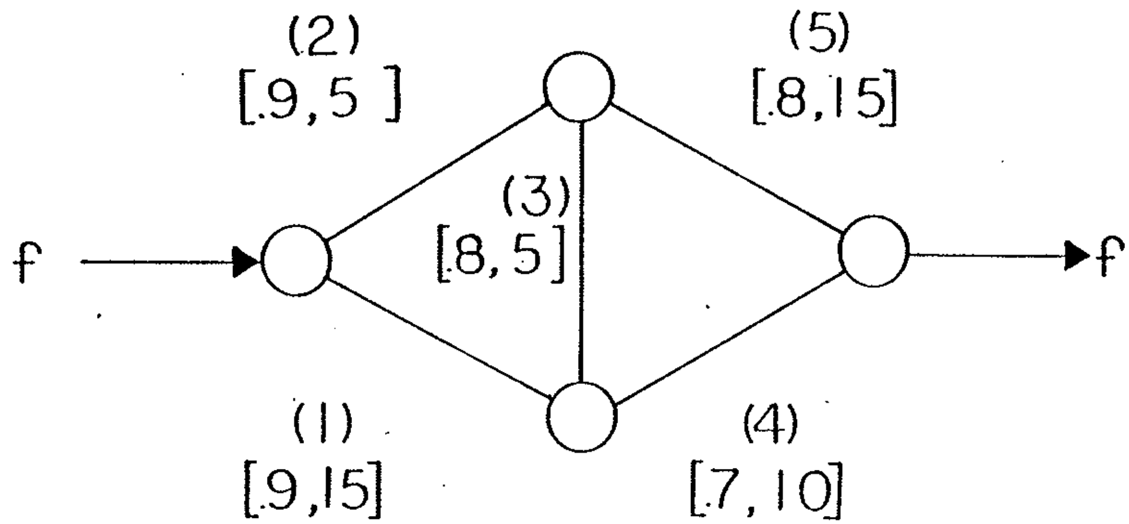

Fig. 1.a

The network representing the water delivery system of Example 1. In square brackets appear the reliability and capacity of the component identified by the number in ( ).

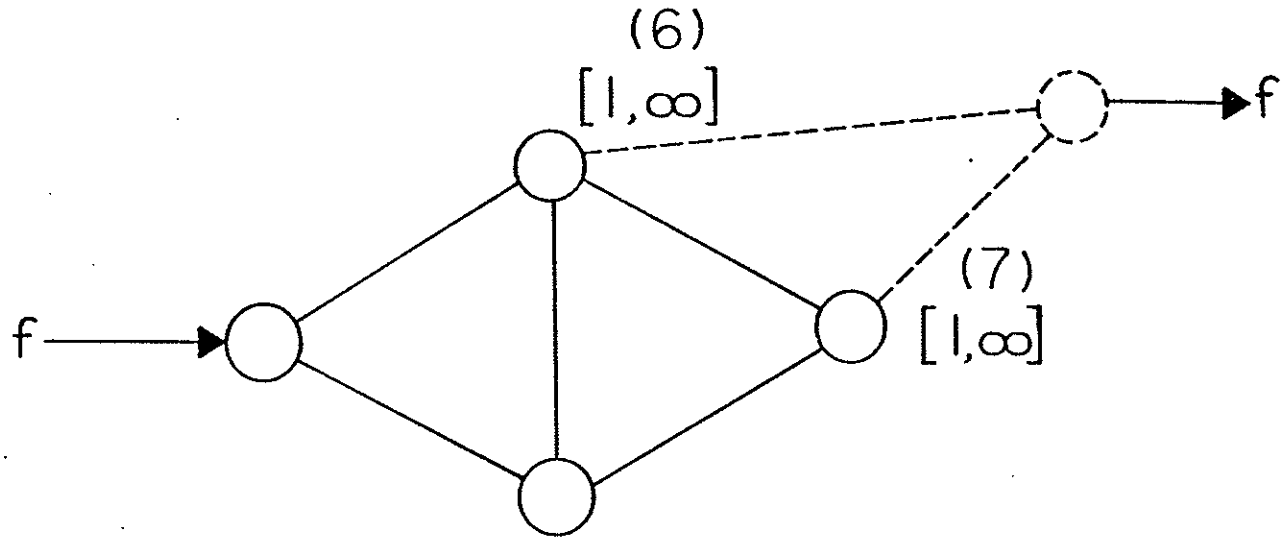

Fig. 1.b

Modifications necessary to calculate source-to-two-terminal reliability.

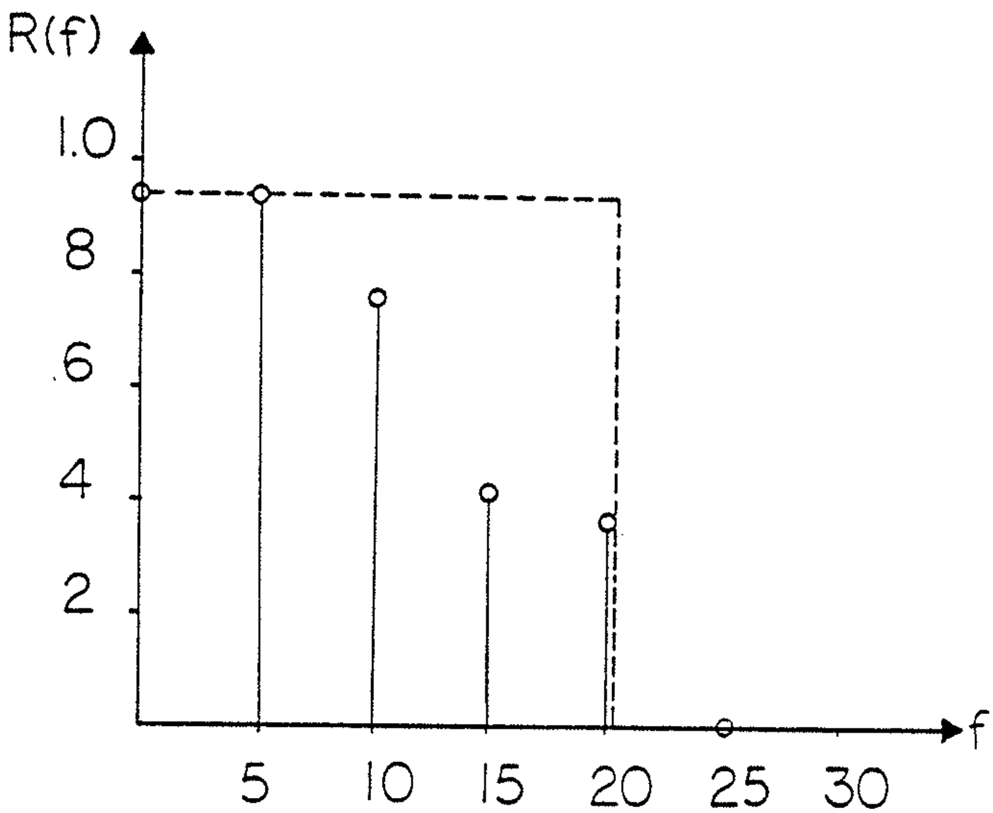

Fig. 2

Variation of system flow reliability
with respect to flow demand.

The expression for R1(10) can be found based on the mentioned cuts.

With $q_i = 1 - p_i$

$$R1(10) = 1.0 - (q_1 + q_4 \cdot q_5 \cdot p_1 + q_3 \cdot q_4 \cdot p_1 \cdot p_5 + q_2 \cdot q_4 \cdot p_1 \cdot p_5 \cdot p_3)$$

Substituting numerical values for the components' reliabilities, one can obtain R1(10) as

$$R1(10) = 1 - .21448 = .78552$$

Repeating the same procedure for f=5,15,20, one obtains the relation between R1(f) and 'f', shown in Figure 2.

Notice that R1(f) approaches a constant value for small "f". This value is the topological reliability of the system. In other words, we can obtain the topological reliability of a certain system from its flow reliability with "f" very small. From such a plot, a system designer can define "flow threshold" and "excess capacity" allowed in the system. The flow threshold is to be taken equal to the flow corresponding to a certain specified value of flow reliability under which the system is considered critical. The excess capacity is the difference between the actual and the threshold flow. It is clear that the reliability expression obtained is a function of: a) the components' reliabilities, b) the components' capacities, c) the topology of the network, d) the flow demand.

It is worthwhile mentioning that the dotted line represents the performance of a single component that could replace the system under investigation. Its reliability is equal to system topological reliability and its capacity is equal to maximum flow that the system can handle.

3. MEASURES OF IMPORTANCE OF A COMPONENT

Given a closed form expression for system probability of failure Q, Q= 1 - R, one can proceed to evaluate the relative importance of system components. R can be the topological or flow reliability of the system (single or multiterminal). Three different measures using concepts of sensitivity will be explored next.

3.1 Structural importance measure, IST [3, 4, 5, 6]

Let Pr (event i) denote the probability of occurrence of event i. Let Pr (component j failed)= q_j. Define the importance of component j in terms of its structural significance for the system as

$$IST_j = \partial Q / \partial q_j = \partial R / \partial p_j$$

Buzacott has shown that

IST_j= Pr(system failure; given that component j has failed) - Pr(system failure; given that component j operates).

The latter form is suitable for iterative computations. This measure is sometimes called probability of boundary conditions. For the system of Example 1 we get, for example

$$IST_1(10) = 1-(q_4 \cdot q_5 + q_3 \cdot q_4 \cdot p_5 + q_2 \cdot q_4 \cdot p_5 \cdot p_3)$$
$$= .872800$$

For practical systems, this measure appears not to be adequate for component ranking. So, other measures need to be used besides the structural importance measure.

3.2 Criticality Importance Measure, ICR [3, 6]

This measure is defined as

$$ICR_j = (\partial Q/\partial q_j)\cdot q_j \;/\; Q = IST_j \cdot q_j \;/\; Q$$

The computation of this factor depends on IST as it appears in earlier equations. It is a per-unit or relative sensitivity. ICR considers the fact that it is more difficult to increase the reliability of a reliable component than to increase the reliability of a less reliable one. For example:

$$ICR_1(10) = (q_1 - q_1 \cdot q_4 q_5 - q_1 \cdot q_3 \cdot q_4 \cdot p_5 - q_1 \cdot q_2 \cdot q_4 \cdot p_5 \cdot p_3)/$$
$$(q_1 + q_4 \cdot q_5 \cdot p_1 + q_3 \cdot q_4 \cdot p_1 \cdot p_5 + q_2 \cdot q_4 \cdot p_1 \cdot p_5 \cdot p_3) = .406938$$

3.3 Fussel-Vesely measure, IFV [3, 6]

A component in a system can be of special importance to system failure by appearing in cut-sets more frequently than other components. A measure to take care of those appearances can be useful. This is the Fussel-Vesely measure of component j and is defined by:

$$IFV_j = Q^* / Q$$

where Q^* is Pr (system failure based on cuts c_i, i is an element of L_j) and L_j is the set of cuts containing component j.

In our example

$$IFV_1(10) = q_1/(q_1 + q_4 \cdot q_5 \cdot p_1 + q_3 \cdot q_4 \cdot p_1 \cdot p_5 + q_2 \cdot q_4 \cdot p_1 \cdot p_5 \cdot p_3) = .46624$$

"The importance rankings produced by the Fussel-Vesely method relate closely to the criticality importance and produce the same rankings and almost the same numbers." [6] Based on this fact we will compute only ICR and IST for all system components as shown in Section 5.

3.4 A note on measures of importance

Plots of IST and ICR measures vs. small changes in individual component reliability demonstrate the relative importance of a certain component in the system as shown in Figures 3, 4. It should be noted that even with those changes in the estimate of a component's probability of failure, the ordering remains the same within the range of changes considered. Applying the same procedure at different flow levels, one can detect critical components at different "loading conditions" of the network.

4. GENERATION OF SYSTEM RELIABILITY EXPRESSIONS

In the previous discussion we considered a very simple network. For networks of realistic size, we need a clearly defined procedure to compute the system's flow or topological reliabilities. Some suitable techniques are mentioned next.

4.1 Flow reliability by Aggarwal et al. [1]

Aggarwal's method solves three subproblems: Finding the minimal paths from a connection matrix, determination of "valid groups" from these paths, and finally derivation of the reliability expression.

Some of the assumptions used are: node capacity is infinite, no node failure is to be considered, a component can be either in an "On" or "Off" state and neither self loops nor cycles are allowed in the network.

The capacity of a path is taken as the minimum of all capacities of its branch segments. A path is considered a "valid group" if, and only if, its capacity is greater than or equal to the amount of flow transmitted. A combination of invalid groups can constitute a valid group if the sum of their capacities can allow this amount of flow to be transmitted. If there is a common branch between these invalid groups, then the capacity of the combination is taken as the capacity of this common branch. Based on these valid groups, a closed form expression can be obtained. The entire procedure is performed through matrix manipulation techniques.

4.2 Flow reliability by Lara-Rosano [9]

The reliability expression is formulated by finding what are called f-cuts. These are the cuts whose removal prevent flow "f" to be transmitted from source to sink. When these cuts are removed from the network, it will not be capable of transmitting a flow of f-units.

If the set of f-cuts is found, then an inclusion-exclusion formula can be applied to get the flow reliability expression.

The method of generating the f-cuts depends on solving an inequality in binary variables which can assume only two states 1 or 0. This is called a pseudo boolean inequality and can be solved by the boolean absorption rule.

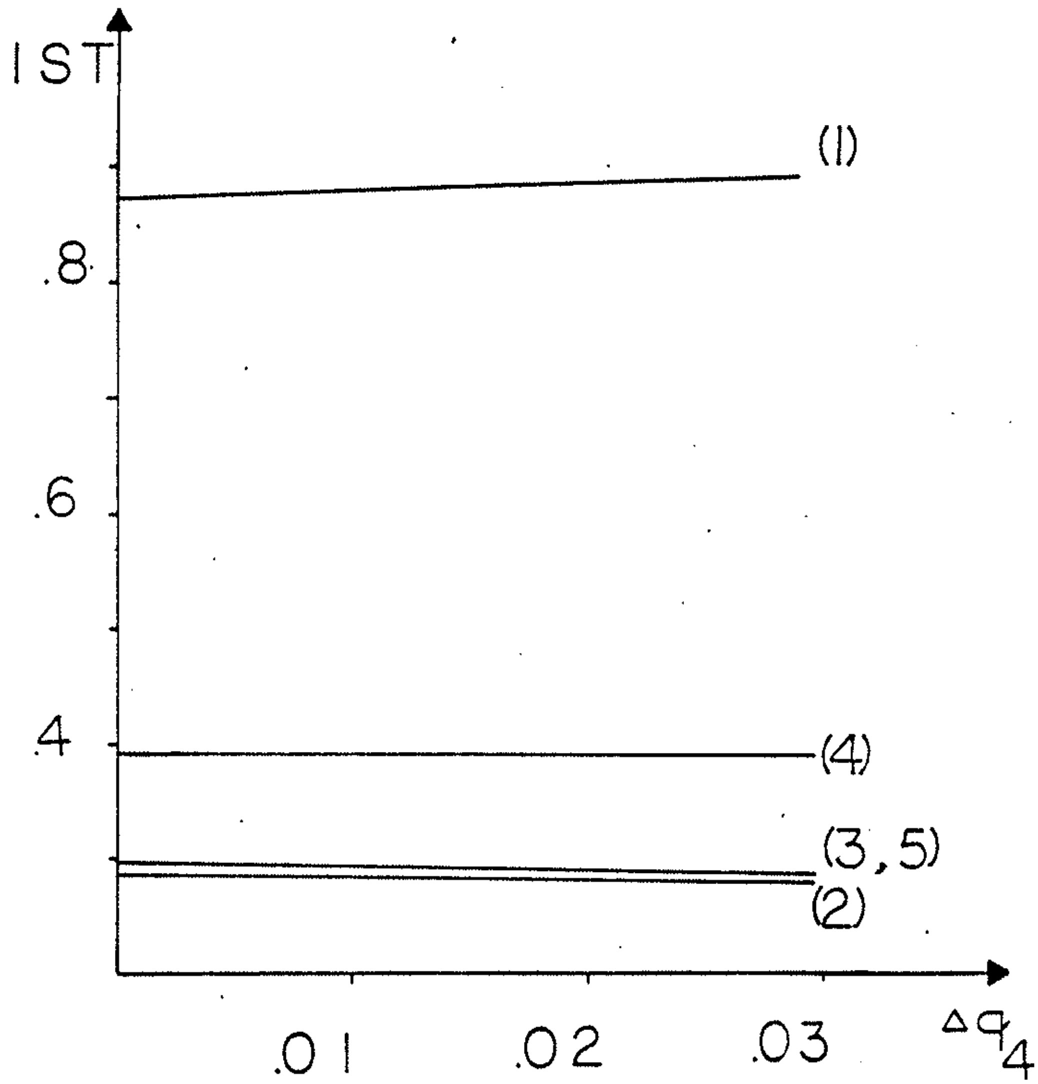

Fig. 3

Variation in structural importance measure (IST) with respect to change in Probability of failure of component 4.

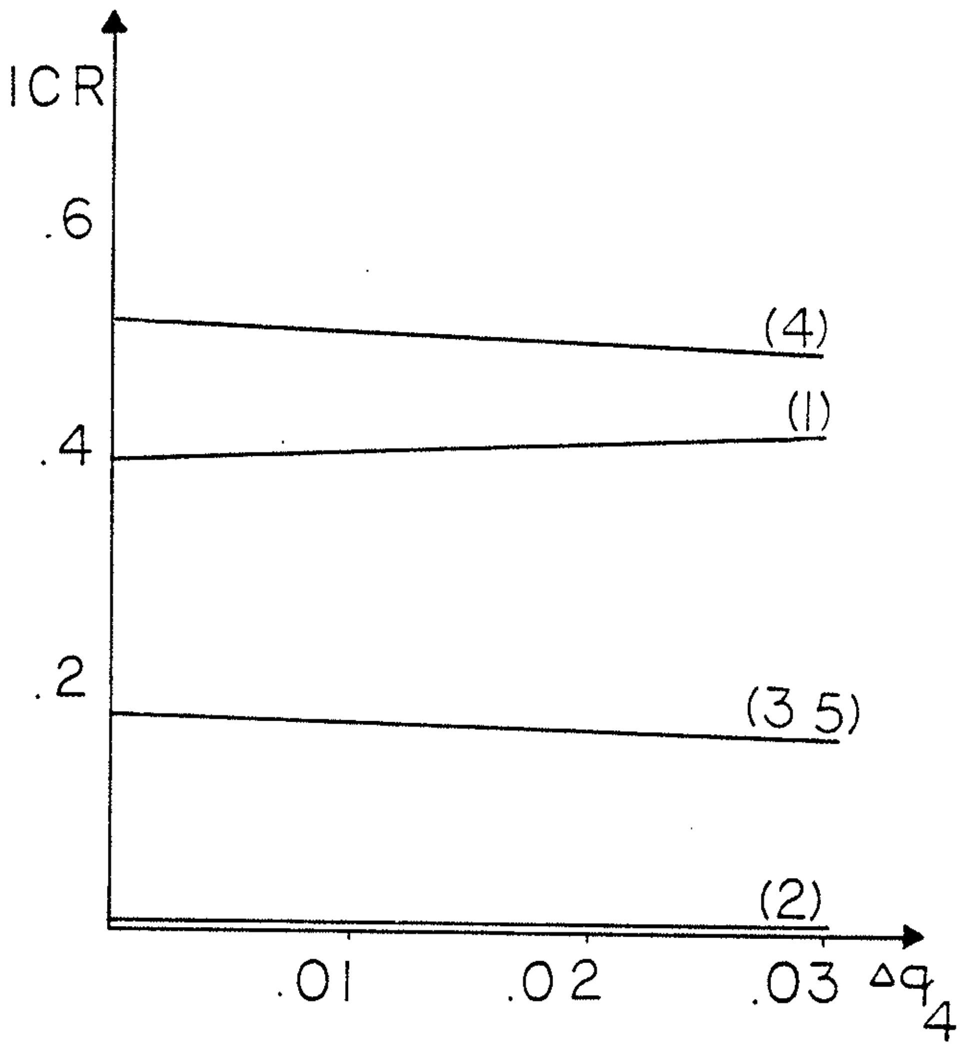

Fig. 4

Variation in criticality importance measure (ICR) and the change in probability of failure of component 4.

4.3 Flow reliability by Lee [10]

The paper presents a method for evaluating reliability of a flow network using the concept of lexicographic ordering. The interested reader can consult reference [10] for a definition and examples regarding this type of ordering. The assumptions used are: all branches are directed, each branch has a finite capacity and is either functioning or has failed, the flow is conserved at each node and the network is reliable if, and only if, a certain amount of flow can be transmitted from source to terminal node. The algorithm which is based primarly on the lexicographic property, finds at each iteration a set of valid states, determines the corresponding probability and then updates the reliability index.

The three methods mentioned above were developed for computing flow reliability. Next we will mention some of the algorithms used for topological reliability calculation. Since the literature in this area is vast, no attempt is made to present an exhaustive list.

4.4 "A unified formula" for computing topological reliability [11]

This approach considers that both branches and nodes in a directed graph are subject to failures. The method can handle different reliability problems such as source to terminal, source to k-terminals, between two nodes and between all node pairs.

Satyanarayana has shown that a source to k-terminals formula can be used to calculate all the mentioned reliabilities. Certain modifications are needed to the graph representing our network.

4.5 A method for computing system topological reliability by Kim et al. [7]

The paper presents a computational technique that is composed of three phases. Phase 1 involves the reduction of all series, parallel, and series-parallel subsystems into an irreducible non-series parallel system. In Phase 2, the set of all minimal paths from source to sink are enumerated. In Phase 3, the system reliability is computed based on the information computed in Phase 2 and the application of an operator [ ] * that removes all the duplication of the probability of success for a given component.

4.6 A technique for computing network reliability by Ahmad [2]

In this work, the reliability expression is found in three steps. The first step involves the construction of a tree. The second step is the calculation of the reliability expression of each branch in the tree by following certain rules. The third step is finding the reliability expression of the network by taking the direct sum of the reliabilities of the branches. The method is suitable for hand computations for moderate size networks.

5. NUMERICAL RESULTS AND DISCUSSIONS

5.1 Importance measures

Consider the system shown in Figure 1.a. From the reliability expression obtained in Example 1 and the definitions of importance measures, a computer program suitable for desk top computers was developed. The results obtained are shown in Table 3 for f=10.

Table 3

Importance measures for flow 'f'=1Ø

Component #	Importance measure		Rank	
	IST	ICR	IST	ICR
1	.8728Ø	.4Ø694	1	2
2	.1728Ø	.Ø8567	5	5
3	.1944Ø	.18128	3	3
4	.3816Ø	.53376	2	1
5	.1944Ø	.18128	3	3

Q(1Ø)=.21448 or R(1Ø) = .78552

From Table (3) we conclude that 1 and 4 are two important components in the system. From Figures 5 and 6 it is seen that any increase in probability of failure of either components 1 or 4 will produce the largest increase in probability of system failure.

Component (4) is less reliable than component (1), so any relative change in its probability of failure will affect the relative change in the system overall probability of failure more than component (1). This is clear from Figure 6. We would, therefore, consider the ICR ranking more appropriate for our needs and component (4) the most critical. Component (2) is the least critical to system failure probability as seen from Table 3 and Figures 5,6. It should appear last in any budget allocation for component hardening. Notice that the importance analysis was carried out at only one flow level, namely 10 units. For a more thorough insight to the system performance, the analysis should cover a range of normal flow levels. This is simply done by changing the reliability expression in the computer program or writing different subroutines for different flow levels. The results in our case are shown in the Appendix.

5.2 Frequency of system failure [3,5]

If a water delivery system can be said to be in an up or down state, then the mean cycle time T is given by:

$$T = u + d$$

where u is the mean up time and d is the mean down time.

The system failure frequency is given by:

$$Fq = 1/T$$

The calculation of F_q can be based on different methods. One of these is based on the structural importance factor. This is sometimes called "probability of boundary conditions". For systems of independent repairable components the failure frequency can be shown to be:

$$Fq = \sum_i fq_i . IST_i$$

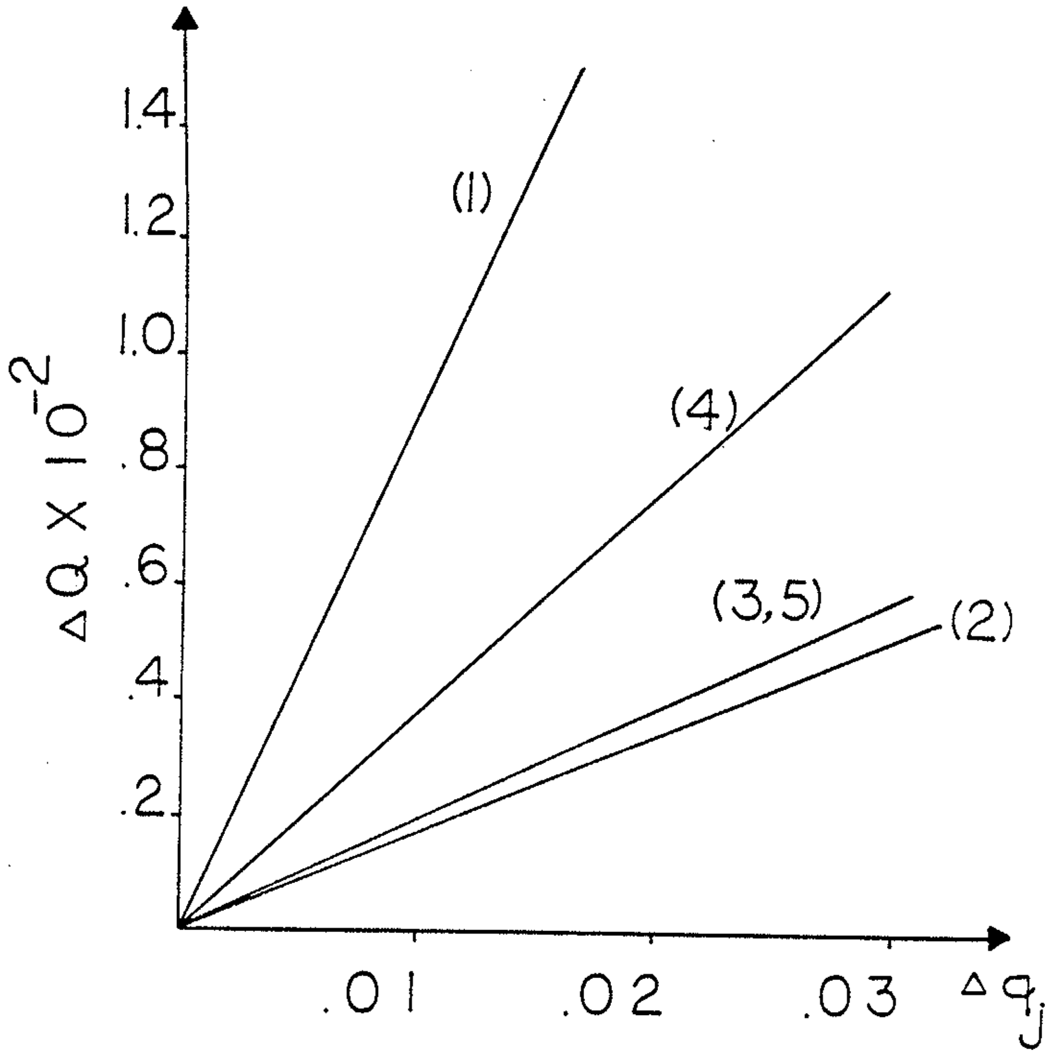

Fig. 5

Change in system probability of failure with change in component probability of failure.

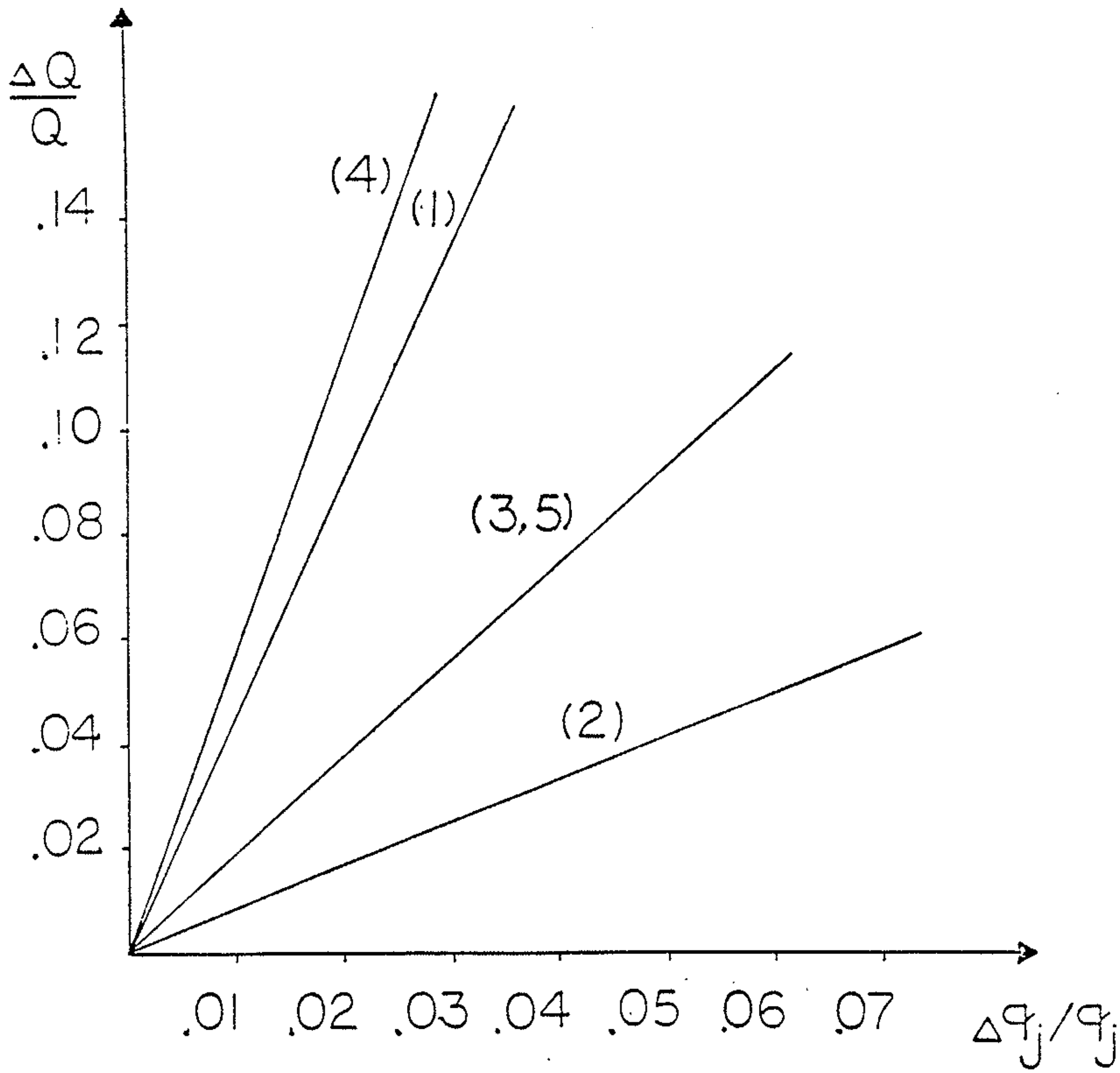

Fig. 6

Relative change in system probability of failure with relative change in component probability of failure.

where fq_i and IST_i are the failure frequency and structural importance factor of the i-th component respectively. A better understanding of the above formula comes from observing that IST_i is nothing but the availability of the i-th component. For more explanation and details about the last point, consult reference [4].

The above technique is simple and straightforward if a closed form reliability expression exists. As an application, let us compute the frequency of system failure shown in Example 1. Assume the failure frequency of the different components is as follows:

Table 4

Frequency of failure of components in Example 1

Component #	Frequency of failure fq_i
1	.1
2	.1
3	.12
4	.15
5	.12

Then Fq= .20846 occurrences/unit time

This value is calculated at a normal flow f=10.

6. SUMMARY AND CONCLUSIONS

Flow and topological reliability models, as applied to water delivery systems, are presented. System performance in case of component failures is better understood through the application of probabilistic approaches, such as the flow reliability model. The parameters that affect the flow reliability are shown to be: the components' reliabilities, the components' capacities, the topology of the network and the flow demand. The model presented allows the system designer to specify "flow threshold" and "excess capacity" permitted in the system. The discrete nature of the relation between the flow demand and the reliability of the system is shown in Figure 3.

Importance analysis is an efficient technique for system design and operation. System optimization, inspection and generation of maintenance schedules can be based on component's rank. In case of emergencies, the knowledge of system component's relative criticality helps making rational decisions regarding the allocation of material, labor and time for repairing different failed components. Three measures of importance are presented. The ranking of different components is best based on the criticality importance measure.

A simple method for estimating the frequency of system failure is discussed. The procedure is explained through the application of a simple example. For moderate size systems the analysis is suitable for desk top computers.

Acknowledgment:

We are grateful to Dennis Brehm for his stimulating discussions and helpful comments.

Notation

d	mean down time
f	flow in the network (in flow units)
f-cuts	the set of minimal cuts that allow f-units of flow to be transmitted from source to sink
fq_i	i-th component frequency of failure
Fq	system frequency of failure
G	a graph that represents the functional behavior of a water delivery system
IST_j	j-th component structural importance measure
ICR_j	j-th component criticality importance measure
p_j	j-th component reliability
q_j	j-th component probability of failure
Q	system probability of failure
R	system reliability in general
$R1(f)$	system source to single terminal flow reliability
$Rk(f)$	system source to k-terminals flow reliability
T	mean cycle time
u	mean up time
$\hat{}$	indicates a small change in the quantity that follows it

APPENDIX

As mentioned before, we need to compute the relative importance measures at different flow levels. The following Tables show the different components' ranking at different flow levels.

Table 5a

Importance measures for flow 'f'=5

Component #	Importance measure IST	Importance measure ICR	Rank IST	Rank ICR
1	.066400	.101467	4	5
2	.086400	.132029	3	3
3	.034200	.104523	5	4
4	.172800	.792176	2	1
5	.250200	.764670	1	2

Q(5)= .06544 or R(5)= .93456

Table 5b

Importance measures for flow 'f'=15

Component #	Importance measure IST	Importance measure ICR	Rank IST	Rank ICR
1	.448000	.075670	4	4
2	.000000	.000000	5	5
3	.504000	.168901	2	2
4	.576000	.289544	1	1
5	.504000	.168901	2	2

Q(15)= .596800 or R(15)= .403200

Table 6

Importance measures for flow 'f'=20

Component #	Importance measure		Rank	
	IST	ICR	IST	ICR
1	.403200	.063284	4	4
2	.403200	.063284	4	4
3	.453600	.142391	2	2
4	.518400	.244098	1	1
5	.453600	.142391	2	2

Q(20)= .637120 or R(20)= .362880

As seen from the above Tables the relative importance of different system components changes with the change of flow. This immediately raises the question "what is the proper way of ranking the system components at all flows?"

One method is by assigning probabilities of occurrence of different flow levels and then computing the expected values of importance measures, using the following equation;

$$X_j(\text{all flows}) = \sum_f X_j(f) \cdot P(f)$$

where $X_j(f)$ is either the structural importance measure or the criticality measure for component j at flow level 'f' and P(f) is the probability of occurrence of flow 'f'.

As an example let us assume the probability of occurrence of different flows is as follows:

Table 7

Probabilities of occurrence of different flow levels

Flow 'f'	P(f)
5	.10
10	.50
15	.22
20	.15
25	.03

The computed structural and criticality importance measures are shown in Table(8).

Table 8

Importance measures for all flows

Component #	Importance measure		Rank	
	IST	ICR	IST	ICR
1	.602081	.239756	1	2
2	.155520	.065531	5	5
3	.279540	.159607	4	4
4	.412560	.446410	2	1
5	.301140	.225622	3	3

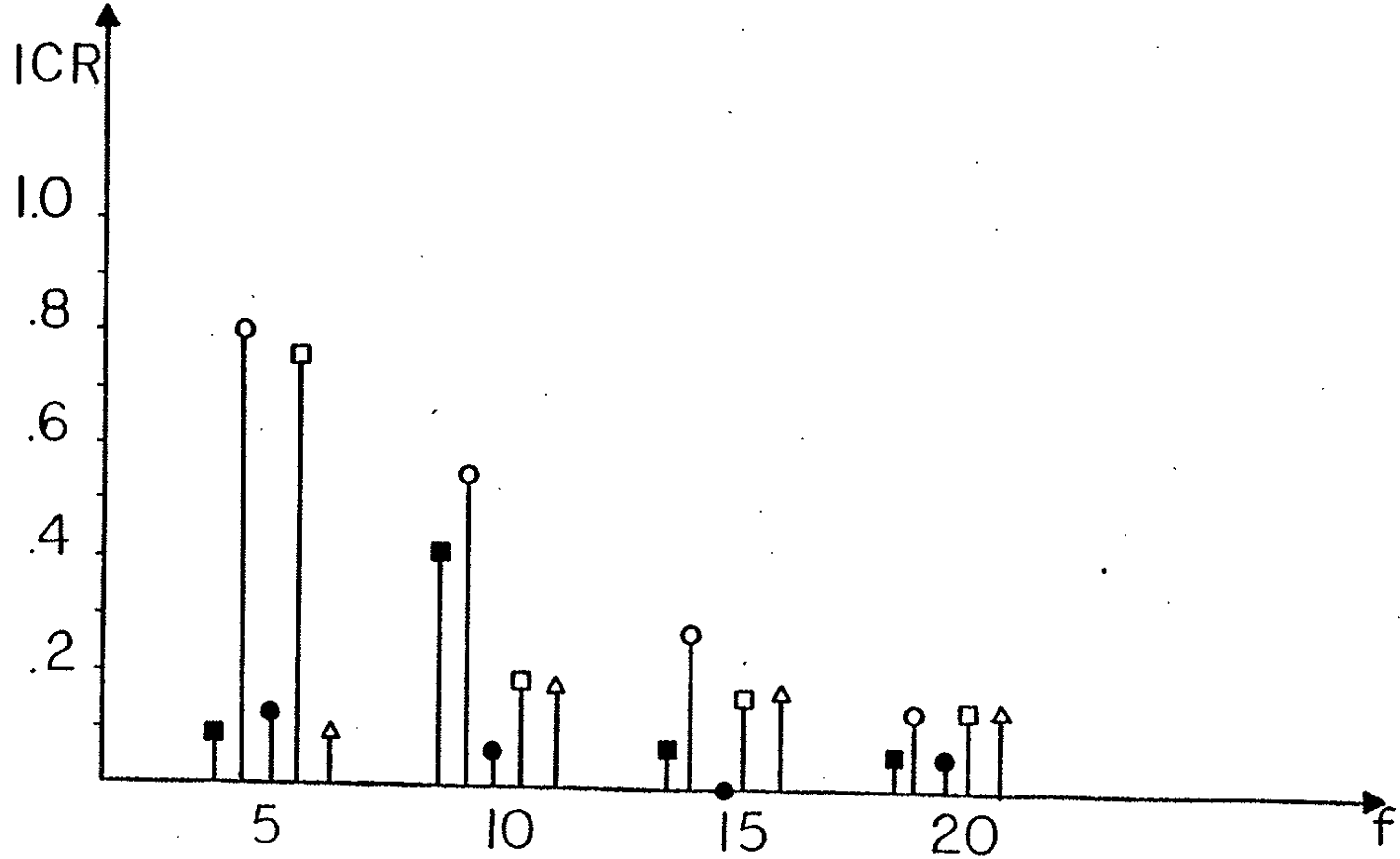

Fig. 7

Relative criticality of system components at different flow levels.

REFERENCES

1. Aggarwal, K.K., Y.C. Chopra and J.S. Bajwa. "Capacity Consideration in Reliability Analysis of Communication Systems." IEEE. Transactions on Reliability. Vol. R-31, No. 2, June 1982. pp. 177-181.

2. Ahmad, S.H. "A Simple Technique for Computing Network Reliability." IEEE. Transactions on Reliability, Vol. R-31, No. 1, April 1982. pp. 41-44.

3. Anders, G.J. "Evaluation of Importance and Related Reliability Measures for Electric Power Systems." IEEE. Transactions on Power Apparatus and Systems. Vol. PAS-102, No. 3, March 1983. pp. 527-535.

4. Barlow R. & Proschan. "Importance of System Component & Fault Tree Events." Stochastic Processes and Their Applications. Vol. 3, 1975. pp. 1-21.

5. Buzacott, J.A. "Network Approach to Finding the Reliability of Repairable Systems." IEEE. Transactions on Reliability. Vol. R-19, No. 4, November 1970. pp. 140-146.

6. Henley, E.J. & H. Kumamoto. "Reliability Engineering and Risk Assessment." Prentice-Hall, 1981. pp. 418-434.

7. Kim, Y.H., E.C. Kenneth and P.M. Ghare. "A Method for Computing Complex System Reliability." IEEE. Transactions of Reliability. Vol. R-21, No. 4, November 1972. pp. 215-219.

8. Lambert, H.E. "Measures of Importance of Events and Cut Sets in Fault Trees." Reliability and Fault Tree Analysis, Theoretical and Applied Aspects of System Reliability and Safety Assessment, SIAM, Philadelphia, 1975. pp. 77-100.

9. Lara-Rosano, Felipe. "Reliability Analysis of Flow Systems." Proceedings 1975 Annual Reliability and Maintainability Symposium. pp. 170-171.

10. Lee, S.H. "Reliability Evaluation of a Flow Network." IEEE. Transactions on Reliability. Vol. R-29, No. 1, April 1980. pp. 24-26.

11. Peiravi, Ali and Roland Schinzinger. "Connectivity and Reachability of Utility-Type Networks." School of Engineering, University of California, Irvine. [Working Paper No. 1] 21 p.

12. Peiravi, Ali and Roland Schinzinger. "Connectivity as a Measure of Power System Integrity." School of Engineering, University of California, Irvine. [Working Paper No. 3] 53 p.

13. Peiravi, Ali and Roland Schinzinger. "Measures of Goodness of Utility Networks." School of Engineering, University of California, Irvine. [Working Paper No. 4] 84 p.

14. Satyanarayana, A. "A Unified Formula for Analysis of Some Network Reliability Problems." IEEE. Transactions on Reliability, Vol. R-31, No. 1, April 1982. pp. 23-32.