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A Compartmental Model of Political Dissent: Application to the French Revolution

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We describe a compartmental model that captures the spread of political dissent between a civilian population and security forces. Both groups are subdivided into "accessories" and "dissenters" with a "dissent bug" spreading from the latter group to the former in the same way a virus spreads from infectives to susceptibles. We provide conditions under which the corresponding system of two non-linear ordinary differential equations has either the origin or a non-trivial value as its unique asymptotically stable equilibrium. The model is illustrated with data from the French Revolution and shows that the unraveling of the old regime is consistent with Chenoweth's rule, which postulates that no revolution has failed if more than 3.5% of the population is involved in "mass non-cooperation". We complement this metric with a dissension ratio equal to the civilian dissenting population divided by the security forces accessories —a ratio with a threshold value approximately equal to five as the old regime was collapsing.

Introduction

The forces that drive the rise and fall of authoritarian regimes have long interested historians and political scientists. There are several examples of popular protests against such regimes in the 21st century alone. (Dirsus 2024; Turchin 2003, Geddes et al. 2018). From the "Color Revolutions" in post-Soviet states to the "Arab Spring" and the Iranian uprisings of early 2026, recent history has shown how mass protests either succeed or fail in toppling authoritarian regimes. Our goal is to propose a simple model that describes and helps understand the dynamics that determine the success or failure of popular uprisings.

Chenoweth (2021) proposes the oddly precise "3.5% rule" that divides success from failure. She defines civilian dissenters as those who have "actively participated in an observable peak event like a battle, a mass demonstration or some other form of mass non-cooperation". She argues that no revolution has

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failed if there are more than 3.5% of such dissenters in the civilian population¹. This criterion could apply to a compartmental model for the dynamics of the Arab Spring consisting of a population of individuals subdivided into those who participate and those who do not participate in the uprising (Lang and De Sterck 2014). The dynamics are driven by a single differential equation combined with exogenous shocks, which capture the nuances of the Arab Spring uprisings.

Still, a model that describes political dissent solely through a restless civilian population ignores the crucial role played by security forces. Surely repressing dissent or on the contrary allowing it to "diffuse" should weigh in on the outcome. As pointed out by Dirsus (2024, p. 142)

[...] the process by which protests spread from one place to another is referred to as "diffusion." This spread can happen [...] through emulation. If protesters in one place see that protesters in another don't get beaten by security forces, they might imitate them.

The model proposed here will capture this diffusion process as well as a two-way emulation between the civilian population and security forces. Dirsus (2024, p. 159) illustrates this point with his description of the mood in the East German security forces facing crowds of anti-communist protesters in 1989:

There were signs of rupture within the security forces [...]. Morale in the party's local militias was so low that a good many of its reservists simply didn't show up. [...] many openly questioned the orders they were given.

Whether it is the demoralized East German STASI secret police in 1989, or Iranian militias killing thousands of demonstrators in January-February 2026, there is little doubt that the role, morale and psychology of security forces cannot be ignored in any dynamic modeling of political dissent.

We will therefore add to the (general) civilian population a compartment of security forces. Both populations will have regime "accessories" and "dissenters." Civilians are accessories through their passivity but are "susceptible" to "dissension" by joining mass protests or engaging in acts of civil disobedience. Security forces are accessories when they are capable of shooting unarmed

¹ There have been exceptions to this rule, e.g. failed protests in Bahrain between 2011 and 2014 despite the active participation of more than 3.5% of the population (Chenoweth 2021; Dirsus 2024, footnote p. 144).

civilians but are "susceptible" to becoming dissenters—e.g. when they become reluctant to arrest or shoot protesters.

We will capture these dynamics with a compartmental model similar to those used to describe the spread of an infection from "susceptibles" (our accessories) to "infectives" (our dissenters) (Hethcote and Van Ark. 1987). Accessories in both populations will catch the dissenting bug in the same way individuals transmit an infectious agent to susceptibles through physical contact or proximity. This will happen endogenously through direct contacts within the group or exogenously through cross-group *emulation* and/or *awareness/perception* of dissent in the other group. We apply the model to data on the French Revolution. We describe extensions and other areas of application of the model.

The model

We let $P = P_a + P_d$ be the (constant) total civilian population, divided into P_a accessories and P_d dissenters, which depend on time t (Figure 1). Similarly, the $S = S_a + S_d$ members of the security forces are the sum of S_a accessories (capable of shooting unarmed civilians) and S_d dissenters (who are more reluctant).

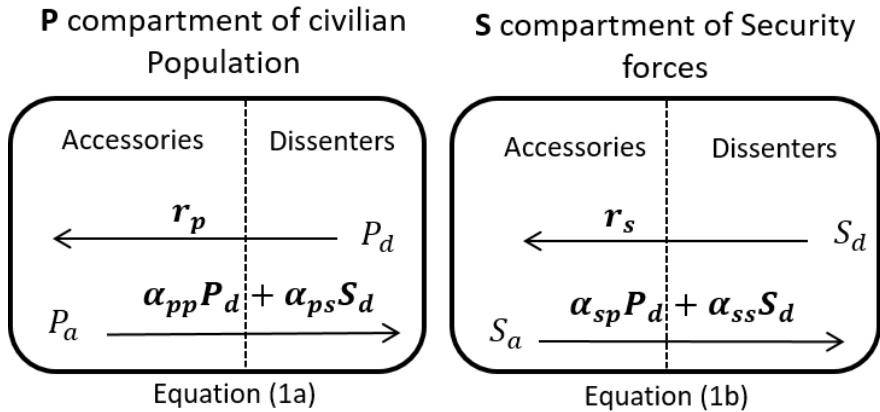


Figure 1: Compartmental representation of flows within the P and S compartments of civilian population and security forces.

In both the political and public health arenas, the simplest way of capturing the diffusion/emulation is by making an assumption of "homogeneous mixing" between accessories and dissenters. This means that there are potentially $P_a P_d$ contacts between civilian accessories and dissenters. A dissension rate α_{pp} captures the fraction of these potential contacts that materialize and result in "bug transmissions", i.e. accessories turning into dissenters. During a time interval dt these contacts thus produce $\alpha_{pp}P_a P_d dt = \alpha_{pp}(P - P_d)P_d dt$ new dissenters in the civilian

population. There is also a cross-group diffusion rate α_{ps} resulting in $\alpha_{ps}P_d S_d dt = \alpha_{ps}(P - P_d)S_d dt$ civilians moved to dissent by the awareness of S_d security dissenters (equation (1a) below). A similar mechanism operates for the security forces.

Exit/return rates r_p and r_s from the dissenting compartments back to the accessories reflect "second thoughts" among dissenters². (The inverses of these rates are the average times spent in the dissenting compartments). We express these flows of Figure 1 as the system of two differential equations in the variables P_d and S_d :

$$\begin{cases} \frac{dP_d}{dt} = (P - P_d)(\alpha_{pp}P_d + \alpha_{ps}S_d) - P_d r_p & (1a) \\ \frac{dS_d}{dt} = (S - S_d)(\alpha_{sp}P_d + \alpha_{ss}S_d) - S_d r_s & (1b) \end{cases}$$

We consider the re-parameterization

$$NEPD \stackrel{\text{def}}{=} P\alpha_{pp} - r_p ; NESD \stackrel{\text{def}}{=} S\alpha_{ss} - r_s ; EXPD \stackrel{\text{def}}{=} P\alpha_{ps} ; EXSD \stackrel{\text{def}}{=} S\alpha_{sp} . (2)$$

The net endogenous population dissension rate $NEPD$ is the net (i.e. positive or negative) balance between new and returning numbers of civilian dissenters endogenously generated by a single civilian dissenter in the early stages of the process (i.e. with $P_d \approx 1 \ll P_a \approx P$). The same observation holds *mutatis mutandis* for $NESD$. $EXPD$ and $EXSD$ are the exogenously generated population dissenters resulting from cross-exposure between civilians and security forces.

The system can now be written in matrix form

$$\begin{pmatrix} \frac{dP_d}{dt} \\ \frac{dS_d}{dt} \end{pmatrix} = \begin{pmatrix} NEPD & EXPD \\ EXSD & NESD \end{pmatrix} \begin{pmatrix} P_d \\ S_d \end{pmatrix} - \begin{pmatrix} \alpha_{pp}P_d^2 + \alpha_{ps}P_d S_d \\ \alpha_{sp}P_d S_d + \alpha_{ss}S_d^2 \end{pmatrix} . \quad (3)$$

Equilibrium results

The origin $(0, 0)$ is an equilibrium point of the system, i.e. both sides of (3) are zero when $P_d = 0 = S_d$. The behavior of the system close to the origin is driven by the linear component of (3), i.e. the Jacobian matrix $J = \begin{pmatrix} NEPD & EXPD \\ EXSD & NESD \end{pmatrix}$. The quadratic terms in (3) capture the limiting/saturation effects due to the finite pools P and S .

² These exit rates are necessary to avoid the triviality of dissenting compartments always filling up to reach 100% of the population and security forces compartments.

We are interested in the long-run behavior of the system for small initial dissenting populations $(P_d(0), S_d(0))$, i.e. the conditions on the parameters under which the vector $\mathbf{P}(\mathbf{t}) \stackrel{\text{def}}{=} (P_d(t), S_d(t))$ of dissenting populations either tends to 0 or diverges as t tends to infinity. In the Appendix we show that $(0, 0)$ is the only equilibrium and an asymptotically stable one (meaning $\mathbf{P}(\mathbf{t})$ tends to 0 and any emerging dissension is "snuffed out") if and only if³

$$NEPD < 0, NESD < 0, EXPD \times EXSD < NEPD \times NESD. \quad (4)$$

$NEPD$ and $NESD$ both negative mean that the return rates r_p and r_s must more than compensate the endogenous dissension rates $EXPD$ and $EXSD$. The third inequality of (4) says that the multiplicatively combined exogenous effects $EXPD \times EXSD$ must be smaller than the multiplicatively combined endogenous ones.

Ignoring again the possibility of equalities we assume that (4) is not satisfied when at least one of the three inequalities is strictly reversed. In this case, dissension takes hold with $\mathbf{P}(\mathbf{t})$ tending to a non-trivial equilibrium (f_p^e, f_s^e) . This equilibrium is more easily identified by rewriting the system of (3) in terms of the two fractional shares of dissenters $f_p \stackrel{\text{def}}{=} P_d/P$ and $f_s \stackrel{\text{def}}{=} S_d/S$:

$$\left\{ \begin{aligned} \frac{df_p}{dt} &= (1 - f_p) \left[(NEPD + r_p)f_p + \frac{S}{P} EXPD f_s \right] - r_p f_p \end{aligned} \right. \quad (5a)$$

$$\left\{ \begin{aligned} \frac{df_s}{dt} &= (1 - f_s) \left[\frac{P}{S} EXSD f_p + (NESD + r_s)f_s \right] - r_s f_s. \end{aligned} \right. \quad (5b)$$

The f_p and f_s nullclines are the loci of values of f_p and f_s for which the derivatives df_p/dt and df_s/dt of (5) are 0:

$$\left\{ \begin{aligned} f_p - \text{nullcline: } f_s &= \frac{f_p(a + b f_p)}{1 - f_p} \stackrel{\text{def}}{=} F_s(f_p) \end{aligned} \right. \quad (6a)$$

$$\left\{ \begin{aligned} f_s - \text{nullclines: } f_p &= \frac{f_s(c + d f_s)}{1 - f_s} \stackrel{\text{def}}{=} F_p(f_s) \end{aligned} \right. \quad (6b)$$

with

$$a = -\frac{NEPD}{EXPD} \frac{P}{S}, b = \frac{NEPD + r_p}{EXPD} \frac{P}{S}, c = -\frac{NESD}{EXSD} \frac{S}{P}, d = \frac{NESD + r_s}{EXSD} \frac{S}{P}. \quad (7)$$

³ To avoid dealing with second order effects we simplify the analysis by ignoring the possibility of equalities in (4).

In the Appendix we show that when (4) is not satisfied there is a single locally stable non-trivial equilibrium (f_p^e, f_s^e) at the intersection of the two nullclines of (6). This equilibrium is found numerically or through the roots of a cubic equation given in the Appendix. We note that the equilibrium depends only on the four "summary" parameters of (7)—even though they are not sufficient to completely define/generate the system of (5).

Application to the French Revolution

Finding time series data on dissenting civilian and security forces populations to test the model was challenging—even for the 21st century events mentioned earlier. Perhaps it is the historical distance (combined with the voluminous research on the subject) that led us to the required data for the French Revolution—a drawn-out 10-year affair stretching from 1789 (with the storming of the Bastille on July 14) to 1799 when Napoleon Bonaparte carried out the 18 Brumaire coup leading to the Consulate.

For the fractional share f_s of dissenting security forces, we use plausible minimum and maximum percentages of armed forces favorable to the Revolution for the six years between 1786 and 1791. For the fractional share f_p of dissenting civilian population we use the proportion of the 83 "departments" (regional units) that were in some state of protest during those same years. These proportions range from 5% to 95%. For each year, we multiply the proportion by a plausible minimum and maximum of 1% and 4% for the estimated percentage of population actively engaged in some form of protest/dissent. For example in 1786, it was estimated that 5% of departments were experiencing protests. This translates into an overall civilian dissenting shares f_p in a range from $5\% \times 0.01 = 0.05\%$ to $5\% \times 0.4 = 0.2\%$. These ranges for observed values of f_p and f_s are represented by the red rectangles in Figure 2. (See Data sources in the Appendix for details).

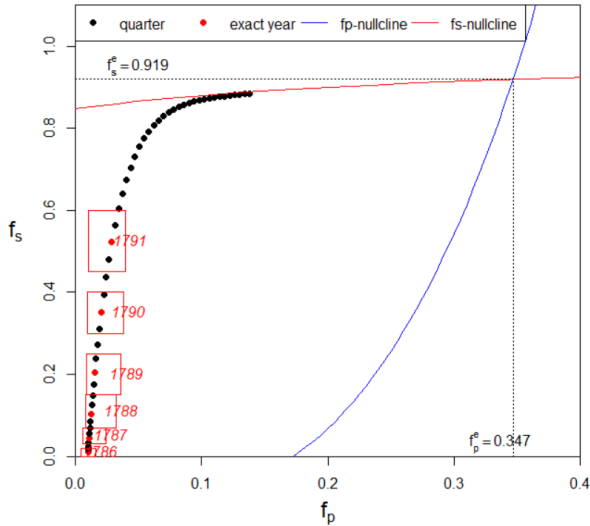


Figure 2. Model fitted to French Revolution data on fractional shares of dissenting civilians and military forces f_p and f_s . Red uncertainty rectangles for both observed variables are at exact years from 1786 to 1791 (time-step is 3 months).

In Table 1 we give the civilian and military populations P and S estimated for that period. We give the six parameter values (two return rates and four dissent rates) that were found to minimize the sum of squared deviations between the (observed) values f_p and f_s at the middle of each red uncertainty rectangle and the corresponding modeled values (red dots). We also provide the "derived" parameters of interest.

Table 1. Results from fitting the model to French Revolution data on shares f_p and f_s of the dissenting civilian and military populations, 1786-1791

<i>Estimated civilian and military populations</i>			
Civil pop. ('000) P	28,000	Milit pop ('000) S	180
<i>Six optimized parameters providing best fit to data</i>			
Civil return rate r_p	0.03891	dissent rate α_{sp}	1.711×10^{-5}
Milit return rate r_s	0.03120	dissent rate α_{pp}	1.680×10^{-6}
dissent rate α_{ps}	2.629×10^{-5}	dissent rate α_{ss}	1.137×10^{-3}
<i>Derived equilibrium shares of dissenters</i>			
civil share f_p^e	0.347	milit share f_s^e	0.919
<i>Derived net endogenous and exogenous dissension rates (eq. (2))</i>			
NEPD	0.008112	NESD	0.173396
EXPD	0.736114	EXSD	0.003079

With *NEPD* and *NESD* both positive the condition in (4) is not satisfied and the system has the non-trivial civilian and military equilibrium shares of dissenters $f_p^e = 0.347$ and $f_s^e = 0.919$. Although the model was run well into the 19th century we do not claim that the shares f_p and f_s converged to these equilibrium values. Indeed, the drawn out collapse of the old regime meant that by the late 1790s there were neither accessories nor dissenters to a regime that was no longer. In that sense the model does not have a collapse condition or mechanism *per se* other than stopping when f_p reaches the threshold of 3.5% —or any other value.

The net endogenous dissenting rate *NESD* in the military is about two orders of magnitude greater than *NEPD*. This is in contrast with an exogenous effect *EXPD* of military dissension on civilians that is roughly two orders of magnitude larger than *EXSD*. This suggests that security forces self-generate dissension more than civilians do, while the civilian population is more susceptible to the dissension in the security forces than the security forces are to dissension in the civilian population. This is consistent with the earlier observation by Dirsus about the large-scale "softening" of the security forces in the face of the 1989 mass protests in East Germany. The collapse of the old regime in France in the 1790s and that of communism in Eastern Europe 200 years later share this trait: the perception/awareness of mass defections within the security forces no doubt convinced the populations that security forces would not turn violent against protesters or those who fled the country en masse.

Discussion

Dissension ratio

While the civilian dissension rate at the regime's end was consistent with Chenoweth's 3.5% rule, the model suggests that the dissension rate within the military at that time was a much larger number around 40%. With very different base civilian and military populations of 28M and 180,000 respectively, it is then worth comparing the number of civilian dissenters P_d to the remaining number of military accessories S_a still willing to confront them. To that end, we define the *dissension ratio* r equal to the number of civilian dissenters per loyal security force personnel i.e. $r \stackrel{\text{def}}{=} P_d / S_a$. In the spirit of Chenoweth's threshold value $f_p^* = 3.5\%$ for the population's dissension rate f_p , we can estimate an approximate threshold value r^* at which the old regime collapsed. As the regime was unravelling in the late 1790s the roughly 3.5% and 40% dissension rates translate into $0.035 \times 28\text{M} = 980,000$ dissenters facing $(1-0.4) \times 180,000 = 108,000$ remaining military accessories. This resulted in a threshold dissension ratio $r^* = 980/180 \approx 9.1$.

Illiberal democracy

The ideas behind the model can be applied to describe other mechanisms for the removal of an unwanted regime. For example, an unhappy civilian population P may be confronted with a democratically elected group R of representatives

sympathetic to an illiberal regime with increasing autocratic tendencies (Sajo *et al.* 2021). The model's feedback loop between the P and R groups could capture the pressure from the civilian population on these representatives who feeling the winds of change may catch the dissenting bug. A constitutional provision allowing e.g. half or two thirds of representatives to vote to destitute an unpopular leader could provide a threshold within the R compartment above which the leader is removed (the parliamentary and peaceful equivalent of "regime collapse").

Multiple rings variant

The model describes the forces that drive dissent in a single outer ring of civilians confronted with an authoritarian regime's political structure. An extension of the model could incorporate one or more intermediate rings of politicians and government officials who support the regime but may also be prone to dissent. The core may consist of an authoritarian leader or tyrant whose fate could depend on forces of dissension rippling through several outer rings with various feedbacks between the rings.

Extensions

The model seems general enough to capture a variety of dynamical processes characterized by a "bug" of some sort spreading within two different populations and sub-populations through endogenous and cross-group emulation. (Table 2).

Table 2. Model applications in the biomedical and social sciences

"Bug"	Two populations	Focus
Political dissent	Civilian / Security	Determinants of regime fall
Electric vehicle adoption	High/low income	Income-sensitivity of EV adoption
Infectious disease spread	Rural/urban	Differential geographic spread of infection
Unionization spread	Citizens/immigrants	Impact of legal status on unionization
Corporate Social Responsibility adoption	Fortune 500/others	Emulation in CSR adoption

An "electric vehicle bug" may spread differentially between low and high-income populations with more or less emulation between the two groups. The exit rate among "adopters" may be of particular interest, as it would reflect misgivings of EV owners concerning costs, range, charging stations, etc. Corporate Social Responsibility might be another example of a "bug" that spreads through emulation/competition between large firms (Fortune 500) and others. In short, the model's scope could potentially extend from the natural to the social sciences and beyond.

Conclusion

A simple compartmental model of civilian populations and security forces with feedbacks between the two groups captures the population dynamics of political dissent. The model shows that small dissenting civilian and security populations will be snuffed out if and only if

$$NEPD < 0, NESD < 0, \quad EXPD \times EXSD < NEPD \times NESD.$$

Bearing in mind the definitions in Eq. (2), the first two inequalities mean that return rates of dissenters with second thoughts must more than offset each population's endogenous dissension rate. The third inequality means that "cross-pollination" of dissent between the two groups measured by the product $EXPD \times EXSD$ must be kept low enough. A strict reversal of any one of the three inequalities means that dissension will take hold. The two dissension rates f_p and f_s then converge to strictly positive values.

Despite its simplicity, the model captures fairly accurately the population dynamics of dissension during the French Revolution. It yields realistic orders of magnitude for threshold values at which the drawn-out affair caused the regime to eventually collapse—with a population threshold dissension rate f_p^* in good agreement with Chenoweth's 3.5% rule. The case of the French Revolution highlights the role of dissension in the security/armed forces: it took a roughly 40% dissension/desertion/exile rate of the military before the regime collapsed. A threshold dissension ratio r^* of about 5 for the number of civilian dissenters per loyal security force personnel complements Chenoweth's rule for a richer pair (r^*, f_p^*) reflecting both forces of dissent and their relative sizes. Mindful of data limitations we will need more empirical research to ascertain the quality of these two statistics as predictive threshold conditions for regime collapse.

In view of the reluctance of East German security forces to crack down on dissenters in 1989, and data permitting, the model could show that the communist regime's collapse also hinged on high dissension rates among the security forces. Conversely, Iran experienced mass protests in January 2026 with estimates of up to 5 million people actively involved in nationwide demonstrations on January 9⁴. With a total population of 92 million, we are well within "3.5% territory" for the proportion dissenting in the general population. At the same time, there have been no reports of significant defections from the security forces. Thus, the model hypothetically applied to Iran in early 2026 might have the origin as a stable equilibrium point: popular protests were doomed and the January demonstrations were snuffed out.

⁴ https://en.wikipedia.org/wiki/2025%E2%80%932026_Iranian_protests

The model does not provide a collapse mechanism other than a stopping rule when these rates become large enough, e.g. Chenoweth's 3.5% condition on the civilian population. A more elaborate version could borrow ideas from catastrophe theory to include additional feedback mechanisms that cause the dissension rates f_p and f_s to converge (possibly in finite time) or bifurcate suddenly to 1 and 0 respectively: Regime collapse is then an entirely dissatisfied population no longer facing any armed opposition. Still, the French Revolution was a long drawn-out affair rather than an instantaneous catastrophe. A slow-motion regime collapse unfolded long before extreme values of 1 and 0 were reached. Perhaps then, a simple model combined with simple stopping rules suffices.

Appendix

i. Under condition (4) the origin is the only equilibrium and it is locally asymptotically stable

Proof. We note that the Jacobian matrix of the system (3) is $J = \begin{pmatrix} NEPD & EXPD \\ EXSD & NESD \end{pmatrix}$.

The origin is locally asymptotically stable if and only if i) the trace $NEPD + NESD$ of J is negative and ii) the determinant $NEPD \times NESD - EXPD \times EXSD$ is positive. Conditions i) and ii) are equivalent to (4) because $EXPD$ and $EXSD$ are positive.

To show that the origin is the only equilibrium we note that (4) is equivalent to $a > 0$, $c > 0$ and $ca > 1$ defined in (7); F_s and F_p of Eqs. (6a)-(6b) are monotonically increasing functions of f_p and f_s respectively with $F_s(0) = 0 = F_p(0)$. The derivatives at 0 are $F'_s(0) = a$ and $F'_p(0) = c$.

Considered in the (f_p, f_s) coordinate system the nullcline F_p of eq. (6b) can be written as the inverse function

$$F_p^{-1}(f_p) = \frac{-c - f_p + \sqrt{(c + f_p)^2 + 4f_p d}}{2d} \quad (8)$$

of $F_p(f_s)$ defined for $f_p \in (0, 1)$; $F_p^{-1}(f_p)$ is the largest root of the quadratic equation $f_s^2 d + f_s(c + f_p) - f_p = 0$ in the unknown f_s derived from (6b). Because $F'_p(0) = c$ the derivative at the origin $F_p^{-1}(0)$ of the inverse is $1/c$. With $ca > 1$ the derivative of F_s is larger than the derivative of F_p^{-1} not just at the origin (because $a > 1/c$) but for all $0 \leq f_p \leq 1$. This is because the derivative $F_p^{-1'}$ increases more and more slowly while F'_s increases faster and faster. This implies that there can be no intersection of the two functions other than at the origin. (See Fig. 3 where the functions F_p^{-1} with $c_1 > 0$ and F_s with $a_1 > 1/c_1$ diverge as f_p grows from 0 to 1.).

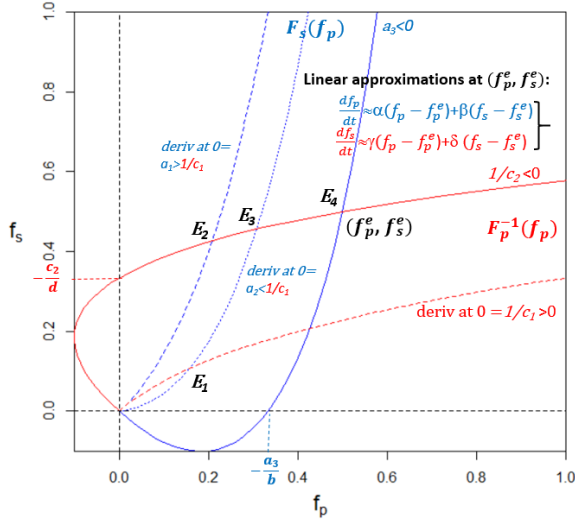


Figure 3. Examples of nullclines F_s and of F_p^{-1} for the proof that when (4) is satisfied the origin is the only equilibrium, i.e. with $a_1 > 1/c_1 > 0$. If (4) is not satisfied, there is a single non-trivial asymptotically stable equilibrium: e.g. E_1 to E_4 , depending on parameter values.

ii. When (4) is not satisfied there is a single asymptotically stable non-trivial equilibrium (f_p^e, f_s^e)

Proof. If (4) is not satisfied we first consider the case $ca < 1$ with both c and a positive. The derivative a of F_s at 0 (e.g. a_2 in Fig. 3) is then smaller than the derivative $1/c$ of F_p^{-1} at 0 (e.g. $a_2 < 1/c_1$). As the derivative of F_s increases and that of F_p^{-1} decreases, there is necessarily a single intersection at an equilibrium point (f_p^e, f_s^e) between 0 and 1, e.g. E_1 . If $c < 0$ (e.g. c_2 in Fig. 3) then $F_p^{-1}(f_p)$ increases monotonically from a positive $-c_2/d < 1$ for $f_p = 0$, crossing F_s exactly once regardless of the value/sign of a (e.g. at E_2, E_3 or E_4 of Fig. 3). Same reasoning if $a < 0$.

To find the root (f_p^e, f_s^e) of the system of Eq. (6) one can eliminate f_s and obtain a cubic equation in f_p alone, namely

$$a_3 f_p^3 + a_2 f_p^2 + a_1 f_p + a_0 = 0 \quad (9)$$

with

$$a_3 = b(bd - 1); \quad a_2 = b(1 - c + 2ad) - a - 1; \quad (10)$$

$$a_1 = a(1 + ad - c) + bc + 2; \quad a_0 = ac - 1. \quad (11)$$

The equilibrium f_p^e is the only feasible root of Eq. (9), i.e. a positive root between 0 and 1 with a corresponding feasible f_s^e obtained from Eq. (6a)⁵.

To prove asymptotic stability we note that the linear approximation of the system (5) close to an equilibrium (f_p^e, f_s^e) is of the form

$$\frac{df_p}{dt} \approx \alpha(f_p - f_p^e) + \beta(f_s - f_s^e) \quad (= 0 \text{ for } f_p - \text{nullcline}) \quad (12)$$

$$\frac{df_s}{dt} \approx \gamma(f_p - f_p^e) + \delta(f_s - f_s^e) \quad (= 0 \text{ for } f_s - \text{nullcline}) \quad (13)$$

with little known on the Jacobian matrix $J_e = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$. Still, the positive coefficients $(1 - f_p)^{\frac{S}{P}} \text{EXPD}$ of f_s in (5a) and $(1 - f_s)^{\frac{P}{S}} \text{EXSD}$ of f_p in (5b) show that β and γ must be positive. Always increasing nullclines in the (f_p^e, f_s^e) space imply that the slopes $-\alpha/\beta$ and $-\gamma/\delta$ of the linear approximation to the two nullclines at (f_p^e, f_s^e) are positive (Fig. 3). This means that $\alpha < 0$ and $\delta < 0$. From the previous discussion we also note that the slope of the f_s -nullcline at (f_p^e, f_s^e) is always less than that of the f_p -nullcline, i.e. $0 < -\gamma/\delta < -\alpha/\beta$ (Fig. 3). Because $\beta, \gamma > 0$ and $\alpha, \delta < 0$ this inequality is equivalent to the determinant $\alpha\delta - \beta\gamma$ of J_e being positive. This combined with the fact that the trace $\alpha + \delta$ is the negative sum of two negative numbers proves that (f_p^e, f_s^e) is a locally asymptotically stable equilibrium (i.e. J_e has a negative trace and positive determinant).

iii. Data source

Armed forces dissenters

Historians describe a rapid deterioration of discipline and spread of dissent within the French army between 1789 and 1791. Soldiers participated in revolutionary agitation, mutinies spread across garrisons, and large numbers of aristocratic officers emigrated. While precise statistics are unavailable, the ranges for observed percentage dissenting within the military were suggested by ChatGPT and are plausible: they rise from negligible levels in the mid-1780s to roughly 50% by 1791

⁵ Although a closed form is always desirable, Cardano's formulas for the roots of a cubic equation are cumbersome and one still needs to find the right root among the three possible values. It is simpler, if less mathematically elegant, to find numerically the intersection (f_p^e, f_s^e) of the two elementary and well-behaved functions F_s and F_p of (6).

—this in good agreement with the rapid increase of dissent within the ranks documented in the sources below.

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Civilian dissenters

Historical datasets show thousands of riots across France in the late eighteenth century, with disturbances spreading geographically during the crisis of 1788–1789. As above, the ranges for the share of administrative regions experiencing unrest were suggested by ChatGPT and are in good agreement with the sources below. The assumption of a 1 to 4% participation rate for affected regions is consistent with research on collective action (Chenoweth and Stephan 2011).

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