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# Learning with a Purpose: The Influence of Goals

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## Abstract

Most learning models assume, either implicitly or explicitly, that the goal of learning is to acquire a complete and veridical representation of the world, but this view assumes away the possibility that pragmatic goals can play a central role in learning. We propose instead that people are relatively *frugal* learners, acquiring goal-relevant information while ignoring goal-irrelevant features of the environment. Experiment 1 provides evidence that learning is goal-dependent, and that people are relatively (but not absolutely) frugal when given a specific, practical goal. Experiment 2 investigates possible mechanisms underlying this effect, and finds evidence that people exhibit goal-driven attention allocation, but not goal-driven reasoning. We conclude by examining how frugality can be integrated into Bayesian models of learning.

**Keywords:** Goals, Learning, Task-effects, Rationality, Frugality, Bayesian inference

## Introduction

Intuitively, what we need to know depends on what we want to do: the information we require from the environment will partially depend on our goals, desires, and intentions. For example, consider reading a recipe. If I am deciding whether to make this dish for a friend with a dairy allergy, then I need to know simply whether the dish contains any milk at all. If I am instead preparing a shopping list so that I can later make the dish for myself, then I need to know how much milk is required, not just whether any at all is involved. In this paper, we examine the extent to which learning is responsive to pragmatic goals (e.g. our desire to succeed at an expected future task).

Many cognitive models of learning assume that individuals are trying to acquire (approximately) complete representations of their environments, so pragmatic goals play essentially no role. For example, most models of causal learning assume that agents are trying to learn the “true” causal structure; most models of language learning assume that people are trying to infer the underlying structure of the language; and most models of category learning assume people are trying to acquire conceptual representations that most closely track the world’s statistical regularities. Under these models, pragmatic goals play essentially no role in learning; instead, the learner always tries to acquire a (relatively) complete and veridical representation of the world. This representation can later be used for a range of practical purposes precisely because it is complete and veridical.

Despite this, previous research suggests that pragmatic goals do impact learning. For example, people acquire different categories from identical data when learning occurs through a categorization task (selecting a category label based on a set of feature values) vs. a feature inference task (inferring a feature value given the values of other features) (e.g., Markman & Ross, 2003; Zhu & Danks, 2007). Also, people learn more in dynamic control tasks when given a general learning goal (learn about the system) rather than a specific task (maintain the system at a specific state) (Burns & Vollmeyer, 2002; Osman & Heyes, 2005). Tasks have even been found to influence low-level processes; for instance, negative priming in selective attention is directed to only task-relevant dimensions of distractor objects (Frings & Wentura, 2006; Maruff et al., 1999; Tipper, Weaver, & Houghton, 1994).

While task effects are common, there has been little study of the extent to which learning is modulated by longer-term pragmatic goals (vs. the task performed *during* learning). Much of everyday learning is driven by the desire to succeed at an expected future task, and it is possible that learning is highly responsive to beliefs about how information will be used in the future. If this is the case, our models of learning cannot ignore the important role that pragmatic goals play in many real-world learning situations.

Our central theoretical proposal is that people’s pragmatic goals direct their learning towards pragmatically relevant information and, perhaps more importantly, away from pragmatically irrelevant information. That is, people are relatively *frugal* learners who encode only the information they need: they acquire goal-relevant representations and ignore goal-irrelevant dimensions of the environment. We first report experimental results suggesting that people are relatively frugal when given a concrete, pragmatic goal (Experiment 1). We then present preliminary evidence about possible mechanisms underlying this frugality (Experiment 2). We finish by arguing that frugality can be a ‘rational’ strategy that can be reconciled with commonly used models of rational learning, including Bayesian inference.

## Experiment 1

Experiment 1 directly tests whether people display frugal learning when provided with a concrete, practical goal. The learning paradigm involved four buttons that probabilistically produced numbers between 1 and 100, where two of the buttons had relatively high means and two

had relatively low means. At the outset, learners were assigned a task to perform *after* learning: they had to choose the button with either the highest (MAX condition) or lowest average (MIN condition), or else simply report the mean for each button (Expected Outcome, or EO condition).

If people are frugal learners, then they should learn just the information necessary for their task and so learn more about task-relevant contrasts than task-irrelevant ones. For instance, the critical decision in MAX is which button has the highest mean. Participants can easily rule out the two low-mean buttons (since their numbers are *much* lower than the other two), and so focus on deciding between the two high-mean buttons. They should thus be more likely to learn the rank order of the two high-mean buttons than that of the two low-mean buttons. Conversely, MIN participants should be more likely to learn the rank order of the low-mean buttons than the high-mean ones. For EO participants, each contrast is equally relevant, so they should learn the rank orders (and hopefully the values) of all four buttons equally well. The EO condition thus acts as a control condition to determine the extent to which participants were able to learn about the values and rank order in this experiment.

## Participants

149 Amazon Mechanical Turk participants (mean age=35.5; 43% female,) were randomly assigned to one goal condition (MIN/MAX/EO), and one of two button order conditions (B-High/C-High). Participants received 50 cents for participation and 50 cents for performance. 29 participants were excluded based on independent criteria (see Results section), leaving 120 participants (mean age=36.9; 43% female) in the final analysis.

## Method

**Instructions** All participants were told that they would be presented with a set of buttons, and that each button produced a number between 1 and 100 when pressed. They were told that the exact number produced by each button would vary, but that different buttons tended to produce higher or lower numbers. Participants were then assigned a task and told that they would be given a bonus based on their success at that task. In the MAX and MIN conditions, participants would later press a single button and receive a bonus dependent on the resulting number: either more if it was higher (MAX), or more if it was lower (MIN). In the EO condition, participants would later estimate the average number produced by each button and receive a bonus based on the accuracy of their estimates.

**Learning Phase** The learning phase for all participants involved passively viewing ten trials in which all buttons were pressed simultaneously and then the results were displayed (Figure 1). The buttons were labeled and colored, and the numbers were displayed in a similarly colored square. Learning was self-paced, though participants had to view each trial for a minimum of three seconds.

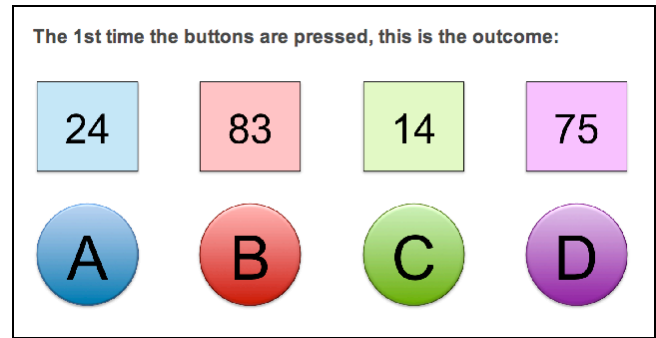


Figure 1: Example learning trial

The button means were 80, 70, 30, and 20, and all had standard deviation of 9.8.<sup>1</sup> The button means varied between button order conditions to ensure that response differences were not due to differences in the position or color of the best/worst buttons. In the B-High (vs. C-High) condition, the A/B/C/D means were: 30/80/20/70 (vs. 70/20/80/30).

**Testing Phase** In the testing phase, all participants (i) chose between all four buttons in accordance with their goal; (ii) chose separately between the high and low pairs of buttons, again in accordance with their goal; and then (iii) estimated each button's average. In the EO condition, participants were randomly assigned to choose the largest or smallest button for the forced choices in (i) and (ii). Numerical estimates for (iii) were recorded using a sliding scale between 1 and 100 with the exact number provided. After all questions had been answered, participants received feedback about the accuracy of their performance on question corresponding to their initial goal, along with the appropriate bonus.

## Results

**Exclusion Criteria** With Mechanical Turk populations, it is particularly important to check if participants understood the instructions and paid attention during the task. We used two exclusion criteria:

- (i) *Choice of a clearly suboptimal button* (e.g., choice of  $\mu=20$  button in the MAX condition). Every observed number for the low-mean buttons is less than every observed number for the high-mean pair, so this behavior implies the participant did not understand the instructions or did not pay attention during learning.
- (ii) *Mean estimate for a 'goal-relevant button'* (i.e., those that should be focal given the goal) *falling outside of the 'acceptable range'* (i.e., more than 5 points outside of the actual observed range of either button in the relevant pair<sup>2</sup>). Mean estimates significantly larger or smaller than any actually observed value indicate either comprehension or attentional failure. We used only the goal-relevant button estimates since lack of attention to

<sup>1</sup> Equal standard deviations imply that the optimal choice in MAX (MIN) is always the button with highest (lowest) mean.

<sup>2</sup> [52,100] for high buttons; [2,50] for low buttons.

goal-irrelevant buttons could be a deliberate (frugal) learning strategy.

Twenty-nine participants were excluded (seven from MAX, nine from MIN, and thirteen from EO).

**Button Choices** Participants made forced choices in accordance with their goal between (i) all four buttons; (ii-a) the two highest buttons; and (ii-b) the two lowest buttons. EO participants were randomly assigned to choose the largest or smallest each time. For analysis, choices were recoded as correct or incorrect. No significant differences in correctness were found between EO participants who chose largest vs. smallest (Fisher’s Exact Tests: all four buttons  $p=1.0$ ; high pair  $p=.66$ ; low pair  $p=.72$ ), so they were pooled for further analyses. There was a significant effect of button order on the accuracy of choices between all four buttons ( $p<.05$ ), but no effect on choices between the high ( $p=.82$ ) or low ( $p=.22$ ) pairs. Only these latter comparisons provide the critical test for frugal learning, so we pool participants from the two order conditions in those analyses below.

Table 1: Percentage of participants choosing the correct button (goal-relevant choices in bold; \* = significantly different from chance). Chi-Square tests compare correct choice frequencies for the high vs. low choice sets.

	% Correct Choices			Chi-Square	
	All	High	Low	$X^2$	$p$
MAX	<b>77.5*</b>	<b>87.5*</b>	67.5*	26.9	<.05*
MIN	<b>97.5*</b>	45.0	<b>97.5*</b>	4.59	<.05*
EO	<b>90.0*</b>	<b>85.0*</b>	<b>75.0*</b>	1.25	.26

Table 1 shows the percentages of correct choices for each choice set and goal condition. Participants in all conditions did significantly better than chance at choosing from all buttons (MAX:  $X^2(1, N=40)=12.1, p<.05$ ; MIN:  $X^2(1, N=40)=36.1, p<.05$ ; EO:  $X^2(1, N=40)=25.6, p<.05$ ). They also were better than chance for choices involving goal-relevant pairs (high for MAX:  $X^2(1, N=40)=22.5, p<.05$ ; low for MIN:  $X^2(1, N=40)=36.1, p<.05$ ; both for EO: high:  $X^2(1, N=40)=19.6, p<.05$ ; low:  $X^2(1, N=40)=10.0, p<.05$ ). Performance on goal-irrelevant pairs varied; choices for the low pair in MAX were significantly better than chance ( $X^2(1, N=40)=8.1, p<.05$ ), but choices for the high pair in MIN were not ( $X^2(1, N=40)=0.4, p=.53$ ). It appears that at least some MAX participants learned goal-irrelevant information, while MIN participants did not. Crucially, participants in both conditions extracted more information about goal-relevant than goal-irrelevant buttons: they were significantly more accurate at choice for the goal-relevant pair than the goal-irrelevant pair (MAX:  $X^2(1, N=80)=4.59, p<.05$ ; MIN:  $X^2(1, N=80)=26.9, p<.05$ ). In contrast, no significant difference between choice performance was found in the EO condition ( $X^2(1, N=80)=1.25, p=.264$ ).

**Estimates** Mean number estimates are displayed in Table 2; the button labels were recoded to reflect the rank order of

their means (1=lowest, 4=highest). A two-way multivariate ANOVA with the number estimates as the dependent variables and the goal and button order as independent variables<sup>3</sup> revealed a significant multivariate effect of goal condition (Pillai’s Trace=.44,  $F(8, 224)=7.90, p<.05$ ); no significant effect of order (Pillai’s Trace=.07,  $F(4, 111)=2.17, p=.08$ ), and a significant goal-order interaction (Pillai’s Trace=.07,  $F(8, 224)=2.47, p<.05$ ).<sup>4</sup> Post-hoc comparisons were conducted using the Tukey HSD test at the  $\alpha=.05$  significance level. For the two higher buttons, MIN participant estimates were significantly lower (and less accurate) than from MAX and EO participants. There were no significant differences between estimates for the second lowest button. For the lowest button, estimates in the MAX condition were significantly higher (and less accurate) than those in MIN, but there were no differences between either of those conditions and the EO condition.

Table 2: Mean estimates of average number for each of the four buttons (bold indicates goal-relevance)

	1 ( $\mu=20$ )	2 ( $\mu=30$ )	3 ( $\mu=70$ )	4 ( $\mu=80$ )
MAX	27.7	31.9	<b>68.9</b>	<b>76.7</b>
MIN	<b>19.6</b>	<b>28.2</b>	60.6	62.1
EO	<b>23.2</b>	<b>31.9</b>	<b>70.0</b>	<b>76.8</b>

Table 3: Mean differences between estimates (bold indicates goal-relevance)

	Low pair (2 – 1)	High pair (4 – 3)
MAX	4.2	<b>7.8</b>
MIN	<b>8.6</b>	1.4
EO	<b>8.7</b>	<b>6.9</b>

It is somewhat informative that estimates were generally more accurate for goal-relevant buttons than goal-irrelevant ones, but we are more interested in whether people learned about *differences* between the buttons in each pair, goal-relevant and goal-irrelevant. (We assume that both differences are goal-relevant in the EO condition.) Table 3 gives mean differences between estimates for the high-mean buttons (4-3) and for the low-mean buttons (2-1). A two-way multivariate ANOVA with the differences as dependent variables yielded a significant main effect of goal condition (Pillai’s Trace=.14,  $F(4, 228)=4.21, p<.05$ ), but no order (Pillai’s Trace=.01,  $F(2, 113)=.28, p=.76$ ) or interaction effects (Pillai’s Trace=.02,  $F(4, 228)=.55, p=.699$ ).

<sup>3</sup> A MANOVA of only the EO condition revealed no significant differences in mean estimates of participants instructed to choose high vs. low buttons (Pillai’s Trace = .182,  $F(4, 33)= 1.83, p=.146$ ), so these participants are pooled for all further analyses.

<sup>4</sup> Estimates were significantly lower for goal-irrelevant buttons in B-HIGH vs. C-HIGH, regardless of whether the goal-irrelevant buttons were high or low. It is quite unclear what might underlie such a difference. More importantly, this interaction does not affect the estimated difference *between* these two buttons, and thus should not influence the substance of our findings.

Post-hoc comparisons (Tukey HSD with  $\alpha=.05$ ) showed that MIN participants reported a significantly smaller difference between the high-mean buttons than MAX and EO participants (between whom there was no significant difference). In contrast, MAX participants reported a significantly smaller contrast between the low-mean buttons than MIN and EO participants (between whom there was no significant difference). Thus the goal-relevance of the buttons had a significant effect on the difference that participants perceived (or failed to perceive) between them. Paired t-tests revealed a significant difference between estimates for the two low-mean buttons in the MAX condition ( $t(39)=2.80, p<.05$ ), but no significant difference between the estimates for the two high-mean buttons in the MIN condition ( $t(39)=0.77, p=.45$ ) reinforcing the conclusion that MAX participants learned some goal-irrelevant information whereas MIN participants did not.

## Discussion

Both participants' choices and estimates of the button means revealed a significant influence of goal on learning. Participants in the MAX condition learned more about the higher buttons than the lower ones, although they still appeared to learn some information about the difference between the two lower buttons. Participants in the MIN condition learned more about the lower buttons than the higher buttons, and this asymmetry was so strong that they were at chance when choosing between the higher buttons and their estimates of the two button means were not significantly different. Moreover, participants in the EO condition learned the contrasts between both the high and low buttons, so relatively 'complete' learning was possible given the evidence. The evidence thus seems to be that people are at least somewhat frugal: they learn more goal-relevant than goal-irrelevant information.

This finding cannot be due simply to task familiarity, in which participants perform better because of repeated practice or advance knowledge of the task. Participants in the EO condition did not know that they would have to make forced choices, and yet performed well. Similarly, participants in the MIN and MAX conditions did not know they would have to estimate the average outcomes, yet they performed quite well at reporting *goal-relevant* button features. It appears that people focus on information relevant for achieving their goal, and can then use that information in a variety of ways. In contrast, they do not collect (as much) information about goal-irrelevant features.

Substantial further questions remain about the extent of frugality in learning, the conditions (if any) that enhance or mitigate frugality, and the mechanisms that drive this effect. The results of this experiment are mixed on the extent of frugality. MAX participants appeared to be moderately frugal, as they learned some goal-irrelevant information but certainly less than was possible (as evidenced by the performance of the EO participants). In contrast, MIN participants appeared to be more radically frugal, learning little more than the information necessary for their goal.

This asymmetry suggests that there may be conditions that encourage frugality. For example, the MIN condition might be more cognitively demanding than the MAX condition, as people might be more used to tracking large numbers than small ones (since higher numbers are usually better).

This experiment demonstrated that people are relatively frugal learners, but also left open the question of the mechanisms underlying this process. A particularly intriguing question is whether it occurs because of how learners allocate attention, because of how they reason and make inferences, or both. Experiment 2 attempts to distinguish between these distinct possibilities.

## Experiment 2

This experiment aimed to test whether goal effects are mediated purely by attention allocation, or also because of differences in *reasoning* about the goal-relevant objects. We used the same learning paradigm as Experiment 1, though with only MAX and MIN goal conditions. In addition, there were two task conditions that required different information for successful completion of the task. In the ONE condition, participants had to choose a single button in testing (as in Experiment 1), so should try to identify the *single* highest or lowest button. In the MANY condition, participants could choose multiple buttons, and so they should identify *all* buttons with average higher/lower than 50. That is, ONE required people to learn rank order but not average value, while MANY required the people to learn (very rough) averages, but not ranks.

Both ONE and MANY require attention on the same buttons (high-mean in MAX, low-mean in MIN), and so attention allocation should be the same. Thus, if goal-directed learning is due solely to attention modulation, then learning should be the same in the two conditions. However, if goal-directed learning involves goal-directed reasoning or encoding during learning, then MANY participants should learn less about the difference between the average values for the goal-relevant buttons, as that difference is irrelevant to that goal.

## Participants

96 Mechanical Turk participants (mean age=36.0; 46% female) were randomly assigned to a goal (MIN/MAX) and task condition (ONE/MANY). Participants were paid 50 cents for participation and 50 cents based on performance. 16 participants were excluded (see Results below), leaving 80 participants (mean age=36.8; 47% female) for analysis.

## Method

Instructions were identical to Experiment 1, except that participants were told that they would be paid based on the amount that the final button outcome was higher/lower than 50. In the MANY condition, participants were also told they would be able to press as many buttons as they wished.

The learning phase was identical to Experiment 1, except that all participants were assigned to the B-High button order. The critical test in this experiment is between the

ONE and MANY participants within each goal condition (MAX/MIN), so controlling for button order between goal conditions was unnecessary. The only difference in the testing phase was that participants in the MANY condition were able to select more than one button in the four-button forced choice.

## Results & Discussion

**Exclusion Criteria** The same exclusion criteria were used. 16 participants were excluded from analysis (one from MAX-ONE, one from MAX-MANY, nine from MIN-ONE, and five from MIN-MANY). The asymmetry in exclusion numbers suggests that the MIN conditions yielded less comprehension or less diligence. However, the critical comparisons are within, rather than across, goal conditions, and so this asymmetry should not impact the results.

**Button Choices** Table 4 compares the percentage of correct button choices for each goal and button pair in the ONE and MANY conditions. Chi Square tests (right hand column) revealed no significant differences between these conditions, even when the choice was goal-relevant in the ONE condition but not the MANY condition (e.g. high pair for MAX, low pair for MIN).

Table 4: Percentage of participants choosing the correct button (\* if sig. different than chance, bold if goal-relevant).

		% Correct Choices		Chi-Square	
		ONE	MANY	$X^2$	$p$
MAX	high pair	<b>90.0*</b>	85.0*	0.23	.63
	low pair	75.0*	60.0	1.03	.31
MIN	high pair	60.0	70.0	0.44	.51
	low pair	<b>80.0*</b>	70.0	0.06	.81

**Estimates** The mean number estimates are shown in Table 5. A two-way multivariate ANOVA with number estimates as the dependent variables and the goal and test task as the independent variables revealed a significant multivariate effect of goal (Pillai's Trace=.26,  $F(4, 73)=6.47, p<.05$ ), but no significant effect of task (Pillai's Trace=.06,  $F(4, 73)=1.12, p=.36$ ) and no interaction (Pillai's Trace=.02,  $F(4, 73)=0.45, p=.36$ ), suggesting that task type did not have an effect on the participant's judgments.

Table 5: Mean estimates of average number for each of the four buttons (bold indicates goal-relevance)

	1 ( $\mu=20$ )	2 ( $\mu=30$ )	3 ( $\mu=70$ )	4 ( $\mu=80$ )
MAX-ONE	23.4	29.5	<b>68.1</b>	<b>76.4</b>
MAX-MANY	25.0	28.4	<b>69.8</b>	<b>75.9</b>
MIN-ONE	<b>19.1</b>	<b>27.6</b>	64.2	66.7
MIN-MANY	<b>22.7</b>	<b>30.0</b>	68.0	62.6

Table 6: Mean differences between estimates (bold indicates goal-relevance)

	Low pair (2 – 1)	High pair (4 – 3)
MAX-ONE	6.1	<b>8.3</b>
MAX-MANY	3.4	6.1
MIN-ONE	<b>8.5</b>	2.5
MIN-MANY	7.3	-5.4

As in Experiment 1, the critical comparison is whether participants learned the *difference* between buttons within the high and low pairs (see Table 6). A two-way multivariate ANOVA with the two comparison variables (4-3 and 2-1) revealed a significant effect of goal (Pillai's Trace=.17,  $F(2, 83)=8.36, p<.05$ ), but no task (Pillai's Trace=.05,  $F(2, 83)=2.15, p=.12$ ) or interaction (Pillai's Trace=.02,  $F(2, 83)=0.81, p=.45$ ) effects, which reinforces the conclusion that participants learned differently between the two goals, but not between the two test phase tasks.

**Discussion** The results of Experiment 2 suggest that goal-dependence of learning arises principally because of attention allocation. We did not find evidence of differential information processing or encoding after attention has been allocated. Instead, it appears that people encode and process the button information similarly whenever the button is goal-relevant. That is, Experiment 2 is suggestive that a key mechanism in the goal-dependence of learning is attention allocation, with people focusing on the goal-relevant information. Of course, drawing any conclusions from a null result is difficult, and more experiments are clearly necessary before any general conclusions can be reached. Nevertheless, these results suggest that attention plays a key (and perhaps essential) role in the influences of goals.

## General Discussion and Conclusion

These experiments clearly demonstrate that people's longer-term goals (i.e., not just learning tasks) influence their learning. Experiment 1 showed that people are relatively frugal about what they learn: they represent goal-relevant information significantly more accurately than goal-irrelevant information. However, this frugality is not absolute, as the results of Experiment 2 (and some results in Experiment 1) suggest that people sometimes acquire richer representations than are strictly necessary for task success. Experiment 2 suggests that attention allocation plays a key role in goal-dependent learning, but that goal-relevant reasoning does not (at least, in this context). Other factors remain to be investigated: perhaps frugality occurs only under high cognitive load, or perhaps involves differential processing in more complex domains. We are far from understanding the conditions and mechanisms responsible for frugal learning. However, the experiments presented here suggest that this is an area ripe for future research.

To the extent that people are frugal in their learning, this poses a significant challenge to the widespread practice of focusing on cognitive models that assume people have

purely epistemic goals (e.g., most rational models of learning and reasoning). However, it is not clear whether this is an inherent weakness of these modeling paradigms, or a byproduct of the lack of attention paid to pragmatic goals in current learning research. Most models of ‘rational’ learning are silent on the normative question of how goals *should* influence learning, and instead focus on how people should make inferences in order to arrive at true (or at least justified) conclusions. Bayesian inference provides a clear (and influential) example, but our discussion here applies equally well to any axiomatic theory of rationality (e.g. deductive inference).

Bayesian models (Oaksford & Chater, 2007; Griffiths, Kemp, & Tenenbaum, 2008) assume that learners (i) have degrees of belief that are compatible with a probability distribution over a set of mutually exclusive and exhaustive hypotheses; and (ii) update this belief distribution by conditioning on new evidence. Crucially, the Bayesian updating procedure (Bayes’s Rule) is goal-invariant; its behavior depends only on the prior probabilities of the hypotheses and the probabilities of the evidence conditional on each possible hypothesis. The Bayesian model assumes that learners are only trying to determine the most probable hypothesis (or hypotheses) from among the previously specified possibilities. Thus, non-epistemic goals can influence learning only through the initial specification of a hypothesis space, or in the assignment of prior probabilities.

Standard practice in Bayesian models of psychological phenomena is to use a hypothesis space covering all possible distinctions (e.g., all possible causal structures, all possible category schemes, etc.), but this choice is decidedly non-frugal and perhaps even sub-optimal. In the (very) long run, selecting a hypothesis space with maximally many distinctions will ensure that you learn as much as you can from the evidence. In the short run, however, such a space may be detrimental because it can increase the variance of the learning method and make overfitting more likely (for an extended discussion see Wellen & Danks, under review).

An alternative is to use a hypothesis space that contains only goal-relevant distinctions. For instance, suppose you are a participant in Experiment 1 and your goal is to choose the button that produces the highest number. In this case, the hypothesis space need only distinguish between ‘worlds’ with different highest mean buttons. It need not encode the precise means or even the rank order of the suboptimal buttons; these are all irrelevant to the test phase choice. A frugal Bayesian learner could thus select a hypothesis space  $H$  with variation on only the relevant dimension, such as:  $H = \{a \text{ is best, } b \text{ is best, } c \text{ is best, } d \text{ is best}\}$ . In the Bayesian framework, however, the choice of hypothesis space occurs before the model can be applied. Hence, although frugality can be captured indirectly, it requires conceptual resources from outside the model.

There are thus ways to (try to) incorporate non-epistemic goals in those models, but that theoretical work remains to be done. More generally, we conjecture that many standard models of learning can incorporate goal effects only through

the way that they represent the learning problem (e.g., the assumed hypothesis space). If that is correct, then any attempts to model goal effects in learning will be forced to include aspects of the situation that have previously been simply stipulated by the theoretician (see also Wellen & Danks, under review). Moreover, there is likely to be a complex interaction between prior beliefs, goals, and evidence, which will further complicate matters. Of course, we are far from understanding how these (and other) factors interact to influence learning, but this paper provides some initial empirical constraints.

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