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Uncertainty in Geospatial Information Representation, Analysis, and Decision Support

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Uncertainty in Geospatial Information Representation, Analysis, and Decision Support

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under the NURI University Research Initiative.
Supported by the National Science Foundation as part of the
“Advancing Geographic Information Science” award to NCGIA.

Introduction

This document provides the research goals, primary investigators (PIs), researchers and overviews of the investigated topics of the NCGIA project “Uncertainty in Geospatial Information Representation, Analysis, and Decision Support”. A bibliography of relevant literature is also included.

Research Goals

The goal of the research is to improve the mechanisms by which users of the defense geospatial information infrastructure are made aware of the presence of uncertainty in data, and its implications for decision-making. Six areas are being addressed in a coordinated approach- elements of uncertainty, measures, models, portrayal, propagation, and higher dimensions. The research is being conducted by five teams, with frequent opportunity for cross-fertilization between them. Underlying our approach to this research are the following broadly defined objectives:

1. to extend the current level of understanding of these six elements through basic research;
2. to apply the results of research under the conditions typical of defense GII- real data sets of large volumes, real environments for visualization, real forms of analysis;
3. to deal with special needs of defense GII- e.g., operational conditions, rapid decisions, particular data types, existing GII standards;
4. to improve access to the results of research by implementing them in practical tools, primarily through collaborative efforts with software vendors, data custodians, and other geographic information scientists and technologists;
5. to address the needs of users as well as creators of data, by including concern for their cognitive abilities and levels of understanding of geospatial concepts.

Project Personnel

Principal Investigators

Daniel Montello (UC Santa Barbara)
Michael Goodchild (UC Santa Barbara)
Kate Beard (University of Maine at Orono)
Keith Clarke (UC Santa Barbara)

Researchers

Alex Keuper (UC Santa Barbara)
Ashton Shortridge (UC Santa Barbara)
Thomas Windholz (University of Maine at Orono)
Paul Teague (UC Santa Barbara)
Peter Fohl (UC Santa Barbara)
Phaedon Kyriakidis (Stanford)
Gary Hunter (University of Melbourne)
Aggrey Agumya (University of Melbourne)

Topic 1: Elements of Uncertainty

Prime: Dan Montello, Michael Goodchild

Research Assistant: Alex Keuper

Objectives

- Identify a set of metadata elements for the various components of the GII system which will reduce uncertainty.
 - Identify for further research a set of uncertainty elements which can be measured and modeled.
-
- Develop model of data life and roles of uncertainty in it; Examine ability of users to comprehend uncertainty concepts
 - Develop methods for evaluating importance of database specification elements in resolving uncertainty
 - Extend treatment of uncertainty to the temporal case; Design and conduct human-subjects experiments

Topic 2: Measures of Uncertainty

Prime: Michael Goodchild, Kate Beard

Research Assistant: Ashton Shortridge

Objectives

- Develop and evaluate measures of uncertainty as well as measures of uncertainty reduction
 - Research the procedures and protocols that will be needed to assess uncertainties in NIMA products, and to test them at realistic scales
 - Develop measures of uncertainty reduction based on metadata elements
 - Research and implement measures of uncertainty as functions of metadata
 - Investigate tradeoffs of computing versus storing measures of uncertainty
 - Develop and implement methods for measurement of the spatial structure of uncertainty, and apply them to real data sets in the GII

Topic 3: Models of Uncertainty

Prime: Kate Beard, Keith Clarke

Research Assistant: Thomas Windholz

Objectives

- Capture the relations among the components of the information system
 - Capture relations among the metadata for each component
 - Capture lineage metadata within and between components for direct support of measurement
 - Support updates within and across components
 - Support queries within and across components
-
- Select the appropriate representational schema for uncertainty
 - Evaluate existing methods for specifying and tracking lineage
 - Formalize the information system components of GII for incorporation and management within a computational system
 - Implement prototype database architecture in collaboration with industry partners

Presentation Notes

1. Introduction to modeling uncertainty in spatial data

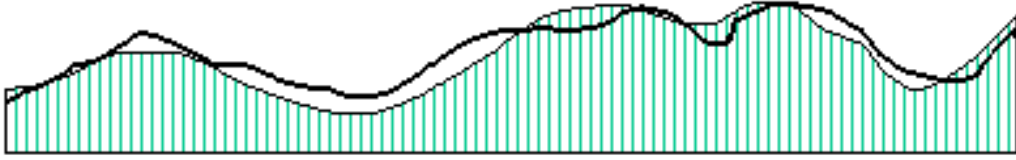
Modeling Spatial Data Uncertainty: Framework

Data --> GIS application/function --> Result

Examples:

- DEM --> path location algorithm --> optimal route
- Land Use map, --> siting procedure --> franchise location

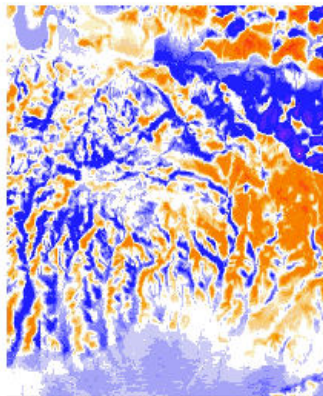
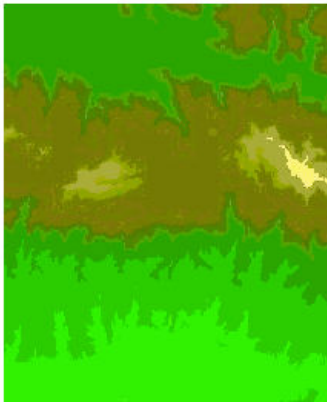
Uncertainty in Spatial Data: Discrepancy between data sets and the spatial characteristic the data set represents. Here, the focus is not on sources of error, but in measuring and modeling it. Below is a terrain profile with a DEM transect (the thick line). Both positive and negative error are present in the DEM.



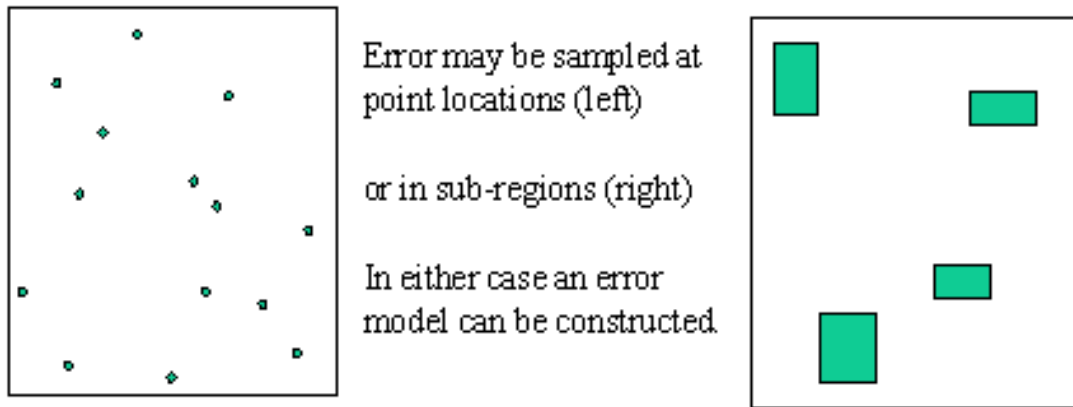
The following image shows an error map, with peaks of positive error and pits of negative error. Error is spatially autocorrelated in the map below; large positive errors tend to be clustered together, as do negative errors.



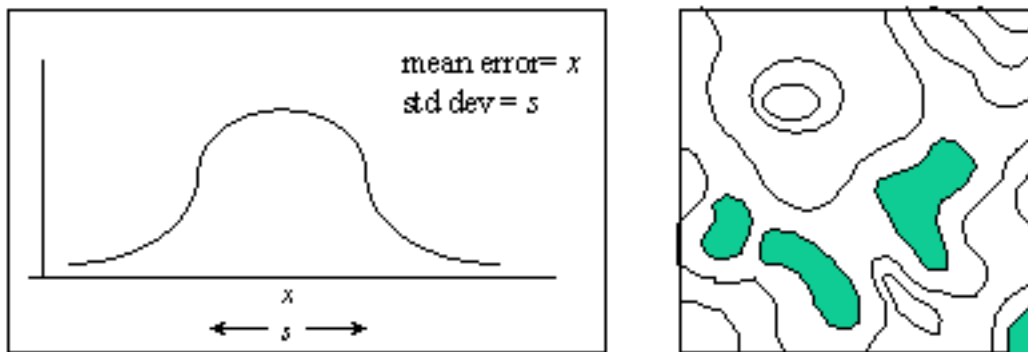
In the example below (Terrain and Error maps of the Santa Ynez Mountains), the error surface is known. But... If the error is not exhaustively specified across an area of interest, uncertainty about the relationship of data to the real world exists.



With some information and some assumptions, this uncertainty may be modeled, and its effect on GIS applications measured. See below:



An uncertainty model is constructed using parameters from the sample error distribution and measures of spatial autocorrelation of the sample error data. This model can be used to generate simulated error surfaces for regions lacking exhaustive error information. A simulated error surface is one with properties matching that of the available error data.



Monte Carlo Simulation:

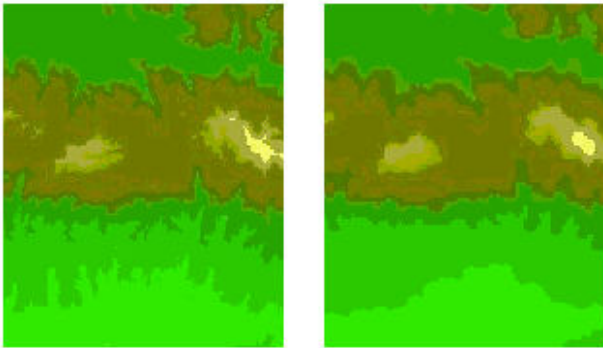
Generating just one simulated error map is not enough. Under Monte Carlo simulation, a large number of equiprobable error surfaces are generated. By running a GIS application of interest on a large number of equiprobable spatial data sets, a distribution of outcomes results. Uncertainty in the spatial data is propagated to the results of a GIS application.



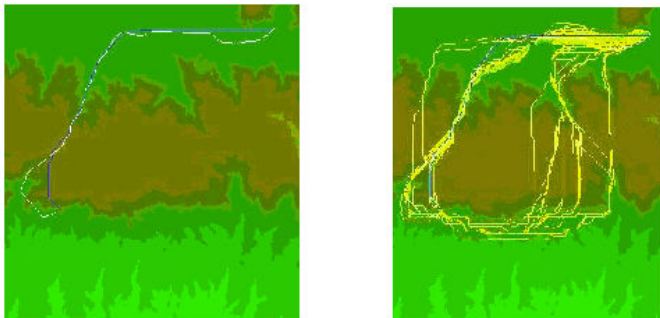
An example of this approach:

- Objective: A route must be chosen to connect two locations on opposite sides of the Santa Ynez Mountains. Specifications indicate that 30m DEM data will be adequate to identify the best route.
- Problem: Only 3 arc-second (90m) DEM data is available for the region in question.
- Question: If we have error information for nearby locations, can we characterize the uncertainty that is introduced due to the use of the coarser 90m DEM?

30m vs. 90m DEMs:



Two paths vs. 247 paths from 247 realizations:



Summary:

- Error models can be developed for spatial data
- These models produce a large set of simulated error realizations
- A GIS application is performed upon these realizations, producing a distribution of results.
- By studying these results, the scientist or decision maker can assess the impact of spatial data uncertainty upon the application
- In the test case shown here, it may well be decided that higher quality data must be collected in order to ensure that the right route is identified.

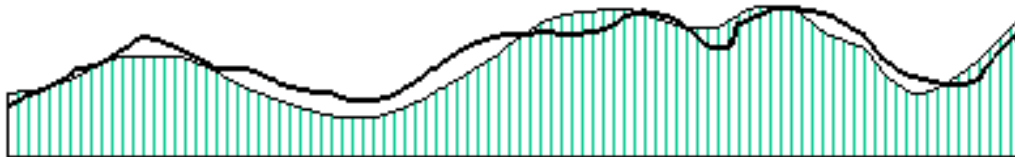
2. Encapsulating simulation models with geospatial data sets

By Michael F. Goodchild, Ashton M. Shortridge, & Peter Fohl (NCGIA, UC Santa Barbara)

Data Uncertainty

Uncertainty is the discrepancy between data and the spatial characteristic represented by the data. Most spatial data is subject to some uncertainty, and this uncertainty affects the reliability of applications using the data. An important goal of data quality reporting is to ensure that users are able to decide if a data set is adequate for their needs.

Below is a terrain profile and a DEM transect (the thick line). Both positive and negative error are present in the DEM.



Uncertainty reporting

Spatial data products include metadata, “that which makes data useful”. Global accuracy/quality measures are included in metadata to give data users a sense of the uncertainty in data. Root Mean Square Error (RMSE) is typical for DEMs and Percent Correctly Classified (PCC) for land classification data. Commonly reported measures like these are not adequate for characterizing spatial data uncertainty, as the example in the following section suggests.

Failure of current accuracy standards

What are users to make of a global accuracy value for a spatial data set?

- Ex: A forester wishes to use a USGS DEM to help identify promising sites for a new fire tower. The forester has calculated the size of the viewshed for a set of locations, and is interested in determining how closely the calculated viewshed matches the actual viewshed at these sites. How can the forester use the 7 meter RMSE for the DEM to determine the quality of the viewshed calculations? In the absence of further assumptions, this statistic is of no benefit.

A new paradigm for reporting uncertainty

Data Producers:

- Replace current accuracy information with an “uncertainty button” in metadata
- Button ties a simulation method to the data quality report, replacing a static measure with a modeling method

Data Users:

- Employ one or more simulated realizations to produce a distribution of potential outcomes
- Analyze this distribution to gain an understanding of how uncertainty in the data affects their applications.

Metadata viewer with an ‘uncertainty button’

Pressing the button creates a set of data simulations based on an uncertainty model developed by the producer. These simulated datasets (here, DEMs) can then be processed in a GIS.

- Ex.: The forester runs the viewshed operation on a number of simulated DEMs and uses the distribution of results to decide if the data is adequate for that application.

Implications

Data producers can use uncertainty models and simulation methods to generate realizations of data incorporating uncertainty information. Data server could generate realizations on-the-fly and distribute them as users require, or a set of realizations could be stored along with the data. Simulation can occur on the user’s machine. The user then performs some process locally on each realization, generates a distribution of results, and can perform rigorous statistical tests to see how uncertainty in the data has affected results.

Summary

Coupling uncertainty models to simulation methods is a general method for describing uncertainty in spatial metadata. Users can use a metadata uncertainty button to generate a set of simulated realizations of the data with uncertainty “added in”. A GIS application is performed upon these realizations, producing a distribution of results. By studying these results, the scientist or decision maker can assess the impact of spatial data uncertainty upon the application. This demonstration used a very simple uncertainty model, but the method itself is generally applicable to any type of spatial data and any uncertainty model.

Topic 4: Portrayal of Uncertainty

Prime: Keith Clarke, Dan Montello

Research Assistant: Paul Teague

Objective

- Review, develop, and test cartographic representational methods for dealing with uncertainty.
 - Develop Tools
 - Test Symbolization & Display Tools
 - Test Methods

This portion of the project began with a comprehensive review of existing techniques for uncertainty portrayal. Bertin's visuable variables, animation, 3dimensional display, and other methods were investigated and a representation matrix was developed. Of these, color, animation, and depth manipulation offer the most to cartographers for uncertainty representation. The Virtual Reality Modeling Language was chosen as the most widely applicable tool for development and testing of these methods. SGI's Cosmo World's VRML authoring system combined with General Reality's CyberEye head-mounted display give researchers the ability to utilize a wide array of symbolization techniques including interaction, navigation, and depth cue information.

Presentation Notes

1. Cartographic Symbolization of Uncertainty

By Keith Clarke and Paul Teague (NCGIA, UC Santa Barbara)

Introduction

- Cartographers concerned with accuracy and reliability of source information
- Hard vs. soft data
- Parallel display of uncertainty
- Integrated display of uncertainty

'Hard' vs. 'Soft' Data

- Hard data of known accuracy, precision, and lineage
- Soft data
 - may contain positional inaccuracy

Uncertainty in Geospatial Information Science (1998)

- may lack precision due to inadequate source scale
- may have attribute or feature uncertainty
- may be of dubious lineage
- GIS and paper maps treat all data as hard

Parallel Display of Uncertainty

- Uses multiple displays or small multiples to communicate error
- Uncertainty portrayal is treated as peripheral information
- Subjected to interpretation after the primary hard data have been read

Integrated Display of Uncertainty

- Seeks to recognize degree of uncertainty associated with features
- Integrates uncertainty into cartographic symbolization - active uncertainty
- Uncertainty integration must not create additional user error

Existing Research and Methods

- Bertin variables
- Animation
- 3D display of uncertainty
- Other methods

Bertin Variables

- Beard, Battenfield, and Clapham (1991)
- MacEachren (1992, 1994)
- McGranaghan (1993)

Beard, Battenfield, and Clapham (1991)

- Mapped representational methods for uncertainty into matrix of data types
- Size, shape, and color suggested for point and line data
- Texture, size, color value, and mixing outlined for categorical data
- Color value, saturation, and possibly size and shape suggested for continuous data
- Hypermedia, animation, and virtual reality worthy of further research

MacEachren (1992, 1994)

- Size and value most suitable for numerical data
- Hue (color), shape, and orientation useful for nominal data
- Favored color saturation (not a Bertin variable) for uncertainty portrayal
- Added variable of focus that could be changed for features by manipulating edge crispness, fill clarity, resolution, or by adding fog
- Tested methods using map pairs, map sequences, and bivariate maps for uncertainty representation

McGranaghan (1993)

- Favored explicitly encoding and attaching data quality information to point, line, and area entities
- Favored creating visual and cognitive ambiguity in a display using Bertin variables as well as focus, realism, time, and interaction
- Work suggested that existing cartographic methods offered promise for the display of uncertainty

Animation

- Projection, animation, time, and sound added as potential variables by Unwin and colleagues (1994)
- Peter Fisher used animation to display the set of possible maps created by multiple equiprobable outcome maps or realizations (1993, 1994, 1996)
 - combined qualitative and quantitative approaches to point, line, and area data
 - used soil maps, imagery, dot maps, and DEMs as test cases
- Ehlschlaeger, Goodchild, and Shortridge used single frame sequential animation of equiprobable stochastic images for line (shortest paths) and DEM data (1994, 1996, 1997)
- Davis and Keller considered animation as separate suite of portrayal methods (1997)

3D Display of Uncertainty

- Vectors, glyphs, and multivariate point symbols have been used in 3D space by researchers of Santa Cruz Laboratory for Visualization and Graphics
- In many cases, uncertainty views are multiple displays separate from the hard data

Other Methods

- Checkerboard grids
- Color variation
- Methods to show geometric pattern of distortion
- Few of these are suitable for display behind a hard map
- Color method shows most promise

DIMENSION OF DATA TYPE	POINT	LINE	AREA	VOLUME	TEMPORAL
NUMERICAL	COLOR ANIMATION SOUND VIRTUAL POSITION SIZE SHAPE THICKNESS GLYPHS TRANSPARENCY	COLOR ANIMATION SOUND VIRTUAL POSITION SIZE SHAPE THICKNESS GLYPHS TRANSPARENCY	COLOR ANIMATION SOUND VIRTUAL POSITION SIZE SHAPE THICKNESS GLYPHS TRANSPARENCY	COLOR ANIMATION SOUND VIRTUAL POSITION TRANSPARENCY GLYPHS	COLOR ANIMATION SOUND GLYPHS TRANSPARENCY
CATEGORICAL	COLOR ANIMATION SOUND GLYPHS SHAPE VIRTUAL POSITION TRANSPARENCY	COLOR ANIMATION SOUND GLYPHS SHAPE VIRTUAL POSITION TRANSPARENCY	COLOR ANIMATION SOUND GLYPHS SHAPE VIRTUAL POSITION TRANSPARENCY	COLOR ANIMATION SOUND GLYPHS VIRTUAL POSITION TRANSPARENCY	COLOR ANIMATION SOUND GLYPHS TRANSPARENCY
CONTINUOUS	COLOR ANIMATION SOUND SIZE TRANSPARENCY	COLOR ANIMATION SOUND SIZE TRANSPARENCY	COLOR ANIMATION SOUND SIZE TRANSPARENCY	COLOR ANIMATION SOUND TRANSPARENCY	COLOR ANIMATION SOUND TRANSPARENCY

Cartographic visual variables (Bertin variables) suitable for use as basis of new representational techniques for the visualization of error and uncertainty on maps

Purpose

- Begin by considering positional error only
- Result of method should be suitable for display behind the hard data without distracting the map reader
- Make use of existing rather than totally new methods to minimize training and user error
- Depict magnitude of error in addition to spatial distribution of both magnitude and direction of error
- Use of color should be numerically sound and perceptually correct

Assumptions

- Assumptions dependent on type and nature of error measurement in use
- Method limited to positional accuracy in three dimensions

Uncertainty in Geospatial Information Science (1998)

- Point, line, and area measurements, and therefore accuracies, are spatially autocorrelated
- It is possible to interpolate local error measurements over space
- To compare two maps even on the same coordinate system, one must be assumed the control and one the measurement

Methodology

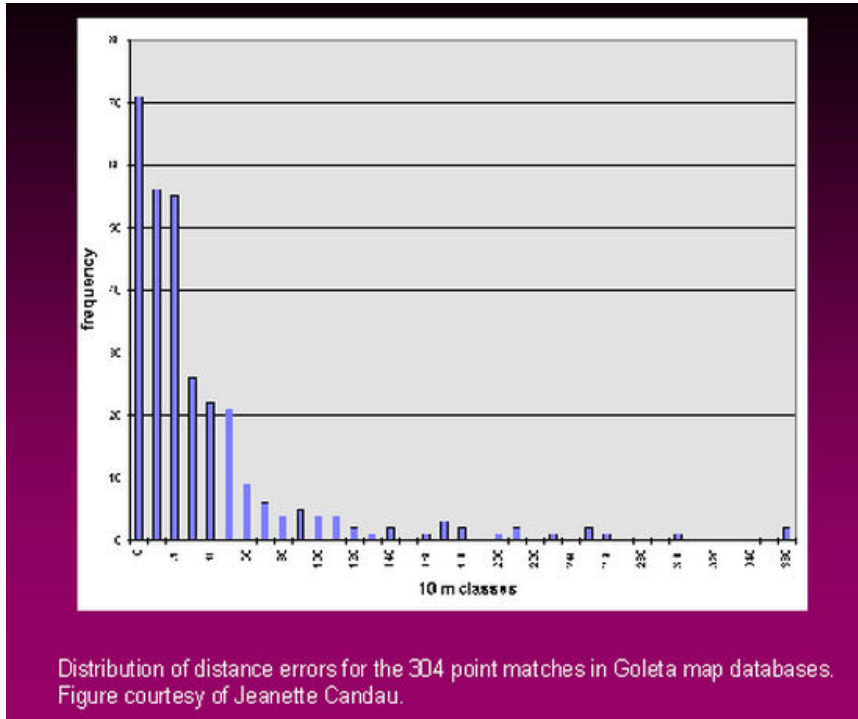
- Color selected based on uniform applicability to static maps
- Test data set selected from VITAL case study
- Maps overdrawn and street intersections used as point data
- Set of comparison points determined
- Least squares six parameter affine fit conducted
- Data entered into a spreadsheet with map A (u,v) set as the source and map B (x,y) as the erroneous data
- 304 points in UTM projection used for fit



Section of the Goleta map databases with highlighted corresponding nodes shown in both their locations, connected by a vector. Map courtesy of Kevin Curtain, NCGIA.

Results

- Magnitude of error ranges from 0 to 366.75 meters with a mean of 41.46 meters and high standard deviation of 53.33 meters
- Distribution clearly not random
- Shows tendency for systematic error

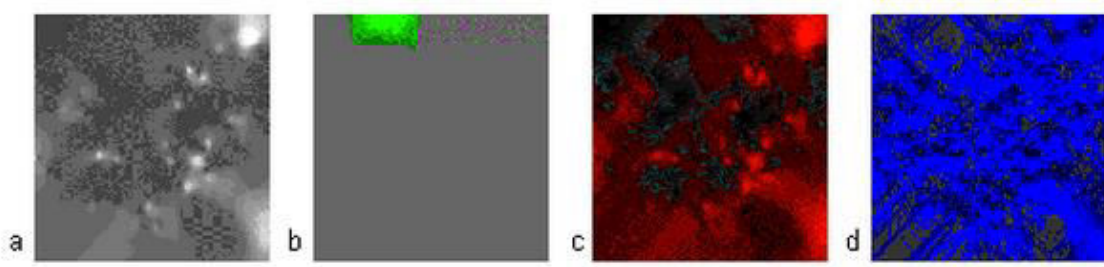


Results (Continued)

- Computed values for the six affine constants
 - $u = 1.000000 x + 0.000000 y + 0.000000$
 - $v = 0.004842 x + 0.986290 y + 51123.742188$
- Perfect on average match between maps in x direction
- Significant mismatch in y direction
- Residual aggregate error vector computed for every point
- Vector represents distance and direction each node would be moved during affine fit of data sets
- At each point in map, distance vector calculated from inverse of six parameter affine transformation
- Values interpolated into grid and decomposed into
 - east-west distance

- north-south distance
- direction, as the aspect of the vector

Below is the Goleta, CA test map transformed into “error” space using the six parameter affine. From the left: a. Raw distance of the rubber-sheeting transformation interpolated from 304 match points. b. Scaled north-south distance c. Scaled north-south distance d. Scaled aspect of the error vector.



And here are: a. a. Magnitude, b. north-south component of error, and c. east-west component of error depicted through hill shading.



Results (Continued)

- Most information-rich backdrop for the data combines the two different (x and y) magnitude components of the error and vector direction information
- Error components scaled and mapped to RGB color scheme
 - east-west component to red
 - north-south component to green
 - direction to blue
- Conversion to HSI and absolute color scaling would be necessary to balance colors perceptually

Conclusion

- Modified Bertin variables of color and animation seem to hold greatest potential for cartography of uncertainty display
- Error surface can be built that yields to some forms of standard cartographic display such as hill shading
- Color combination method misrepresents the variable of hue by failing to use the Munsell color scheme
- Methods explored all reveal the segregated random and systematic directional nature of the error in the case study

2. Cartographic Representation of Positional Error in Linear Features Using Virtual Worlds

By Paul D. Teague (NCGIA, UC Santa Barbara)

Overview

- Background
- VRML and cartographic presentation
- Case study: Goleta, California
- Shortcomings of VRML for cartography
- Conclusion

Objectives

- Part of broader NIMA work on mapping positional and attribute uncertainty
- Create an environment to develop and test cartographic presentation methods
- Combine traditional cartographic portrayal techniques, animation, and 3-dimensional display

Background

- Uncertainty portrayal- existing research
 - Bertin's variables
 - Animation
 - 3-dimensional display
- What is VRML?
 - 3-d interchange format
 - Defines objects and user interaction

Definitions

- VRML
- Accuracy

Uncertainty in Geospatial Information Science (1998)

- Positional uncertainty
- Attribute uncertainty
- Assume underlying model (feature or field)

Methods for Portrayal

- Extended set of Bertin's variables
- Animation
- 3-D display: Manipulation of depth

Research Environment

- Use SGI's CosmoWorlds for VRML authoring
- Use CosmoPlayer for VRML browsing and web interface
- Test data consist of Census TIGER derivatives, aerial photography, and a portion of the Goleta DEM

Test Environment

- Adapt Head-Mounted Display for use with SGI hardware
- Create external control mechanism for VRML browser
- Define user abilities (interaction) in virtual environment to optimize information display

VRML and Cartographic Presentation

- Tools defining user interaction
- Symbolization tools

Tools Defining User Interaction

- Navigation information
- Level of Detail (LOD)
- Sensors
- Viewpoints

Symbolization Tools

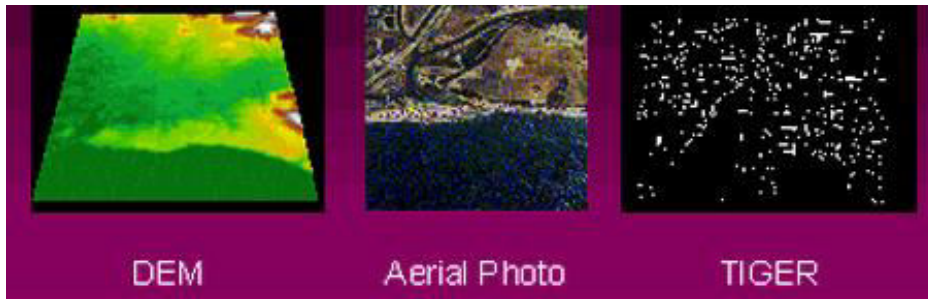
- Animation
 - Interpolators
 - Frame rate
 - Frame order
- Hyperlinks
- Sound

Case Study

- Source data
- Data assembly in VRML
- Interaction defined

- Symbolization of uncertainty

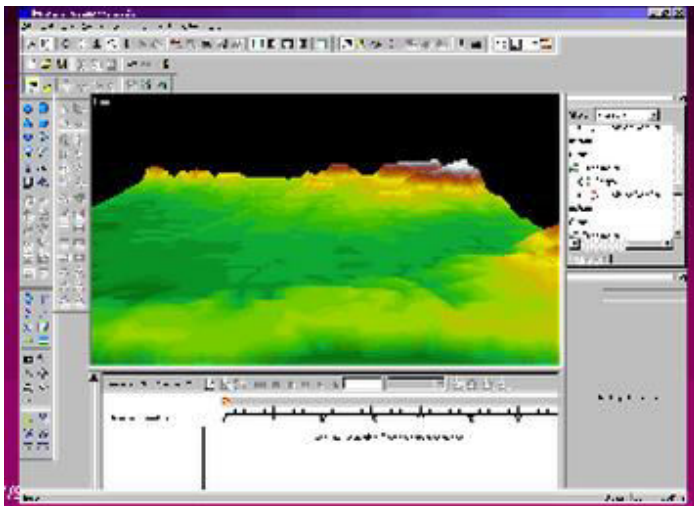
Sources of Test Data



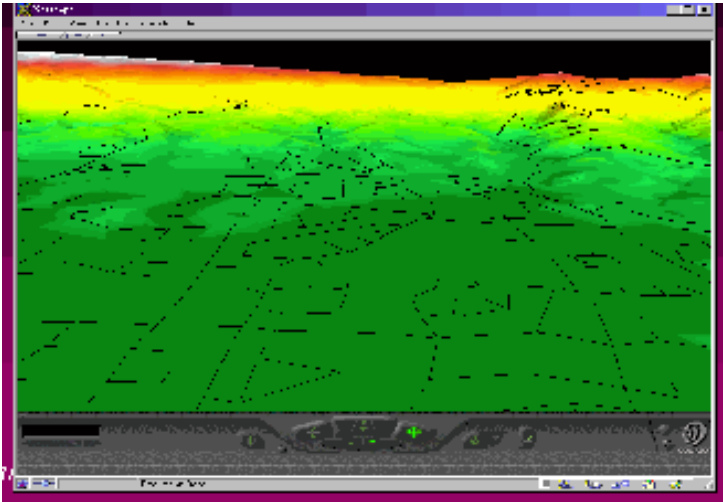
Assembled Test Data



CosmoWorlds VRML Authorizing Package



CosmoPlayer Interface



Shortcomings of VRML

- Graphic rendering
- Need geo-standards, e.g, coordinates
- VRML limited to single precision floating point values
- Limited to four levels of detail

Conclusion

- VRML valuable
- Encourages links with fields of Psychology and Computer Science
- Future determined by ability of VRML to accommodate geospatial information
 - GeoVRML Working Group
 - VRML standard internet 3-d format?

Future Work

- Conduct human subjects tests
- Create external control supporting Head-Mounted Display
- Implement multiple levels of detail to allow scale-dependent symbolization

Topic 5: Propagation of Uncertainty

Prime: Michael Goodchild, Keith Clarke

Research Assistant: Ashton Shortridge

Objective

- Determine the effects of uncertainty on decisions by propagating it through the various stages of the decision-making process
- Evaluate the status of efforts to implement propagation techniques for uncertainty in geospatial data
- Extend our understanding of the effects of uncertainty propagation on decision-making, and integrate with other research results

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