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Authors
Schurr, Friederike
Tam, Brian P.
Maloney, Laurence T.

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Learning Patterns in Noise: Environmental Statistics Explain the Sequential Effect

Friederike Schüür (fs62@nyu.edu), Brian Tam (bpt218@nyu.edu), Laurence T. Maloney (ltm1@nyu.edu)
Department of Psychology, 6 Washington Place
New York, NY 10003 USA

Abstract

Effects of trial history, or sequential effects, are typically observed in perceptual, motor, and decision making tasks and explained by subjects' irrational sensitivity to local patterns in stimulus history. We propose that in 2 alternative forced choice reaction time tasks (2AFC), sequential effects are a consequence a rational agent engaging in probability learning but with an inappropriate world model for 2AFC. We manipulate subjects' world model and show expected changes in sequential effects. Sequential effects are at least in part driven by subjects' beliefs about their environment.

Keywords: sequential effects; two alternative forced choice reaction time tasks; Bayesian modeling

Sequential Effects

Subjects display sensitivity to local patterns in stimulus history in perceptual (Howarth & Bulmer, 1956; Maloney, Martello, Sahm, & Spillmann, 2005), motor (Cho et al., 2002; Remington, 1969; Soetens, Boer, & Hueting, 1985), and decision making tasks (Ayton & Fischer, 2004; Gilovich, Vallone, & Tversky, 1985). In two alternative forced choice reaction time tasks (2AFC), for example, subjects' reaction times (RTs) depend not only on the current stimulus but also on the sequence of preceding stimuli (Cho et al., 2002; Remington, 1969), a phenomenon known as *sequential effects* (SQE). In addition, participants typically respond faster to an alternation of stimuli after a run of alternations compared to a repetition of stimuli after a run of repetitions (Soetens, et al., 1985). We refer to this finding as *alternation bias* in SQE. While alternation bias seems more common, *repetition bias* has been observed, too (Cho et al., 2002). We here ask what processes give rise to biased SQE.

SQE are in part determined by the time interval between subsequent stimuli (inter-trial interval). If this interval is short (< 500 ms), SQE are driven by automatic facilitation (Bertelson, 1961; Soetens et al., 1985). Responses to repeated stimuli benefit from residual activation left by previous stimulus-response cycles and consequently, RTs to repeated stimuli are faster while responses to alternating stimuli are slower (Soetens et al., 1985). If the inter-trial interval is long (> 500 ms), SQE are driven by subjective expectancy. Subjects use the sequence of preceding stimuli to predict the next, upcoming stimulus and consequently, a run of alternations induces expectancy for more alternations while a run of repetitions induces expectancy for more repetitions (Soetens et al., 1985).

But in typical 2AFC tasks, the sequence of preceding stimuli does not predict the next, upcoming stimulus – a repetition of stimulus X does not increase the probability of stimulus X compared to stimulus Y. In other words, stimulus history has no predictive value and should not affect subjects' expectancy (or RTs if the inter-trial interval exceeds 500ms). Why do we find persistent SQE (after > 4000 trials) (Soetens et al., 1985) in 2AFC tasks?

Previously, SQE in 2AFC tasks with long inter-trial intervals were cast as instances of irrational sensitivity to local patterns in stimulus history, presumed to give rise to other suboptimal behavior, like the gambler's and hot-hand fallacy in decision making (Ayton & Fischer, 2004; Gilovich et al., 1985). Instead, we propose that SQE effects are driven by subjects' attempts to learn the probability of occurrence of the two stimuli in 2AFC with a world model that, while ecologically plausible, does not match the true generative model of the task.

In 2AFC tasks, which out of two stimuli is going to appear on each trial is sampled from a Bernoulli distribution. With probability p one stimulus will appear and its alternative with probability $1-p$. In common 2AFC, probability p is constant throughout the experiment (or at least throughout an experimental block). The true generative model is thus a Bernoulli distribution with constant probability p. Participants could learn probability p by using stimulus history to update estimated probability $\hat{p}$ using, for example, Bayesian updating (Gerhard, Wolfe, & Maloney, under review).

But participants may believe that instead, probability p changes over time. In other words, instead of a stable world with constant probability p – the true generative model of 2AFC – participants may believe in a dynamic world with changing probability p_t. We propose that such belief could give rise to biased SQE: biased SQE are a consequence of an agent engaged in probability learning with an incorrect world model. As participants cannot possibly know the correct world model of 2AFC prior to taking part in 2AFC, a belief in a changing world is, while incorrect, not irrational. Under this account, SQE are a consequence of *conditionally rational* behavior: given one incorrect assumption – an inappropriate world model – subsequent behavior (SQE) is rational (see Green, Benson, Kersten, & Schrater (2010) for a similar approach to explain probability matching).

Two previous studies suggested that SQE may be due an inappropriate world model and developed a computational model to explain commonly observed SQE in 2AFC (Wilder, Jones, & Mozer, 2009; Yu & Cohen, 2009). We developed a modified 2AFC task to test for effects of an inappropriate world model on SQE accompanied by a Bayesian model. Participants took part in a 2AFC task before and after a training session (Figure1). Participants were instructed to press a left or right button with their left or right index finger in response to a stimulus, which appeared either left or right to central fixation. On each trial, stimuli were equally likely to appear left or right ($p_{LR} = 0.5$) and were equally likely to repeat or alternate ($p_{RA} = 0.5$). Crucially, p_{LR} and p_{RA} did not change over time.

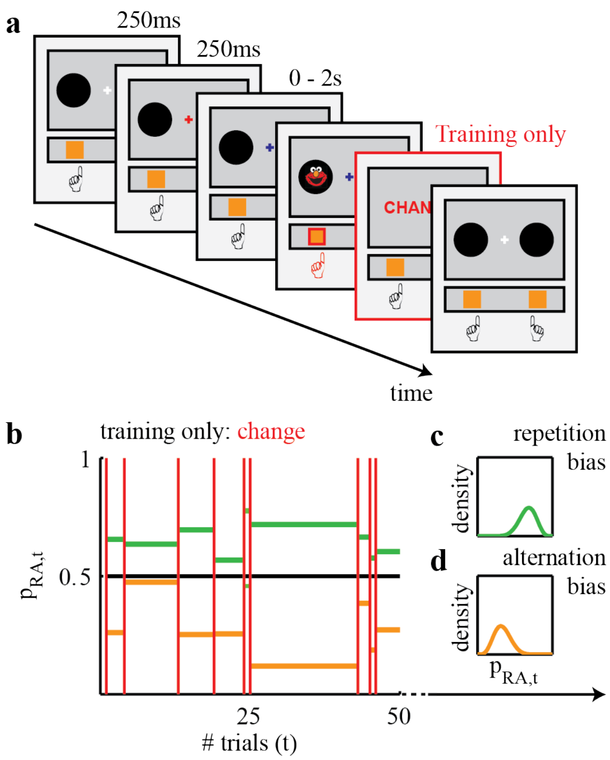

Figure 1: Experimental design. **a** Participants completed a 2AFC task. The stimulus could appear left or right of fixation and participants were instructed to press a button with their left or right index finger. During pre- and post-measurement, the probability of left / right and of repetition / alternation was 0.5. During training, repetition probability was resampled on 18% of all trials. Each change was signaled to the subjects. **b** The probability of repetition was resampled from a Beta-distribution biased towards repetition **c** (high values, green) or biased towards alternation **d** (low values, orange).

During training, we put participants into an environment with changing p_{RA}. Participants continued responding to stimuli presented to the left or right of central fixation with a left or right index finger button press but 18% of all trials, p_{RA} was (re-)sampled from a Beta-distribution $\mathrm{B}(a,b)$,which was biased either towards repetitions (a = 12, b = 6) or alternation (a = 6, b = 12). Each change in p_{RA} was explicitly signaled to the subject. Belief in change was induced to produce SQE and belief in a biased world after change (or biased "re-set prior") was induced to produce *biased* SQE (see **Computational Model & Hypotheses**). We expected to find biased SQE before and after training, given the numerous reports of biased SQE in 2AFC. We aimed to *change* participants' bias in SQE in line with the bias they received during training – towards alternation bias for alternation training and repetition bias for repetition training. Such change would suggest that biased SQE are – at least in part – driven by participants' (inappropriate) world model: manipulating their world model during training changes biased SQE in the expected direction in a post- compared to pre-measurement.

Computational Model and Hypotheses

If participants believe that probability p_t changes over time, they should estimate current probability $\hat{p}_t$ based on the outcome of all trials since the final change in probability and discard the outcome of all trials prior to this change. The decision which trials to include in estimating $\hat{p}_t$ is easy when participant know when change happened, or alternatively, when they know the run length r since change. But in most situations, change is not explicitly signaled to participants and participants need to estimate change c or alternatively run length $\hat{r}$ (Wilson, Nassar, & Gold, 2010).

The full Bayesian model of probability updating in changing environments requires maintaining a distribution over all possible r, which grows as participants complete more trials. In addition, participants need to estimate the hazard rate h_t – the probability of change on trial t - and maintain a distribution across all possible hazard rates (and functions) for optimal Bayesian probability updating in a changing world (Nassar, Wilson, Heasly, & Gold, 2010). Probability updating can very quickly become computationally expensive if not intractable.

Nassar and colleagues (2010) developed a reduced Bayesian model to make probability-updating algorithms more tractable. They designed their model for probability updating in a changing environment with constant probability of change (or constant hazard rate h) and in their model, trial outcomes were supposed to be generated from a normal distribution. We adapted their reduced model to fit our task with an increasing hazard rate $h_{\hat{r}}$ and a Bernoulli distribution with p_{RA} (see Appendix for details). We used this model to compute probability estimates $\hat{p}_{RA}$ of an agent that beliefs in a changing world (incorporated in the model) and completes a 2AFC task (incorporated in the input to the model). Based on the agent's $\hat{p}_{RA}$ we subsequently computed his expectation γ_t (or posterior, see **Appendix**) for the upcoming trial. We then grouped γ_t based on preceding trial history: whether it was preceded by three alternations AAA, three repetitions RRR, or any of the six other possible combinations: AAR, ARA, …

In Figure 2 we plot $1-\gamma_t$ grouped by preceding trial history (x-axis). The curves depend on three parameters: the probability of change p_c and a_0 and b_0 of the Beta-distribution $B(a_0, b_o)$, which incorporates the agents belief in what the world is like after change prior to new, incoming evidence. Simulations show that if $a_0 > b_0$ and $p_c > 0$ SQE are repetition biased and if $a_0 < b_0$ and $p_c > 0$ SQE are alternation biased. During training, we lead participants to believe that $p_c > 0$ and either $a_0 > b_0$ (repetition bias group) or $a_0 < b_0$ (alternation bias group) and expected to observe a corresponding change in bias from pre- to post-measurement.

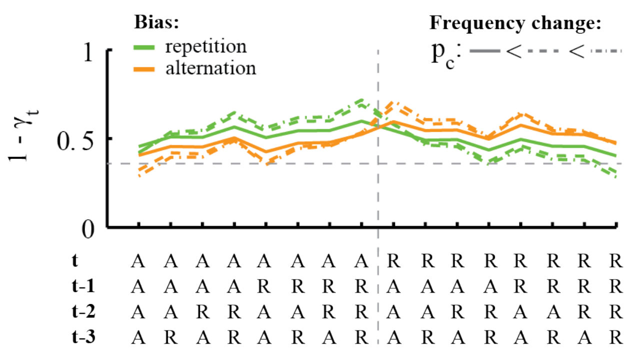

Figure 2: Effects of an agent's belief in frequency of change (solid, dashed, and dashed-dotted lines represent increasing frequency) and bias in its environment after change (green: repetition biased / orange: alternation biased) determined by p_c, a_0, and b_0.

Methods

Participants

25 participants took part in the experiment (12 female, mean age: 22.3 years, age range: 19 - 24 years, 2 left handed) and completed a single session of 60 minutes. They were compensated for time and effort ($10) and received an additional bonus of $4. Participants were told they would get rewarded for fast responses but we rewarded all participants for their fastest 25% of all trials so they all received the same bonus, unbeknownst to them. Informed consent was obtained prior to testing. An internal ethics review board at New York University approved of experimental procedures.

Procedure and Apparatus

Participants were randomly allocated to receive either repetition training (N = 12) or alternation training (N = 13). They were seated at approximately 40 cm viewing distance from a 19'' Dell computer screen in a dimly lit room and asked to wear BOSE QuietComfort 15 Acoustic Noise Cancelling headphones to reduce background noise and to allow them to listen to incorporated auditory feedback. The experiment was run on a Mac Mini (Mac OS X Version 10.7.5) programmed in MatLab 7.5 (http://www.mathworks.com/) and Psychtoolbox 3 (Brainard, 1997; Pelli, 1997). Participants responded by pressing the c-key with their left index finger or the m-key with their right index finger on a standard QWERTY keyboard.

Experimental design

Participants completed a pre-training, training, and post-training (Figure 1). During the pre- and post-training, participants took part in a 2AFC task based on the arcade game "Whack-a-Mole (elMo)!". At trial onset, participants saw a box with two holes – a gray square with two black circles equidistant from a white, central fixation cross. 250ms after trial onset, the white fixation cross turned red and then, after an additional 250ms, blue. Once the fixation cross had turned blue, the stimulus – Sesame Street's Elmo – appeared to the left of right of fixation with probability $p_{LR} = 0.5$ and with repetition probability $p_{RA} = 0.5$ after a time interval chosen from a truncated exponential distribution (mean = 500ms, max. 2000ms). We chose the exponential to reduce temporal expectancy (Luce, 1991). The initial color change of the fixation cross – or count down – ensured that inter-trial intervals exceeded 500ms to ensure that we measured subjective expectancy and not automatic facilitation (Soetens et al., 1985).

During training, participants completed the same task with one important modification. The probability of repetition p_{RA} was resampled on 18% of all trials from a Beta-distribution with a = 12 and b = 6 in the repetition biased group and a = 6 and b = 12 in the alternation biased group. Each time p_{RA} changed, this change was signaled explicitly to the subjects. The word 'CHANGE' was displayed on the gray box prior to each trial with a new re-sampled probability. Participants were not told p_{RA} after change.

We chose to manipulate the probability of repetition versus alternation, instead of the probability of left versus right, for two reasons. First, studies on SQE typically look at effects of trial history coded in as repetition versus alternations instead of left versus right (Cho et al., 2002; Remington, 1969; Soetens et al., 1985; Yu & Cohen, 2009). Second, by manipulating p_{RA}, we kept $p_{LR} = 0.5$ and therefore, any biases in SQE we find cannot be due to differences in left versus right hand action preparation, for example.

Data analysis

We measured RTs, defined as the time interval between stimulus-onset and button press, as a measure of subjective expectancy during pre- and post-training. Trials with incorrect responses (left button press for a stimulus presented to the right of fixation and vice versa) were removed from the data (5.8%). We refrained from analyzing error trials, due to their small number. Reaction times were normalized for each participant and the pre- and post-training separately. We then classified each trial according to its current stimulus and trial history. As we manipulated the probability of alternation and repetition, trials were classified according to whether a trial was a repetition R or alternation A trial and whether a trial was preceded by three alternations AAA, three repetitions RRR, or any of the six other possible combinations: AAR, ARA, … We computed the mean RTs for each trial group and analyzed mean RTs using a 2 x 2 x 2 x 8 mixed design ANOVA with bias as a between subject factor (repetition vs. alternation), and measurement (pre- vs. post), final event (R vs. A) and trial history (AAA, AAR, ARA, …, RRR) as within subject factors.

Results

We found a significant 3-way interaction between group, measurement, and final event ($F(1,24) = 6.39$, $p = 0.012$). Prior to training, we found a bias towards alternations A (mean = -0.132, SE = 0.022) compared to repetitions R (mean = 0.124, SE = 0.024; final event: $F(1,24) = 24.26$, $p < 0.001$; Figure 3). After training, bias was different for each group (final event * training group: $F(1,24) = 8.89$, $p = 0.005$). If repetition trained participants experienced an alternation, then it took them longer to respond (mean = 0.076, SE = 0.063), compared to alternation trained participants (mean = -0.092, SE = 0.045; $t(24) = -2.23$, $p = 0.036$). And conversely, repetition trained participants marginally significantly responded faster when they experienced a repetition (mean = -0.011, SE = 0.057) compared to alternation trained participants (mean = 0.118, SE = 0.043; $t(24) = 1.86$, $p = 0.076$). Experiencing a dynamic environment with a bias either towards repetitions or alternations determines the bias in SQE in a subsequent stable environment.

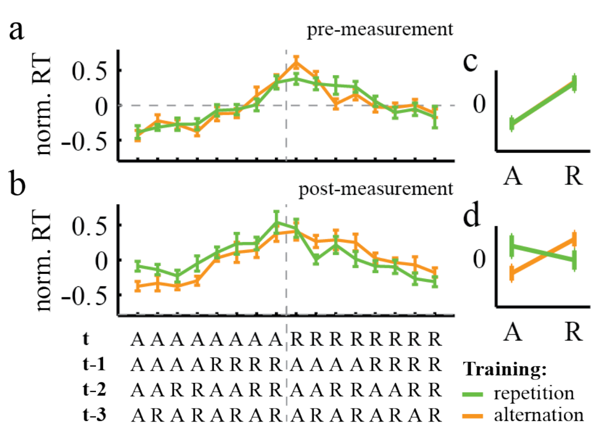

Figure 3: Results. **a** Prior to training, we find alternation biased SQE. **b** After training, we find a change in bias from alternation to repetition in repetition trained participants (green lines). Alternation trained participants (orange lines) maintained their alternation bias. **c** bias in SQE prior to training and **d** bias in SQE after training averaged across trial history t-1 to t-3.

Discussion

SQE are a pervasive phenomenon in 2AFC and are typically explained by an irrational sensitivity to local patterns in trial history, which is supposed to give rise to other suboptimal behavior, too, such as the gambler's and hot-hand fallacy in decision making. We propose instead that SQE are conditionally rational: they arise because subjects attempt to learn the probability of occurrence of the two stimuli in 2AFC but with an inappropriate world model. Instead of constant stimulus probability, they believe in change. We trained participants in a changing world with a repetition or alternation bias and observed a change in participants' repetition or alternation bias in SQE consistent with the repetition or alternation training they received. Our data support the conclusion that biased SQE are at least in part driven by an inappropriate world model: SQE are conditionally rational.

Two previous studies proposed that participants' belief in a changing world gives rise to SQE. Yu and Cohen (2009) developed a Bayesian model of probability updating. Like our model, probability updating was based on stimulus repetitions and alternations (2nd order) in trial history. Their model produced SQE similar to the one's described in the literature, primarily Cho and colleagues (2002). Crucially, the authors did not explicitly measure the effects of training participants to believe in a particular world model on SQE and their model could account for bias in SQE only through ad hoc choice of a reset-prior (a prior belief in what the world will most likely be like after change) skewed towards, in their case, repetitions. Our results indicate that such biases can be altered by relatively small amounts of training.

Wilder and colleagues (Wilder et al., 2009) also developed a Bayesian model of probability updating to explain previously observed SQE based on stimulus repetitions and alternations (2nd order) *and* stimulus location (1st order). Like Yu and Cohen (2009), Wilder and colleagues (2009) explained bias in SQE through ad hoc choice of a biased reset-prior. They state that bias in SQE changes from experiment to experiment, is difficult to predict, and should not be cast as part of a computational theory of SQE. Instead, it reflects attentional and perceptual mechanism. We assume they hereby mean that bias reflects automatic facilitation and not subjective expectancy, to use Soetens' et al. terminology (Soetens et al., 1985). We observed a predicted chance in bias after a manipulation of participants' world model, which speaks against this interpretation. The bias in SQE should be part of a computational theory of SQE.

Cho and colleagues (2002) conducted a 2AFC experiment and developed a computational model to explain the repetition biased SQE they observed. Their model explains SQE as the consequence of special pattern detectors. According to the authors, subjects have two detectors: (a) a relatively simple repetition detector, which increases our expectation to observe a stimulus again when it has just occurred and (b) a more complex alternation detector which counts observed alternations and increases the expectation to another alternation in proportion to the number of already observed alternations. But other than being able to account for their data, the model does not explain why we have certain pattern detectors and not others (Cho et al. (2002) list six possible detectors). Crucially, their model cannot easily explain why the training participants received in our experiment altered bias in SQE.

Green and colleagues took a similar approach to ours to explain a different phenomenon, namely probability matching in sequential binary decision tasks (2010). They proposed that participants' belief in an inappropriate world model for sequential binary decision tasks causes probability matching. In sequential, binary decision tasks, participants have to choose one of two options. One option has a higher probability of winning (70% versus 30%). The optimal strategy for this task is to determine which option has a higher probability of winning and then choose that option exclusively. Instead, participants tend to choose the option with 70% success probability 70% of the time and its alternative 30% of the time. While this probability matching behavior is suboptimal, Green and colleagues showed that given a particular albeit inappropriate world model for the task, probability matching is optimal. The authors asked participants to complete sequential, binary decision tasks and manipulated them to believe in different world models. This manipulation changed probability matching behavior – a strong support for their claim. Probability matching is conditionally rational. We conclude similarly that biased SQE are conditionally rational.

Simpler models, exponential down-weighting of trial history (Anderson & Carpenter, 2006), for example, can explain SQE but cannot explain the change in *bias* in SQE that we observed. SQE indicate that subjects are sensitive to recent but not distant trial history. The change in bias in SQE, however, indicates that subjects are sensitive to what they experienced many trials back (during training), too. Exponential down weighting of evidence cannot explain this dependence on temporally distant and at the same time recent information. One could augment a model that explains SQE by exponential down weighting of trial history with a bias but, to compete with our explanation, there would have to be a rational explanation for this bias.

Our findings thus show that an inappropriate world model at least in part gives rise to biased SQE. This shows that in 2AFC, participants try to learn the generative process of the task – the process, which determines how outcomes, in this case repetition versus alternation, are generated. Learning such a generative model is what distinguishes model-based from model-free learning, according to Daw and colleagues (Daw, Niv, & Dayan, 2005; Doll, Simon, & Daw, 2013; Otto, Gershman, Markman, & Daw, 2013) and Green and colleagues (Green, Benson, Kersten, & Schrater, 2010). We demonstrate that a seemingly simple behavioral phenomenon (SQE) is at least in part driven by model-based learning, which supports the recently proposed ubiquity of model-based learning algorithms (Doll et al., 2013).

In summary, we proposed that biased SQE are a consequence of participants' selection of an inappropriate world model for 2AFC. We manipulated participants' beliefs and observed predicted changes in bias of SQE. Our predictions were based on a Bayesian model of probability updating, which estimates probability of change and estimated run length to derive trial-by-trial estimates of the probability of observing a repetition versus alternation.

Appendix

The predictive distribution is computed with respect to expected run length $\hat{r}_t$ (Nassar, Wilson, Heasly, & Gold, 2010). On each trial, the agent computes the probability that a change c occurred using Bayes rule:

$$p(c \mid X_t) = \frac{p(X_t \mid c)p(c)_t}{p(X_t \mid c)p(c)_t + p(X_t \mid \hat{p}_{RA,t})(1 - p(c)_t)} \tag{1}$$

In the repetition bias group $p(X_t \mid c) = \max \mathrm{B}(a_0, b_0)$ with $a_0 > b_0$ for a repetition and $p(X_t \mid c) = 1 - \max \mathrm{B}(a_0, b_0)$ with $a_0 > b_0$ for an alternation. In the alternation bias group $p(X_t \mid c) = \max \mathrm{B}(a_0, b_0)$ with $a_0 < b_0$ for an alternation and $p(X_t \mid c) = 1 - \max \mathrm{B}(a_0, b_0)$ with $a_0 < b_0$ for a repetition. $p(c)_t$ depends on current, estimated run length $\hat{r}_t$ and increases with increasing $\hat{r}_t$:

$$p(c) = 1 - (1 - p_c)^{\hat{r}_t} \tag{2}$$

Change becomes more likely as participants complete more trials without intervening change (a uniform distribution has an increasing hazard function). $p(X_t \mid \hat{p}_{rep,t})$ is the predictive distribution if a change point did not occur and depends on $\hat{r}_t$ and the number of alternations A and repetition R in $\hat{r}_t$.

The expected or mean value of the predictive distribution is based on two possibilities: (a) a change point occurred, in which case $\hat{p}_{RA,t} = \max \mathrm{B}(a_0, b_0)$ with $a_0 > b_0$ for the repetition bias group and $\hat{p}_{RA,t} = \max \mathrm{B}(a_0, b_0)$ with $a_0 < b_0$ for the alternation bias group, or (b) no change point occurred, in which case the recent outcome X_t is used to update $\hat{p}_{RA,t}$. If X_t is a repetition, the number of repetitions R_t in estimated run length $\hat{r}_t$ is increased by one: $R_t = R_{t-1} + 1$. If X_t is an alternation then $A_t = A_{t-1} + 1$ and in the repetition bias group (note that $R_t + A_t = \hat{r}_t$):

$$\hat{p}_{RA,t} = \max B(a_0 + R_t, b_0 + A_t) \tag{3}$$

with $a_0 > b_0$ in the repetition bias group and $a_0 < b_0$ in the alternation bias group. The posterior distribution is a weighted average of these two possibilities:

$$\gamma_t = p(c \mid X_t)p(c)_t + (1 - p(c)_t)\hat{p}_{RA,t} \tag{4}$$

Expected run length is updated on each trial based on the probability that change occurred (in which case it is reset to one) and based on the probability that there was no change (in which case it is incremented by one):

$$\hat{r}_{t+1} = (\hat{r}_t + 1)(1 - p(c)_t) + p(c)_t \tag{5}$$

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