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Promoting Development of Geometry Concepts: Interfacing Multiple Embodied Representations with a Computer Game

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Abstract

In this study we examined the effectiveness of a computer-based geometry learning tool with 4th grade students. In both an experimental and control condition children constructed a series of common four-sided figures to fit a set of visual constraints. Additionally, children in the experimental condition were required to validate the presence of parallel segments, congruent segments, or right angles embedded in their figures with the assistance of a visual depiction of the property's spatial significance. Following training we conducted six identification tasks in which participants attempted to discriminate two valid members of a given polygon class from four displayed polygons. In all six instances, varying polygon, instruction, and feedback type, children in the experimental condition were more likely to correctly identify both class members than children in the control condition.

Keywords: embodied cognition, mathematical development, education, cognitive development

Introduction

Core mathematical abilities may emerge from either innate processes or general human experiences (Lakoff & Nuñez, 2000). Yet, the development of mature concepts and skills depends on one's cultural experiences – such as schooling. For example, while number sense may emerge from general spatial processes (Dehaene, 1997), the development of mature numerical representation is moderated by exposure to formal and informal activities (Siegler & Ramani, 2008). Furthermore, Siegler and Ramani (2008) successfully demonstrated that persistent SES-based disadvantages could be overcome by effective use of appropriate tools (i.e., linear board games).

Cognitive research in geometry may follow a similar trajectory. The core systems approach – which has been applied successfully in numeracy – has recently been invoked to explain the cognitive basis of Euclidean geometry. Specifically, Spelke, Lee, and Izard, (2010) claim that geometric concepts are grounded in two core cognitive systems, typically associated with spatial navigation and object perception. The latter system is hypothesized to support the concepts of lengths and angles, which are critical to school-based geometric activities.

Yet, while lengths and angles are certainly perceivable attributes of objects, other qualities, such as bilateral symmetry (Quinlan & Humphreys, 1993), may be more salient. In some cases these perceptually salient features may interfere with formal classifications of geometric objects. For example, Mach (1886/1959) demonstrated that a square may be perceived as a diamond when its diagonal is oriented vertically.

Similarly, in a study that applied nonmetric multidimensional scaling to a set of four-sided figures – a technique in which perceived dissimilarities between objects are spatially represented for analysis – perceived attributes confounding lengths and angles emerged, including "dispersion" (irregularity), "elongation", and "jaggedness" (Behrman & Brown, 1968). Likewise, Shepard and Chipman (1970) found similar dimensions in participants' categorizations of U.S. state shapes.

While a variety of factors affect natural object perception, perhaps the more narrow set of educationally-relevant geometric figures (e.g. square, equilateral triangle) better afford conceptualization by normative (Euclidean) class-based properties (e.g. parallel lines). However, interview data demonstrates that young children categorize polygons based on informal perceptual attributes, such as "slanty", "pointy", or "skinny" (Clements, Swaminathan, Hannibal, & Sarama, 1999). As the Mach (1886/1959) square-diamond demonstration suggests, informal concepts play a role in geometric representation into adulthood.

Yet, over two thousand years of geometry as a formal discipline suggests that humans are not bound by superficial percepts. Abstractly, geometric concepts may be conceived in terms of strict hierarchy, facilitating deductive reasoning. Such organization suggests an internal representation similar to the discrete, hierarchical, propositional network of Quillian (1966).

Yet, rather than viewing the development of geometry concepts as a shift from perception to abstraction, one may conceive of development in terms of perceptual

reorganization and learning (Goldstone & Barsalou, 1998; Goldstone, Landy, & Son, 2010). From this perspective, experts do not rely upon perception any less than novices, but are trained to perceive structurally-relevant attributes, while overlooking irrelevant features. We suspect this is the case in geometry as children shift from informal perceptual features (e.g. "pointy" shapes) to more normative ones (e.g. right angles).

Geometry learning

While the shift from informal (or core) concepts to formal, Euclidean concepts is the goal of geometry education, many current practices are ineffective or misleading (Clements, 2004). Reliance on a limited range of exemplar shapes and activities often leaves children without the concepts or skills to perform more complex tasks, such as geometric proof.

Digital technology, on the other hand, facilitates the display of a wide range of figures in a variety of contexts, thereby promoting development of more general concepts. Further, technology may elicit specific behaviors through systems of constraints, feedback, and goal structures, thereby promoting development of precise concepts. Yet, what specific mechanisms will direct learners' behaviors towards meaningful features of the shape concept?

Lakoff and Nuñez (2000), hypothesize that basic mathematical concepts are supported by "grounding metaphors" embodied in physical experience with the natural world. We suggest that one such geometric metaphor is "polygons as a surface face of physical objects." Lakoff and Nuñez further suppose that advanced mathematical knowledge depends upon linking multiple metaphors in a spatially-grounded context. A digital learning system may play a role in either assisting in the development of a primary conceptual representation or the interfacing (i.e., "linking") of multiple representations.

In the study that follows we examine the process of interfacing multiple embodied representations in a polygon construction task. Specifically, children in both a control and experimental condition were tasked with construction of specific polygons within some physical bounds. However, in the experimental condition, children validated the inclusion of target properties in their constructed figure (e.g. parallel lines for a parallelogram) by manipulating a software tool depicting the property as a hand gesture. Additionally, the experimenters showed children how to use their own hands to mimic the on-screen gesture, in line with research demonstrating gesture's (potential) causal role in mathematical learning (Broaders, Wagner, Zachary, & Goldin-Meadow, 2007). The visual depiction and physical gesture each provide a means of embodying the mathematical concept in the perceptual and/or motor systems, respectively – in contrast to approaches that situate mathematical concepts in symbolic or verbal systems.

In this initial study we compare our embodied, property validation approach to a non-validation procedure to provide causal evidence for our system's role in successfully shifting children's perception/conception of polygons. As such, children in the experimental condition should be more likely than children in the control condition to correctly identify non-prototypical members of a polygon class.

Method

Participants

Twenty-one fourth grade children were recruited from an after-school program located in a low-SES, predominantly Hispanic neighborhood of New York City. The children were randomly assigned to either an experimental or control condition. The experimental condition consisted of ten children (M = 9.4 years, SD = .16, 40% female, 90% Hispanic, 10% African American). The control condition also consisted of ten children (M = 9.6 years, SD = .28, 50% female, 100% Hispanic). Two children (one from each condition) were native Spanish speakers, but could communicate sufficiently in English and showed little difficulty understanding the tasks. Additionally, one child, assigned to the experimental condition, was not included in the study due to prolonged absence.

Materials and Procedure

Study-related tasks were conducted in the context of a weekly after-school robotics program directed by the experimenters. Robotics activities were not essential to the tasks of this study, although they did inform the visual design of the game. Generally, robotics activities were conducted after children completed study-related activities.

An approximately 1.5 hour block was divided into two 45 minute sessions. In each session all children from either the control or experimental condition were "pulled-out" to a separate classroom to perform either the learning or assessment tasks. Because of the prolonged nature of the after-school program we took the opportunity to test these tools across three curricular units, focusing on three polygon properties: parallel segments, congruent (adjacent) segments, and right angles.

In each unit the children engaged in a series of self-paced polygon construction activities (10 for parallel segments, 6 each for congruent adjacent segments and right angles). Children performed the construction task on individual computers, in parallel with all other (present) participants in his or her condition. Immediately following completion of all construction activities within a unit, the assessment task (polygon identification) was administered, individually, apart from the other children.

Polygon construction game

The central learning tool was intended to provide children with multiple instances of polygons in a game-like context. The general goal of the game was to navigate a virtual agent (robot) through an obstacle course, collecting "goal" objects, avoiding "danger" objects, and returning to the

starting position (i.e., closing the polygon) via a series of lines and angles. The placement of goals and dangers was directed to promote the construction of specific polygons, including trapezoids, parallelograms, kites, rhombi, rectangles, and squares. In each case a dense set of circumnavigable objects were placed in the screen's center, in the general shape of the intended polygon. Additional goals and dangers were positioned away from this central set to restrict paths from large deviations. The particular game objects and cover story were varied throughout the sessions to maintain high motivation.

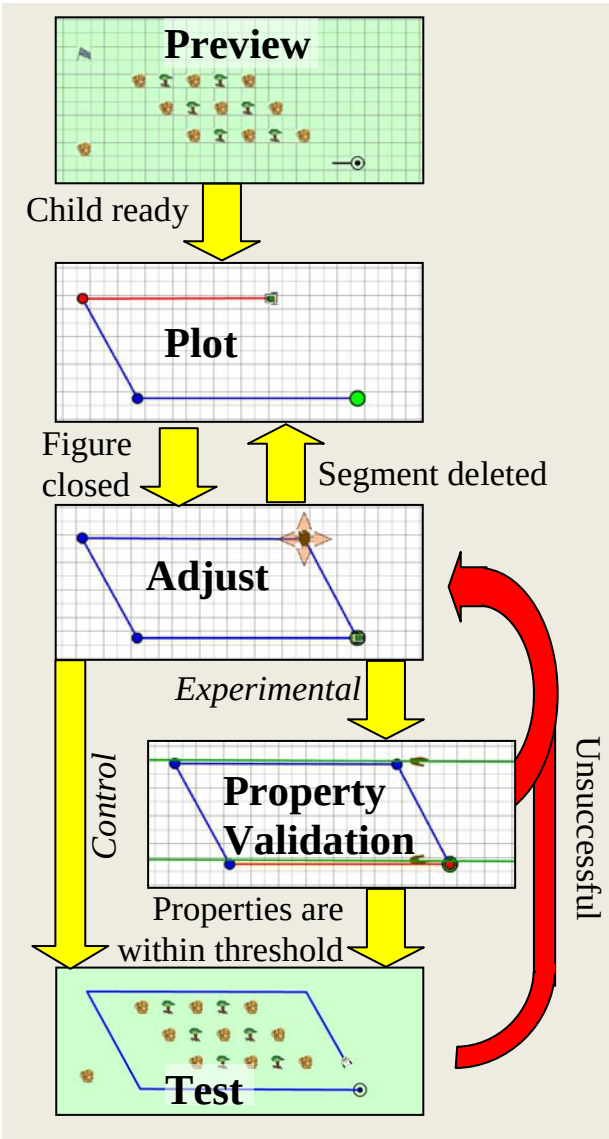

Figure 1. Polygon construction game flow, with cropped screenshots from a parallelogram task. Yellow arrows demonstrate progress in the intended direction. Red arrows demonstrate mistake-based

The game design is summarized in figure 1, above. In either condition the children first previewed the obstacle course on a grid. Children were verbally instructed to try and "memorize" the shape that would "fit" the course. In a left side panel, not shown in figure 1, the name of the intended polygon and a list of properties were displayed (e.g. 4 right angles).

At the child's discretion, he or she clicked a continue button, clearing the screen of all game objects. The child then engaged in an iterative process of plotting line segments and angles over a grid by manipulating the mouse in corresponding straight lines and arcs, respectively. Pressing the delete key removed the previously constructed shape component. To assist in the construction of an appropriate figure the initial placement and orientation of the first segment were fixed and could not be deleted.

A closed polygon could then be adjusted by dragging and dropping vertex points – displayed as small circles – within the game screen. During this phase, pressing the delete key removed the polygon's final segment and resumed plotting.

The two phases of shape construction (plotting, adjusting) were intended to afford multiple intuitive strategies. Although a single mechanism may have sufficed (for example, by providing an initial, random polygon to be adjusted), our priority was to elicit the child's intuitive representation of shape, which could be embodied by either the path-plotting or shape-transforming activity. As such, there was some variability in strategy, as some children engaged in precise plotting, while others relied more heavily upon adjusting.

Once a figure was closed, the child could proceed to either the testing phase (in the control condition) or the property validation phase (in the experimental condition), in which participants confirmed the presence of parallel segments, congruent segments, and/or right angles embedded in the polygon, according to the polygon's class. Specifically, the child either clicked pairs of sides to validate parallelism or congruency, or clicked vertex points to validate right internal angles. If the figure successfully met all property criteria (within a small threshold: 3% grade for parallel lines, 20 pixels for congruency, and 4° for right angles, inclusive), the participant continued to the testing phase. Else, the child returned to the adjusting phase.

Further, to facilitate comprehension, the spatial significance of the property was visually depicted (see figure 2). For parallelism two hands moved in parallel at the same slope as one side and were matched for slope against a second side. For congruency two hands marked the distance of one side and were matched against the length of a second side. For right angles two perpendicular hands were matched against the angle at a vertex.

The display of hands, for each of these checks, was included to provide the children with a justification that could be applied outside of the game context.

Finally, in either condition the children tested their figure design on the obstacle course. In this phase the virtual agent was placed on the starting point, the participant adjusted its initial heading, and clicked to set the agent in motion upon the constructed path. A trace of the agent's path, i.e., the constructed figure, followed its motion. If the agent

successfully returned to the starting position, while collecting all goals and avoiding all dangers, the level was completed and the child received a reward stamp on a personalized document tracking his or her progress. If the agent was unable to navigate the course successfully the participants would return to the adjusting phase for revision.

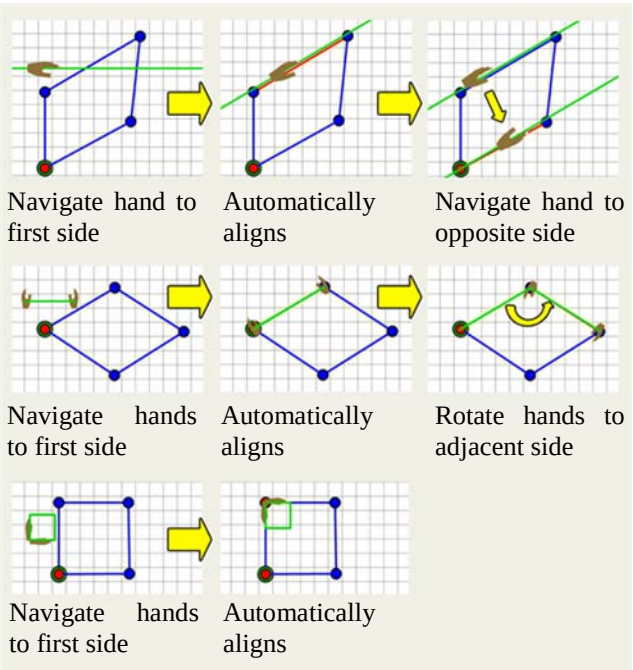

Figure 2. Visual depictions in property validation phase. Displays validation of parallel sides, congruent sides, and right angles (from top to bottom).

Polygon identification task

As stated above, the intended purpose of the polygon construction game was to shift children's perception away from informal attributes of shapes to formal properties. To assess this shift children performed the task of identifying (i.e., clicking) two examples of a given polygon from four displayed polygons (see figure 3). Following each learning unit (parallel segments, congruent adjacent segments, right angles) the child performed 60 trials in two blocks. The blocks assessed the child's ability to identify trapezoids, parallelograms, rhombi, (mixed) isosceles triangles and isosceles trapezoids, rectangles, and (mixed) right triangles and right trapezoids.

To generate the stimuli for each task we began with the image of three valid members of the shape's class, in a prototypical orientation. We then distorted these images in one of two dimensions such that they remained visually similar to the class members, yet class invalid. In the case of isosceles and right trapezoids exemplars were distorted along one dimension, creating half as many figures (but the same total when combined). Finally, the pool of figures was doubled by rotating each image a quarter-turn. In the case of parallelograms, rhombi, and rectangles distortions were designed such that properties of a parent or grandparent class were retained. Specifically parallelograms were distorted into trapezoids, rhombi into kites and parallelograms, and rectangles into trapezoids and parallelograms (see figure 4 for the rhombus example).

Individual trials consisted of two sets of valid-invalid pairs of polygons that applied the same invalidating alteration. For example, in figure 3, the upper polygons are valid trapezoids while the lower polygons have been altered by changing the slope of the shortest side.

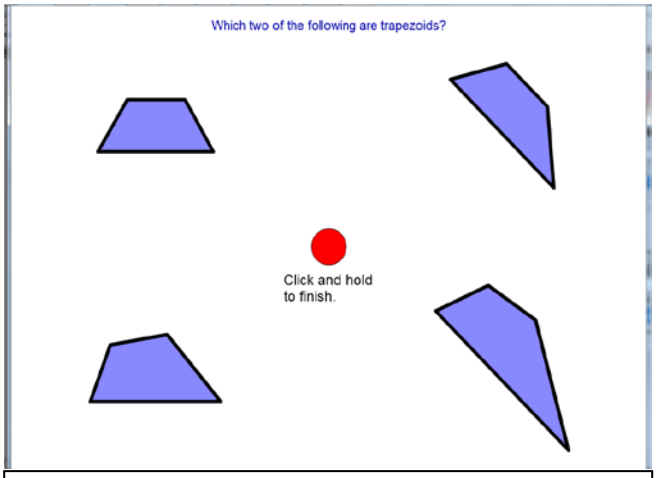

Figure 3. Trapezoid trial in the polygon identification task.

The rational for this trial structure – as opposed to a simpler choice between two figures – was to make the dimensions of alteration between the figures explicit. For example, in figure 3 the top-left trapezoid may be considered prototypical. The top-right trapezoid is non-isosceles and rotated – and therefore non-prototypical. On the other hand, the bottom-left figure is an irregular distortion of the prototypical trapezoid, but visually similar. Clearly, the difficult choice lies in these two options. But making the more prototypical (and the non-prototypical, invalid) figure available we encourage the participants to think about the rules governing class inclusion.

A block of trials paired all possible combinations of two valid polygons with their invalid partners. With six valid polygons in each set (including the rotated figures), the number of combinations is 15. Paired with two invalid partners the number of trials is 30. In the case of the mixed triangle/trapezoid trials, with a single invalid partner, two sets of 15 trials were combined to produce a block of 30.

To explore this task's robustness across different conditions planned variations of feedback and instruction were embedded across the six blocks. For the trapezoid, parallelogram, rhombus, and rectangle blocks (i.e., those figures that had been trained) instructions included a text display of the relevant rule of inclusion (e.g., "A trapezoid has one pair of parallel lines"). Additionally, in these cases, two figures were displayed – one valid example and one

invalid example – upon which the participants were asked to click the target components of the valid figure (e.g., the parallel sides of the trapezoid). In the case of the mixed triangle/trapezoid blocks, only the text rule of inclusion was provided (e.g., a triangle or trapezoid is "right" if it contains at least one right angle). The lack of a visual example in this case was intended to demonstrate how well the child could transfer the visual interpretation of the previous block to the current block.

Following each trial participants were provided feedback, stating the number of correctly identified figures. This feedback was intended to reinforce the rule of inclusion rather than memory for specific figures. As an additional indicator of effect robustness the assessment blocks following the third unit (rectangles, right triangles/trapezoids) did not include feedback.

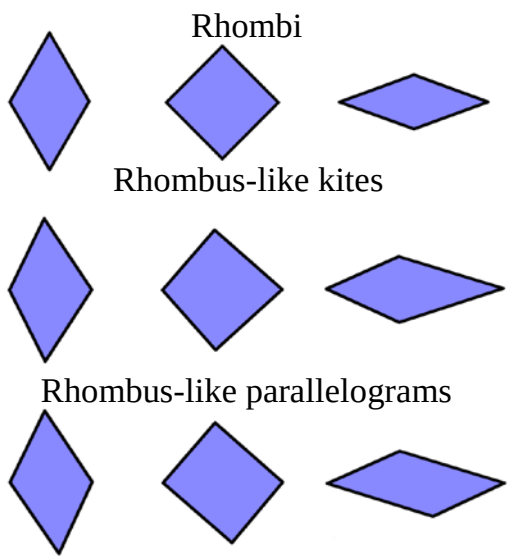

Figure 4. Examples of rhombus stimuli in polygon identification task. Top row shows valid rhombi. Second row shows rhombi manipulated to form kites. Third row shows rhombi manipulated to form parallelograms. Another nine images (not shown) are 50° rotations of these figures.

Results

All children completed all 22 construction tasks. The number of sessions significantly differed between conditions (experimental: Mean 8.8; control: Mean 6.7, $t(18)=3.9$, $p < .001$).

For the polygon identification task we classified each trial according to the number of correctly selected polygons (0, 1, or 2). For each participant, within a single block, total 2-correct, 1-correct, and 0-correct trials were then computed (e.g., in a block of 30 trials a subject has 15 2-correct, 10 1-correct, and 5 0-correct trials). Alternatively, the number of correctly selected polygons across an entire block may be calculated to produce a single statistic (e.g., out of 60 correct polygons displayed the participant selected 40). However, we suspect that this metric does not fully capture performance. For example, a child might attend to the correct dimension, but in the wrong direction, resulting in a 0-correct trial. Following this error, given feedback, he or she may have corrected this directional error and followed with a 2-correct trial. On the other hand, a child with two 1-correct trials in a row is likely attending to the wrong feature of the polygon. Although both children correctly selected two figures in total, the nature of their performance is quite different.

To avoid the assumption of normality in the trial classification data (e.g., rhombus results are skewed to the left due to a ceiling effect) a nonparametric, one-tailed Mann-Whitney U test was applied to compare counts of 2-correct and 1-correct trials between conditions.

As table 1 displays, each comparison between experimental and control is significant at the $\alpha < .05$ level. This is true in both easier blocks in which the majority of participants, in both conditions, correctly identified both polygons in more than half of the trials (i.e., rhombus), and difficult blocks in which the majority of participants, in both conditions, misidentified at least one polygon in more than half of the trials (i.e., right triangle/trapezoid).

		Median trial counts (out of 30)			
Exam	Block	2-Correct	U	1-Correct	U
1st	Trapezoid	E: 18 C: 12.5	74*	E: 12 C: 17	79*
	Parallelogram	E: 23.5 C: 14.5	82**	E: 6 C: 13	79.5*
2nd	Rhombus	E: 24 C: 17.5	80.5**	E: 6 C: 12	80.5**
	Isosceles Trap., Tri.	E: 22 C: 4	83.5**	E: 8 C: 23.5	81**
3rd	Rectangle	E: 22.5 C: 14.5	89**	E: 6.5 C: 15	88.5**
	Right Trap., Tri.	E: 7 C: 2.5	72.5*	E: 18 C: 23	72.5*

* $p < .05$ ** $p < .01$

Table 1. Median distribution of trials with one and two correctly chosen polygons and associated Mann-Whitney U tests.

Discussion

Dehaene and Izard (2006) found that indigenous Amazonians, without formal geometric knowledge, successfully discriminate parallel and perpendicular segments from sets of arbitrarily oriented segments. Undoubtedly, U.S. children are capable of classifying based on these attributes of shapes. In this study, only the children in the experimental condition reliably did so.

Yet, these preliminary results are merely suggestive in nature. By testing our embodied validation condition against a non-validation condition we conclude that our software was effective – but we cannot be sure why. One explanation is that children in the experimental condition developed a higher perceptual sensitivity to small deviations from parallelism, perpendicularity, and congruity. Additionally, the children in the experimental condition may have developed a greater sense of how to apply these property concepts to polygons. On the other hand children in the control condition likely applied a weak representation of the property or misunderstood how to apply the property as a basis of polygon classification.

However, the use of a non-validation control raises several alternative, information processing-based, explanations. Specifically, because the control group's shapes were unconstrained by polygon class-based properties, constructed figures may have deviated markedly from the intended polygon. In this case the children would have been exposed to a sparser set of valid polygons than the experimental condition, making the implicit encoding of the relevant property more difficult. Anecdotally, while it was rarely the case that children greatly deviated from the intended polygon – which was generally prevented by the placement of mandatory path goals and peripheral obstacles – children often engaged in a form of "guess-and-check" in which tests were followed by minor adjustments, retesting, and so on. This strategy, at least in its most rapid form, was prevented by the property validation procedure in the experimental condition. Learning artifacts and strategies will be analyzed in future work.

Secondly, because of the additional difficulty inherent in the experimental condition, those children engaged in approximately a third more game sessions. While we expected some difference in training duration, given a fixed number of activities, additional measures may be taken to equate these groups more closely in both number of activities and time-on-task.

Specifically, in ongoing work we address both of these concerns by introducing a property validation phase in the control group. Rather than promoting the spatial characteristics of the property, children in this control are attended towards a numerical representations of the property (e.g., ensuring that a right angle measures 90°). This additional step lengthens the duration of training and ensures the construction of accurate polygons – likely more accurate than those created in the experimental condition. This refinement of the learning task will determine whether the experimental group's success was founded on greater exposure to accurate polygons or due to the interactive, embodied nature of the property validation task.

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