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# Strategies and Tactics in Sentential Reasoning

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## Abstract

We propose a theory of the spontaneous reasoning strategies that individuals develop. These strategies depend on component tactics based on mental models. Reasoners vary their use of tactics in ways that are not deterministic. This variation leads different individuals to assemble different strategies, which include constructing incremental diagrams corresponding to mental models, and pursuing the consequences of a single model step by step. The number of models required by the premises predisposes reasoners towards certain strategies, e.g., multiple models tend to elicit incremental diagrams. Similarly, the connectives in premises also bias reasoners towards certain strategies, e.g., conditional premises tend to elicit reasoning step by step from a single model.

## Introduction

Psychologists have tended to neglect the strategies that individuals develop spontaneously to make complex inferences (cf. Schaeken, De Vooght, Vandierendonck, and d'Ydewalle, 2000). By a *strategy*, we mean a systematic sequence of elementary mental steps, i.e., tactics, that an individual follows in making an inference. Pioneering studies of strategies examined relational reasoning in which the task is, say, to infer who is tallest in a series of individuals. The results suggested that reasoners develop a variety of strategies (e.g. Wood, 1969; Quinton and Fellows, 1975). However, there has been a dearth of studies of strategies in sentential reasoning, which hinges on negation and connectives such as "if", "or", and "and". Some theorists have argued that sentential reasoning relies on a single deterministic strategy based on formal rules of inference (cf. Rips, 1994; Braine and O'Brien, 1998). We suspect that theorists have postulated a single deterministic strategy because their experiments have used too simple premises for strategies to differ, and because they have failed to gather evidence about reasoner's strategies. Indeed, we and our colleagues have proposed that naïve reasoners generally develop a variety of strategies (e.g. John-

son-Laird and Byrne, 1990; Byrne and Handley, 1997; Bucciarelli and Johnson-Laird, 1999).

## Experiment 1: A taxonomy of strategies

How can experimenters best observe the strategies that reasoners use in sentential reasoning? In our view, studies of strategies should examine inferential problems that are sufficiently time-consuming to force the participants to think, but not so difficult that they make many errors. We therefore used sentential problems based on three premises, but each set of premises was compatible with only two alternative possibilities. The task was to evaluate a given conclusion and to think aloud (cf. Ericsson and Simon, 1984). Here is a typical example of a problem:

Either there is a blue marble in the box or else there is a brown marble in the box, but not both. Either there is a brown marble in the box or else there is white marble in the box, but not both. There is a white marble in the box if and only if there is a red marble in the box. Does it follow that: If there is a blue marble in the box then there is a red marble in the box?

Henceforth, we use the abbreviations: "iff" for biconditionals of the form "if and only if", "ore" for exclusive disjunctions of the form "either \_ or else \_, but not both", and "or" for inclusive disjunctions of the form "\_ or \_, or both".

Our theory of strategies is based on mental models (Johnson-Laird and Byrne, 1991), and each mental model represents a possibility. All the problems in the experiment called for two mental models. The premises of the example above yield the following two models of the possible contents of the box, shown on separate lines:

blue	white	red
brown		

As the models show, the putative conclusion follows from the premises.

*Method.* Eight Princeton undergraduates, who had no training in logic, carried out twelve inferences, which each had a conclusion to be evaluated. The problems were based on three or four premises. Half of them had valid conclusions and half of them had invalid conclusions. The premises were mainly biconditionals and exclusive disjunctions, and the conclusions were conditionals except for two problems, which had exclusive disjunctions as conclusions. As in the example above, the contents of the problems concerned different colored marbles. The problems were presented in a different random order to each participant.

The participants were allowed to use pencil and paper. They were told to think aloud as they tackled each inference, and we video-recorded what they said, wrote, and drew. The camera was above them and focused on the paper on which they wrote, and they rapidly adapted to its presence.

*Results.* None of the participants made any errors in evaluating the given conclusions, though they were not always right for the right reasons. We transcribed the tapes verbatim apart from repetitions of words, filled pauses, and hesitations. These protocols also included a record of the step by step drawings of diagrams. We were able to make sense of almost all of what the participants said, drew, and wrote. Most participants used two or more distinct strategies, but two of them stuck to the same strategy throughout the experiment. What the protocols did not reveal were either the processes in developing a strategy, or the mechanisms underlying the tactical steps. We were able, however, to categorize the protocols from every participant for every problem into one of the strategies in the taxonomy in Table 1 below.

The taxonomy distinguishes five main strategies. It is based on all our experiments, but it may be necessary to add further strategies: no-one can ever know when the classification is complete. The five strategies were:

1. The incremental models strategy. Reasoners draw a diagram that integrates all the information from the premises. The diagram corresponds to a set of models (see the example above). Some participants drew the models in vertical columns down the page. Others arranged them horizontally. One participant merely drew circles around the propositions in the premises themselves to pick out one of the two models. Participants work through the premises in an order that allows them to increment their diagrams.

2. The step strategy. Reasoners pursue the step by step consequences of either a categorical proposition or a supposition. They accordingly infer a sequence of what logicians refer to as “literals”, where a *literal* is a proposition that does not contain any sentential connectives: it may be an atomic proposition, *A*, or its negation, *not A*. Consider the following problem, stated in an abbreviated form:

- Pink iff black.
- Black ore gray.
- Gray iff blue.

Does it follow that if not pink then blue?

One participant's complete verbatim protocol, illustrating the strategy, is:

Assuming we have no pink:  
 There is no pink. [He crosses out “pink” in premise.]  
 So there is no black. [Crosses out “black” in premises.]  
 There is gray. [Circles “gray” in premise.]  
 There is blue. Yes. [The conclusion follows.]

3. The compound strategy. Reasoners take two compound assertions, i.e., assertions containing a sentential connective, and draw a compound conclusion from them, e.g.:

Pink ore brown. [Reads premise]  
 Pink and white. [Points to diagram of premise:  
 pink → white]  
 If brown then not white. [A compound inference.  
 Writes: brown, ~~white~~]  
 White ore brown. [The required conclusion]

The strategy consists in a sequence of such compound inferences that yield an ultimate conclusion.

4. The chain strategy. Reasoners construct a chain of *conditionals* leading from one constituent of a compound conclusion to its other constituent. They make an immediate inference from any premise that is not a conditional to convert it into an appropriate conditional (see Richardson and Ormerod, 1997). Here is an example of a protocol:

[Crosses out terms in diagrams:  
 If not pink then not green. ~~pink~~ = ~~green~~  
 If not green then red. ~~green~~ or red  
 If red then white. red = white  
 Yes. [I.e. If not pink then white]

The valid use of the strategy to prove a biconditional or exclusive disjunction calls for two chains, but reasoners usually rely on just a single chain.

5. The concatenation strategy. Reasoners sometimes concatenate the premises to form a complex intermediate conclusion. They then draw an immediate inference from it to the required conclusion. For example, one participant concatenated the premises:

- A and B.
- B iff C.
- C iff D.

to yield : A and (B iff C iff D). She then made an immediate inference to the required conclusion: A and D.

For the twelve problems in Experiment 1, we calculated the total number of times each strategy occurred in the protocols, and then expressed these numbers as percentages of the total number of occurrences of strategies. The results were as follows:

Incremental models strategy:	34% of overall use.
Supposition and step strategy:	21% of overall use.
Compound strategy:	19% of overall use.
Chain strategy:	25% of overall use.
Concatenation strategy:	0% of overall use.

The most salient feature of the protocols was that different participants used different strategies.

### A theory of reasoning strategies

A *deterministic* process is one in which each step depends solely on the current state of the process and whatever input it may have (Hopcroft and Ullman, 1979). Following Harman (1973), however, we assume that reasoning is not a deterministic process that unwinds like clockwork. Our first assumption is accordingly:

1. The principle of *nondeterminism*: thinking in general and sentential reasoning in particular is governed by constraints, but there is seldom just a single path it must follow. It varies in a way that can be captured only in a nondeterministic account.

Experiment 1 corroborated the principle of nondeterminism, and it did so at two levels. At a high level, the participants developed diverse strategies. At a low level, there was considerable variation within strategies.

Our second assumption is:

2. The principle of *strategic assembly*: naïve reasoners assemble reasoning strategies bottom up as they explore problems using their existing inferential tactics. Once they have developed a strategy, it can control their reasoning in a top-down way.

A corollary of the principle is that individuals are most unlikely to develop a reasoning strategy working “top down” from a high-level specification. Granted the principle, it also follows that the space of possible strategies is defined by the different ways in which inferential tactics can be sequenced in order to make inferences. Hence, an exhaustive enumeration of tactics provides the recursive basis for all possible strategies.

If the mechanism underlying reasoning depends on mental models, then each inferential tactic must be based on models. We therefore postulate a third assumption:

3. The principle of *model-based tactics*: inferential tactics are based on mental models.

The mechanisms for constructing models are, in turn, constrained by the nature of the human mind, which reflects innate constraints and individual experiences.

Our first test of the three principles was to show that mental models can underlie all the strategies and tactics in our taxonomy. The *incremental models* strategy is isomorphic to the cumulative construction of a single set of models based on the premises. The *step* strategy is based on a categorical premise or a supposition. Although the strategy is similar to the one strategy that Rips (1994) proposes, the model theory allows a greater freedom in the use of suppositions – a freedom that corresponds to their use by naïve reasoners. The main inferential step is to use a literal to update a set of models based on a premise in order to draw another literal as a conclusion. A premise, such as: Black ore gray, yields two models:

black  
          gray

and the supposition, Not black, eliminates the first model and yields the literal conclusion: gray. The *compound*

Table 1: The model-based tactics underlying each of the five strategies: + indicates the use of a tactic, and (+) indicates its optional use.

Tactics	The five strategies				
	Increment models	Step	Compound	Chain	Concatenation
Make a supposition	(+)	+			
Concatenate premises		(+)	(+)		+
Construct models	+	+	+	+	+
Update models	+	+	+		
Immediate inference from models	(+)		(+)	+	+
Formulate intermediate conclusion from models		+	+		
Evaluate or formulate a conclusion from models	+	+	+	+	+

strategy relies on a series of compound inferences based on models. The *chain* strategy depends on the construction of a chain of conditionals. The chain has one explicit mental model and one implicit mental model. To prove a conditional of the form:

If A then D.

individuals can construct a chain leading from D to A, e.g.:

If D then not-C.  
If not-C then B.  
If B then A.

Such a strategy is invalid. So, why do reasoners construct this chain? The answer is that the conclusion holds in the mental models of the chain:

d        ¬c        b        a

Hence, mental models underlie the strategy. The *concatenation* strategy appears at first sight to rely on purely syntactic operations, and therefore to violate the principle of model-based tactics. In fact, the strategy depends critically on mental models. Given a pair of premises of the form:

A iff B.  
B ore C.

there are two ways in which to concatenate a conclusion:

1. (A iff B) ore C.
2. A iff (B ore C).

Which of these two conclusions follows from the premises? In fact, neither conclusion is valid. Yet, eight out of the eight participants in Experiment 3 who concatenated conclusions from the relevant premises generated conclusion 2. It is the one conclusion that has the same mental models as the premises. Ten participants in Experiment 2 used the tactic of concatenating a conclusion on one or more occasions. On 82% of occasions, the resulting conclusions were compatible with the mental models of the premises, and nine of the ten participants concatenated more conclu-

sions of this sort than not (Sign test,  $p < .02$ ). Concatenation is not blindly syntactic. It tends to be accepted only if it yields the same mental models as the premises. Table 1 presents the taxonomy of strategies and their underlying model-based tactics.

**Experiment 2: Development of strategies**

The theory predicts that the nature of the inferential problems given to reasoners should influence their development of strategies. According to the principle of strategic assembly, the characteristics of particular problems should trigger certain strategies “bottom up”. One instance of this prediction concerns the effects of number of models. Problems that include a categorical premise or a conjunction of them yield a single model. Hence, individuals can use a categorical premise as the starting point of their reasoning, and the step strategy is the easiest way to proceed because it places a minimal load on working memory. With multiple-model problems, the optimal way to keep track of the possibilities is to use the incremental models strategy. Multiple models, however, should also yield a greater number of errors. The aim of the present experiment was to test these predictions.

*Method.* Twenty Princeton undergraduates acted as their own controls and evaluated given conclusions to 36 problems presented in three blocks: twelve one-model inferences, twelve two-model inferences, and twelve three-model inferences. Typical problems were of the form:

<i>One-model</i>	<i>Two-model</i>	<i>Three-model</i>
A and B.	A iff B.	A iff B.
B ore C.	B ore C.	B iff C.
C iff D.	C iff D.	C or D.
A and not D?	A iff D?	A or D?

The participants evaluated the conclusions, and we used the same think-aloud and video-recording procedure as before.

Table 2: The percentages of the different strategies for the three sorts of problems in Experiment 2. The balances of the percentages (5%) were uncategorizable strategies.

	The strategies		
	Incremental models	Step	The other strategies
One-model premises	21	69	3
Two-model premises	26	56	15
Three-model premises	49	45	2

*Results.* As the model theory predicts, errors increased with the number of models: there were 8% of errors with one-model problems, 15% of errors with two-model problems, and 20% of errors with three model problems (Page’s  $L = 251.5$ ,  $p < 0.05$ , one-tailed). Table 2 presents the percentages of the different strategies for the different sorts of problem. The participants were sensitive to the properties

of the particular problems. As the theory predicts, they relied increasingly on the incremental models strategy as the problems required a greater number of models (Page’s  $L = 254.5$ ,  $p < .05$ , one-tailed). They tended to use the step strategy with one-model problems, but the use of the strategy declined with an increasing number of models. The results accordingly corroborated the principle of strategic assembly: reasoners develop strategies “bottom-up” depending on the sort of problem that they encounter.

**Experiment 3: Formulating conclusions**

This experiment was similar to Experiment 2, except that the participants had to draw their own conclusions.

*Method.* Twenty four Princeton undergraduates acted as their own controls and carried out four one-model inferences, four two-model inferences, and four three-model inferences, in counterbalanced orders. For each problem, they wrote down their answer to the question, “What, if anything, follows?” and we used the same procedure as before.

*Results.* The participants developed diverse strategies, and the realization of any particular strategy varied from trial to trial even for the same participant. As the model theory predicts, the percentages of invalid conclusions, modal conclusions about possibilities, and conclusions that failed to take into account all the premises, each increased significantly with the number of models. Table 3 presents the percentages of the different strategies in the experiment. As predicted, the use of the incremental models strategy increased with the number of mental models required by the premises. With one-model problems, the participants were likely to use the step strategy, but there was an increase in the use of the incremental models strategy with multiple-model inferences. This trend was reliable (Kendall’s coefficient of concordance,  $W = 0.228$ ,  $X^2 = 10.94$ ,  $p < .01$ , two-tailed).

Strategies should influence the form of the conclusions that reasoners draw. With incremental models, it is difficult to see what is common to a number of alternative possibilities, and so reasoners should tend to describe each possibility separately and to combine these descriptions in a disjunction. The other strategies, however, are unlikely to yield conclusions of this sort. These strategies focus on a single possibility, such as a supposition. We examined this prediction by dividing the participants in Experiment 3 into two post hoc groups. In the *model* group (9 participants), more than half of the participants’ identifiable strategies yielding conclusions were the incremental models strategy. In the *non-model* group (15 participants), more than half of the participants’ identifiable strategies yielding conclusions were some other sort. For the model group, 63% of the problems solved with the model strategy had a conclusion that was a disjunction of possibilities, but for the non-model group only 11% of the problems solved with a non-model strategy had such a conclusion (Mann-Whitney test,  $z = 2.87$ ,  $p < .005$  one-tailed). Different strategies do yield different sorts of conclusion.

Table 3: The percentages of the different strategies for the three sorts of problems in Experiment 3. The balances of the percentages (11% overall) are uncategorizable strategies.

	The strategies			
	Incremental models	Step	Compound	Chain
One-model premises	14	80	5	3
Two-model premises	33	22	20	9
Three-model premises	36	25	14	7
Overall	28	41	13	7

### Experiment 4: Strategies and premises

The principle of strategic assembly implies that the form of the premises should influence the development of strategies. A way to elicit the incremental models strategy should be use to disjunctive premises, which are naturally represented as sets of possibilities. A way to elicit the step and chain strategies is to use conditional premises, which have only a single explicit model required by these strategies. These effects should occur even when the premises are otherwise logically equivalent. Once individuals have developed a strategy, it should have a “top down” residual effect on their subsequent performance. It should be used for problems that would not normally trigger its use. The experiment tested these predictions.

*Method.* Twenty Princeton undergraduates acted as their own controls and drew their own conclusions to two sets of problems: four disjunctive problems and four logically equivalent conditional problems. Half the participants received the four disjunctive problems in a random order followed by the four conditional problems in a random order; and half the participants received the two blocks of problems in the opposite order.

*Results.* Table 4 presents the percentages of the different strategies for the two sorts of problems, and it gives the data separately for the two blocks of trials. As the theory predicts, the participants were more likely to use the incremental models strategy (56%) for the disjunctive problems than for the conditional problems (23%; Wilcoxon test  $T = 66, n = 11, p < .0005$ ). The table shows that the participants who first carried out the conditional problems rarely developed the incremental models strategy (10% of these problems), but their use of the strategy increased reliably for the disjunctive problems (55% of problems, Sign test,  $p < .02$ , two tailed). In contrast, those who first carried out the disjunctive problems often developed the incremental models strategy, and did not reliably reduce its use with the conditional problems. This difference between the two groups was reliable (Mann-Whitney  $U = 21, p < .05$ , two tailed). An obvious explanation for the differential transfer is that the incremental models strategy is simpler to use with any sort of sentential connective, whereas the step and chain strategies call for additional immediate inferences to convert

Table 4: The percentages of the different strategies for (a) the disjunctive problems and (b) the conditional problems in Experiment 4. The balances of the percentages are trials with erroneous responses or uncategorizable strategies.

(a) Disjunctive problems	The strategies	
	Incremental models	Step, Compound, and Chain
Presented first	58	35
Presented second	55	35
Overall	56	35

(b) Conditional problems	The strategies	
	Incremental models	Step, Compound, and Chain
Presented first	10	90
Presented second	35	60
Overall	23	75

disjunctive premises into conditionals.

The experiment corroborated the principle of strategic assembly. The nature of the sentential connectives biases reasoners to adopt particular strategies. The incremental models strategy, though it places a greater load on working memory, is more flexible than the other strategies, which are more finely tuned to conditional premises.

### General Discussion

Unlike some cognitive domains, such as arithmetic (Lemaire and Siegler, 1995), accounts of sentential reasoning have neglected strategies (for reviews, see Evans, Newstead, and Byrne, 1993; Garnham and Oakhill, 1994). Studies have failed to use appropriate methods to discover strategies; and in consequence theorists have often assumed that reasoners rely on a single deterministic strategy. We have tried to remedy the neglect and to advance a new theory of strategies in reasoning. Naïve reasoners use at least five distinct strategies. As the theory predicts, each strategy is built from tactical steps that rely on the manipulation of models (see Table 1). The *incremental models* strategy keeps track of all the mental models compatible with the premises. The *step* strategy pursues the step by step consequences of one model – either one derived from a categorical assertion in a premise or one created by a supposition. The *compound* strategy combines the models of compound premises to infer what is necessary or possible. The *chain* strategy pursues a model in a sequence of conditionals, which may be inferred from the premises, leading from one constituent of a conclusion to another. The *concatenation* strategy forms a conclusion by concatenating the premises, but normally only if the resulting conclusion has the same mental models as the premises. Because it relies on mental models, it gives rise spontaneously to illusory inferences (cf. Johnson-Laird and

Savary, 1999; Goldvarg and Johnson-Laird, 2000; Johnson-Laird et al., 2000; Yang and Johnson-Laird, 2000).

The model theory explains how people develop reasoning strategies. They are equipped with a set of inferential tactics. As they reason, the variation in their performance, leads them to assemble these tactics in novel ways so that they yield a reasoning strategy. As a result, they develop different strategies. All their strategies, however, depend on tactics based on mental models. The properties of inferential problems can accordingly influence the development of particular strategies. The problems in Experiments 2 and 3 called for one, two, or three models. As the theory predicts, the participants tended to use the conjunction in one-model problems as the starting point for the step strategy, which places a minimal load on working memory. As the number of models increased, they were more likely to use the incremental models strategy, which keeps track of the different possibilities compatible with the premises. Experiment 4 also bore out the theory's account of strategic assembly. Disjunctive premises, as predicted, tended to elicit the incremental models strategy, whereas conditional premises tended to elicit other strategies. The participants increased their use of incremental models on switching to disjunctive premises, but they did not decrease its use on switching to conditional problems. Although incremental models load working memory, the strategy is more flexible than those that are optimal for conditional premises.

What would have refuted our theory? At the lowest level, that of inferential mechanisms, the theory would have been refuted if there had not been an increase of difficulty with the number of models required by the problems. This phenomenon has been observed in previous studies (see Johnson-Laird and Byrne, 1991), but not before in inferences based on three sentential connectives. At the tactical level, the theory would have been refuted if reasoners used tactics incompatible with manipulations of models. Suppose, for example, that the concatenation had not been sensitive to the mental models of the premises, then a tactic would have been controlled purely by syntactic considerations, and it would have been contrary to the theory. At the strategic level, the theory would have been refuted if reasoners had uniformly developed a single deterministic strategy (cf. Rips, 1994). The moral of our results is clear. They support the three principles of the model theory of reasoning strategies.

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