Modeling Spatial Ability in Mental Rotation and Paper-Folding

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Abstract

Spatial ability tests like mental rotation and paper-folding provide strong predictions of an individual's achievement in science and engineering. What cognitive skills are involved in them? We use a computational model to analyze these tasks, asking how much information must be processed to perform them. The models demonstrate that in some cases stimuli can be vastly simplified, resulting in consistent performance regardless of stimulus complexity. The ability to produce a scaled-down representation of a complex stimulus may be a key skill underlying high spatial ability.

Keywords: spatial ability; mental rotation; paper-folding; cognitive modeling.

Introduction

There is strong evidence linking spatial ability to academic achievement. Children who perform well on spatial ability tests are more likely to study STEM disciplines (Science, Technology, Engineering, and Mathematics) and to go into a STEM profession (Shea, Lubinski, & Benbow, 2001; Wai, Lubinski, & Benbow, 2009). This effect holds even when controlling for verbal and mathematical ability, suggesting that spatial ability is an independent component of intelligence. If we are to improve STEM achievement, it is critical that we better understand the skills that compose spatial ability and how they can be taught.

Traditionally, spatial ability has been evaluated using tasks such as mental rotation and paper-folding. In mental rotation (Figure 1A, 1B), individuals are shown two shapes and asked whether a rotation of one shape could produce the other. In paper-folding, they are shown a line-drawing of paper and asked to imagine the results of unfolding (Figure 1C) or folding up (Figure 1D) the paper. Both tasks appear to measure *spatial visualization*, the ability to manipulate mental representations of images (McGee, 1979). There is evidence that the tasks are linked, with training on one improving performance on the other (Wright et al., 2008). However, many questions remain about what skills enable people to perform them quickly and accurately.

Here, we study the mental rotation and paper-folding tasks using a computational model. The model operates directly on 2D line drawings (sketches), automatically generating representations, transforming them, and evaluating the results of the transformation. We use the model to analyze the tasks, asking how much information must be encoded and carried through the transformations to perform each task consistently. This analysis allows us to address a longstanding debate about the effects of shape complexity on mental rotation. It also provides hypotheses about the skills supporting fast, efficient mental rotation, and thus the skills underlying spatial ability.

We begin with background on mental rotation and the question of shape complexity. We show how paper-folding appears to violate many researchers' conclusions, as it involves simple shapes but requires great deliberation and effort. We next present our computational model, which builds on previous cognitive models of perception, comparison, and visual problem-solving (Falkenhainer, Forbus, & Gentner, 1989; Lovett & Forbus, 2011). We apply the model to the two tasks, determining the amount of information that must be carried through the transformations, and showing why paper-folding is a more difficult task. Finally, we discuss the results and consider the ramifications for spatial ability in general.

Background

Mental Rotation

Mental rotation is frequently used to evaluate spatial ability (Vandenberg & Kuse, 1978). Typically the distractors—the shapes that aren't a valid rotation—are mirror reflections. When they are presented sequentially, there is often a cue indicating what the orientation of the second shape will be (e.g., Cooper & Podogny, 1976; Figure 1B). A common finding across task variations is that the response time is proportional to the angle of rotation between the shapes. That is, response times increase linearly with angular distance. This finding has led to the claim that people use a mental space, analogous to the physical space, and that they rotate their representation through this space just as an object might rotate physically (Shepard & Cooper, 1982).

One common question concerns how shapes are rotated through mental space. Are they rotated piecemeal, with one part rotated at a time, or are they rotated holistically, with every part rotated together (Bethell-Fox & Shepard, 1988)? These two possibilities produce different predictions about how shape complexity interacts with rotation speed. If shapes are rotated piecemeal, then people should rotate complex shapes more slowly, because there are more parts to rotate. If shapes are rotated holistically, then shape complexity may not affect rotation speed.

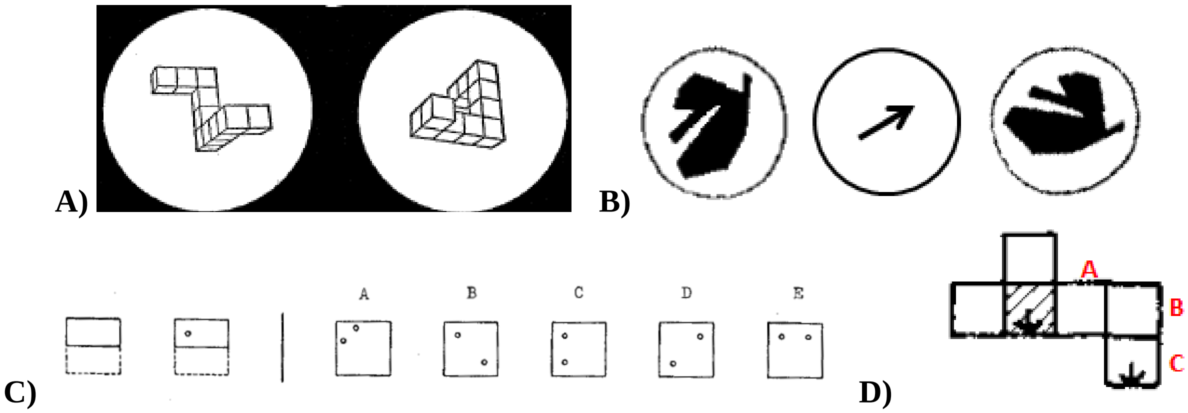

Figure 1. Mental rotation (A, B) and paper-folding (C, D) tasks (A: Shepard & Metzler, 1971; B: Cooper & Podogny, 1976; C: Ekstrom et al., 1976; D: Shepard & Feng, 1972).

The results on shape complexity provide evidence for both piecemeal and holistic rotation. Overall, it appears that rotation speed depends more on other factors, such as the familiarity of the objects (Bethell-Fox & Shepard, 1985; Yuille & Steiger, 1982), the similarity of the distractors (Folk & Luce, 1987), and the strategy and overall ability of the participant (Yuille & Steiger, 1982; Heil & Jansen-Osmann, 2008). These findings suggest that dealing with shape complexity may itself be a spatial skill. Skilled participants may apply heuristics to simplify shapes for rapid rotation. However, when these heuristics fail, they are reduced to rotating one piece at a time, which is slower.

One straightforward heuristic for simplifying a shape is to ignore parts of it. When Yuille and Steiger (1982) told participants they could complete a mental rotation task using only the top halves of the shapes, participants rotated the shapes more quickly. Alternatively, participants might utilize *scalable* representations (Schultheis & Barkowsky, 2011) that support dynamic variation of detail based on task demands. Both the degree and the type of detail may vary. For example, while we can imagine both the locations and orientations of objects in space, a task might require considering only one of these. In this paper, we use the term *spatial smoothing* for any process that removes spatial detail, producing a simpler representation.

Participants may smooth out the details in complex shapes, producing representations with equal complexity to those of simpler shapes. However, when the distractors are particularly similar to the base shapes, participants may require additional detail, and so they may use more complex representations that are more difficult to rotate.

This hypothesis leads immediately to two predictions: 1) When similarity of distractors is kept constant and relatively low, people should rotate shapes at the same rate regardless of shape complexity. 2) As distractors become more similar, people should rotate shapes more slowly, particularly when the shapes are complex. There is evidence supporting both predictions (1: Cooper & Podgorny, 1976; 2: Folk & Luce, 1987).

Paper-Folding

In contrast with mental rotation, paper-folding has seen relatively little study. This is surprising, given that it is also often used to evaluate spatial ability (Ekstrom et al., 1976). Here, we focus on a version of paper-folding that emerged at about the same time as mental rotation (Shepard & Feng, 1972). While this version is used less frequently in spatial ability evaluations, there is direct evidence linking it to mental rotation (Wright et al., 2008).

Figure 1D shows an example. The letters have been added for illustrative purposes and are not part of the stimulus. In this task, participants are shown six connected squares, representing the surfaces of a cube that has been unfolded. Two edges are highlighted by arrows, and one square is grayed out, indicating it is the base of the cube. Participants are asked whether the highlighted edges would align if the squares were folded back into a cube.

Unlike mental rotation, this task requires a sequence of rotations. For example, Figure 1D requires three rotations. One solution (Figure 2) would be: 1) Rotate squares A, B, and C up, so that they stick out from the plane. 2) Rotate squares B and C down to make the top surface of the cube. 3) Rotate square C farther down, making the front surface of the cube. At this point, the two arrows align perfectly.

Surprisingly, even though each of these three rotations seems simple, they appear to be piecemeal rotations. Participants' response times are not a function of the number of rotations performed, but of the number of times every square is rotated. In this case, three squares are rotated (Figure 2B), then two squares (2C), then one square (2D),

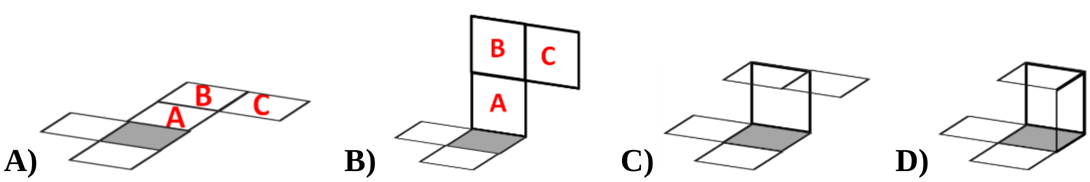

Figure 2. Possible solution for Figure 1D.

so the overall number of squares rotated is 3 + 2 + 1 = 6. The response times reflect the six rotations, suggesting participants rotate a single square at a time.

Why should participants require piecemeal rotation for such apparently simple shapes? We propose that, unlike many mental rotation tasks, little spatial smoothing can be performed. The precise location and orientation of every surface rotated is critical to performance. In Figure 2A, the location and orientation of square A determines where the second rotation occurs, and the location and orientation of square B determines where the third rotation occurs.

If this proposal is true, it may shed light on how and when spatial smoothing can be applied, and what happens when it cannot be used. Understanding this requires determining how much spatial information must be rotated in each task. To better answer this question, we developed a computational model of the tasks.

Model

The spatial ability model is built within CogSketch, a sketch understanding system. Below, we present CogSketch and its framework for cognitive modeling. We then describe how the model performs mental rotation and paper-folding.

CogSketch

CogSketch is an open-domain sketch understanding system (Forbus et al., 2011). Users sketch a scene by drawing one or more objects. It is the user's responsibility to manually segment a sketch into objects, indicating when they have finished drawing one object and begun on the next.

Given a set of objects, CogSketch automatically generates a representation of the scene. While CogSketch does not model the process of visual perception, its representations are a model of those produced by human perception. The representations are based on two psychological claims: 1) Spatial representations include a *qualitative* or categorical component and a *quantitative* or metric component (Kosslyn et al., 1989; Forbus, et al 1991). When possible, people use the qualitative component during reasoning. CogSketch computes qualitative spatial relations between objects, e.g., indicating that one object is **right of** another or that two objects **intersect**. 2) Spatial representations are hierarchical, meaning they can represent a scene at different levels of abstraction (Palmer, 1977; Marr & Nishihara, 1978). CogSketch can represent objects and the relations between them, or it can represent the edges of an individual object and the relations between those edges. To produce an edge-level representation, CogSketch segments an object's contour into edges at points of maximum curvature (e.g., Figure 3A; Lovett et al., 2012). Once edges have been computed, it generates qualitative spatial relations between the edges, e.g., indicating that two edges are **parallel** or that a corner between edges is **convex**.

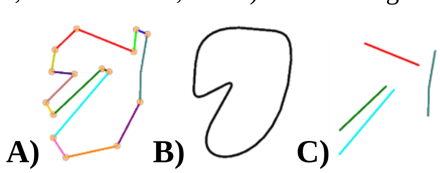

Figure 3. A: Shape segmented into edges. B: Result of Gaussian smoothing. C: Result of selecting 4 longest edges.

CogSketch models visual comparison using the Structure-Mapping Engine (SME) (Falkenhainer et al 1989), a domain-general cognitive model based on Gentner's (1983) structure-mapping theory. SME compares two qualitative representations by aligning their common relational structure, highlighting commonalities and differences. For example, suppose SME is comparing two shapes like the one in Figure 3A. Each representation will contain entities (symbols representing each edge), attributes (features of the edges, such as straight vs. curved), first-order relations between edges (e.g., indicating that a corner between edges is convex), and higher-order relations between other relations (e.g., indicating that two corners are adjacent along the shape). By aligning the common relations, SME can determine the corresponding edges in the two shapes.

Modeling in CogSketch CogSketch possesses two key features that support modeling psychological experiments. First, in addition to sketching by hand, users can import shapes from another program such as PowerPoint. Perfectly straight line drawings from an experiment can be replicated in PowerPoint and imported into CogSketch, providing it with the same stimuli as those shown to human participants.

Second, CogSketch includes a *Spatial Routines* language for writing cognitive models. Spatial Routines, which builds on Ullman's (1984) concept of visual routines, provides modelers with a set of cognitive operations. These include visual perception, visual comparison, and spatial transformation operations. Modelers can parameterize these operations and combine them in different ways to produce a spatial routine. Each routine is both a theoretical model of how people perform a task and a fully automated computational model. The computational model can be run on visual stimuli, and its performance can be compared to human responses to evaluate the theoretical model.

We have previously modeled visual problem-solving tasks such as geometric analogy (Lovett & Forbus, 2012), Raven's Progressive Matrices (Lovett, Forbus, & Usher, 2010), and an oddity task (Lovett & Forbus, 2011).

Mental Rotation

We modeled a classic sequential mental rotation task because it presents the clearest evidence for efficient, holistic rotation. In this task (Figure 1B; Cooper & Podogny, 1976), participants are presented with three stimuli in sequence: 1) They see the base shape. 2) They see an arrow indicating what the target shape's orientation will be. They are encouraged to mentally rotate the base shape to that orientation and press a button when they are done. 3) They see the target shape, and they indicate whether it is the same as the rotated base shape. The amount of time participants spend on step 2) indicates the rotation time. The key finding of the experiment was that rotation time did not increase as the shape complexity increased from a 6-sided polygon to a 24-sided polygon. Our model is designed to evaluate whether spatial smoothing can explain this finding.

Input In CogSketch, sequences of images can be input using a *sketch lattice*, a grid which divides the sketch space. For this model, we used a three-cell lattice to represent the three phases of each experimental trial. Stimuli were reproduced in PowerPoint. The experimenters traced over images of the original stimuli, ensuring that the number of sides in the new polygons was the same as in the original.

Representations CogSketch uses edge-level representations to perform two-dimensional shape transformations and comparisons. The *qualitative* edge-level representations describe spatial relations between edges, as summarized above. The *quantitative* representation includes for each edge: 1) The **location** of its center. 2) Its two-dimensional **orientation**. 3) Its **length**. 4) Its **curvature**.

When a shape is scaled, rotated, or reflected, each individual edge is transformed. This has little effect on the qualitative representation, but it can change each of the four features in the quantitative representation.

Shapes are compared in a two-step process. 1) Qualitative representations are compared using the Structure-Mapping Engine. This identifies the corresponding edges in the two shapes. 2) Each corresponding edge pair's four quantitative values are compared. If every pair is quantitatively the same, the shapes are identical.

Strategy Given a stimulus such as Figure 1B, the model automatically constructs an edge-level representation of each shape. It detects the orientation of the arrow and rotates the base (leftmost) shape accordingly. It then compares the rotated base shape to the target shape to determine whether they are identical.

Spatial Smoothing Recall that scalable representations allow two forms of spatial smoothing: spatial detail may be smoothed out, or certain types of spatial information may be removed entirely. There are many possible ways to smooth data, e.g. apply a Gaussian filter to the entire shape (Figure 3B). Such an approach would lose critical information about the nature of the edges making up the object. Here, we use a simple *sampling* strategy: we remove all but the four longest edges (Figure 3C). This operation produces representations of equal size for all shapes, regardless of their initial complexity, which is what we desire.

The quantitative representations contain four types of spatial information. We propose that spatial smoothing might remove three, leaving only a single type. In our evaluation, we test whether the task can be performed using only edge **locations** or using only edge **orientations**.

Paper Folding

We modeled the paper-folding task shown in Figure 1D.

Input Each paper-folding stimulus was recreated in PowerPoint. The square representing the base of the cube was given a solid gray fill (CogSketch can recognize elements by their color). CogSketch was given three objects: the unfolded cube and the two arrows pointing to the critical edges.

Representations This model required the development of a new representational level: surfaces. Surfaces are closed shapes making up the sides of three-dimensional objects. Each square of the unfolded cube is a separate surface. Surfaces can be computed easily using CogSketch's existing ability to find closed cycles of edges.

This model does not need to find corresponding elements, so it requires only *quantitative* representations. Each surface is represented with: 1) The **location** of its center. Locations are now in three-dimensional space, unlike with the previous model. 2) Its **orientation**, i.e., the orientation of a vector orthogonal to the surface. Three-dimensional orientations are unit vectors containing (x, y, z) components, unlike the single value required for two dimensions. 3) A list of **edges** going around the surface. Each edge has its own individual location and orientation.

Three-dimensional rotations are performed about an axis in three-dimensional space. For example, in Figure 2B, surfaces A, B, and C are rotated about the edge connecting surface A to the base of the cube.

Strategy Given a stimulus, the model segments the object into edges and surfaces. Using the arrows, it identifies the two critical edges and their associated surfaces. In Figure 1D, the critical surfaces are the base of the cube and surface C. It folds each critical surface back into the cube shape via two spatial operations: 1) Trace along adjacent surfaces from the critical surface to the base surface. For surface C, this would produce the following trace: C->B->A->base. 2) Rotate 90° about the edge between surfaces in the reverse trace order. First rotate surface A about the edge between the base and surface A. Because surfaces B and C are connected to A, they will also be rotated (Figure 2B). Next, rotate surface B about the edge between surfaces A and B. Because C is connected, it will also be rotated. And so on.

The model performs these two operations on each critical surface. In Figure 1D, the second critical surface is not rotated because it is already the base. The model takes the resulting shape and evaluates whether the two critical edges are aligned, i.e., have the same location and orientation.

Spatial Smoothing In this task, the location and orientation of most edges is irrelevant to the task; only the two critical edges matter. If an edge lies along only one surface, it can be ignored. If an edge lies between two surfaces (e.g., the edge between surfaces A and B), it is important for determining the axis of rotation. However, because this task involves perfectly regular square shapes, a heuristic can be used: when rotating between two surfaces, place the axis of rotation halfway between the surfaces' centers, and orient it within the plane of those surfaces, perpendicular to the line connecting their centers. Due to this heuristic, all edges can be ignored except the two critical edges.

This means the following is being considered: the location and orientation of each rotated surface (and of the base), and the location and orientation of each critical edge. It may be possible, again, to consider only the location or orientation of the critical edges. However, both a surface's location and its orientation must be used in computing axes of rotation.

Simulation

Mental Rotation

The original experiment (Cooper & Podogny, 1976) used five base shapes which varied in complexity from a 6-sided figure to a 24-sided figure. On each trial, participants were cued to rotate shapes some multiple of 60° using the rotated arrow. They were then presented with a target shape at the new orientation. This could be: a) the same shape; b) a mirror-reflected shape; or c) a shape with some of the points permuted from the base shape. While distractors of type b) are commonly used, the distractors of type c) were added to test how carefully participants were rotating the shapes.

In this simulation, we ran the model on all six base shapes. However, we used only a single rotation value (60°), and only the mirror-reflected distractors. The single rotation was used because other rotations are mathematically equivalent and should not place additional demands on the model. The mirror-reflected distractors were used because they are the most common distractors found across different mental rotation tasks. In Future Work, we consider the challenge of recognizing permuted distractors.

Results Recall that the model spatially smoothed each shape, removing all but the four longest edges. This proved sufficient for recognizing that same shapes were the same and mirror-reflected shapes were different. Furthermore, when only edge orientations or only edge locations were used, either was sufficient for performing the task. We can conclude that when the distractors are sufficiently different, very little information must be rotated to perform mental rotation, and the complexity of the shapes is irrelevant.

Paper-Folding

Shepard and Feng (1972) identify ten different classes of paper-folding problems, based on the number of folds and the number of squares per fold. For example, Figure 1D is a class I problem, in which 3 + 2 +1 squares must be rotated for one critical edge and no squares must be rotated for the other. Their paper provides one example of each class.

In this simulation, we ran the model on the single example of each class. Other instances of a class are mathematically equivalent. As in the original study, there was one nonmatch problem (where folding did *not* cause the critical edges to align) for each match problem. Nonmatch problems were created by randomly rotating an arrow so that it pointed to an adjacent edge in the same square.

Results Recall that the model rotated each surface's center and orientation. The two critical edges were rotated also, but all other edges were ignored. This proved sufficient for solving all problems—the model correctly distinguished between matches and nonmatches. Furthermore, when only edge orientations or only edge locations were used to compare the critical edges, either was sufficient.

Discussion

Having successfully modeled both tasks, we can now consider how much data must be transformed to perform mental rotation and paper-folding. The mental rotation model required only four values: the orientations or locations of the four longest edges.

Now, suppose the model were rotating two surfaces during paper-folding. This would require five values: the location *and* orientation of the two surfaces, and the location *or* orientation of the critical edge. Furthermore, these values are three-dimensional, whereas the mental rotation values were two-dimensional. In the computational model, three-dimensional values are far more complex—for example, an orientation is a vector with (x, y, z) components. Spatial data is likely implemented differently in the brain, but there still may be increased processing demands for three dimensions (see Jolicoeur et al., 1985 for a discussion; but see Shepard & Metzler, 1988).

These results support our initial hypothesis. Rotation rate appears to depend less on shape complexity than on the degree and type of detail required by the task. Paper-folding requires that more information be transformed, even when only two surfaces are being rotated and the surfaces are perfect squares. We propose that paper-folding overwhelms people's spatial working memory, forcing them to rotate one surface at a time in a piecemeal fashion.

Conclusions and Future Work

This paper demonstrates how visual stimuli can be encoded, transformed, and compared. The computational model builds on existing cognitive models of visual representation and comparison. While we wish to avoid strong conclusions about how spatial information is represented and transformed in the brain, the model provides valuable information on the constraints of the modeled tasks.

In particular, the model suggests much of the spatial detail in a shape can be ignored during transformation. The detail needed depends on the task. In a task like mental rotation, where a single transformation is applied to all edges, very little detail is required. In a task like paper-folding, where the results of one rotation determine the axis for the next rotation, more detail must be carried through each transformation. Of course, even in mental rotation more detail will be required as the distractors become more similar to the shapes being rotated (Folk & Luce, 1987; Yuille & Steiger, 1982).

These findings are important for understanding spatial ability. Fast, holistic spatial transformations require spatial smoothing. Thus, a key component of spatial ability must be spatial abstraction: the ability to identify and encode critical spatial details while ignoring irrelevant features.

Questions remain about how skilled rotators perform spatial abstraction. The present approach of selecting the four longest edges, while effective, is only one heuristic. Others might include studying the distractors to determine which parts of a shape are most diagnostic (Yuille & Steiger, 1982) and segmenting shapes into larger-scale parts (Hoffman & Richards, 1984). In the future, we would like to study a larger stimulus set with distractors that vary in their similarity to the base shapes (Cooper & Podogny, 1976; Folk & Luce, 1987). By evaluating different spatial smoothing heuristics in the model, we can better understand the skills supporting spatial abstraction and spatial ability.

Acknowledgments

This work was supported by NSF SLC Grant SBE-0541957, the Spatial Intelligence and Learning Center (SILC).

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