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A Unified, Resource-Rational Account of the Allais and Ellsberg Paradoxes

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Abstract

Decades of empirical and theoretical research on human decision-making has broadly categorized it into two, separate realms: decision-making under risk and decision-making under uncertainty, with the Allais paradox and the Ellsberg paradox being a prominent example of each, respectively. In this work, we present the first unified, resource-rational account of these two paradoxes. Specifically, we show that Nobandegani et al.'s (2018) *sample-based expected utility* model provides a unified, process-level account of the two variants of the Allais paradox (the common-consequence effect and the common-ratio effect) and the Ellsberg paradox. Our work suggests that the broad framework of resource-rationality could permit a unified treatment of decision-making under risk and decision-making under uncertainty, thus approaching a unified account of human decision-making.

Keywords: decision-making under risk; decision-making under uncertainty; Allais paradox; Ellsberg paradox; resource-rationality; sample-based expected utility model

1 Introduction

When choosing among several alternatives, either the objective probabilities associated with the possible outcomes of each alternative are fully known, or these objective probabilities are partially or fully unknown. The former is known as decision-making under risk, while the latter is studied under the rubric of decision-making under uncertainty (Knight, 1921; Weber & Camerer, 1987; Camerer & Weber, 1992).

Decades of empirical and theoretical research on human decision-making has extensively studied decision-making under risk and decision-making under uncertainty, predominantly treating them as two distinct modes of decision-making, with potentially different cognitive underpinnings (e.g., Camerer & Weber, 1992; Bonatti et al., 2009; Johnson & Busemeyer, 2010; Buckert et al., 2014; De Groot & Thurik, 2018). However, recent work has called for a *unified* treatment of decision-making under risk and decision-making under uncertainty (e.g., Hsu et al., 2005; Deany & Ortoleva, 2017), potentially understanding them as limiting cases of a broader framework (Hsu et al., 2005).

Initially introduced as two major violations of expected utility theory, the Allais paradox (1953) and the Ellsberg paradox (1961) have been a driving force for developing models of decision-making under risk and decision-making under uncertainty, respectively. However, as Deany and Ortoleva (2017) point out, among the remarkably large number of decision-making models developed in the literature, few of them can explain *both* paradoxes (we discuss these models in the Discussion section).

As a step toward a unified treatment of these two types of decision-making, here we investigate whether the the broad framework of resource-rationality (Nobandegani, 2017; Lieder & Griffiths, 2020) could provide a unified account of the Allais paradox and the Ellsberg paradox. That is, we ask if these two paradoxes could be understood in terms of optimal use of limited cognitive resources. Answering this in the affirmative, we show that a resource-rational process model, *sample-based expected utility* (SbEU; Nobandegani et al., 2018), provides a unified, process-level account of the two variants of the Allais paradox (the common-consequence effect and the common-ratio effect) and the Ellsberg paradox.

This paper is organized as follows. We first elaborate on the computational underpinnings of SbEU (Sec. 2). We then introduce the Allais paradox (Sec. 3) and the Ellsberg paradox (Sec. 4), and present our simulation results. We conclude by discussing the implications of our work for a unified treatment of decision-making under risk and under uncertainty, and, more broadly, for human rationality.

2 Sample-based Expected Utility Model

SbEU is a resource-rational process model of risky choice that posits that an agent rationally adapts their strategy depending on the amount of time available for decision-making (Nobandegani et al., 2018). Concretely, SbEU assumes that an agent estimates expected utility

$$\mathbb{E}[u(o)] = \int p(o)u(o)do, \qquad (1)$$

using self-normalized importance sampling (Hammersley & Handscomb, 1964; Geweke, 1989), with its importance distribution q^* aiming to optimally minimize mean-squared error (MSE):

$$\hat{E} = \frac{1}{\sum_{j=1}^{s} w_j} \sum_{i=1}^{s} w_i u(o_i), \quad \forall i: \; o_i \sim q^*, \, w_i = \frac{p(o_i)}{q^*(o_i)}, \qquad (2)$$

$$q^*(o) \propto p(o)|u(o)| \sqrt{\frac{1+|u(o)|\sqrt{s}}{|u(o)|\sqrt{s}}}. \qquad (3)$$

MSE is a standard measure of estimation quality, widely used in decision theory and mathematical statistics (Poor, 2013).

In Eqs. (1-3), o denotes an outcome of a risky gamble, $p(o)$ the objective probability of outcome o, $u(o)$ the subjective utility of outcome o, $\hat{E}$ the importance-sampling estimate of expected utility given in Eq. (1), q^* the importance-sampling distribution, o_i an outcome randomly sampled from q^*, and s the number of samples drawn from q^*.

SbEU assumes that, when choosing between a pair of risky gambles A, B, people consider whether the expected value of the utility difference $\Delta u(o)$ is negative or positive (w.p. stands for "with probability"):

$$A = \begin{cases} o_A & \text{w.p. } P_A \\ 0 & \text{w.p. } 1 - P_A \end{cases} \tag{4}$$

$$B = \begin{cases} o_B & \text{w.p. } P_B \\ 0 & \text{w.p. } 1 - P_B \end{cases} \tag{5}$$

$$\Delta u(o) = \begin{cases} u(o_A) - u(o_B) & \text{w.p. } P_A P_B \\ u(o_A) - u(0) & \text{w.p. } P_A(1 - P_B) \\ u(0) - u(o_B) & \text{w.p. } (1 - P_A)P_B \\ 0 & \text{w.p. } (1 - P_A)(1 - P_B) \end{cases} \tag{6}$$

In Eq. (6), $u(\cdot)$ denotes the subjective utility function of a decision-maker. Consistent with past work (Nobandegani et al., 2018; Nobandegani et al., 2019b; Nobandegani, Destais, & Shultz, 2020, Nobandegani & Shultz, 2020a), in this paper we use the following utility function:

$$u(x) = \begin{cases} x^{0.85} & \text{if } x \geq 0, \\ -2|x|^{0.95} & \text{if } x < 0. \end{cases} \tag{7}$$

Nobandegani et al. (2018) showed that SbEU can explain the well-known fourfold pattern of risk preferences in outcome probability (Tversky & Kahneman, 1992) and in outcome magnitude (Markovitz, 1952; Scholten & Read, 2014). Notably, SbEU is the first rational process model to score near-perfectly in optimality, economical use of limited cognitive resources, and robustness, all at the same time (see Nobandegani et al., 2018; Nobandegani et al., 2019).

Relatedly, recent work has shown that SbEU provides a resource-rational mechanistic account of cooperation in one-shot Prisoner's Dilemma games (Nobandegani et al., 2019b), inequality aversion in the Ultimatum game (Nobandegani et al., 2020), emotions in both one-shot Prisoner's Dilemma and the Ultimatum games (Lizotte, Nobandegani, & Shultz, 2021), and human coordination strategies in coordination games (Nobandegani & Shultz, 2020a), thus successfully bridging between game-theoretic and risky decision-making. SbEU can also account for violation of betweenness in risky choice (Nobandegani, da Silva Castanheira, Shultz, & Otto, 2019a) and the centuries-old St. Petersburg paradox in human decision-making (Nobandegani & Shultz, 2020b,c). SbEU also provides a resource-rational process-level explanation of several contextual effects in risky and value-based decision-making (da Silva Castanheira, Nobandegani, Shultz, & Otto, 2019; Nobandegani et al., 2019a). There is also experimental confirmation of a counterintuitive prediction of SbEU: deliberation makes people move from one cognitive bias, the framing effect, to another, the fourfold pattern of risk preferences (da Silva Castanheira; Nobandegani, & Otto, 2019). Importantly, SbEU is the first, and so far the only, resource-rational process model that bridges between risky, value-based, and game-theoretic decision-making.

3 The Allais Paradox

Introduced as a violation of expected utility theory, the Allais paradox (1953) has been a driving force for developing models of decision-making under risk (Kahneman & Tversky, 1979; Katsikopoulos & Gigerenzer, 2008; Dean & Ortoleva, 2017). The Allais paradox has two variants: the common-consequence effect and the common-ratio effect. We present each of these variants and demonstrate that SbEU provides a unified, resource-rational, process model of them.

3.1 The Common-Consequence Effect

As its name implies, the common-consequence effect (CCE) concerns choosing between two risky gambles that share a common consequence, with known objective probability. As a canonical example of the CCE, imagine choosing between the following two risky gambles (Kahneman & Tversky, 1979):

$$A = \begin{cases} z & \text{w.p. } 66\% \\ \$2,500 & \text{w.p. } 33\% \\ 0 & \text{w.p. } 1\% \end{cases} \tag{8}$$

$$B = \begin{cases} z & \text{w.p. } 66\% \\ \$2,400 & \text{w.p. } 34\% \end{cases} \tag{9}$$

As can be seen, if you choose A or B, you will get z dollars with probability 66% either way (hence, the term common-*consequence* effect).

When choosing between A, B, according to expected utility theory, preference should not be affected by the value of z. The rationale behind this is as follows. Note that the gap between the expected utility of gamble A, $EU(A)$, and the expected utility of B, $EU(B)$, does *not* depend on z:

$$\begin{aligned} EU(A) - EU(B) &= \big(u(z) \times 0.66 + u(\$2,500) \times 0.33\big) - \\ &\quad \big(u(z) \times 0.66 + u(\$2,400) \times 0.34\big) \\ &= u(\$2,500) \times 0.33 - u(\$2,400) \times 0.34. \end{aligned}$$

Therefore, varying z will not change the sign of $EU(A) - EU(B)$. (If $EU(A) - EU(B) < 0$, gamble B should be chosen, and if $EU(A) - EU(B) > 0$, gamble A should be chosen.) Hence, a decision-maker who follows expected utility theory should make the same choice, regardless of the value of z.

However, empirical evidence reveals that the value of z does affect choice: when $z = 0$ (Condition 1), the majority of participants (83%) chose gamble A, with the trend reversing

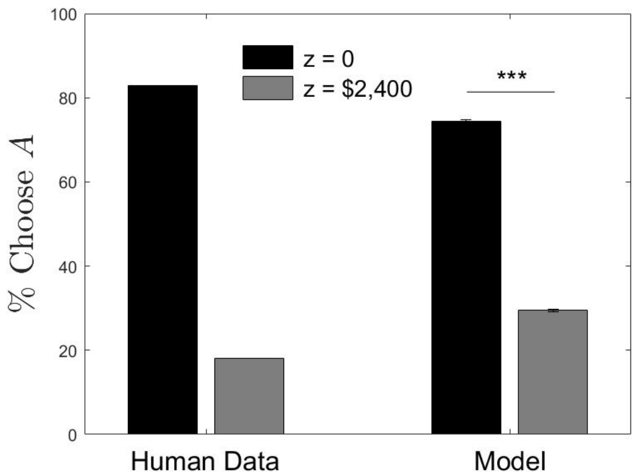


Figure 1: Comparing human data (Kahneman & Tversky, 1979) with SbEU model prediction for the Allais paradox (the common-consequence effect). For model prediction, Chi-squared test confirms that gamble A should be chosen more often when $z = 0$ than when $z = \$2,400$ ($\chi^2_{(1)} = 40467$, $p < 10^{-15}$), consistent with the empirical data (Kahneman & Tversky, 1979). We simulate 100,000 participants with $s = 2$. Error bars indicate binomial 95% CI. *** $p < 10^{-15}$.

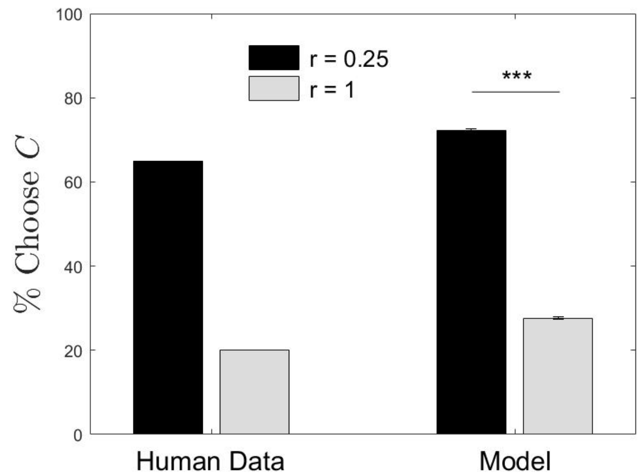


Figure 2: Comparing human data (Kahneman & Tversky, 1979) with SbEU model prediction for the Allais paradox (the common-ratio effect). For model prediction, Chi-squared test confirms that gamble C should be chosen more often when $r = 0.25$ than when $r = 1$ ($\chi^2_{(1)} = 39955$, $p < 10^{-15}$), consistent with the empirical data (Kahneman & Tversky, 1979). We simulate 100,000 participants with $s = 2$. Error bars indicate binomial 95% CI. *** $p < 10^{-15}$.

when $z = \$2,400$ (Condition 2), in which case only a minority (18%) chose gamble A (Kahneman & Tversky, 1979); see Fig. 1.

Consistent with the empirical data, SbEU predicts that the majority of people (74.44%) should choose gamble A when $z = 0$ ($\chi^2_{(1)} = 23896$, binomial 95% $CI = [74.17\%, 74.71\%]$, $p < 10^{-15}$), with the trend reversing when $z = \$2,400$, in which case only a minority (29.49%) should choose gamble A ($\chi^2_{(1)} = 16820$, binomial 95% $CI = [29.21\%, 29.78\%]$, $p < 10^{-15}$). We simulate 100,000 participants with $s = 2$; see Fig. 1.[1]

3.2 The Common-Ratio Effect

The common-ratio effect (CRE) concerns choosing between two risky gambles that each would yield a non-zero payoff with probability proportional to a positive common factor $0 < r \leq 1$. As a canonical example of the CRE, imagine choosing between the following two risky gambles (Kahneman & Tversky, 1979):

$$C = \begin{cases} \$4,000 & \text{w.p. } 0.8r \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

$$D = \begin{cases} \$3,000 & \text{w.p. } r \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

As can be seen, the probability that any of these gambles yields a non-zero payoff depends on a common factor r.

When choosing between C, D, according to expected utility theory, preference should not be affected by the value of $r \in (0, 1]$. That is, a decision-maker should make the same choice, regardless of the value of r. The rationale behind this is as follows. Note that the gap between the expected utility of gamble A, $EU(A)$, and the expected utility of B, $EU(B)$, is given by:

$$\begin{aligned} EU(C) - EU(D) &= u(\$4,000) \times 0.8r - u(\$3,000) \times r \\ &= \big(u(\$4,000) \times 0.8 - u(\$3,000)\big) \times r. \end{aligned}$$

Therefore, varying $r \in (0, 1]$ does not change the sign of $EU(C) - EU(D)$. (If $EU(C) - EU(D) < 0$, gamble D should be chosen, and if $EU(C) - EU(D) > 0$, gamble C should be chosen.) Hence, a decision-maker who follows expected utility theory should make the same choice, regardless of the value of $r \in (0, 1]$.

However, empirical evidence reveals that the value of r does have an effect on choice: when $r = 0.25$ (Condition 1), the majority of participants (65%) chose gamble C, with the trend reversing when $r = 1$ (Condition 2), in which case only a minority (20%) chose gamble C (Kahneman & Tversky, 1979); see Fig. 2.

Consistent with the empirical data, SbEU predicts that the majority of people (72.31%) should choose gamble C when $r = 0.25$ ($\chi^2_{(1)} = 19917$, binomial 95% $CI = [72.04\%, 72.59\%]$, $p < 10^{-15}$), with the trend reversing when $r = 1$, in which case only a minority (27.62%)

[1]In the original paper introducing the paradox, Allais (1953) considered two slightly different gambles, which, nevertheless, followed the general template of the gambles given in Equations (8-9). The SbEU model can also explain change of preference for the original gambles of Allais (1953).

should choose gamble C ($\chi^2_{(1)} = 20040$, binomial 95% $CI = [27.34\%, 27.9\%]$, $p < 10^{-15}$). We simulate 100,000 participants with $s = 2$; see Fig. 2.

4 The Ellsberg Paradox

Introduced as a violation of expected utility theory, the Ellsberg paradox (1961) has been a driving force for developing models of decision-making under uncertainty (e.g., Gilboa & Schmeidler, 1989; Ghirardato et al., 2003; Dean & Ortoleva, 2017). A canonical example of the Ellsberg paradox concerns an urn containing 90 balls: 30 of the balls are red; the remaining 60 are either black or yellow in unknown proportions. The balls are well mixed so that each individual ball is as likely to be drawn as any other. There are two experimental conditions. In Condition 1, participants are asked to choose between the following two gambles:

A) You receive \$100 if you draw a red ball
B) You receive \$100 if you draw a yellow ball

And in Condition 2, participants are asked to choose between the following two gambles (about a different draw from the same urn):

A) You receive \$100 if you draw a red or black ball
B) You receive \$100 if you draw a yellow or black ball

As can be seen, the objective probability of winning \$100 in gamble A of Condition 1 is fully known (it is 1/3), while the objective probability of winning \$100 in gamble B of Condition 1 is only imperfectly known (it could be anything between 0 and 2/3). Likewise, in gamble B of Condition 2, the objective probability of winning \$100 is fully known (it is 2/3), while the objective probability of winning \$100 in gamble A of Condition 2 is only imperfectly known (it could be anything between 1/3 and 1).

According to (subjective) expected utility theory, a decision-maker should make the same choice in both conditions (i.e., either to choose A in both conditions or choose B in both conditions). Hence, choosing A in one condition and B in the other constitutes a violation of expected utility theory. The rationale behind this is as follows. Let $p_{\text{red}}, p_{\text{black}}, p_{\text{yellow}}$ denote a decision-maker's subjective probability of drawing a red, black, and yellow ball from the urn, respectively. In Condition 1, the gap between the expected utility of gamble A, $EU(A)$, and the expected utility of B, $EU(B)$, is given by:

$$\begin{aligned} EU(A) - EU(B) &= u(\$100) \times p_{\text{red}} - u(\$100) \times p_{\text{yellow}} \\ &= \big(p_{\text{red}} - p_{\text{yellow}}\big) u(\$100). \end{aligned}$$

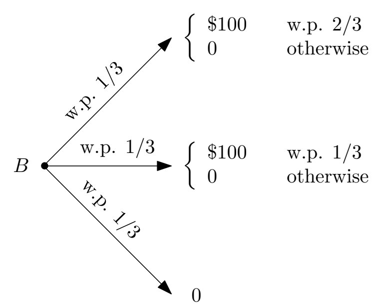
And, in Condition 2, the gap between the expected utility of gamble A, $EU(A)$, and the expected utility of B, $EU(B)$, is given by:

$$\begin{aligned} EU(A) - EU(B) &= u(\$100)(p_{\text{red}} + p_{\text{black}}) - \\ &\quad\; u(\$100)(p_{\text{yellow}} + p_{\text{black}}) \\ &= \big(p_{\text{red}} - p_{\text{yellow}}\big) u(\$100). \end{aligned}$$

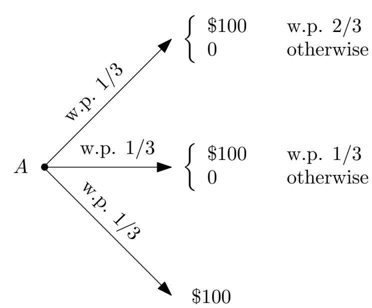
Therefore, the gap between the expected utility of gamble A, $EU(A)$, and the expected utility of B, $EU(B)$, is the same in both conditions, implying that the same choice should be made in both conditions (i.e., either A should be chosen in both conditions or B should be chosen in both conditions).

However, empirical evidence reveals that people do change their choice across the two experimental conditions (e.g., Weber & Tan, 2012). In Condition 1, the majority of participants (59.78%) chose gamble A, while, in Condition 2, only a minority (28.49%) chose gamble A (Weber & Tan, 2012); see Fig. 3.

Next, we show that SbEU can account for the Ellsberg paradox. We first model the two ambiguous gambles (gamble B of Condition 1 and gamble A of Condition 2), for which the probability of winning \$100 is only imperfectly known, as two-stage gambles. Specifically, to formally capture the state of maximal uncertainty regarding the objective probability of winning \$100 in these two gambles, we assume that the agent induces a *uniform* prior distribution on the range of possible probability values. That is, for gamble B of Condition 1, the agent assumes that the probability of winning \$100 is equally likely to take on any value between 0 and 2/3. Similarly, for gamble A of Condition 2, the agent assumes that the probability of winning \$100 is equally likely to take on any value between 1/3 and 1. For ease of analysis, next we assume that the agent's uniform prior distribution is defined over a finite set of states (here, we assume three canonical states), allowing us to model gambles B of Condition 1 as:



and gamble A of Condition 2 as:



The top branch of gamble B corresponds to the case where all of the 60 balls (whose composition are unknown) are yellow, implying that $p_{\text{yellow}} = 2/3$; the middle branch of gamble B corresponds to the case where half of the 60 balls are yellow,

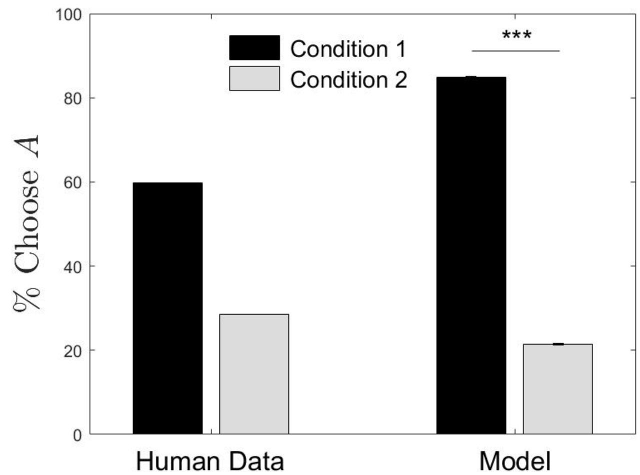


Figure 3: Comparing human data (Weber & Tan, 2012) with SbEU model prediction for the Ellsberg paradox. For model prediction, Chi-squared test confirms that gamble A should be chosen more often in Condition 1 than in Condition 2 ($\chi^2_{(1)} = 80554$, $p < 10^{-15}$), consisted with the empirical data (Weber & Tan, 2012). We simulate $100,000$ participants with $s = 2$. Error bars indicate binomial 95% CI. *** $p < 10^{-15}$.

implying that $p_{\text{yellow}} = 1/3$; and finally, the bottom branch of of gamble B corresponds to the case where none of the 60 balls are yellow, implying that $p_{\text{yellow}} = 0$. In the same vein, the top branch of gamble A corresponds to the case where half of the 60 balls (whose composition are unknown) are black, implying that $p_{\text{black}} = 1/3$ and hence $p_{\text{red}} + p_{\text{black}} = 2/3$; the middle branch of gamble A corresponds to the the case where none of the 60 balls are black, implying that $p_{\text{black}} = 0$ and hence $p_{\text{red}} + p_{\text{black}} = 1/3$; and finally, the bottom branch of gamble A corresponds to the case where all of the 60 balls are black implying that $p_{\text{black}} = 2/3$ and hence $p_{\text{red}} + p_{\text{black}} = 1$.

To simulate choice behavior, we first apply SbEU on the risky gambles residing at the end of each branch.[2] Doing that reduces each of the two two-stage gambles A, B given above to a classic one-stage risky gamble of the type we deal with in Sec. 2 and Sec. 3.

Consistent with the empirical data (Weber & Tan, 2012), SbEU predicts that the majority of people (84.8%) should choose gamble A in Condition 1 ($\chi^2_{(1)} = 48430$, binomial 95% $CI = [84.57\%, 85.02\%]$ $p < 10^{-15}$), with the trend reversing in Condition 2, in which case only a minority (21.46%) should choose gamble A ($\chi^2_{(1)} = 32593$, binomial 95% $CI = [21.20\%, 21.71\%]$, $p < 10^{-15}$). We simulate $100,000$ participants with $s = 2$; see Fig. 3.

[2]As the risky gambles residing at the end of each branch involve only a single non-zero outcome, to evaluate their expected utility using SbEU, we first decompose them as a sure amount (equal to their expected value, EV) plus a zero-EV risky gamble, and subsequently use SbEU to evaluate their expected utility.

5 Discussion

Originally introduced as two major violations of expected utility theory, the Allais paradox (1953) and the Ellsberg paradox (1961) have been a driving force for developing models of decision-making under risk and decision-making under uncertainty, respectively (see Dean & Ortoleva, 2017).

Decades of empirical and theoretical research on human decision-making has broadly categorized it into two separate domains: decision-making under risk and decision-making under uncertainty. However, recent work has called for a unified treatment of these purportedly distinct domains (Hsu et al., 2005; Deany & Ortoleva, 2017), possibly viewing them as limiting cases of a broader framework (Hsu et al., 2005).

In this work, we present the first unified, resource-rational account of the Allais paradox and the Ellsberg paradox. Specifically, we show that a single parameterization of *sample-based expected utility* model (SbEU; Nobandegani et al., 2018) provides a unified, process-level account of the two variants of the Allais paradox (the common-consequence effect and the common-ratio effect) and the Ellsberg paradox, demonstrating that the paradoxes can be understood in terms of optimal use of limited cognitive resources. As such, our work takes an important step toward effectively bridging between decision-making under risk and decision-making under uncertainty, using the broad framework of resource-rationality (Nobandegani, 2017; Lieder & Griffiths, 2020).

Among the remarkably large number of decision-making models developed in the literature, few of them can explain *both* the Allais paradox and the Ellsberg paradox (e.g., Wakker's (2001) and Deany and Ortoleva's (2017)). SbEU differs from those models, both conceptually and in terms of breadth of explanation. At the conceptual level, the main contrast between SbEU and those is that SbEU retains the basic assumption that agents maximize expected utility — à la expected utility theory. More specifically, SbEU assumes that agents maximize expected utility as best as they can, given their cognitive resources. As such, SbEU can be seen as an algorithmic implementation of the integration of two broad theories: expected utility theory and resource-rationality. In terms of breadth of explanation, SbEU is markedly superior to those models, as it accounts for a wider range of empirical phenomena (see Sec. 2 for details), in a unified fashion.

As noted earlier, SbEU has already bridged between risky, value-based, and game-theoretic decision-making, explaining a range of empirically well-established regularities in each of these domains (Sec. 2). As we show in this work, SbEU can also account for two well-known paradoxes in decision-making under risk and under uncertainty (the Allais paradox and the Ellsberg paradox, respectively), thus bridging between these two realms of decision-making. Together, these bridges suggest that integration of expected utility theory and resource-rationality, as two broad theories of cognition, is a fruitful direction for uncovering the algorithmic foundations of human decision-making, and will likely bring us closer to developing a unified account of human decision-making. We see our work as a step in this important direction.

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