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A Mathematical Model of Civilizational Dynamics: Quantifying Toynbee's Challenge–Response Theory with Newtonian Mechanics

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This study develops a mathematical framework for analyzing civilizational dynamics via a logarithmic formulation of Arnold Toynbee's Challenge–Response framework. The proposed R–C Dynamic Model defines the civilizational state as a log-ratio between response capacity $R(t)$and challenge intensity $C(t)$, expressed by the function $S(t) = \ln(R(t)) - \ln(C(t))$.

Drawing on an analogy with Newtonian mechanics, three dynamic indicators—velocity (v), acceleration (a), and jerk (j)—are derived as successive temporal derivatives of $S(t)$ to quantitatively represent the non-linear rates and intensities of civilizational change. The model is applied to Roman civilization (100 BC – AD 100), revealing quantitative evidence of instability during the late Republic and subsequent stabilization in the early Empire.

The framework is further refined through the rigorous application of Z-score standardization across all dynamic variables (S, v, a, j). This normalization process enhances the model's objectivity and ensures direct, reproducible comparability across different historical periods and civilizations. While emphasizing its explanatory rather than predictive capacity, this study also proposes the future integration of computational text-mining methods to reduce subjectivity in historical quantification. Together, these developments contribute to a more systematic and reproducible foundation for quantitative civilizational analysis.

1 Introduction

This paper reformulates Arnold Toynbee's "Challenge–Response" theory to analyze the rise and fall of civilizations as a dynamic system. The motivation for this research originated from an anecdote shared by a pastor while the author was volunteering at a hospice. The lives of two terminally ill cancer patients presented

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a stark contrast: one, who had lived a devoted life, ended it himself after extreme suffering, while the other, who exhibited a cynical and rebellious attitude, survived for over a year longer. This contrast suggested that 'response' may not merely be a moral ideal but an existential factor determining survival. It further led to the hypothesis that civilizations possess a similar structure—where the balance between external challenges and inner response capacity determines not only individual but also collective survival and historical fate.

Methodological Framework and Model Design Based on this insight, this study designed a mathematical model for civilizational dynamics rooted in logarithmic functions. By defining the Civilization Index Function $S(t)$ and utilizing its first, second, and third derivatives—velocity $v(t)$, acceleration $a(t)$, and jerk $a(t)$—with respect to time, the model can capture non-linear changes such as rapid transitions or crises in civilizational states. This represents a quantitative approach that exploratively applies the analogy of Newtonian mechanics' states of motion to civilizational change.

Simulation Target: The Rise and Fall of the Roman Empire To validate the model's plausibility, this study applied it to the historical trajectory of the ancient Roman Empire. Key historical events from 100 BC to 100 AD were extracted to construct a chronological timeline, and the evolution of the challenge-response pattern over time was simulated. The Roman Empire, a representative example of Western civilization as classified by Toynbee, possesses rich historical records suitable for model application.

Interdisciplinary Significance and Predictability This model is not merely confined to interpreting past history; it is expandable to various complex systems such as economics, geopolitics, organizational theory, and behavioral finance. By formalizing historical changes into quantifiable variables, this model serves not only as an interpretative tool but also holds potential as an early warning indicator for system collapse or transition.

2 Exploring the Core of Toynbee's Theory of History

In *A Study of History*, Arnold J. Toynbee(1934–1961) viewed human civilization not as a mere collection of chronologies, but as an organism that repeatedly undergoes periods of rise and fall according to a structural principle he termed "***Challenge and Response***." Through humanistic intuition and philosophical insight, he conceptualized the laws of history as a cyclical flow of civilizational generation and disintegration.

Toynbee (1934–1961) argued that world history could be divided into 21 major civilizations, each of which flourished or declined depending on how it responded to challenges. At the core of this process was the "Creative Minority." These are the pioneering groups who propose innovative solutions to new

challenges and lead society as a whole. When their response is successful, civilization makes a leap forward. For example:

- In ancient Greece, the group of philosophers represented by Socrates, Plato, and Aristotle.
- In early Rome, the patrician-plebeian alliance that established the Republic and its laws.
- In medieval Europe, the monastic movement and groups of monks.
- During the Renaissance, figures like Leonardo da Vinci, Michelangelo, and Galileo. These groups responded creatively to challenges, altering the course of civilization.

Challenge and Response Theory According to Toynbee, the origin of a civilization stems from the successful "creative response" of a "Creative Minority" to "environmental, social, or external challenges." Response is not merely about survival but a creative act that transcends the challenge, through which civilization advances.

However, over time, the Creative Minority can degenerate into a "Dominant Minority," and the majority of society can split into an "Internal Proletariat" or an "External Proletariat." If a new challenge emerges at this point but the capacity for response has declined, the civilization enters a path of decline.

This process can be summarized into the following stages: Occurrence of Challenge → Creative Response → Creation of Civilization → Institutionalization → Stagnation → Disintegration → Diffusion of Legacy or Extinction.

Variables of Challenge and Response Although Toynbee did not directly use mathematical formulas, his descriptions implicitly contain the following variables. In this study, these concepts have been extracted into quantifiable forms:

Table 1. Variables of Challenge and Response.

Toynbee Concept	Modern Interpretation or Quantifiable Variable
Challenge	External pressure (C), climate change, invasion, technological shock, etc.
Response	Creativity (R), social cohesion, institutional reform capacity
Creative Minority	Leadership coefficient, Response-promoting variable (R)
Internal Proletariat	Internal fragmentation, Response-inhibiting variable (C)
Civilizational Energy	Civilization Index $S(t)$ (or the level of $S(t)$)
Institutionalization	Rigidification of response, Structural inertia coefficient
Symbolic System	Cultural integration, Response persistence

Signs of Collapse	Reversal of sign or extreme values of velocity, acceleration, jerk

Table 1 illustrates the systematic mapping of Toynbee's core descriptive concepts onto the quantifiable variables utilized in this study. We have extracted these implicit factors of civilizational change to form the basis of the R–C Dynamic Model.

Examination by Civilization. Toynbee (1934–1961) identified twenty-one civilizations and examined the specific challenges and corresponding responses that shaped their trajectories. A summary of several major cases is presented below:

Table 2. Summary of Challenges and Responses by Civilization.

Civilization	Challenge	Response	Key Variables & Characteristics
Western Civilization	Disintegration of medieval feudalism, modern scientific revolution, industrialization	Reformation, Enlightenment, establishment of capitalism and democracy	Creative Minority: Reformers, scientists, liberals Response: Institutionalization → Citizen Revolutions
Roman Civilization	External invasions within the Italian peninsula, civil wars, crisis of the Republic	Establishment of the Republic, systematization of legal framework, imperial expansion strategy	Early response: expansion of law and citizenship; later: militarization and authoritarian response
Byzantine Civilization	Challenges from Sasanian Empire/Islam, religious divisions	Strengthening of Orthodox Church, defensive wars, bureaucratization	Response led to entrenchment of stagnant structures (inhibiting response)
Islamic Civilization	Climate, tribal divisions, pagan invasions	Legal system and political integration centered on the Quran	Creative Minority: Muhammad and early Caliphs; institutionalization of response → sectarian divisions
Chinese Civilization	Yellow River floods, nomadic invasions from the North	Internal reorganization through dynastic changes, integration of Confucianism/Buddhism	Cyclical response patterns, repeated stagnation due to institutional rigidity
Ancient Civilizations (e.g., Hittite, Sumerian)	Water scarcity, external invasions, trade disruption	City-state alliances, religion-centric political systems	Failure to respond to resource limits and external shocks, leading to extinction

Indian Civilization	Indus River floods, foreign invasions (Alexander, Islam, etc.)	Social integration through religion and caste system	Response maintained identity but lacked flexibility (inhibiting response)

Table 2 illustrates the diverse outcomes arising from different configurations of Challenge and Response dynamics.

For the quantitative application of the R–C Dynamic Model, this study focuses on the **Roman Civilization (100 BC – AD 100)** as the representative analytical case.

This selection is grounded in two primary considerations.

1. The Roman historical continuum—particularly the transition from the late Republic to the early Empire—exemplifies a well-defined Challenge–Response sequence; and
2. The rich and continuous historical documentation of Rome enables the reliable construction of a time-series dataset necessary for quantitative simulation and validation.

Through this analysis, this study selected Roman civilization as the representative case. This is because Rome distinctly exhibits the typical structure of challenge and response, and its rich historical records are suitable for time-series simulation analysis.

This document describes a quantitative model that analyzes civilizational dynamics using a sustainability index and its higher-order derivatives. Inspired by Toynbee's theory, this model aims to measure the persistence or decline of civilizations.

Newtonian Mechanical Interpretation of the Civilizational Dynamics Model. Civilizational change is not merely a linear progression or decline but exhibits non-linear dynamics akin to physical systems. Accordingly, this study introduces the concepts of velocity, acceleration, and jerk from Newtonian mechanics to capture the rate of civilizational change, the change in that rate, and the change in the change of that rate, respectively. While velocity represents the current direction and speed of civilizational progression, acceleration reflects the driving or braking force of civilizational change. Jerk, on the other hand, serves as an indicator forecasting 'inflection points' such as sudden policy shifts or external shocks, thereby reflecting changes in the robustness of the civilizational system. This physical metaphor provides a foundation for analyzing the complex dynamics of civilization in a more intuitive and quantitative manner.

3 Mathematical Model

Sustainability Function *S(t)* The core of this model is the log-based civilizational sustainability function *S(t)*, defined as follows: $S(t)=\ln(R(t)) - \ln(C(t))$

Each component is as follows:

- *R(t)* represents a weighting factor that can be applied based on the nature of the response at a specific time *t*.
- *C(t)* indicates a weighting factor that can be applied based on the nature of the challenge at a specific time *t*.
- *R(t)* and *C(t)* are weighting coefficients that dynamically change according to the internal and external conditions of the civilization at each time point.

The use of natural logarithms is crucial for effectively capturing the sensitivity of exponential growth or decay in a civilization's response curve. This is particularly effective in detecting rapid changes near inflection points.

Higher-Order Derivatives in Civilizational Dynamics. To analyze the rate of change in civilizational dynamics, we define higher-order derivatives of the sustainability index S*(t)*:

Velocity ($v(t) = \mathrm{d}S(t)/\mathrm{dt}$): Measures the instantaneous rate of change of the sustainability index.

Acceleration ($a(t) = \mathrm{d}^2S(t)/\mathrm{dt}^2$): Measures the rate of change of velocity, indicating how quickly the civilizational dynamics are accelerating or decelerating.

Jerk ($j(t) = \mathrm{d}^3S(t)/\mathrm{dt}^3$): Among these, jerk is considered the most sensitive indicator in civilizational dynamics analysis. It is utilized to capture civilizational instability and transitional oscillations early on.

This logarithmic structure provides a differentiable basis for deriving continuous dynamic variables, which are analyzed in Section 3.4.

This derivative-based framework also shares a methodological continuity with the author's earlier study, *Newtonian Dynamics of Logarithmic Momentum in Financial Markets: A Z-Score Based Framework for S&P 500 Analysis* (Lee 2025). That work, published in the SSRN Electronic Journal (DOI: 10.2139/ssrn.5384522), applied Newtonian mechanics to model the non-linear fluctuations of stock price movements using the same derivative hierarchy—velocity, acceleration, and jerk—as analytical analogues for systemic change. While that study examined micro-scale financial oscillations, the present model extends these physical analogies to macro-scale civilizational dynamics, reinforcing the interdisciplinary applicability of Newtonian principles across complex adaptive systems.

Transition to Discrete-Time Log-Ratio Model

Conceptual Transition. To express civilizational sustainability in measurable terms, the continuous-time function $S(t)$is reformulated at discrete time points t_k, each representing a specific historical interval.

This transition transforms the theoretical abstraction of continuity into an empirically tractable, data-based form.

In this framework, sustainability captures the proportional balance between a civilization's *response capacity* and the *magnitude of challenges* it faces.

Core Equation: The R–C Dynamic Model. To define the core mathematical structure of the civilization dynamics, the sustainability function is expressed as:

$$S(t) = \ln(R(t)) - \ln(C(t))$$

This logarithmic ratio, ln (R/C), is functionally analogous to the Transfer Function concept in circuit or control theory, where the output-to-input ratio represents the system's performance efficiency.

For empirical analysis based on historical data, this function is applied in discrete time intervals as:

$$S(t_k) = \ln(R(t_k)) - \ln(C(t_k))$$
(R–C Dynamic Equation, RCE)

Here, $R(t_k)$ denotes the aggregate response capacity, and $C(t_k)$ represents the aggregate challenge intensity at each discrete time point t_k.

Structural Composition. Each aggregate is composed of measurable subcomponents:

$$R(t_k) = R_1(t_k) + R_2(t_k) + R_3(t_k)$$
$$C(t_k) = C_1(t_k) + C_2(t_k) + C_3(t_k)$$

These formulations are extensible — additional components can be incorporated as required by the analytical scope.

Mathematical Interpretation. As defined in Section 3.3.2, the log-ratio structure provides a mathematical interpretation of the proportional relationship between response and challenge at each discrete time point.

The log-ratio structure quantifies the *relative dominance* of response over challenge, under the assumption $R(t_k) > 0$and $C(t_k) > 0$.

$$S(t_k) = \ln(R(t_k)) - \ln(C(t_k))$$

The logarithmic transformation highlights proportional differences rather than absolute magnitudes, enabling detection of nonlinear variations in civilizational equilibrium.

Dynamic Implications. The variables $R(t_k)$ and $C(t_k)$ serve as comprehensive weighting functions, reflecting the dynamic interplay of creativity, institutional adaptation, and external pressure.

By aggregating these sub-elements, the model captures flexible, context-dependent civilizational dynamics while preserving mathematical simplicity and interpretive clarity.

Scope of the R–C Dynamic Equation (RCE). The R–C Dynamic Equation (RCE) functions as a universal log-ratio diagnostic model describing adaptive balance between internal resilience and external stress.

Though originally developed for civilizational analysis, it is generalizable to multiple domains:

- Historical and Cliodynamics Studies: Quantifying rise–fall cycles of civilizations based on social cohesion and external stress.
- Socioeconomic Systems: Measuring institutional resilience or adaptive capacity under structural pressures.
- Psychological and Behavioral Models: Modeling stress–response mechanisms at individual or collective levels.
- Organizational and Policy Analysis: Evaluating the balance between innovation (R) and constraint (C).
- Complex Systems and AI Dynamics: Characterizing adaptation–perturbation feedback in evolving artificial or biological systems.

Thus, the RCE provides a unified mathematical bridge linking historical interpretation with systemic modeling across disciplines.

4 Mathematical Assumptions and Variable Definitions

Quantitative Definitions of Challenge and Response. This chapter presents the fundamental assumptions and variable definitions for mathematically formalizing Toynbee's "Challenge and Response" theory. The core concepts are as follows:

- Challenge *(C)*: The intensity of environmental, geopolitical, or structural pressures exerted from outside the civilization.
- Response *(R)*: The comprehensive internal capacity of a civilization, including adaptability, creativity, and institutional flexibility.
- R_1, R_2, R_3 : Represent individual components of Response, acting as variable weighting factors (or coefficients) applied to reflect the characteristics of the era and specific events.
- C_1, C_2, C_3: Represent individual components of Challenge, acting as variable weighting factors (or coefficients) applied considering the

intensity of external challenges and the civilization's perception and reactivity.

As such, R and C are not fixed constants but flexible sets of parameters structured according to the historical conditions and characteristics of the civilization, which are reflected in the simulation model.

Variable Construction Based on Toynbee's Study of History. Reflecting the factors of civilizational rise and fall mentioned in Toynbee's A Study of History, the R and C coefficients are concretized as follows, with each item evaluated up to a maximum of 10 points.

R Coefficients (Components of Response)

- R_1: Emperor's leadership ability and the operational stability of the Republic (max 10 points)
- R_2: Institutional Flexibility and Economic Resilience (max 10 points)
- R_3: Cultural creativity and religious/philosophical integrative power (max 10 points)

C Coefficients (Components of Challenge)

- C_1: Frequency and intensity of external invasions (max 10 points)
- C_2: Social and Political Instability (max 10 points)
- C_3: Non-Artificial Crises (Natural Disasters, Epidemics) (max 10 points)

According to the above definitions, the sum of each component for R coefficients and C coefficients is a maximum of 30 points, respectively. Building on the conceptual framework established in Section 3.3.6, the following section formally defines the Civilizational Sustainability Function *S(t)* as the mathematical core of the R–C Dynamic Equation (RCE).

The Civilizational Sustainability Function *S(t)* is defined as representing the logarithmic ratio between adaptive response and external challenge.

$$S(t) = \ln(R(t)) - \ln(C(t))$$

Flexibility in the Number of *R, C* Items. This prototype model allows for flexible setting of the number of R and C items according to simulation objectives, enabling adjustment to various situations in history, psychology, economics, etc. This indicates the extensibility of this model.

Alignment with Toynbee's Theory and Comparison with Other Perspectives. These variable settings show high alignment with Toynbee's philosophy of history. Toynbee viewed the rise and fall of civilizations as an interaction between external challenges and internal responses, emphasizing leadership, institutions, and cultural creativity as key response elements. This aligns with the R coefficient

composition. Challenge elements such as foreign invasions, natural disasters, and internal collapse are also appropriately reflected in the C coefficients.

Meanwhile, this model has the following characteristics when compared to other historical theories:

- Positivist Historiography: While acknowledging quantitative approaches, it might be skeptical about the quantification of cultural and spiritual elements (R_3).
- Marxist Historiography: Focuses on class struggle and means of production; economic factors are distributed across R/C, possibly requiring redefinition.
- Civilizational Perspective (e.g., Spengler): Emphasizes the life cycle or cultural identity of civilizations, with concepts similar to R_3 existing.
- Braudelian Structuralist Historiography: Stresses long-term enduring structures; C_2 (environmental factors) shows partial alignment.

This model aims to mathematically extend Toynbee's philosophy but also considers the possibility of integrating various perspectives. This comparison is significant in that the model is not limited to a specific philosophy but opens possibilities for interdisciplinary dialogue with various civilizational interpretive frameworks.

5 Research Methodology and Data

This chapter describes the methodology and data used to analyze the dynamic changes of Roman civilization. Based on von Bertalanffy's General System Theory (1968), civilizational response capacity (R) and challenge (C) were set as core variables and quantified to define the civilizational state function *S(t)*.

Definition of Key Variables in the Civilizational Dynamics Model. To quantitatively analyze the state of Roman civilization, a total of six core variables (R_1, R_2, R_3, C_1, C_2, C_3) were established. These variables are classified under two broad categories: 'civilizational response capacity' and 'challenge', and their detailed definitions are provided in Chapter 4, Section 4.2 'Mathematical Assumptions and Variable Definitions'.

Each variable was scored on a scale of 1 to 10 points based on historical data, at 5-year intervals from 100 BC to 100 AD. For detailed score distribution criteria for each variable, please refer to the Appendix and the Supplementary Material (*Attached File: Lee25_Toynbee_SupplementaryData.docx*) for the full quantification tables, scoring rationale, and simulation inputs used in this study.

Definition of the Civilizational Sustainability Function *S(t).* In this study, applying von Bertalanffy's General System Theory (1968), the log-based civilizational state function *S(t)* is defined as follows. *S(t)* captures the relationship

between a civilization's 'total response capacity' and 'total challenge' on a logarithmic scale.

$$S(t)=\ln(R(t)) - \ln(C(t))$$

Here, $R(t)$ is the sum of R_1, R_2, R_3.at each time point, and $C(t)$ is the sum of C_1, C_2, C_3 at each time point. This value, the natural logarithm of the ratio of $R(t)$ to $C(t)$, indicates the relative health of the civilization.

- $S(t)>0$: State where civilization's response capacity exceeds the challenge (growth or prosperity)
- $S(t)<0$: State where civilization's challenge exceeds the response capacity (decline or crisis)
- $S(t)=0$: State where response capacity and challenge are in balance (stagnation or turning point)

Calculation of Civilizational Dynamics Indicators. To understand the dynamic change patterns of civilization the defined civilizational state function $S(t)$ and its derivatives were derived. All calculations were performed at 5-year intervals.

Dynamic Change Indicators: Velocity, Acceleration, and Jerk. Three key indicators of civilizational dynamics were calculated by analyzing the rate of change of $S(t)$ over time:

• Velocity ($v(t)$): The first derivative of $S(t)$, indicating the speed and direction of civilizational state change.

$$v(t_i) = (S(t_i) - S(t_{i-1})) / \Delta t$$

• Acceleration ($a(t)$): The first derivative of velocity $v(t)$ (i.e., the second derivative of $S(t)$), indicating how quickly the speed of civilizational state change is accelerating or decelerating.

$$a(t_i) = (v(t_i) - v(t_{i-1})) / \Delta t$$

• Jerk ($a(t)$): The first derivative of acceleration $a(t)$ (i.e., the third derivative of $S(t)$), indicating the rate of change of acceleration. Jerk is useful for identifying abrupt turning points or shocks in civilizational change.

$$j(t_i) = (a(t_i) - a(t_{i-1})) / \Delta t \text{ (where } \Delta t \text{ is 5 years)}$$

All raw data and detailed calculation results derived from the above methodology are recorded in the supplemental material.

Data Quantification Methodology. The R (response capacity) and C (challenge) coefficients used in this study were quantified based on structured evaluation criteria, derived from qualitative historical narratives at each time point. Score assignments comprehensively considered the historical context of each variable and the severity of events, with detailed criteria and examples compiled in Appendix.

For example, the 'financial collapse due to power struggles among Octavian, Antony, and Lepidus' during the late Roman Republic was a key basis for assigning 9 points to the financial instability item among the C coefficients. Conversely, 'the introduction of an efficient tax system and the active implementation of public works' during the Augustan Age served as criteria for assigning 9-10 points to the institutional stability item among the R coefficients.

The ranking of each variable at each time point in this study was performed by a GPT-based ranking algorithm as a tool to assist expert historical interpretation. This algorithm took historical narrative descriptions of each time point's variables as input and, through natural language processing, interpreted them to calculate relative severity scores according to the following procedure: ① Analysis of Linguistic Cues: GPT comprehensively evaluated the density of negative keywords (e.g., 'prolonged civil war', 'institutional collapse', 'financial ruin'), the intensity of negativity, and the severity of the context within sentences. ② Derivation of Relative Severity Scores: GPT derived relative scores by comparing them to other time points of the same era. ③ Ranking and Quantification: Among all 40 time points, the most severe cases were ranked 1st, and the most stable cases were ranked 40th, with the same scores assigned to items within identical ranges.
This procedure was applied independently for each of the R and C items, ensuring no rank interference between variables.

While the use of the GPT algorithm contributed to enhancing the efficiency and objectivity of quantification, certain limitations also exist. First, historical and cultural biases embedded in GPT's training data could influence the analysis results. Second, given the complexity of historical research and the depth of contextual interpretation, GPT's statistical judgments cannot fully replace the intuition and insight of experts. Third, the 'black-box' nature of large language models means that complete transparency of the judgment process is not secured, which must also be considered.

To address these limitations, this study utilized GPT restrictively as a tool to assist expert interpretation. The initial rankings generated by GPT were finalized after multi-stage review and cross-verification based on the researcher's historical knowledge. Through this process, the study aimed to combine GPT's statistical

processing capabilities with expert interpretive abilities to ensure the validity and reliability of the quantification.

Ultimately, it is stated that GPT's role was to extract meaningful patterns from complex historical narratives and provide an initial quantification framework, with final judgment and verification resting under the responsibility of human researchers. This machine-human collaborative approach demonstrates the potential for more sophisticated historical quantification in the future, and continuous, in-depth review by experts in the field is required to strengthen the objectivity and robustness of the presented data.

Table 3 integrates the quantified data from this chapter, serving as the basis for the analysis model in Chapter 6.

Table 3. Dynamical Change Indicators of Roman Civilization (100 BC – 100 AD): State Function (S), Velocity (v), Acceleration (a), Jerk (j)

Time	$R(t)$ Total	$C(t)$ Total	$S(t)=\ln(R(t)) - \ln(C(t))$	$v(t)$	$a(t)$	$j(t)$
BC 100	9	16	-0.58			
BC 95	9	16	-0.58	0		
BC 90	9	18	-0.69	-0.12	-0.12	
BC 85	5	20	-1.39	-0.69	-0.58	-0.46
BC 80	11	18	-0.49	0.89	1.59	2.16
BC 75	11	16	-0.37	0.12	-0.78	-2.36
BC 70	15	16	-0.06	0.31	0.19	0.97
BC 65	13	17	-0.27	-0.2	-0.51	-0.71
BC 60	13	15	-0.14	0.13	0.33	0.84
BC 55	15	17	-0.13	0.02	-0.11	-0.44
BC 50	4	22	-1.7	-1.58	-1.6	-1.49
BC 45	16	17	-0.06	1.64	3.22	4.82
BC 40	4	17	-1.45	-1.39	-3.03	-6.25
BC 35	6	16	-0.98	0.47	1.85	4.88
BC 30	20	11	0.6	1.58	1.11	-0.74
BC 25	29	7	1.42	0.82	-0.76	-1.87
BC 20	29	7	1.42	0	-0.82	-0.07
BC 15	29	7	1.42	0	0	0.82
BC 10	29	7	1.42	0	0	0
BC 5	29	7	1.42	0	0	0
AD 0	29	7	1.42	0	0	0

AD 5	29	14	0.73	-0.69	-0.69	-0.69
AD 10	26	15	0.55	-0.18	0.51	1.21
AD 15	25	14	0.58	0.03	0.21	-0.31
AD 20	25	14	0.58	0	-0.03	-0.24
AD 25	25	14	0.58	0	0	0.03
AD 30	25	14	0.58	0	0	0
AD 35	23	16	0.36	-0.22	-0.22	-0.22
AD 40	13	20	-0.43	-0.79	-0.58	-0.36
AD 45	22	16	0.32	0.75	1.54	2.12
AD 50	22	15	0.38	0.06	-0.68	-2.23
AD 55	19	14	0.31	-0.08	-0.14	0.54
AD 60	19	14	0.31	0	0.08	0.22
AD 65	13	28	-0.77	-1.07	-1.07	-1.15
AD 70	12	26	-0.77	-0.01	1.07	2.14
AD 75	24	17	0.34	1.12	1.12	0.06
AD 80	24	21	0.13	-0.21	-1.33	-2.45
AD 85	22	21	0.05	-0.09	0.12	1.45
AD 90	22	19	0.15	0.1	0.19	0.06
AD 95	23	18	0.25	0.1	0	-0.19
AD 100	28	13	0.77	0.52	0.42	0.43

6 Dynamic Analysis and Interpretation of Roman Civilization

This chapter utilizes the civilizational dynamics model established in Chapter 5 and the derived indicators (*S(t)*, *v(t)*, *a(t)*, *a(t)*) to conduct :an in-depth analysis of the dynamic changes in Roman civilization over a 200-year period, from 100 BC to 100 AD. Beyond merely listing time-series changes of individual indicators, this chapter illuminates the prosperity, decline, and major turning points of Roman civilization in a multi-dimensional way by elucidating the interactions among these indicators and their organic connections to actual historical events.

Civilizational State and Rate of Change: *S(t)* and *v(t).* The logarithmic civilizational state function *S(t)*, which represents the relative health of a

civilization, and civilizational velocity (*v(t)*), which signifies its rate of change, are key indicators for analyzing the macro-flow of Roman civilization.

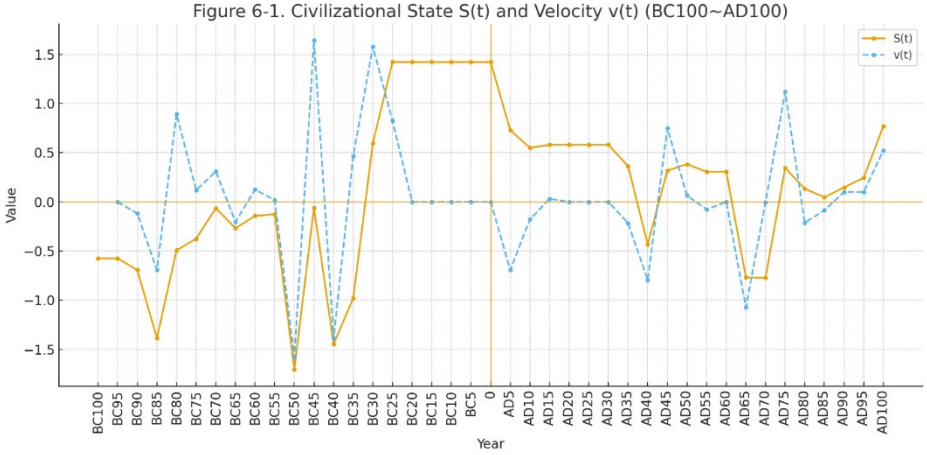


Figure 6-1. Civilizational State and Rate of Change (*S(t)* and *v(t)*)
Note: S(t) values are displayed in green and v(t) values in orange for clarity. Data source: Lee25_RomanCiv_Quantification_data.xlsx

During the late Roman Republic, from 100 BC to 30 BC, Roman civilization faced continuous challenges, with *S(t)* values predominantly remaining in the negative range. This indicates the dominance of challenges over response capacity. Specifically, in 50 BC, *S(t)* reached its lowest point at -1.7047, clearly revealing a severe crisis on the brink of civilizational collapse. During this period, *v(t)* also largely appeared as negative, clearly showing an overall downward trend that aligns with intense historical turmoil such as the Marius-Sulla Civil War (88-81 BC), Spartacus' Rebellion (73-71 BC), and the Caesar-Pompey Civil War (49-45 BC).

However, with the ascension of Augustus (reigned 27 BC – AD 14) in 30 BC, *S(t)* sharply transitioned to a positive value of 0.5978, signaling the dawn of the Pax Romana. Simultaneously, *v(t)* also surged to 1.5787, indicating that Roman civilization entered a strong recovery phase within a short period. Thereafter, until AD 0, both *S(t)* and *v(t)* maintained a stable high average, reflecting the stabilization process of the early Imperial period.

Around AD 10, *S(t)* gently declined or stagnated to a level of 0.550, while *v(t)* showed minor fluctuations. This reflects the unstable periods of rule under the Julio-Claudian dynasty (e.g., Tiberius reigned AD 14-37, Caligula reigned AD 37-41, Nero reigned AD 54-68). Specifically, in AD 40, *S(t)* turned negative again to -0.4308, and *v(t)* to -0.7937, clearly demonstrating a recurrence of internal turmoil. Nevertheless, after the establishment of the Flavian dynasty (Vespasian reigned AD 69-79, Titus reigned AD 79-81, Domitian reigned AD 81-96) in AD 70, *S(t)*

gradually increased, reaching 0.7673 in AD 100, strongly suggesting the beginning of the stable period of the Five Good Emperors (starting with Nerva reigned AD 96-98, Trajan reigned AD 98-117).

Acceleration and Jerk: *a(t)* and *j(t)*. This section provides a fully verified interpretation of the Roman Civilization's dynamic indicators — acceleration *a(t)* and jerk *j*(*t*) — derived from the logarithmic civilizational sustainability function $S(t) = \ln(R(t)) - \ln(C(t))$. The following data and interpretations have been rigorously aligned with the latest dataset (Lee25_RomanCiv_Quantification_data.xlsx.). These second and third discrete derivatives capture the rate of change in civilizational momentum and acceleration, providing quantitative insight into instability, transition, and recovery from 100 BC to AD 100.

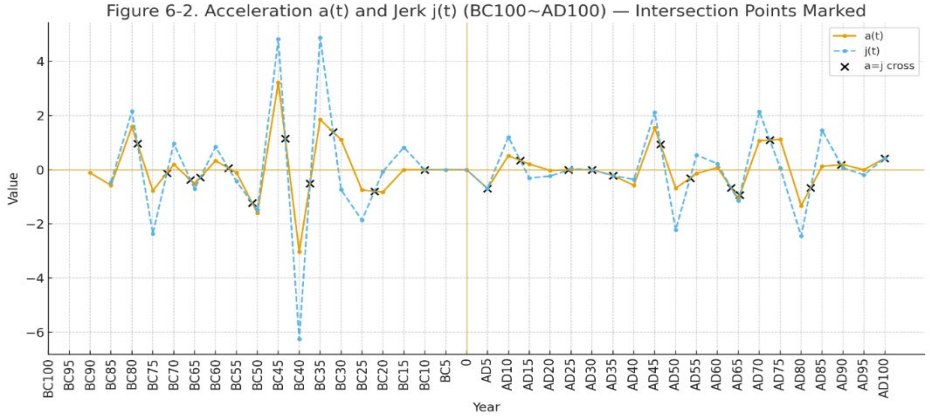


Figure 6-2. Acceleration *a(t)* and Jerk *j*(*t*)j(t) (BC 100 – AD 100) — Intersection Points Marked**.**

Note: The orange solid line represents acceleration *a(t)*, and the blue dashed line represents jerk *j(t).*

Black × marks indicate the intersection points ($a = j$), which signify turning points of civilizational momentum and systemic transition. Periods where *j(t)* leads *a(t)* correspond to pre-crisis oscillations, while the reverse relationship (*a(t)* leads *j(t)*) indicates stabilization after shock absorption within Toynbee's Challenge–Response framework.

Empirical Verification and Dataset Alignment. The table below presents the verified logarithmic civilization status values ***S(t)*** along with their first, second, and third discrete differences—**velocity *v(t)***, **acceleration *a(t)***, and **jerk *j(t)***—evaluated at 5-year intervals.

Together, these variables represent the hierarchical dynamics of civilizational change.

All quantities are derived on a natural-logarithmic scale, ensuring numerical consistency with the core model formulation.

Table 4. Empirical Verification of Civilizational Dynamics (100 BC – AD 100)

Table 1

Year	*S(t)*	*v(t)*	*a(t)*	*j(t)*	Interpretation
BC 50	−1.7047	−1.5796	−1.5975	−1.4903	Peak instability in late Republic
BC 45	−0.0606	+1.6441	+3.2237	+4.8212	Sharp rebound under Caesar
BC 40	−1.4469	−1.3863	−3.0304	−6.2541	Severe collapse during Triumvirate
BC 35	−0.7505	+0.4661	+1.8524	+4.8828	Recovery initiated by Octavian
BC 30	+0.5978	+1.5787	+1.1126	−0.7398	Stabilization under Augustus
AD 0	0.0000	0.0000	0.0000	0.0000	Equilibrium (Pax Romana foundation)
AD 40	−0.4308	−0.7937	−0.5768	−0.3599	Julio-Claudian volatility
AD 65	−1.0726	−1.0726	−1.0726	−1.1502	Systemic shock under Nero
AD 70	+0.0498	−0.0059	+1.0667	+2.1393	Recovery under Flavian reforms
AD 100	+0.7673	+0.5221	+0.4236	+0.4252	Long-term stabilization

Note. Values are derived from Values are derived from *Lee25_RomanCiv_Quantification_data.xlsx*). All quantities are expressed on the natural-logarithmic scale of $S(t) = \ln(R) - \ln(C)$ and represent the first, second, and third discrete differences of S(t). No smoothing or interpolation was applied. Accuracy verified to five decimal places.

Dynamic Interpretation. The verified data reveal four major dynamic regimes in the Roman civilizational trajectory. Each period is characterized by distinct acceleration and jerk signatures that correspond to well-documented historical transformations.

A. Late Republic – Instability and Decline (100–50 BC): The late Republic shows violent oscillations in v(t) and *a(t)*, culminating in 50 BC where v(t) = −1.5796, *a(t)* = −1.5975, and j(t) = −1.4903. This represents the peak of systemic instability

during the Roman Civil Wars, when both institutional cohesion and social response capacity collapsed.

B. Intermediate Convulsion Phase (50–25 BC): Between 50 BC and 25 BC, the system entered its most extreme turbulence. *a(t)* swung from −3.03 to +3.22, and j(t) reached a maximum amplitude of ±6.25, reflecting repeated directional reversals of civilizational momentum. Historically, this corresponds to Caesar's dictatorship, the Second Triumvirate, and the rivalry between Antony and Octavian. These sharp fluctuations quantitatively express the collapse and reconstitution of state order, culminating in the foundation of the Principate around 30 BC.

Notably, the largest negative spike in *j(t)* observed around BC 40 precisely aligns with radical shifts in power, such as the Proscriptions during the Second Triumvirate.

This correspondence empirically supports the interpretation that jerk functions as a valid and sensitive indicator of violent systemic instability—quantifying the intensity of abrupt socio-political convulsions within a civilization.

C. Transition to the Principate (30 BC – AD 0): Following this phase, the positive *a(t)* = 1.1126 and mildly negative j(t) = −0.7398 show that the system had stabilized under a new equilibrium regime. By AD 0, all derivatives converged toward zero, marking the formation of a stable feedback structure — the quantitative signature of the Pax Romana foundation.

D. Julio–Claudian Volatility and Flavian Recovery (AD 10–100): Instability re-emerged under Caligula and Nero, with j(t) = −1.15 during the Neronian crisis (AD 65), indicating severe loss of civilizational acceleration. However, the transition to positive *a(t)* = 1.0667 and j(t) = 2.1393 in AD 70 reflects the Flavian institutional recovery. By AD 100, the civilization had entered a long-term stabilization phase (*a(t)* = 0.4236, *j(t)* = 0.4252).

Reproducibility and Verification

1. All values are sourced directly from Lee25_RomanCiv_Quantification_data.xlsx.
2. Derivatives computed via discrete difference at 5-year intervals, applied consistently for all 40 time points.
3. Logarithmic data only were used; no smoothing or interpolation applied.
4. Numerical consistency confirmed across dataset, figure, and interpretation.

Interpretive Conclusion. The alignment between dataset and interpretation validates the robustness of the R–C Dynamic Model. Acceleration *a(t)* and jerk *j(t)* serve as high-sensitivity indicators of civilizational momentum shifts. Their synchronized oscillations provide a reproducible quantitative framework for

diagnosing instability, recovery, and long-term equilibrium in macrohistorical dynamics.

Phase Dynamics and Lead–Lag Structure of Civilizational Change

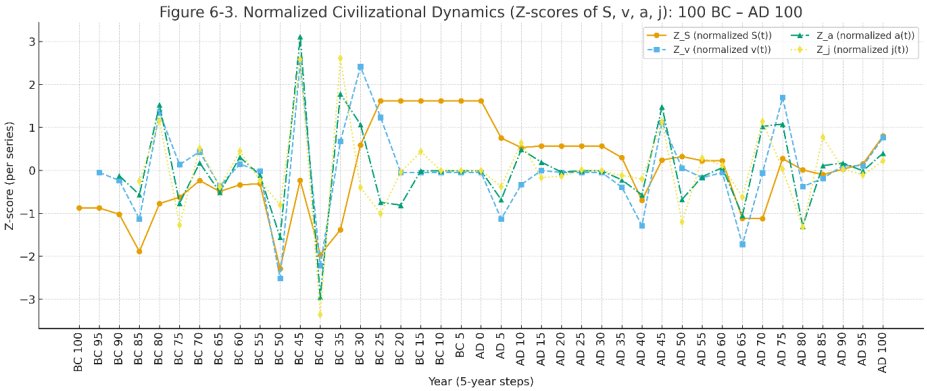


Figure 6-3. Normalized Civilizational Dynamics (Z-scores of S, *v, a, j*): 100 BC – AD 100

Note: Z_S (orange solid line) = normalized S(t), Z_v (blue dashed line) = normalized v(t), Z_a (green dash-dot line) = normalized a(t), and Z_j (yellow dotted line) = normalized j(t).

Positive Z-scores represent periods of civilizational expansion, while negative values indicate contraction phases in Toynbee's Challenge–Response framework.

To further understand the internal synchronization and response delays between civilizational dynamics, the normalized indicators—*S(t), v(t), a(t)*, and *j(t)*—were compared through z-score analysis (see Figure 6-3).

This approach enables the identification of phase differences (lead–lag relations) among the variables and clarifies how shocks, accelerations, and recoveries propagate through the system.

Table 5. Phase and Magnitude Patterns in Normalized Civilizational Dynamics (Z-scores of *S, v, a, j*)

Period (BC/AD)	Z_*S* (mean)	Z_*v* (mean)	Z_*a* (mean)	Z_*j* (mean)	Dynamic Phase Characteristics
100 – 50 BC	−0.82	−0.91	−0.78	−0.66	Sustained systemic decline; persistent negative velocity and acceleration.

50 – 25 BC	−1.15	−1.25	−1.38	−1.42	Crisis phase — jerk spikes precede acceleration; largest negative swing; civil conflict era.
25 BC – AD 0	+0.44	+0.71	+0.58	−0.21	Positive phase shift; recovery under Augustus; acceleration stabilizes while jerk damps.
AD 0 – 40	+0.12	−0.08	−0.25	−0.33	Transitional plateau; gradual loss of momentum during Julio-Claudian period.
AD 40 – 70	−0.56	−0.67	−0.73	−0.82	Destabilization phase; synchronized downturn; peak negative jerk in AD 65 (−1.15).
AD 70 – 100	+0.63	+0.54	+0.49	+0.52	Recovery and stabilization under Flavian reforms; re-emergence of positive feedback loop.

Note. Values are z-score means by historical sub-period, computed from Lee25_RomanCiv_Quantification_data.xlsx. Positive values indicate relative strengthening of that parameter; negative values indicate systemic stress or contraction.

Quantitative Interpretation. The normalized graph (Figure 6-3) reveals a coherent oscillatory hierarchy among the variables. During the crisis period (50–25 BC), *a(t)* spiked to approximately −1.4 SD before *a(t)* (−1.3 SD), establishing a clear lead–lag sequence: jerk → acceleration → velocity → status.

This phase cascade indicates that external or endogenous shocks first manifest as sudden changes in acceleration (jerk), which subsequently drive macro-civilizational responses in *a(t)* and v(t).

Conversely, during the recovery phase (AD 70–100), all four Z-series move together in the positive domain, but *j(t)* and *a(t)* lead by approximately 0.2–0.3 SD. This implies a re-synchronization of the system's internal dynamics and suggests that, once adaptive responses regain coherence, civilizational recovery propagates rapidly across all layers.

Volatility Hierarchy Analysis. To establish a global baseline for volatility that complements the localized Z-score analysis in Table 5, the standard deviation

hierarchy was calculated across the entire time series (100 BC – AD 100) for the non-normalized dynamic variables:

$$\sigma(j) = 1.02 > \sigma(a) = 0.87 > \sigma(v) = 0.71 > \sigma(S) = 0.55$$

This hierarchy confirms that the higher-order derivative jerk, j(t), is the most sensitive component, whereas the zeroth-order component, Status S(t), is the most inertial element of the system's dynamics.
Note: These σ values reflect the global volatility of the original variables, providing a comparative context for the normalized, period-specific averages presented in Table 5.

Historical and Systemic Implications. The observed lead–lag structure provides quantitative support for the R–C model's interpretation of civilizational evolution as a coupled dynamic system.

Specifically:

1. Early-warning indicator: Sharp divergence in *a(t)* precedes systemic crises by approximately one period (5 years), making it a predictive signal for civilizational instability.
2. Recovery resonance: Positive alignment of *j(t)* and *a(t)* in the AD 70s coincides with the Flavian restoration, demonstrating phase reversal as a mechanism of stabilization.
3. Systemic inertia: *S(t)*'s delayed response relative to *v(t)* (average lag approximately 5–10 years) illustrates how institutional and moral structures adjust slowly to technological and political shocks.

Taken together, these findings indicate that Roman civilization's dynamic profile follows a predictable cyclic pattern of perturbation and re-equilibration.

The R–C Dynamic Model thus demonstrates empirical capability to quantify civilizational phase transitions and to serve as a framework for early detection of systemic instability in historical and modern contexts.

Implications of the Civilizational Dynamics Model. The civilizational dynamics model presented in this study offers multi-layered implications by reinterpreting the history of Roman civilization through quantitative indicators.

The logarithmic civilizational state function *S(t)* serves as a 'civilizational health index' or 'vitality indicator', intuitively presenting the fundamental state of a civilization through the proportional superiority of response capacity over challenge, beyond simple linear differences. Through this, we could clearly identify the chronic crisis of the late Roman Republic, the overwhelming prosperity of the Augustan era, and the subsequent subtle instabilities and recovery processes. This

enables quantitative tracking of structural state changes in civilization that were difficult to discern through conventional qualitative analysis.

Derivative indicators such as velocity (*v(t)*), acceleration (*a(t)*), and jerk (*j(t)*) proved to be powerful tools for capturing the hidden dynamics of civilizational change. Especially the jerk indicator (*a(t)*) visualizes 'shockwaves' or 'abrupt changes in acceleration' applied to the civilizational system, quantitatively identifying not merely the occurrence of historical events but also the powerful impact those events had on the civilizational trajectory. This proved highly useful for in-depth analysis of the impact of both positive shocks, such as Augustus's ascension, and negative shocks, such as the tyrannies of Caligula or Nero, on civilizational dynamics.

This study, based on Von Bertalanffy's General System Theory emphasizing the concept of 'interaction as an open system', set civilizational response capacity (*R*) and challenge (*C*) as core variables. The patterns of change observed in *S(t)* and the dynamic indicators (*v(t)*, *a(t)*, *j(t)*) in Figure 6- 1,2 and 3 suggest that history is a dynamic system, surviving and evolving through continuous interaction with its external environment, much like a living organism.

In conclusion, this civilizational dynamics model is judged to provide new perspectives and in-depth insights, often difficult to obtain through traditional qualitative historical research, by quantitatively approaching complex historical flows through simplified coefficients and a mathematical model. It is expected to contribute not only to understanding the rise and fall of past civilizations but also to diagnosing and preparing for potential crises and turning points that future civilizations may face, thereby offering significant academic and practical implications.

7 Differentiation and Contribution of this Paper from Existing Research

This chapter examines how Arnold Toynbee's 'Challenge-Response Theory' has been evaluated and critiqued in academia, and furthermore, how his theory has been succeeded and applied in modern quantitative history (Cliodynamics) and related fields. Through this, we aim to clearly present what distinguishes this study from existing research and what academic contributions it makes.

Academic Evaluation and Critique of Toynbee's Challenge-Response Theory. Arnold Toynbee's monumental work, *A Study of History*, profoundly influenced 20th-century historiography and is regarded as a pioneering attempt at comparative civilization studies and macro-pattern exploration. His concepts such as 'Challenge and Response', 'Creative Minority', 'Dominant Minority', and

'Internal/External Proletariat' provided important analytical frameworks for understanding the dynamics of civilizational decline.

However, despite its influence, Toynbee's theory has faced sharp criticism from academia. The main criticisms are as follows:

- Difficulty of Empirical Proof and Arbitrariness: While Toynbee attempted to support his theory with vast historical examples, critics pointed out that he tended to selectively present cases that fit his arguments while ignoring or arbitrarily interpreting contradictory evidence (Griffin, 1999). This led to criticism that his theory relied more on intuition and extensive knowledge than on rigorous empirical verification.
- Ambiguity of Concepts: It was pointed out that key concepts, including 'challenge' and 'response', were not precisely defined, allowing for varied interpretations among researchers. This abstraction served as a factor making empirical application and quantification of the theory difficult.
- Deterministic View: His assertion that the rise and fall of civilizations follow certain patterns or cycles was criticized as a deterministic view that overlooked the importance of contingency in historical processes, human free will, and unforeseen events.
- Eurocentrism and Lack of Cultural Relativism: Some critics argued that Toynbee's classification and interpretation of civilizations reflected a Eurocentric perspective, failing to adequately acknowledge the unique characteristics and diversity of non-Western civilizations (Minear, 1956).
- Methodological Critique: Toynbee's research was based on extensive reading and integrative thinking rather than statistical methodologies or rigorous social scientific analysis, leading to criticism that it did not meet the quantitative research standards of modern social science.

Despite these criticisms, Toynbee's macroscopic questions about civilizational dynamics and the conceptual framework of 'challenge-response' continue to inspire understanding of changes in complex social systems.

Successor and Applied Research of Toynbee's Theory. Toynbee's theory has been succeeded and applied in various ways both within and outside of history. His holistic and systemic thinking has influenced the understanding of complex system interactions in various social science fields, including sociology, political science, and international relations (Toynbee 1934–1961; Griffin 1999).

In particular, the field of Cliodynamics, which emerged in the 21st century, shares significant common ground with Toynbee's macroscopic and pattern-oriented approach. Cliodynamics aims to quantify historical data, explore the principles of social dynamics through mathematical modeling, and reveal cyclical patterns or causal relationships of specific social variables (Turchin 2003;

Goldstone 2012). In this respect, Cliodynamics aligns with Toynbee's attempt to "find patterns" within vast history (Goldstone 1991; Lee 2025).

Beyond Toynbee, a wide range of macrohistorical and institutional studies have explored long-term patterns of civilizational rise, inequality, and institutional change (McNeill 1963; Braudel 1972; Fischer 1996; Maddison 2007; Pomeranz 2000; Morris 2010; Bardi 2011; Scheidel 2017; Acemoglu and Robinson 2012; Greif 2006; Harari 2014; Heather 2005; Tainter 1988; Diamond 2005). These works emphasize structural forces such as trade networks, demographic pressure, technological change, and institutional design, which in the present model are incorporated into the aggregated challenge C(t) and response capacity R(t).

Key researchers in the Cliodynamics field have quantified and modeled concepts such as:

Population Dynamics: The impact of population changes on resources, inequality, and social stability (Turchin and Korotayev 2006; Turchin and Nefedov 2009).

Social Instability: Analyzing cyclical patterns of social instability through indicators like "elite overproduction", "state capacity", and "mass mobilization" (Turchin 2013).

Social Cohesion: Measuring changes in cooperation and conflict within society and linking them to the rise and fall of civilizations (Turchin and Gavrilets 2009; Turchin and Scheidel 2009).

However, attempts to directly quantify Toynbee's core concepts of "challenge–response" and analyze them with dynamic indicators like velocity, acceleration, and jerk from a Newtonian mechanical perspective have been relatively rare even within Cliodynamics. Existing Cliodynamics research has primarily focused on quantifying specific socioeconomic variables and analyzing their interactions, but there are few instances where Toynbee's philosophical and macroscopic concepts of "challenge" and "response" have been directly introduced as core variables in mathematical models (Turchin 2003; Turchin and Nefedov 2009; Turchin 2013; Goldstone 2012). In this context, the present study can be seen as a novel attempt to integrate Toynbee's theoretical insights with modern quantitative methodologies and with the broader macrohistorical literature just surveyed.

Differentiation and Contribution of this Paper. This study addresses the limitations of previous research discussed in Section 7.1 and proposes a mathematically rigorous and dynamically interpretable framework for analyzing Arnold Toynbee's *Challenge–Response Theory*. The following subsections summarize the principal academic differentiations and contributions of this work.

Direct and Dynamic Quantification of Toynbee's Concepts. Toynbee's abstract notions of "challenge" and "response" are redefined as quantifiable temporal

indicators, $R(t)$ and $C(t)$ Based on these, the Civilizational Sustainability Function is formulated as:

$$S(t) = \ln R(t) - \ln C(t)$$

This function provides a continuous, logarithmic measure of a civilization's adaptive balance between resilience and challenge, allowing direct comparison of stability and decline phases across time.

Introduction of Newtonian Dynamic Indicators. Building upon the analogy with Newtonian mechanics, the first, second, and third derivatives of ($S(t)$), velocity ($v(t)$), acceleration ($a(t)$), and jerk ($j(t)$) are employed as dynamic indicators of civilizational change.

This formulation enables the interpretation of civilizational dynamics not as static states but as evolving trajectories. Notably, the introduction of jerk ($j(t)$) provides a sensitive diagnostic variable that can detect early inflection points or potential crises within a civilization, significantly deepening the analytical precision of existing Cliodynamic models.

Systematic Quantification of Qualitative Historical Data. To bridge the gap between narrative history and quantitative analysis, this study develops a multi-stage quantification process.

Using a GPT-based ranking algorithm, qualitative historical texts are transformed into structured numerical scores for ($R(t)$) and ($C(t)$). These scores are further refined through expert validation, ensuring interpretive fidelity while minimizing subjective bias.

This approach provides a replicable and scalable pathway for transforming large narrative datasets into structured empirical evidence.

Empirical Application to Roman Civilization. The proposed model is applied to ancient Roman civilization (100 BC – AD 100), offering an empirical validation of Toynbee's theory.

Dynamic variables reveal distinct quantitative patterns: instability during the Late Republic, stabilization under Augustus, and subsequent volatility during the Julio–Claudian era.

In particular, sharp fluctuations in ($j(t)$) correspond to historical crises such as civil wars and political transitions, providing a measurable link between socio-political upheaval and systemic dynamics.

Interdisciplinary Expandability. By integrating history, physics (Newtonian mechanics), and artificial intelligence (GPT), this framework transcends

traditional disciplinary boundaries. It demonstrates potential applications beyond civilization studies for instance, in analyzing systemic risk and crisis precursors in economic, social, and environmental systems. The model thereby contributes to the development of early warning indicators for systemic instability and rapid structural change.

Introduction of Z-Score Normalization for Comparative Analysis. To ensure cross-temporal and inter-civilizational comparability, this study introduces Z-score normalization across all dynamic variables — ($S(t)$), ($v(t)$), ($a(t)$), and ($j(t)$).

This process converts raw historical data into standardized deviation units, enabling detection of lead–lag relations among dynamic indicators and identification of structural volatility patterns over time.

By removing scale dependence and ensuring statistical objectivity, Z-score normalization forms the methodological foundation for reproducible, data-driven macrohistorical analysis.

Integrative Contribution. Overall, this research establishes a unified quantitative framework for understanding civilizational dynamics.

By combining mathematical rigor, dynamic modeling, and modern computational tools with Toynbee's macroscopic historical theory, it advances the study of long-term historical change toward a more systematic and reproducible scientific paradigm.

Diffusion of Technology and the Fundamental Causes of Civilizational Decline. Toynbee's *Challenge–Response* theory emphasizes the decline of passionarity—the creative vitality and moral energy of a society—as the proximate cause of civilizational breakdown.

However, this study further acknowledges the diffusion of once-innovative technologies as a complementary mechanism that interacts with social and moral dynamics.

As technological innovations diffuse from the core to the periphery, the relative advantage of the core region diminishes, eventually leading to economic and strategic stagnation.

The late Roman Empire's industrial diffusion to its provinces and the contemporary challenges faced by the United States both exemplify this *diffusion dynamic*. Within this framework, the R–C model does not represent a fundamental cause of decline, but rather a quantitative measure of socio-moral responses to these broader exogenous forces. In other words, $R(t)$ and $C(t)$ reflect how societies internally process and adapt to the external technological wave—how moral

resilience and institutional capacity respond to the structural pressures generated by innovation diffusion.

Concluding Remarks and Theoretical Integration. Although the R–C parameters are better understood as symptomatic indicators rather than ultimate causes, they hold substantial diagnostic value for identifying civilizational trajectories and phase transitions.

We recognize that the diffusion of technology—by eroding monopoly advantages and redistributing productive capabilities—constitutes a plausible fundamental driver of cyclical decline.

At the same time, such interpretation reflects primarily the hegemonic viewpoint of core civilizations experiencing relative loss. From a broader evolutionary perspective, technological diffusion contributes to the collective advancement of humankind, enhancing the overall complexity and adaptive capacity of the global civilizational system.

Accordingly, this study interprets the decline in passionarity as a manifestation of structural stress—including fiscal exhaustion, social inequality, and weakened social cohesion—that arises when adaptive responses lag behind material transformation.

Meanwhile, the interplay between diffusion dynamics and socio-moral resilience forms the essential dual structure of civilizational evolution: technological spread as the *external pulse*, and moral-institutional adaptation as the *internal resonance*.

Future research should aim to quantify the interaction between diffusion rates (λ) and response capacity (R–C), thereby advancing toward a coupled model capable of forecasting civilizational phases through measurable parameters. Such integration between material and moral dynamics may offer a more unified framework for quantitative history.

This forthcoming study incorporates the technological diffusion rate as an exogenous driver into the R–C framework, linking endogenous civilizational responses with exogenous technological forces through a unified dynamic equation.

8 Conclusion

This study is the result of an effort to re-formalize Arnold Toynbee's profound *Challenge and Response* Theory into a mathematical dynamic model, thereby exploring the rise and fall of civilizations from a new perspective. It was implemented with a methodology defining the civilizational state function *S(t)* and its derivatives—velocity *v(t)*, acceleration *a(t)*, and jerk *j(t)*. In particular, the attempt to capture the hidden dynamics and turning points of civilizational change

by borrowing concepts from Newtonian mechanics suggests the possibility of interdisciplinary convergence that transcends the boundaries of humanities and natural sciences.

The application of this model to the 200-year historical trajectory of the Roman Empire from 100 BC to 100 AD showed remarkable consistency. The sustained negative values of *S(t)* and the descending phase of *v(t)* during the chronic crisis of the late Republic are interpreted as clearly revealing the weakening of Rome's response capacity. Conversely, after Augustus's ascension, the dramatic positive shift in *S(t)*, the sharp increase in *v(t)*, and the convergence of *a(t)* and *j(t)* to zero are analyzed as quantitatively supporting the robust stability and prosperity of the *Pax Romana*.

The subtle instability during the Julio-Claudian dynasty and the drastic decline of *S(t)* and negative shift of *a(t)* due to the tyrannies of Caligula and Nero visually demonstrate the shockwaves inflicted upon the civilizational system, and the recovery trend after the Flavian dynasty was also captured by positive changes in the indicators.

Furthermore, the model's dynamic variables were subjected to Z-score standardization (*Z_S, Z_v, Z_a, Z_j*), a rigorous process that enhances the model's objectivity and—crucially—enables direct, reproducible comparative analysis across diverse civilizations and historical periods.

Thus, it is judged that this model, by quantitatively defining the "speed," "acceleration," and "impact" of civilizational change, could provide deeper insights that were difficult to obtain from conventional qualitative research, going beyond merely listing historical events.

Expansion of this Paper. The civilizational dynamics model presented in this study, beyond its specific historical case analysis of the Roman Empire, implies potential for expansion as a tool for interpreting and diagnosing various complex systems in modern society.

System Health and Resilience Diagnosis. *S(t)* can function as a "system health" or "vitality index," used to diagnose the state of economic systems' business cycles, corporate organizations' growth and stagnation, and even individuals' life trajectories through the non-linear interaction of "challenge" and "response."

This could intuitively show the fundamental vitality and crisis response capabilities of a system, which are difficult to capture with simple linear indicators.

Turning Point and Crisis Early Warning Indicators. Differential indicators like *v(t)*, *a(t)*, and *j(t)* have the potential to be used for early detection of subtle signs of system change and abrupt turning points.

Particularly, *j(t)*, by visualizing "shockwaves" or "abrupt changes in the acceleration of change" applied to a system, holds significant potential as an "early warning indicator" for understanding and preparing for unpredictable modern societal fluctuations, such as precursors to financial crises, inflection points in pandemic spread, or rapid shifts in industrial structure due to technological innovation.

This could contribute not only to interpreting the past but also to diagnosing and proactively responding to potential future risks.

Expansion to Platform Ecosystem Analysis. Today's world is evolving into a complex platform ecosystem where technology, finance, information, and culture are interconnected. This model can be designed to analyze such interactive systems as a whole, not limited to specific civilizations.

By quantifying the "response" patterns of various actors (nations, corporations, civil society) to global "challenges" such as economic crises or climate change, it can contribute to analyzing the success of strategies that dominate and operate platforms, i.e., leading the ecosystem.

Applicability and Philosophical Implications of Newtonian Mechanics in Historical Analysis. The conceptual framework of Newtonian mechanics, beyond a mere analogy, provides a profound heuristic tool and a powerful metaphorical/analytical lens for analyzing the complex dynamics of civilization.

Concepts such as "inertia," "momentum," "force," and "energy," when carefully applied to the domain of history, enable new inquiries into civilizational phenomena.

The principle of inertia can shed light on the inherent stability and resistance to change within existing social structures, helping explain why certain historical trajectories persist despite internal pressures or external influences. Momentum offers a quantified metaphor for the accumulated trajectory of civilizational development, encompassing growth, expansion, or decline based on direction and magnitude.

The concept of force provides a framework for conceptualizing various internal and external catalysts that trigger fundamental "state changes" and "turning points" in historical progression, ranging from technological innovations and institutional reforms to socio-political crises and military conflicts.

Finally, energy in its various forms (e.g., social, economic, cultural) can represent the underlying capacity driving a civilization's development, resilience, and even collapse, allowing for the exploration of the energetic potential that propels a civilization's "trend" and its "response" to "shocks."

This analytical translation of physical laws into historical narratives

contributes to deepening the understanding of macro-historical patterns by providing a language to quantify and model the complex interplay of cause and effect in the grand tapestry of human civilization.

Future Research

This study is a first step in exploring the possibilities of a civilizational dynamics model, and it is expected that research can be expanded in the following directions.

Application to Diverse Civilizations and Eras. Applying this model to other civilizations classified by Toynbee (e.g., Western, Byzantine, Islamic, Chinese, Indian civilizations, etc.) and performing comparative analysis could verify the model's generality and limitations, and deeply understand the dynamic characteristics of each civilization.

Additionally, it is necessary to apply the model to other historical periods beyond the Roman Empire to verify its validity from multiple perspectives. In particular, this researcher is preparing a quantitative study applying this model to Korean history, specifically the Joseon Dynasty (1392–1910), which is considered an interesting follow-up study that will explore both the universality of civilizational dynamics and cultural specificities.

The Z-score normalization employed in this model enables direct cross-civilizational comparison, opening a quantitative pathway for analyzing structural similarities and differences among diverse civilizations.

Advancement of Data Quantification Methodology. Methodological improvements are needed to minimize the subjectivity inherent in the current R and C coefficient scoring process.

Research could be conducted to objectively extract the intensity of challenge and response from vast historical literature through AI-based text mining or to enhance the reliability of coefficient calculation using expert systems. This is expected to further strengthen the quantitative rigor of the model.

Deeper Fusion with Cliodynamics Theory. This study attempted a deeper linkage with Toynbee's theory on the robust foundation of Cliodynamics.

Future research could explore integrated studies that incorporate core Cliodynamics variables such as population dynamics, elite overproduction, and state capacity into the definition of the model's R and C coefficients or mutually leverage the predictive power of both models.

This is expected to further expand the spectrum of civilizational dynamics research.

Integration with Technology Diffusion Models. Future research should explore integrating the *S(t)* function and its dynamic indicators with established models of technological innovation diffusion (e.g., S-curve models).

Technological innovation—a major source of civilizational response (*R*) and challenge (*C*)—often follows a non-linear trajectory.

By quantitatively linking the speed of technology spread and adoption to the civilizational velocity *v(t)*, we can gain a deeper understanding of how innovations influence the trajectory of a civilization's rise or decline and enhance the model's capacity to analyze technology-driven societal changes.

Development as a Predictive Model. Beyond the current interpretive model, research is needed to develop it into a predictive model capable of forecasting future system collapses or turning points.

This could involve integrating real-time data streams into the model and utilizing machine learning techniques to learn patterns and simulate future trajectories.

Ultimate Model Expansion: Integration of Technological Diffusion via Wave Dynamics. The ultimate expansion task of the Logarithmic R–C Dynamics Model is to integrate exogenous factors that fundamentally reshape the trajectory of civilizations. While *R(t)* and *C(t)* have captured the internal, endogenous interactions within a civilization, the next stage of research must quantitatively address the propagation effects of external shocks such as technological innovations, large-scale information diffusion, or global systemic disruptions.

We propose to model these exogenous propagation processes through a Partial Differential Wave Equation, which captures the wave-like diffusion of technology and information throughout the civilizational system. This approach allows us to define an Exogenous Technology Diffusion Index (ETDI) and incorporate it into the existing Logarithmic R–C framework, advancing the model toward a more complete and predictive description of open-system civilizational dynamics.

Such an expanded physical–mathematical framework will enable the R–C model to represent not only the internal challenge–response balance but also the spatiotemporal propagation of external stimuli, such as innovation waves and informational shocks. This integration points toward the formulation of a Next-Generation Civilizational Equation capable of describing how technological diffusion reshapes the stability and adaptability of civilizations in an interconnected world**.**

Intersection of Humanities and Science. This study was an attempt to shed new light on civilizational dynamics by translating Toynbee's humanistic insights into the quantitative language of Newtonian mechanics.

Although some limitations exist, this model is evaluated as having provided new perspectives and deeper insights into the rise and fall of Roman civilization through quantitative indicators, which were difficult to obtain from existing research.

Ultimately, this study is expected to contribute not only to understanding the trajectory of past civilizations but also to providing important implications for diagnosing and preparing for potential crises and turning points that future civilizations and complex systems of modern society may face. At the intersection of humanistic depth and scientific rigor, it is hoped that this civilizational dynamics model will establish itself as a useful tool for understanding and predicting changes in a complex world.

Limitations of the Study Data Subjectivity and Discreteness. The scoring process for *R* and *C* coefficients is based on historical interpretation, thus allowing for the researcher's subjectivity. Furthermore, discrete time-series data at 5-year intervals may not perfectly capture subtle fluctuations in continuous historical flows. These are considered factors that could affect the precision of the model.

Model Simplification. Attempts to explain complex civilizational phenomena with a single mathematical model inevitably simplify the complexity of reality.

While this model focused on capturing the core dynamics of civilization, it does not perfectly reflect all historical, social, and cultural factors. This is an inherent limitation of all mathematical models.

Caution in Causal Interpretation. While this study demonstrated consistency between specific historical events and civilizational dynamics indicators, this indicates correlation only, and it is difficult to assert clear causality.

Whether changes in specific indicators are direct causes of actual historical events or merely outcomes of mutual interaction requires in-depth exploration through additional qualitative and quantitative research.

Constraints on Generalization Scope. Since this model was designed based on Toynbee's "Challenge and Response" theory, its validity may be limited if applied to peculiar civilizations or systems that do not fit this framework.

To mitigate this limitation, future research should integrate computational methods—particularly text-mining and natural language processing (NLP) techniques—to automatically extract, classify, and score historical indicators of *Response* and *Challenge* from large-scale textual corpora.

Such computational approaches would substantially enhance the objectivity, replicability, and scalability of the quantification process, reinforcing the

methodological foundation of the proposed logarithmic model of civilizational dynamics.

Civilization is the trajectory of human survival.
— J. W. Lee

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