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Authors

Newton, Adam T.
Waring, Rylan J.
Penner-Wilger, Marcie

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Adam T. Newton (anewton9@uwo.ca)
Rylan J. Waring (rwaring2@uwo.ca)
Marcie Penner-Wilger (mpennerw@uwo.ca)
Department of Psychology, King's University College at Western University, London, ON, N6A 2M3, Canada

Abstract

There is debate in the numerical cognition literature concerning symbolic and nonsymbolic number representation systems as foundations for more complex mathematical skills. The purpose of this study was to investigate the relation between these number representation systems and calculation fluency. The present study used 51 university students. Participants completed symbolic and nonsymbolic magnitude comparison and ordinality tasks on an iPad as well as a pen-and-paper version of the addition and subtraction-multiplication subtest of the Kit of Factor-Referenced Cognitive Tests (French, Ekstron, & Price, 1963). Data reductions were performed and a symbolic and a nonsymbolic factor were constructed. A multiple regression analysis revealed that the symbolic factor was a significant predictor of calculation fluency, but the nonsymbolic factor was not. Two separate repeated measures ANOVAs revealed 3-way interactions between task, distance, and format for both accuracy and response time. These results support the view that the two systems develop separately.

Introduction

Basic numeracy skills are essential for success across the lifetime. The understanding and processing of numerical quantity and relations between numbers have been associated with a variety of positive life outcomes including academic achievement, occupational salary, and homeownership (Bynner, & Parsons, 1997; Finnie & Meng, 2001). This paper addresses a current debate in numerical cognition over which number representation system contributes to mathematical ability: symbolic number representation or nonsymbolic number representation. Symbolic number representation refers to the presentation of numbers as abstract symbols, such as Arabic digits (e.g., 1, 2, 3) or as words (e.g., "six"), Given that current theories of numerical cognition suggest that complex mathematical skills (such as multi-digit arithmetic) develop from foundational skills in basic numeracy (Butterworth, 2005), it is important to determine the relative importance of both the symbolic and nonsymbolic representation systems with regards to the development of basic numeracy skills. Some researchers argue that the symbolic number representation system is mapped onto the nonsymbolic representation system (Dehaene, 1992; Mundy & Gilmore, 2009; Verguts & Fias, 2004) whereas other researchers argue these two systems develop separately (Bulthé, De Smedt, & Op de Beeck, 2014; Holloway & Ansari 2009; Lyons, Ansari, & Beilock, 2012). The present study investigated the symbolic and nonsymbolic controversy in numerical cognition using magnitude comparison and ordinality tasks.

On the mapping view, symbolic numbers are mapped onto an existing nonsymbolic number representation system as the acquisition of symbols occurs. Mundy and Gilmore (2009) assessed the predictive relation between symbolic comparison, nonsymbolic comparison, and the accuracy of the mapping between these two systems to predict performance on a test of school mathematics. The researchers found that performance on all three tasks predicted mathematical performance. They argue that these results indicate that the strength of the mapping between the representation systems as the best predictor of mathematical ability. In a neural simulation study, Verguts and Fias (2004) demonstrated how artificial neurons in an unsupervised learning model could map symbolic representation onto existing nonsymbolic number representation systems through repeated pairing of stimuli. Additionally, Halberda, Mazzocco, and Feigenson (2008) found that children's nonsymbolic number approximation skills at age 14 were related to their earlier calculation skills. These behavioural and neural simulation data suggest the nonsymbolic system is the basis for the development of the symbolic system and complex calculation skill.

In contrast to this view, other research suggests that these two number representation systems develop separately from each other, with the symbolic system not arising through the innate nonsymbolic system (Bulthé et al., 2014; Holloway & Ansari 2009). Holloway and Ansari (2009) examined whether the ability of children aged 6-8 to compare symbolic and nonsymbolic magnitudes was related to individual differences in standardized math scores. They found that children's ability to discriminate relative magnitude in symbolic trials was associated with their ability to perform simple arithmetic. This association however, was not seen in nonsymbolic trials, providing evidence for distinction between the two number representation systems. Similar results have also been found in adult participants. Lyons et al. (2012) compared performance on symbolic, nonsymbolic, and mixed comparison tasks in university students and found that it was substantially more difficult to compare a mix of symbolic (digits) and nonsymbolic (dots) quantities than it was to compare two nonsymbolic quantities. Their results suggest that symbolic numerals do not provide direct access to the numerosities the numerals represent. In other words, seeing the number 5 may not bring about a conception of 5 objects, suggesting an indirect cognitive link between the two systems in adulthood. Recent neuroimaging data also supports a two systems view. Bulthé et al. (2014) conducted a multi-voxel pattern analysis fMRI study with the aim of identifying neural correlates that underlie symbolic and nonsymbolic magnitude processing. They found no significant neural patterns for nonsymbolic to symbolic magnitudes, which they took to suggest a divergence between the neural representations of symbolic and nonsymbolic magnitudes.

Both of these views provide distinct predictions and explanations concerning the relation between performance on nonsymbolic tasks, symbolic tasks, and arithmetic tests. The mapping view proposes that symbolic representation is built on nonsymbolic representation, which both underlie more complex arithmetic. As such, during complex arithmetic tasks, both representation systems would be involved. Therefore, the mapping view suggests that performance on both symbolic and nonsymbolic tasks will predict mathematical ability – and indeed there is evidence to support this view (Mundy & Gilmore, 2009). Additionally, the mapping view suggests that symbolic representation mediates the relation between nonsymbolic representation and mathematic ability, as complex mathematics is built on symbolic representation, which is in turn built on nonsymbolic representation (Verguts & Fias 2004). Conversely the two separate systems view would suggest that only symbolic representation predicts mathematic ability as symbolic and nonsymbolic number representation systems involve two separate cognitive mechanisms. This relation is demonstrated with children by Holloway and Ansari (2009). These researchers argue that although each system is a foundational numerical mechanism, only symbolic representation underlies arithmetic.

The two number representation views provide contrasting interpretations of the distance effect. The distance effect is the finding that participants will have slower response times (RTs) during a comparison task when the numerical distance between stimuli is small (e.g., 4-5) compared to when numerical distance is large (e.g., 3-9). The distance effect is an index of the strength of the relation between external representations (symbolic or nonsymbolic) and mental representations of number, with a smaller distance effect reflecting a stronger relation (Dehaene, Dupoux, & Mehler, 1990; Moyer & Landauer, 1967). The distance effect results from fuzzy mapping between external and internal representations of number (Butterworth & Reigosa, 2008; Holloway & Ansari, 2009). Otherwise stated, integers that are close together will share more features in their mental representations than integers further apart in magnitude. Therefore, comparison becomes more difficult as numerical distance between stimuli decreases. As such, individuals with larger distance effects would have less distinct mental representations of number. It is expected that the size of the distance effect observed among participants will be predictive of their calculation fluency, such that individuals with a smaller distance effect will show greater calculation fluency and individuals with larger distance effects will show poorer calculation fluency.

A reverse distance effect has been observed in symbolic ordinality tasks, where participants are asked to make judgements about whether a set of digits are in order. As the numerical distance between numbers increases, participants are generally less accurate and slower than when the numerical distance between stimuli is small (e.g., 1, 2, 3). Lyons and Beilock (2009) argue that this observed reverse distance effect may arise due to increased familiarity with ascending numbers with a small numerical distance, as these sequences occur most commonly and because numerical symbols are strongly ordinal (also see Turconi, Campbell, & Seron. 2006). Given the weak association between the two number representation systems observed by Lyons et al. (2012) during comparison tasks, the two separate systems view suggests that the reverse distance effect observed in symbolic ordinality tasks will be reduced or appear as a standard distance effect in nonsymbolic ordinality tasks – as nonsymbolic stimuli are not frequently presented in ordinal sequences and because symbolic stimuli in comparison tasks do not seem to bring about a quick and accurate representation of their nonsymbolic equivalents (Lyons et al., 2012).

The mapping view might predict that the distance effects for ordinality may be similar for symbolic and nonsymbolic stimuli and suggest that RTs for nonsymbolic trials would be slower than in symbolic trials, similar to the trend observed in comparison tasks (Mundy & Gilmore 2009, Verguts & Fias, 2004). In this way, the mapping view predicts an additive relation between symbolic and nonsymbolic stimuli. According to the mapping view, in the ordinality task, nonsymbolic stimuli may link to symbolic representations, which in turn link to numerosity.

Lyons and Beilock (2009) suggest two interrelated aspects of numerical representations: a sense of quantity and of relative order. Many studies have examined the relation between quantity and numerical representation through comparison tasks (e.g., Holloway & Ansari, 2009; Lonnemann, Linkersdörfer, Hasselhorn, Lindberg, 2011). Tasks looking at relative order (ordinality tasks) require an additional operation in relation to magnitude comparison tasks. Rather than simply comparing one number to another (e.g., 3 to 4) to determine which is numerically larger, an individual must determine if the full sequence of three numbers are in ascending or descending order (e.g., 3-4-5). As such, it can be argued that an ordinality task requires both a sense of quantity and of relative order and thus provides a foundational measure of numeracy.

The present study examined effects of task (number comparison vs. ordinality), distance (small vs. large), and format (symbolic vs. nonsymbolic) on accuracy and response time. Additionally, the symbolic and nonsymbolic RTs for both the ordinality and magnitude comparison tasks were used to analyze the predictive relation between the number representation systems and calculation fluency. If symbolic and nonsymbolic number representation are supported by separate systems, the results should show that performance on symbolic tasks, but not nonsymbolic tasks, predicts mathematical ability. Moreover, a task x format x distance interaction, wherein a reverse distance effect is observed for the nonsymbolic format in the ordinality task would provide further support for the separate systems view.

Method

Participants

The participants for the study consisted of 51 undergraduate students (*M* = 19.8 ± 1.0 years, Range = 18-23 years), from King's University College and Western University (27 male, 24 female). All participants completed their elementary and secondary education in Canada. Participation for this study was on a voluntary basis.

Materials

Magnitude Comparison Task. Two single digit numbers (ranging from 1 to 9) were presented on an iPad screen, and participants were asked to choose the numerically larger number as fast as they could without making any errors. Problems appeared in two different formats: symbolic (Arabic digits) and nonsymbolic (dots). On nonsymbolic tasks, surface area of the dots was congruent, (larger number with a larger surface area than the smaller number), non-congruent (larger number with a smaller surface area than the smaller number), or matched (both numbers take up the same surface area), each for a third of the trials. Stimuli remained on the screen for 7800ms or until the participant made a choice, and the time between trials was 1000ms. Participants performed two blocks of 54 trials (one symbolic, one nonsymbolic) and the presentation order of these blocks was counterbalanced based on participant number. The order of the problems presented in each block was randomized. As in Holloway and Ansari (2009), small distances in this task were those in which displayed numerosities differed by one (e.g., 2 and 3) and large distances were those in which displayed numerosities differed by five (e.g., 2 and 7).

Ordinality Task The ordinality task employed in this experiment was a modified version of the task used by Lyons and Beilock (2009). In this task, participants were presented with three boxes, each containing a distinct set of either digits or dots, for 1000ms. After 1000ms, the stimuli disappeared, and the participant was asked to determine if the presented stimuli were in order (ascending or descending). For example, stimuli sets of "1, 2, 3", "6, 5, 4" and "3, 5, 7" were all considered as "in order", whereas a stimuli set of "3, 7, 5" was considered to be "not in order". From the time of stimuli presentation, participants were given up to 7800ms (including the 1000ms the stimuli remained on the screen) to make their response, and the inter-trial interval was 1000ms. Participants performed two blocks of trials (one symbolic, one nonsymbolic), with each set of trials performed twice for a total of 214 trials. The presentation of the first block was counterbalanced based on participant number. The order of the problems placed in each block was randomized. As in Lyons (2013), small distances in this task were those in which displayed numerosities differed by one (e.g., 1, 2, 3) and large distances were those in which numerosities differed by 3 (e.g., 3, 6, 9).

Calculation Fluency Participants completed the addition and subtraction-multiplication subtests of the Kit of Factor-Referenced Cognitive Tests (French, Ekstrom & Price, 1963). Each subtest of this paper-and-pencil task consisted of two-pages of multi-digit arithmetic problems (two pages of three digit addition problems, and two pages containing both two-digit subtraction problems and two-digit multiplication problems). Participants were instructed to solve the problems as quickly and accurately as possible and were given two minutes per page. Calculation fluency was measured as the total number of correct solutions on both tests, and reflects an individual's ability to quickly and accurately execute simple arithmetic procedures on multi-digit problems.

Procedure

Participants were seated in a quiet room in front of an iPad. Once comfortable, participants completed the magnitude comparison and ordinality tasks on an iPad. Following the iPad tasks, the iPad was removed and participants completed the Kit of Factor-Referenced Cognitive Test. These tasks were completed in one session lasting approximately one hour.

Results

Data Reduction

Two Principal Components Analyses (PCA) were used to examine tasks indexing symbolic RTs and tasks that index nonsymbolic RTs. PCA was used for the present data reduction because this data reduction is largely exploratory in nature. Four criteria were employed to guide the decision making process for the analyses: (1) percent of variance explained (should exceed 60%), (2) factor loadings (should exceed .40), (3) visual inspection of scree plots, and (4) eigenvalues exceeding 1, in accordance with de Winter, Dodou, and Wieringa's (2009) research on factor analyses with small sample sizes. Tasks indexing symbolic RT (i.e., symbolic magnitude comparison RT and symbolic ordinality ascending trials RT) were entered into a PCA and yielded a one-factor solution that accounted for 79.6% of the variance among the measures with the following loadings: symbolic magnitude comparison RT (.89) and symbolic ordinality ascending trials RT (.89), both of these loadings exceed .40. Both a visual analysis of the scree plot and the initial eigenvalue for component one (1.59) supported the extraction of a single factor. Factor scores were used as the symbolic RT measure in the subsequent multiple regression analysis.

Measures indexing nonsymbolic RT (i.e., nonsymbolic magnitude comparison RT and nonsymbolic ordinality ascending trials RT) were entered into a PCA. A one-factor solution emerged that accounted for 71.4% of the variance among the measures with the following loadings: nonsymbolic magnitude comparison RT (.85) and nonsymbolic ordinality ascending trials RT (.85), both of these values exceed .40. A visual analysis of the scree plot and the initial eigenvalue for component one (1.43). Factor scores were used as the nonsymbolic RT measure in the subsequent multiple regression analysis.

Distinguishing Symbolic and Nonsymbolic Systems

Pearson's bivariate correlation was used to analyze the relation between calculation fluency, the symbolic factor, and the nonsymbolic factor. There was a significant correlation between the symbolic factor and calculation fluency, $r = -.39$, $p = .005$. This correlation indicates that faster RTs were associated with higher calculation fluency. There was no observed correlation between the nonsymbolic factor and calculation fluency, $r = -.01$, ns. Finally, there was a significant correlation between the symbolic factor and the nonsymbolic factor, $r = .54$, $p < .001$. As RTs for the symbolic factor increased, so did RTs for the nonsymbolic factor. Pearson's Correlations for calculation fluency, symbolic and nonsymbolic comparison (RT and accuracy), and symbolic and nonsymbolic ordinality (RT and accuracy) can be found in Table 1.

Data were analyzed using multiple regression to determine whether performance on symbolic and nonsymbolic magnitude comparison and ordinality tasks predicted calculation fluency. The results of the multiple regression indicated the two predictors explained 21% of the variance in calculation fluency, $R^2 = .21$, $F(2,49) = 6.35$, $p = .004$. It was found that the symbolic factor was a significant predictor of calculation fluency, $\beta = -.54$, $t = -3.56$ $p = .001$. The nonsymbolic factor was not found to be a significant predictor of calculation fluency, $\beta = .28$, $t = 1.87$,
ns. The symbolic factor accounted for 21% of the unique variance in calculation fluency whereas the nonsymbolic factor accounted for 7% of the unique variance in calculation fluency.
These results support the hypothesis that the symbolic and nonsymbolic systems are distinct and that only the symbolic system predicts calculation fluency.

Accuracy

To assess replication of the results found by Lyons and Beilock (2009), a 2(task: magnitude comparison, ordinality) x 2(distance: small, large) x 2(format: symbolic, nonsymbolic) repeated measures factorial ANOVA was performed with percent error as the dependent variable. There was a main effect of task; participants made fewer errors while performing the magnitude comparison task ($M = 2.4\%$, $SD = .32\%$) than while performing the ordinality task ($M = 16.1\%$, $SD = 1.84\%$), $F(1,50) = 55.85$, $p < .001$, $\eta^2 = .53$, power = 1.0. There was a main effect of format; participants made fewer errors when presented with symbolic stimuli ($M = 6.0\%$, $SD = .99\%$) than when presented with nonsymbolic stimuli ($M = 12.4\%$, $SD = 1.84\%$), $F(1,50) = 38.29$, $p < .001$, $\eta^2 = .43$, power = 1.0. There was no significant main effect of distance, $F(1,50) = 1.92$, *ns*, $\eta^2 = .04$, power = .27. This result, however, is due to a qualitative three-way interaction described below.

There was an interaction between task and format, $F(1,50) = 40.04$, $p < .001$, $\eta^2 = .45$, power = 1.0. The effect of format was greater for the ordinality task than for comparison. There was an interaction between format and distance, $F(1,50) = 18.96$, $p < .001$, $\eta^2 = .28$, power = .99. Interpretation of this interaction is aided by considering the significant task x format x distance interaction shown in Figure 1, $F(1,50) = 15.99$, $p < .001$, $\eta^2 = .24$, power = .98. For magnitude comparison there was a standard distance effect for both formats, whereas for ordinality there was a standard distance effect for nonsymbolic stimuli and a reverse distance effect for symbolic stimuli. These results provide a replication of Lyons and Beilock (2009).

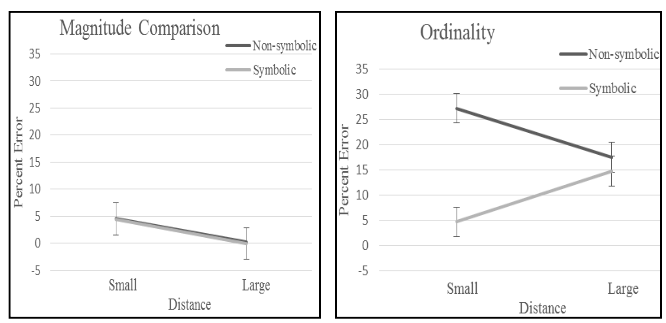

Figure 1: Interaction between task, format, and distance with accuracy as dependent variable. Confidence intervals were constructed using the procedure from Jarmasz and Hollands (2009).

Table 1: Correlations among student measures (N = 51)

	1	2	3	4	5	6	7	8
1. Calculation Fluency								
2. Symbolic Compare RT	-.31*							
3. Symbolic Ordinal RT	-.39**	.60**						
4. Symbolic Compare DE	-.17	.26	.24					
5. Symbolic Ordinal DE	.25	-.02	-.41**	-.04				
6. Non-Symbolic Compare RT	.04	.48**	.28*	.01	-.06			
7. Non-Symbolic Ordinal RT	-.06	.39**	.50**	-.19	-.05	.43**		
8. Non-Symbolic Compare DE	.09	.35*	.14	-.11	-.16	.66**	.30*	
9. Non-Symbolic Ordinal DE	.08	.18	.10	.20	.07	.06	-.03	.18

*$p < .05$, **$p < .01$

Response Time

To assess replication of the results found by Lyons and Beilock (2009), a 2(task: magnitude comparison, ordinality) x 2(distance: small, large) x 2(format: symbolic, nonsymbolic) repeated measures factorial ANOVA was performed with RT (ms) as the dependent variable. There was a main effect of task, participants responded more quickly during the magnitude comparison task (M = 893, SD = 34) than during the ordinality task (M = 1357, SD = 38), $F(1,50) = 154.14$, $p < .001$, $\eta^2 = .76$, power = 1.0.

There was also a main effect of format, participants were more quick to respond to symbolic stimulus (M = 875, SD = 27) than to nonsymbolic stimulus (M = 1375, SD = 45), $F(1,50) = 155.55$, $p < .001$, $\eta^2 = .76$, power = 1.0. There was a significant main effect of distance. When the numerical distance was small, participants were slower to respond (M = 1255, SD = 41) than when the numerical distance was large (M = 995, SD = 29), $F(1,50) = 72.68$, $p < .001$, $\eta^2 = .59$, power = 1.0.

There was an interaction between task and distance, $F(1,50) = 97.07$, $p < .001$, $\eta^2 = .66$, power = 1.0. The distance effect was greater for magnitude comparison than for ordinality. An interaction was observed between format and distance, $F(1,50) = 110.78$, $p < .001$, $\eta^2 = .69$, power = 1.0. Interpretation of this interaction is aided by considering the significant task x format x distance interaction shown in Figure 2, $F(1,50) = 16.82$, $p < .001$, $\eta^2 = .25$, power = .98. For magnitude comparison there was a standard distance effect for both formats, whereas for ordinality there was a standard distance effect for nonsymbolic stimuli and a reverse distance effect for symbolic stimuli. These results provide a replication of Lyons and Beilock (2009).

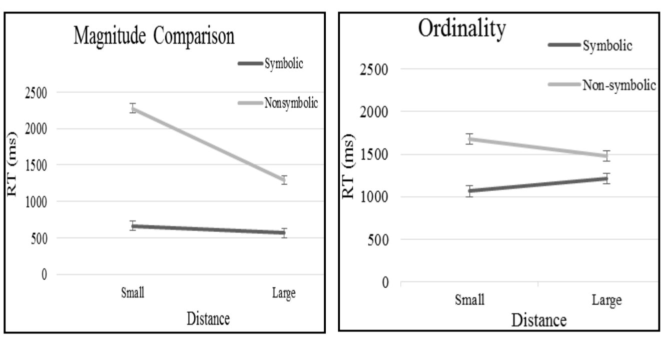

Figure 2: Interaction between task, format, and distance with RT as dependent variable. Confidence intervals were constructed using the procedure from Jarmasz and Hollands (2009).

Discussion

There is controversy in the numerical cognition literature concerning how the symbolic and nonsymbolic number representation systems underlie the development of more complex numerical skills, like multi-digit arithmetic (Bulthé et al., 2014; Dehaene, 1992; Holloway & Ansari 2009; Mundy & Gilmore, 2009). The results of the present study suggest that, in adulthood, the symbolic number representation system plays a more central role in predicting calculation fluency. After combining RT scores for symbolic magnitude comparison and ordinality tasks into a symbolic factor and combining RT scores for nonsymbolic magnitude comparison and ordinality tasks into a nonsymbolic factor, a multiple regression analysis revealed that the symbolic factor was predictive of calculation fluency, but the nonsymbolic factor was not. These results do not support a relation between the nonsymbolic number representation system and complex mathematics skill but do support a relation between the symbolic number representation system and calculation fluency. These results are similar to those found by Lyons et al. (2012), who suggest that symbolic representation begins to overshadow nonsymbolic representation as development progresses. These results provide evidence for the existence of two separately developing number representation systems.

As the two separate systems view would suggest, the distance effects were different between tasks and across formats. We observed a much stronger distance effect for nonsymbolic stimuli in the comparison task. For the ordinality task, we observed a reverse distance effect for symbolic stimuli, as found in Lyons and Beilock (2009). In their study, the reverse distance effect was accounted for by increased familiarity with sequences in small numerical distances commonly appearing in order (e.g., 1, 2, 3). In our study, nonsymbolic trials yielded a standard distance effect. This result supports the familiarity hypothesis proposed by Lyons and Beilock (2009). Additionally, the correlation between symbolic ordinality RT and calculation fluency (see Table 1) is consistent with another hypothesis proposed by Lyons and Beilock (2011), wherein the development of ordinal understanding can be seen as a stepping stone to the understanding of symbolic numerical framework. In a symbolic numerical framework, each number carries a distinct and exact numerical value, contrasting the approximate nature of the nonsymbolic number system, and allows for the realization of numerical quantities that would be nearly impossible to recognize and/or comprehend with the nonsymbolic number system.

This correlation with calculation fluency was not seen in nonsymbolic ordinality trials, which is consistent with the view that the two systems determine ordinality in different ways, one through a set of iterative comparisons (nonsymbolic) and one through consideration of the entire ordered sequence (symbolic; Lyons, 2013). This difference is in line with the view that the two systems develop separately, as it is expected that the two systems would use the same processes when determining order if they developed together.

The positive relation between symbolic, but not nonsymbolic, task performance and calculation fluency is consistent with the two separate systems view. This view is further supported by the replication of a reverse distance effect in symbolic stimuli, in contrast to a standard distance effect in nonsymbolic stimuli, in the ordinality task. This finding provides support for the claim that the ordinality task is performed in different ways depending on the system engaged (Lyons, 2013). Only the symbolic, and not the nonsymbolic system, appears capable of considering an entire ordered sequence. This process may be key to understanding numerosity in a symbolic framework (Lyons, 2011). As such, the findings of this paper suggest that the development of such a symbolic framework does not result from a mapping of symbolic information onto the nonsymbolic system, as each system uses different mechanisms for determining order. Given that the need for a symbolic number system stems from the requirement to accurately recognize and comprehend numbers in a manner that the nonsymbolic system lacks the affordances to do (due to its approximate nature), it makes sense that the development of a new symbolic system would not map information onto a system that cannot accurately comprehend it. Future studies should investigate in particular the emergence of symbolic ordinality capabilities in children and the relation of these capabilities to symbolic, nonsymbolic, and general mathematical abilities.

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