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# Fostering the Understanding of Multi-Representational Examples by Self-Explanation Prompts

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## Abstract

Multiple representations in learning materials are usually employed in order to foster understanding. However, they also impose high demands on the learners (e.g., need for integration). By embedding multi-representations in worked-out examples, cognitive capacity is released that can be used for self-explanations on the integration and understanding of multiple representations. The effects of two types of self-explanation prompts were investigated by conducting an experiment comprising three conditions (domain: mathematics). The learners ($N = 62$) received either (1) self-explanation prompts, (2) self-explanation prompts in a scaffolding-fading procedure, or (3) no prompts. Both types of self-explanation prompts fostered procedural and conceptual knowledge. With respect to procedural knowledge, the different self-explanations prompts did not differ in their effectiveness. However, conceptual knowledge and especially knowledge indicating the integration of multiple representations was particularly fostered by scaffolded self-explanation prompts. Thus, for enhancing conceptual understanding, such self-explanation prompts should be provided because they scaffold the learners to reach their zone of proximal development.

**Keywords:** multiple representations; self-explanations; worked-out examples

## Learning with Multiple Representations

### Potentials of Multiple Representations

Multiple representations in learning materials (e.g., combinations of pictorial and arithmetical representations) are commonly used because they promise unique potential in fostering understanding. By combining different representations with different properties, learners are not limited by the strengths and weaknesses of one particular representation (cf. Ainsworth, Bibby, & Wood, 2002). Furthermore, it is expected that if you provide learners with a rich source of different representations of a domain, then they build references across these representations (Ainsworth, 1999). In this context, Kaput (1989, pp. 179-180) states that the "cognitive linking of representations creates a whole that is more than the sum of its parts…it enables us to see complex ideas in a new way and apply them more effectively." In their cognitive flexibility theory, Spiro and his colleagues (e.g., Spiro & Jehng, 1990) argue that the ability to construct and switch between multiple representations is fundamental to successful learning. Mayer (e.g., Mayer & Sims, 1994) describes a theory of multi-media learning, which states that learners acquire more procedural and conceptual knowledge when they receive multiple representations.

According to a functional taxonomy of Ainsworth (1999), multiple representations are provided for three main purposes: (1) to support different ideas and processes, (2) to constrain representations, and (3) to promote a deeper understanding. The last aspect is focused in this research.

In sum, learners cannot only learn how different individual representations with their strengths and weaknesses operate. They can also gain an understanding how the representations relate to each other. The latter is often a unique contribution to learning.

### Problems of Multiple Representations

A major problem in employing multiple representations for learning is that very often the expected learning outcomes do not occur (e.g., de Jong et al., 1998). This is due to the fact that learners are faced with complex learning demands when they are presented with a novel multi-representational system (Ainsworth, 1999): (a) They must learn the format and operators of each representation, (b) understand the relation between each representation and the domain it represents, and (c) learn how the representations relate to each other. Particularly with the latter, the learners experience difficulties. Very often they just concentrate on one type of representation or fail to link different representations to each other so that the positive effects that were intended by the use of multiple representations do not occur to the expected extent (e.g., Ainsworth, Bibby, & Wood, 1998). If learners have difficulties in mapping their knowledge between representations, the benefits of multiple representations may never arise (cf. Ainsworth et al., 2002).

On the one hand, multiple representations offer unique possibilities of fostering understanding. On the other hand, they impose high demands on the learners. This state of affairs suggests that multiple representations have to be implemented in a learning approach which reduces demands on the learner – such as learning with worked-out examples. Thereby, cognitive load is relieved. The opportunity arises to use this free cognitive capacity for integrating and deeply understanding the multiple representations.

# Multiple Representations in Worked-Out Examples

Worked-out examples consist of a problem formulation, solution steps, and the final solution itself. Learning from worked-out examples is a very effective method for cognitive skill acquisition in well-structured domains such as mathematics (for an overview, see Atkinson et al., 2000) because the learners are relieved from finding a solution on their own. Thereby – in terms of the cognitive load theory – *extraneous load* (load not directly relevant to learning) is reduced (cf. Paas, Renkl, & Sweller, 2003). Thus, the learners can concentrate on understanding the solution (which can be presented in a multi-representational format) and the underlying principles. Thereby, *germane load* (load imposed by processes aimed to gain understanding) is enhanced.

However, studies on worked-out examples which were presented in a *split-source format* – that included multiple representations – showed that materials requiring learners to split their attention among multiple information sources imposed a heavy extraneous load and, as a consequence, eliminated the worked-out example effect (e.g., Tarmizi & Sweller, 1988). This phenomenon was labeled the *split-attention effect*. However, worked-out examples in which the multiple representations were integrated (*integrated format*) enhanced learning in comparison to conventional problem solving and split-source worked-out examples. Thus, multi-representational solutions in worked-out examples should be presented in an integrated format. Thereby, mapping between representations is easier, which makes cognitive resources available for productive learning processes such as self-explanations (germane load).

The classical study of Chi et al. (1989) analyzed individual differences with respect to how intensively learners self-explained the solution steps of worked-out examples (from the domain of physics). They found that learners who explained the worked-out examples more actively to themselves learned more. Renkl (1997) showed that even when the study time was held constant, self-explanation activity was related to learning outcomes.

In sum, the quality of self-explanations is a major determinant of what is learned from studying worked-out examples. However, learners show clear individual differences in processing worked-out examples. Most learners do not actively self-explain worked-out examples, that is, they do not productively use their free cognitive capacity for germane load (Renkl, 1997). This suggests that self-explaining has to be instructionally supported, by making the link between representations salient (e.g., integrated format) and by prompting self-explanations.

## Prompting Self-explanations

Renkl et al. (1998) found that spontaneous self-explanations during worked-out example study were not as effective as self-explanations that were enhanced by prompting. Prompts elicit self-explanation activities that the learners are capable of doing but which they spontaneously do not show. Thus, it is sensible to design prompts that foster self-explanations in order to ensure that the free capacity that is available for studying multi-representational examples is effectively used for integrating and understanding the representations.

## Scaffolding Self-Explanations

It has to be taken into account that relying only on self-explanations has several disadvantages – even when self-explaining is elicited by prompts. The quality of the self-explanations elicited by self-explanation prompts were in many cases far from being optimal. Sometimes the learner is not able to self-explain a specific solution step, the self-explanation is only partially correct, or the given self-explanations are even incorrect (Renkl, 2002). This can lead to incomplete or incorrect knowledge that, at worst, can severely impede further learning. Thus, the challenging task is to find ways to support self-explanations further than is possible with prompts.

The instructional method of scaffolding offers a promising starting point. Collins, Brown, and Newman (1989) refer to scaffolding as a support for the learners that relieve them of parts of an overall task that the learner cannot yet manage, for instance, explaining why the multiplication rule has to be applied in probability theory. The intention is, however, to hand over responsibility to the learners as soon as possible. The latter implies a fading process. Fading consists of the gradual removal of support until students are working on their own.

According to Vygotskian's approaches, scaffolding is related to the zone of proximal development (Vygotsky, 1978). This is the region of activity in which learners can perform successfully given the aid of a supporting context. Thus, it is sensible to support learners by scaffolding on knowledge construction that would be out of reach for the learners without assistance.

Yet, studies on different scaffolding procedures show mixed results. In a qualitative study, Chi (1996) demonstrated that a tutor's actions of co-construction of knowledge (which included self-explanations of the tutee) led to the learners' deep understanding. In an experimental study, Hilbert, Schworm, and Renkl (2004) fostered learning either by self-explanation prompts or by instructional support which changed during the course of learning from instructional explanations to self-explanation prompts. However, the transition from instructional explanations to self-explanation prompts was equally effective as giving only self-explanation prompts. Thus, constructing an effective scaffolding method is not a trivial task.

In sum, a combination of worked-out examples and multiple representations might be very effective: It can be argued that the employment of worked-out examples in an integrated format reduces extraneous cognitive load which enables the learners to use "free" cognitive capacity for self-explanations (germane load) on the integration and understanding of multiple representations. This in turn may bring to bear the advantages of learning with multiple representations. In this research, the effects of using open self-explanation prompts (questions to induce self-explanations) and scaffolded self-explanation prompts ("fill-in-the-blank" explanations) to foster the understanding of multi-representational examples are investigated. Probability theory was chosen as the learning domain. Procedural

knowledge (problem-solving performance) and conceptual knowledge (knowledge about concepts and principles) were assessed as learning outcomes.

## Hypotheses

Specifically, the following hypotheses were tested:

(1) Self-explanation prompts (scaffolded and open) foster *procedural knowledge* acquired from multi-representational examples.

(2) Scaffolded self-explanation prompts have additional effects on *procedural knowledge* when compared to open self-explanation prompts.

(3) Self-explanation prompts (scaffolded and open) foster *conceptual knowledge* acquired from multi-representational examples.

(4) Scaffolded self-explanation prompts on multi-representational examples have additional effects on *conceptual knowledge* when compared to open self-explanation prompts.

Furthermore, a focus of our learning environment was on understanding the multiplication rule. Thus, we were especially interested whether conceptual knowledge of the multiplication rule could be enhanced.

## Methods

### Sample and Design

The participants of this study were 42 female and 20 male students of the University of Freiburg, Germany. The mean age was 25 years ($M = 25.02$, $SD = 6.12$). A one-factorial experimental design with three groups was conducted (see Table 1).

Table 1: Design and sample size.

| Scaffolded self-explanation prompts | Open self-explanation prompts | No prompts |
|---|---|---|
| $n = 20$ | $n = 22$ | $n = 20$ |

In a computer-based learning environment all learners worked on eight worked-out examples which included multiple-representational solution procedures. Additionally, participants of the condition "scaffolded self-explanation prompts" received "fill-in-the-blank" explanations in the first worked-out examples (e.g., "There are □ times □ branches. Thus, all possible outcomes are included."). In following isomorphic examples, this support was faded out and they received open self-explanation prompts. The answers had to be typed into corresponding boxes. In the condition "open self-explanation prompts", the learners were provided only with open self-explanation prompts (e.g., "Why do you calculate the total acceptable outcomes by multiplying?"). The group "no prompts" (control group) included no additional support; the learners were just provided with a text box in order to take notes.

## Learning Environment

Probability theory (specifically: complex events) was chosen as the learning domain because it is suited for the use of different representation codes (pictorial and arithmetical). In addition, it is relatively difficult for learners. Eight worked-out examples were presented in a computer-based learning environment. Specifically, four principles of the topic complex events were addressed in the worked-out examples. On each principle, the learners were provided with two isomorphic worked-out examples. The participants were allowed to regulate the processing speed of the worked-out examples on their own. The worked-out examples were presented with multiple-representational solution procedures: a pictorial, tree-like solution and an arithmetical solution (see Figure 1). The learners were supported in integrating the information from the tree (e.g., the ramifications) with the respective arithmetical information (e.g., the multiplication signs). This was accomplished by having the corresponding information from the different representations simultaneously flashing in the same color – "information pair" after "information pair". At the end, a colored freeze image was presented. One focus of our learning environment was the understanding of the multiplication rule. This rule has to be applied to calculate the probability of the complex events. Usually, the learners understand *that* the multiplication rule has to be applied, but they rarely understand *why* the fractions have to be multiplied. For many learners, the latter is not apparent. However, it is "encapsulated" in the multi-representational solution. The learner can "unpack" it by integrating the information of the multiplication sign of the arithmetical code with the ramifications in the tree-diagram (for the numerator in Figure 1: there is twice one branch; for the denominator there are five times four branches).
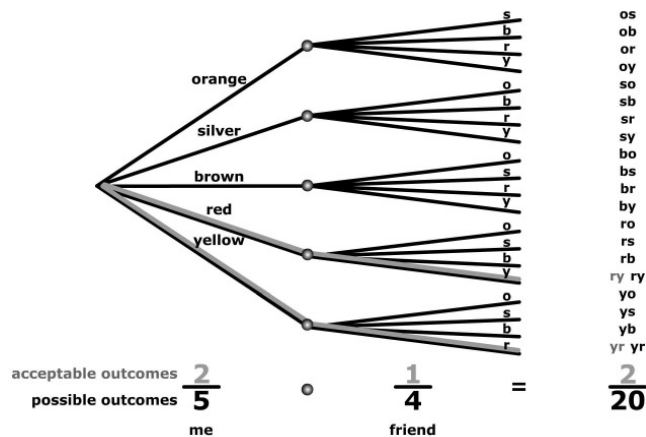


Figure 1: Multi-representational solution procedure in an integrated format (originally, it was colored).

## Procedure

The experiment was conducted in individual sessions. First, the participants were asked to fill in a demographic questionnaire. Afterwards, the learners worked on a pretest. Then, they entered the computer-based learning environment and worked individually in front of a computer. In

order to provide or re-activate basic knowledge that allowed the participants to understand the following worked-out examples, an instructional text on basic principles of probability was provided. Afterwards, the participants studied eight worked-out examples. During this phase, the experimental manipulation was realized, that is, the participants were provided with the scaffolded self-explanation prompts, open self-explanation prompts, or no prompts. Finally, the participants completed a post-test on procedural and conceptual knowledge. The experiment lasted approximately two hours ($M$ (in minutes) = 128.63, $SD$ = 31.30).

## Instruments

**Pretest: Assessment of Prior Knowledge** A short pretest on complex events containing six problems examined the topic-specific prior knowledge of the participants. The maximum score for the pretest was six points.

**Post-test: Assessment of Learning Outcomes** The learning outcomes were measured with a post-test which contained 14 problems (seven problems on procedural knowledge and seven problems on conceptual knowledge).

*(1) Procedural Knowledge (Problem-Solving Performance).* Procedural knowledge contains actions or manipulations that are valid within a domain (de Jong & Ferguson-Hessler, 1996), e.g. multiplying two fractions to calculate the probability of a complex event. This category included four near transfer items (same structure as the worked-out examples presented for learning but different surface features, such as the cover story) and three far transfer items (different surface features and also different structure, which means that a modified solution procedure had to be found). An example of a near transfer item is "Bicycle number-locks usually have four digits. What is the probability that one guesses the right digit sequence on the first guess?" In each task, 0.5 points could be achieved if the numerator of the solution was correct and 0.5 points if the denominator was correct. These scores were summed up to a total score of procedural knowledge. Thus, a maximum score of seven points could be achieved in this category.

*(2) Conceptual Knowledge.* Conceptual knowledge refers to static knowledge about facts, concepts, and principles that apply within a domain (de Jong & Ferguson-Hessler, 1996). In particular, it includes understanding about "what is behind the solution procedure". This category contained seven open questions which required written explanations on conceptual knowledge of the principles presented in the learning phase. For example, the learners were to explain *why* the multiplication rule has to be applied (e.g., "Why are the two fractions multiplied?"). As the rationale for the multiplication rule can be figured out relatively easily when the pictorial and the arithmetical representations are integrated, this post-test measure also tapped on the quality of representation integration. Two independent raters scored the open answers by using a 6-point rating scale ranging from 1 *(no conceptual understanding)* to 6 *(very clear conceptual understanding)*. A very clear conceptual understanding was indicated by a correct answer with a high degree of reasoning and elaboration. Inter-rater reliability was very good (intra-class-coefficient .90).

## Results

Table 2 presents the mean scores and standard deviations for the three experimental groups on the pretest as well as on the procedural and conceptual knowledge. Additionally, knowledge of the multiplication rule (which was part of the conceptual knowledge) is reported. The measures on learning outcomes were subjected to a priori contrasts that correspond to the hypotheses (i.e., one-tailed $t$ tests). An alpha-level of .05 was used for all statistical tests.

With respect to the students' topic-specific prior knowledge, an ANOVA revealed no significant differences, $F < 1$. Hence, there was no a priori difference between groups with respect to prior knowledge.

Table 2: Means and standard deviations (in parentheses) on the pretest and on the dependent measures.

| | Pretest | Procedural knowledge | Conceptual knowledge | Multiplication Rule |
|---|---|---|---|---|
| Scaffolded self-explanation prompts | 2.30 (1.41) | 4.55 (1.20) | 3.63 (1.02) | 3.57 (1.65) |
| Open self-explanation prompts | 2.52 (1.69) | 4.41 (1.05) | 2.98 (.87) | 2.00 (1.08) |
| No prompts | 2.35 (1.86) | 3.63 (1.36) | 2.58 (.77) | 1.85 (.89) |

*(1) Effects of Self-Explanation Prompts on Procedural Knowledge.* Descriptively, we obtained higher means in the groups with self-explanations prompts (scaffolded self-explanation prompts and open self-explanation prompts) for procedural knowledge. To test this difference, we aggregated the two groups with self-explanation prompts and compared them with the control group. A $t$ test yielded a significant and medium to strong difference for procedural knowledge in favor of the self-explanation prompts conditions, $t(60) = 2.62$, $p = .005$, $d = .68$. Hence, the participants who had received self-explanation prompts performed significantly better on procedural knowledge compared with those learners who had received no such prompts.

*(2) Effects of Scaffolded vs. Open Self-Explanation Prompts on Procedural Knowledge.* To test for additional effects of scaffolded self-explanation prompts on procedural knowledge when compared to open self-explanation prompts, a $t$ test was performed. However, it failed to reach statistical significance, $t(40) = .41$, $p = .688$. Thus, the two conditions with self-explanation prompts did not differ with respect to procedural knowledge. Hence, with respect to procedural knowledge, scaffolded and open self-explanation

prompts fostered procedural knowledge. Yet, the two self-explanation prompts groups did not differ in this respect.

*(3) Effects of Self-Explanation Prompts (Scaffolded and Open) on Conceptual Knowledge.* Descriptively, we obtained the highest mean in the condition scaffolded self-explanation prompts, followed by the mean of the group open self-explanation prompts. The lowest mean revealed for the group no prompts. A *t* test comparing the groups with self-explanation prompts against the control group yielded a significant and strong effect, $t(60) = 2.84$, $p = .003$, $d = .80$. Evidently, the participants of the conditions with self-explanation prompts outperformed their counterparts of the group no prompts with respect to conceptual knowledge. However, did scaffolded and open self-explanation prompts have diverse effects on conceptual knowledge?

*(4) Effects of Scaffolded vs. Open Self-Explanation Prompts on Conceptual Knowledge.* A *t* test which tested whether the group scaffolded self-explanation prompts outperformed the group open self-explanation prompts revealed a significant and medium to strong effect, $t(40) = 2.23$, $p = .016$, $d = .68$. Thus, scaffolded self-explanation prompts had additional effects on conceptual knowledge in comparison to open self-explanation prompts. In sum, with respect to conceptual knowledge, scaffolded and open self-explanation prompts were effective. Obviously, especially scaffolded self-explanation prompts fostered this type of knowledge.

A special focus of our learning environment was to understand *why* the multiplication rule has to be applied. This type of knowledge also indicates to what extent the different representations were integrated because it can hardly be understood by studying just one representation but by mapping the multiplication sign of the arithmetical code with the ramifications in the tree-diagram (in the denominator in Figure 1, there are five *times* four branches which represent the possible combinations). Therefore, we tested whether scaffolded and open self-explanation prompts fostered understanding of the multiplication rule. Descriptively, we obtained the highest mean in the condition scaffolded self-explanation prompts, whereas the means of the conditions open self-explanation prompts and no prompts were relatively low. A *t* test which tested whether the groups with self-explanation prompts outperformed the group no prompts revealed a significant and medium to strong effect, $t(60) = 2.36$, $p = .011$, $d = .70$. Thus, the participants of the conditions with self-explanation prompts outperformed their counterparts of the group no prompts with respect to understanding of the multiplication rule.

To test whether the condition scaffolded self-explanation prompts fostered understanding of the multiplication rule more effectively than the group open self-explanation prompts, a *t* test was performed. A significant and strong effect was obtained, $t(40) = 3.67$, $p < .001$, $d = 1.13$. Hence, the overall pattern of performance indicates that especially scaffolded self-explanation prompts fostered the integration of multiple representations.

In sum, self-explanation prompts on multi-representational examples fostered procedural and conceptual knowledge. With respect to procedural knowledge, it did not make a difference whether the learners were provided with scaffolded or with open self-explanation prompts. However, with respect to conceptual knowledge (especially: understanding of the multiplication rule), an overall effect of the self-explanation prompts can be mainly ascribed to the scaffolded self-explanation prompts.

## Discussion

In summary, our study revealed four essential contributions for the problem of supporting effective self-explanations during learning with multi-representational examples: (1) Self-explanation prompts foster procedural and conceptual knowledge. This result adds to the growing body of evidence that shows that prompting self-explanations is crucial with respect to learning outcomes in example-based learning. In particular, we were able to show that prompting self-explanations is also very effective in understanding multi-representational examples. (2) With respect to procedural knowledge, it is equally effective to use open or scaffolded self-explanation prompts. (3) Yet, with respect to conceptual knowledge, especially scaffolded self-explanation prompts are effective. (4) In particular, this is true for integrating multiple representations, as indicated by the understanding of the multiplication rule. This rule can be understood by integrating the multiplication sign of the arithmetic equations and the ramifications of the tree diagram. Thus, our findings also suggest that scaffolded prompts particularly support the integration of multiple representations.

The question arises *why* especially scaffolded self-explanation prompts were effective with respect to conceptual knowledge, whereas with respect to procedural knowledge, providing open self-explanation prompts were sufficient. Conceptual understanding, for example, understanding of the multiplication rule, is more demanding than gaining procedural knowledge – in particular because such type of conceptual understanding is seldomly addressed in mathematics lessons in school or at university. Nevertheless, it is crucial for further learning. The finding that scaffolded self-explanation prompts (as opposed to open self-explanation prompts) showed to be effective with respect to conceptual knowledge may be related to the zone of proximal development (Vygotsky, 1978). The scaffolded self-explanation prompts fostered the integration of the multiple representations and the conceptual understanding that was both slightly out of reach for learners without this assistance. For instance, the learners were not able to self-explain the rationale of the multiplication rule – even if they were prompted. The "fill-in-the-blank" explanations provided the learners with the pieces of information they needed to integrate and conceptually understand the multi-representational examples (e.g., "There are □ times □ branches. Thus, all possible outcomes are included."). Conceptual understanding refers in particular to an understanding about what the logic of (here: multi-representational) solution procedures is. Obviously, the scaffolds supported the learners to look behind the multi-representational solutions and in understanding the relation between the multiple representations and the domain (Ainsworth, 1999).

Our findings suggest that scaffolded self-explanation prompts have to be provided if understanding the learning

contents is slightly out of reach for learners without this assistance. Yet, to diagnose the dimensions of the zone of proximal development is a difficult task (Ainsworth et al., 1998). Nevertheless, we should be able to identify its lower boundary by analyzing the learner's unscaffolded performance. With this information, it should be possible to construct scaffolded prompts on knowledge that is out of reach for the unsupported learner and which therefore falls within the learner's zone of proximal development.

By providing only "fill-in-the-blank" explanations instead of complete instructional explanations and by fading out the scaffolds in the following isomorphic examples, it was assured that the learners did not superficially and passively but rather actively process the new information. Yet, as the scaffolded self-explanation prompts included additional information compared to the open self-explanation prompts, it might be that not the scaffolding-fading procedure was fostering learning but only the additional information. However, usually instructional explanations in worked-out examples proved to be rather inefficient (Hilbert et al., 2004; Renkl, 2002). Thus, it is not probable that the pure, partial instructional explanation in the scaffolded self-explanation prompts was the crucial factor. Nevertheless, further studies should explore the specific contribution of the additional information in the scaffolded self-explanation prompts.

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