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A Resource-Rational Process Model of Violation of Cumulative Independence

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Abstract

Human decision-making is filled with numerous paradoxes and violations of rationality principles. A particularly notable example is violation of cumulative independence (VoCI). Recently, there has been a surge of interest in theorizing and developing a resource-rational foundation for many such phenomena. Here we ask whether VoCI could be given a resource-rational basis too. To what extent could VoCI be explained in terms of the optimal use of limited cognitive resources? In this work, we look at VoCI through the lens of modern psychological theories of bounded rationality, presenting the first resource-rational account of VoCI. We discuss the implications of our work for risky decision-making, and more broadly, human rationality.

Keywords: cumulative independence; resource-rationality; risky choice; resource-rational process models

1 Introduction

Over the past years, extensive empirical work has revealed that human decision-making is filled with numerous paradoxes and apparent violations of rationality principles. Examples include the Allais paradox (Allais, 1953), the St. Petersburg paradox (Bernoulli, 1738), the Ellsberg paradox (Ellsberg, 1961), the decoy effect (Huber et al., 1982), and violations of transitivity (Loomes et al., 1991), stochastic dominance (Birnbaum, 2005), betweenness (Camerer & Ho, 1994), the sure-thing principle (Jeffrey, 1982), and cumulative independence (Birnbaum & Navarrete, 1998).

In particular, mounting empirical evidence shows that violation of cumulative independence (VoCI) is both substantial and systematic (e.g., Birnbaum & Navarrete, 1998; Birnbaum et al., 1999; Birnbaum, 1999, 2006).

Recently, there has been a surge of interest in theorizing and developing a resource-rational foundation for many such paradoxes and violations of rationality principles (e.g., Dasgupta, Schulz, & Gershman, 2017; Nobandegani, da Silva Castanheira, Shultz, & Otto, 2019b; Nobandegani & Shultz, 2020c, 2020d; Nobandegani, Shultz, & Dubé, 2021), viewing them through the lens of modern psychological theories of bounded rationality (e.g., Griffiths, Lieder, & Goodman, 2015; Gershman, Horvitz, & Tenenbaum, 2015; Nobandegani, 2017; Bhui, Lai, & Gershman, 2021).

Following this new line of research, we ask whether VoCI also could be given a resource-rational basis. To what extent could VoCI be explained in terms of the optimal use of limited cognitive resources? We present here the first resource-rational account of VoCI. Specifically, we show that a resource-rational process model, *sample-based expected utility* (SbEU; Nobandegani et al., 2018) can account for a broad range of empirical results on VoCI. Here, we particularly focus on Birnbaum and Navarrete (1998), which is, to our knowledge, the most extensive empirical study of VoCI.

We begin by formally defining CI (Sec. 2) and discussing how SbEU works (Sec. 3). We then present our simulation results, comparing SbEU model predictions to human data (Sec. 4). We conclude by discussing the implications of our work for risky decision-making, and more broadly, human rationality.

2 Cumulative Independence

Before defining CI, we introduce two notations. First, a shorthand notation for representing risky gambles (Birnbaum & Navarrete, 1998): A generic n-branch gamble P given by: (w.p. stands for "with probability")

$$P = \begin{cases} x_1 & \text{w.p. } p_1 \\ x_2 & \text{w.p. } p_2 \\ \vdots & \\ x_n & \text{w.p. } p_n = 1 - \sum_{i=1}^{n-1} p_i \end{cases} \tag{1}$$

$$\text{where} \quad x_1 \leq x_2 \leq x_3 \leq \ldots \leq x_{n-1} \leq x_n$$

can be alternatively represented, in a vector format, as $P = (x_1, p_1; x_2, p_2; \ldots, x_{n-1}, p_{n-1}; x_n, p_n)$. Second, $A \succ B$ means gamble A is preferred to gamble B, and $A \prec B$ means gamble B is preferred to gamble A.

There are two types of cumulative independence: lower cumulative independence (LCI) and upper cumulative independence (UCI). Assuming $0 < z < x' < x < y < y' < z'$ and $p + q + r = 1$, LCI corresponds to the following condition being satisfied (Birnbaum & Navarrete, 1998):

$$S = (z, r; x, p; y, q) \succ R = (z, r; x', p; y', q) \Rightarrow \\ S'' = (x', r; y, p+q) \succ R'' = (x', r+p; y', q) \tag{2}$$

while, UCI corresponds to the following condition being satisfied (Birnbaum & Navarrete, 1998):

$$S' = (x, p; y, q; z', r) \prec R' = (x', p; y', q; z', r) \Rightarrow \\ S''' = (x, p+q; y', r) \prec R''' = (x', p; y', q+r) \tag{3}$$

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In verbal terms, LCI and UCI indicate that choice preference among two risky gambles should remain unchanged when those two gambles undergo a particular form of transformation—involving removal of common consequences and swapping of probabilities—formalized in (2) and (3). For example, according to LCI, if you prefer gamble S over gamble R, then you should also prefer a particular transformation of S, i.e. S'', over a particular transformation R, i.e. R''. Therefore, choosing S together with R'' would count as violation of LCI, as this choice pattern violates the LCI condition given in (2). Likewise, choosing R' together with S''' would count as violation of UCI, as this choice pattern violates the UCI condition given in (3).

3 Resource-Rational Process Model

Extending an earlier model of decision-making (Lieder, Griffiths, & Hsu, 2018) to the realm of meta-reasoning, *sample-based expected utility* (SbEU; Nobandegani et al., 2018) is a resource-rational process model of risky choice which maintains that people rationally adapt their strategy depending on the amount of time available for decision-making. Concretely, SbEU assumes that people estimate expected utility

$$\mathbb{E}[u(o)] = \int p(o)u(o)do, \tag{4}$$

using self-normalized importance sampling (Hammersley & Handscomb, 1964; Geweke, 1989), with its importance distribution q^* aiming to optimally minimize mean-squared error (MSE):

$$\hat{E} = \frac{1}{\sum_{j=1}^{s} w_j} \sum_{i=1}^{s} w_i u(o_i), \quad \forall i: \; o_i \sim q^*, \; w_i = \frac{p(o_i)}{q^*(o_i)}, \tag{5}$$

$$q^*(o) \propto p(o)|u(o)| \sqrt{\frac{1+|u(o)|\sqrt{s}}{|u(o)|\sqrt{s}}}. \tag{6}$$

MSE is a standard measure of estimation quality, widely used in decision theory and mathematical statistics (Poor, 2013). In Eqs. (4-6), o denotes an outcome of a risky gamble, $p(o)$ the objective probability of outcome o, $u(o)$ the subjective utility of outcome o, $\hat{E}$ the importance-sampling estimate of expected utility given in Eq. (4), q^* the importance-sampling distribution, o_i an outcome randomly sampled from q^*, and s the number of samples drawn from q^*.

In this work, we assume that, when choosing between a pair of risky gambles A, B, people consider whether the expected value of the utility of the difference between the two gambles, $\Delta u(o)$, is positive or negative:

$$A = \begin{cases} o_A & \text{w.p. } P_A \\ 0 & \text{w.p. } 1-P_A \end{cases} \tag{7}$$

$$B = \begin{cases} o_B & \text{w.p. } P_B \\ 0 & \text{w.p. } 1-P_B \end{cases} \tag{8}$$

$$\Delta u(o) = \begin{cases} u(o_A - o_B) & \text{w.p. } P_A P_B \\ u(o_A) & \text{w.p. } P_A(1-P_B) \\ u(-o_B) & \text{w.p. } (1-P_A)P_B \\ u(0) & \text{w.p. } (1-P_A)(1-P_B) \end{cases} \tag{9}$$

In Eq. (9), $u(\cdot)$ denotes the subjective utility function of a decision-maker. Fully consistent with past work (Nobandegani et al., 2018; Nobandegani et al., 2019a; Nobandegani, Destais, & Shultz, 2020a, Nobandegani & Shultz, 2020b), in this paper we use the following utility function:

$$u(x) = \begin{cases} x^{0.85} & \text{if } x \geq 0, \\ -|x|^{0.95} & \text{if } x < 0. \end{cases} \tag{10}$$

Also, in line with prospect theory (Kahneman & Tversky, 1979), we here assume that people perform a variant of *cancellation* on gambles having 3 or more branches, as a form of editing, prior to evaluating the gambles. The purpose of editing is to obtain a simplified representation of gambles prior to further evaluation (Kahneman & Tversky, 1979).[1] In this variant of cancellation, the common outcomes between two gambles are fully removed from those gambles.

In our simulations (Sec. 4), we also assume that people draw between 1 to 6 samples when deciding. Specifically, we adopt a uniform distribution and assume that one-sixth of the population draw one sample (i.e., $s = 1$; see Eqs. (5-6)), one-sixth of the populations draw two samples (i.e., $s = 2$), one-sixth of the population draw three samples and so on. This is consistent with mounting evidence suggesting that people draw only a few samples in probabilistic judgment and reasoning (e.g., Vul et al., 2014; Battaglia et al., 2013; Lake et al., 2017; Gershman, Horvitz, & Tenenbaum, 2015; Hertwig & Pleskac, 2010; Griffiths et al., 2012; Gershman, Vul, & Tenenbaum, 2012; Bonawitz et al., 2014; Nobandegani et al., 2018; Nobandegani et al., 2020a).

Recent work has shown that SbEU provides a unified account of a wide range of empirical findings across risky, value-based, and strategic decision-making (Nobandegani et al., 2018; Nobandegani et al., 2019a, 2019b; Nobandegani et al., 2020a; Nobandegani & Shultz, 2020b, 2020c, 2020d; Lizotte, Nobandegani, & Shultz, 2021), and also bridges between decision-making under risk and decision-making under uncertainty (Nobandegani et al., 2021). There is also a counterintuitive prediction of SbEU which is empirically confirmed: deliberation makes people move from one cognitive bias, framing effect, to another bias, the fourfold pattern of risk preferences (da Silva Castanheira et al., 2019). Notably, SbEU is the first rational process model to score near-perfectly in optimality, economical use of limited cognitive resources, and robustness, all at the same time (Nobandegani et al., 2018; Nobandegani et al., 2019c).

4 Simulation Results

In this section, we simulate the empirical results on VoCI in Birnbaum and Navarrete (1998), which is, to our knowledge,

[1] As such, editing is broadly consistent with resource-rationality. We elaborate on this in the Discussion section (Sec. 5).

the most extensive empirical study of VoCI.

4.1 Lower Cumulative Independence (LCI)

Birnbaum and Navarrete (1998) performed 27 tests of violation of LCI. Each test comprised two trials. In one trial, participants had to choose between two 3-branch risky gambles: S and R. In the other trial, participants had to choose between two 2-branch risky gambles: S'' and R'' (see the LCI condition given in (2)). The parameters of these 4 gambles were systematically manipulated across the 27 tests (see Appendix for details). Accordingly, possible choice patterns are: SS'', SR'', RS'', and RR''.

As Fig. 1(a) shows, SbEU model predictions for these choice patterns correlate highly with the empirically observed data (Pearson $r = .8021$, $p < 10^{-4}$). We simulate 6000 participants in each trial of each test.

4.2 Upper Cumulative Independence (UCI)

Birnbaum and Navarrete (1998) performed 27 tests of violation of UCI. Each test comprised two trials. In one trial, participants had to choose between two 3-branch risky gambles: S' and R'. In the other trial, participants had to choose between two 2-branch risky gambles: S''' and R''' (see the UCI condition given in (3)). The parameters of these 4 gambles were systematically manipulated across the 27 tests (see Appendix for details). Accordingly, possible choice patterns are: $S'S'''$, $S'R'''$, $R'S'''$, and $R'R'''$.

As Fig. 1(b) shows, SbEU model predictions for these choice patterns again correlate highly with the empirically observed data (Pearson $r = .8030$, $p < 10^{-4}$). We simulate 6000 participants in each trial of each test.

5 Discussion

Decades of research has revealed that human decision-making is filled with numerous biases, paradoxes, and violations of rationality principles (e.g., Allais, 1953; Ellsberg, 1961; Loomes et al., 1991; Birnbaum, 2005; Camerer & Ho, 1994; Jeffrey, 1982; Birnbaum & Navarrete, 1998), seriously calling into question human rationality.

Viewing these puzzling behaviors through the lens of modern psychological theories of bounded rationality, a new line of research has provided a resource-rational foundation for many of these violations of rationality principles (e.g., Nobandegani et al., 2019b; Dasgupta et al., 2017; Nobandegani et al., 2021), explaining them in term of the optimal use of limited cognitive resources (e.g., Griffiths et al., 2015; Gershman et al., 2015; Nobandegani, 2017; Bhui et al., 2021).

Pursuing this new line of research, in this work we focus on a notable violation of rationality principles in risky decision-making, *cumulative independence*, and ask: could violation of cumulative independence (VoCI) be given a rational basis? Specifically, could VoCI be understood in terms of the optimal use of limited cognitive resources?

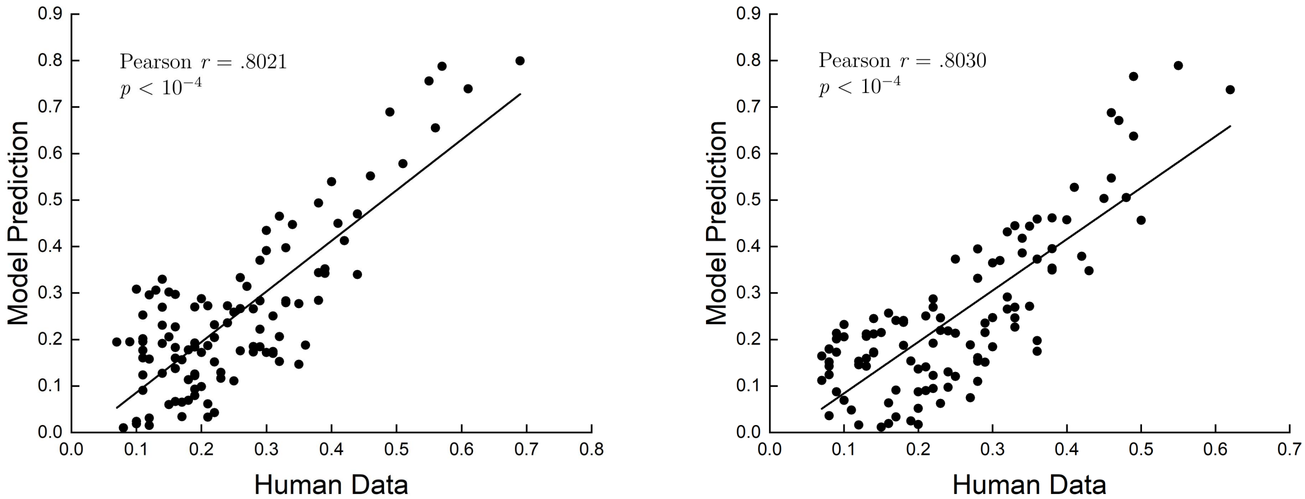
In this work, we present the first resource-rational account of VoCI. Concretely, we show that a resource-rational process model, sample-based expected utility (SbEU), can account for the empirical results of Birnbaum and Navarrete (1998), the most extensive empirical study of VoCI to date.

Although the SbEU model predictions correlate highly and significantly with the empirical data (see Fig. 1), the quantitative fit is not perfect. Several factors may explain this. In the work presented here, we make only minimal assumptions on the part of the simulated participants. To be consistent, we use the exact same utility function (Eq. 10) used in past work, without optimizing it to improve model fit. Also, for the sake of simplicity, we assume that *every* simulated participant implements cancellation, but this might not be the case. Presumably, human decision-makers make use of various editing rules (not just cancellation), and some might not use editing at all. Future work should investigate the effect of all these assumptions on the quality of model fit, and optimize the corresponding parameters (i.e., the utility function, number of samples, editing rules) to improve model fit. The observation that SbEU can adequately account for the empirical data of Birnbaum and Navarrete (1998) *in spite of* making such minimal assumptions provides even stronger evidence that resource rationality might play an important role in shaping the algorithmic foundations of VoCI.

The purpose of editing is to obtain a simplified representation of gambles prior to further evaluation (Kahneman & Tversky, 1979). As such, editing is broadly consistent with resource-rationality as it acknowledges the representational constraints that people are naturally faced with (see Bhui & Gershman, 2018). To show that editing is fully consistent with resource-rationality, future work should investigate whether people boundedly-optimally allocate their representational bandwidth in editing. These future investigations could be guided by, and fruitfully benefit from, recent theoretical advances in heuristics which provide optimality results and strong robustness guarantees on well-known heuristics (e.g., Nobandegani & Shultz, 2019).

All rank-dependent and rank-and-sign-dependent theories of decision-making (e.g., Quiggin, 1982; Lopes, 1990; Luce & Fishburn, 1991, 1995; Wakker, Erev, & Weber, 1994), including cumulative prospect theory (CPT) (Tversky & Kahneman, 1992), satisfy cumulative independence and therefore fail to explain VoCI (Birnbaum & Navarrete, 1998). The work presented here explains VoCI in terms of the optimal use of limited cognitive resources, suggesting that expected utility maximization based on only a few samples might be responsible for the empirically observed VoCI in human decision-making. Nonetheless, to provide direct evidence for this account, future experimental work should test participants under various time pressure and/or cognitive load conditions and verify if observed VoCI rate is consistent with SbEU model predictions.

An intimately related phenomenon to VoCI is violation of stochastic dominance (VoSD) (e.g., Birnbaum & Navarrete, 1998; Birnbaum, 2004a, 2004b; Birnbaum et al., 1999; Birnbaum, 1999, Birnbaum & Martin, 2003; Birnbaum, 2005), which all rank-dependent and rank-and-sign-dependent the-



(a) (b)

Figure 1: **Comparing human data (Birnbaum & Navarrete, 1998) with SbEU model predictions. (a) LCI:** The x-axis shows the human participants' probability of choosing a pattern (SS'', SR'', RS'', or RR'') in the Birnbaum and Navarrete's (1998) LCI experiment, and the y-axis shows the SbEU model predictions for the corresponding choice patterns. **(b) UCI:** The x-axis shows the human participants' probability of choosing a pattern ($S'S'''$, $S'R'''$, $R'S'''$, or $R'R'''$) in the Birnbaum and Navarrete's (1998) UCI experiment, and the y-axis shows the SbEU model predictions for the corresponding choice patterns.

ories of decision-making fail to account for (Birnbaum & Navarrete, 1998).[2] Interestingly, Birnbaum and Navarrete (1998) provided empirical evidence for both VoCI and VoSD in the same population. This weakly suggests that similar psychological processes might underlie these two phenomena. Providing further evidence for this hypothesis, recent work has shown that SbEU can also explain VoSD (Xia, Nobandegani, Shultz, & Bhui, 2022), thus providing a unified, resource-rational account of VoCI and VoSD.

Another closely related phenomenon is violation of branch independence (VoBI) (Birnbaum & Beeghley, 1997; Birnbaum & McIntosh, 1996; Birnbaum & Navarrete, 1998). Future work should investigate whether VoBI could also be given a resource-rational basis. The observation that the Allais paradox, as a notable instance of violation of VoBI, can be given a resource-rational account elevates this possibility (Nobandegani et al., 2021).

In this work, we examine VoCI through the lens of modern psychological theories of bounded rationality (Griffiths et al., 2015; Gershman et al., 2015; Nobandegani, 2017; Bhui et al., 2021), providing a resource-rational algorithmic foundation for VoCI. Given the broad empirical coverage of SbEU across risky, value-based, and strategic decision-making (see Sec. 3), this result is particularly interesting as it brings us a step closer to developing a unified, boundedly-optimal account of human decision-making. The work presented here is a step in this important direction.

[2] Stochastic dominance directly follows from the three assumptions of *outcome monotonicity*, *transitivity*, and *coalescing*. Cumulative independence directly follows from those three assumptions plus *comonotonic independence* (Birnbaum & Navarrete, 1998).

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Appendix

LCI Experiment

Below, we use the same parameters used in the LCI condition given in (2) in the main text.

$(r, p, q) = (.50, .25, .25)$:
Choice 1: $(z, x', x, y', y) = (\$2, \$11, \$52, \$56, \$97)$
Choice 2: $(z, x', x, y', y) = (\$3, \$10, \$48, \$52, \$98)$
Choice 3: $(z, x', x, y', y) = (\$2, \$11, \$45, \$49, \$97)$
Choice 4: $(z, x', x, y', y) = (\$2, \$10, \$40, \$44, \$98)$
Choice 5: $(z, x', x, y', y) = (\$4, \$11, \$35, \$39, \$97)$
Choice 6: $(z, x', x, y', y) = (\$5, \$12, \$30, \$34, \$96)$

$(r, p, q) = (.80, .10, .10)$:
Choice 7: $(z, x', x, y', y) = (\$2, \$11, \$52, \$56, \$97)$
Choice 8: $(z, x', x, y', y) = (\$3, \$10, \$48, \$52, \$98)$
Choice 9: $(z, x', x, y', y) = (\$2, \$11, \$45, \$49, \$97)$
Choice 10: $(z, x', x, y', y) = (\$2, \$10, \$40, \$44, \$98)$
Choice 11: $(z, x', x, y', y) = (\$4, \$11, \$35, \$39, \$97)$
Choice 12: $(z, x', x, y', y) = (\$5, \$12, \$30, \$34, \$96)$

$(r, p, q) = (.60, .30, .10)$:
Choice 13: $(z, x', x, y', y) = (\$2, \$11, \$52, \$56, \$97)$
Choice 14: $(z, x', x, y', y) = (\$3, \$10, \$48, \$52, \$98)$

Choice 15: $(z,x',x,y',y) = (\$2,\$11,\$45,\$49,\$97)$
Choice 16: $(z,x',x,y',y) = (\$2,\$10,\$40,\$44,\$98)$
Choice 17: $(z,x',x,y',y) = (\$4,\$11,\$35,\$39,\$97)$
Choice 18: $(z,x',x,y',y) = (\$5,\$12,\$30,\$34,\$96)$
Choice 19: $(z,x',x,y',y) = (\$3,\$10,\$25,\$29,\$98)$

$(r,p,q) = (.60,.10,.30)$:
Choice 20: $(z,x',x,y',y) = (\$4,\$10,\$61,\$65,\$98)$
Choice 21: $(z,x',x,y',y) = (\$3,\$12,\$56,\$60,\$96)$
Choice 22: $(z,x',x,y',y) = (\$2,\$11,\$52,\$56,\$97)$
Choice 23: $(z,x',x,y',y) = (\$3,\$10,\$48,\$52,\$98)$
Choice 24: $(z,x',x,y',y) = (\$2,\$11,\$45,\$49,\$97)$
Choice 25: $(z,x',x,y',y) = (\$2,\$10,\$40,\$44,\$98)$
Choice 26: $(z,x',x,y',y) = (\$4,\$11,\$35,\$39,\$97)$
Choice 27: $(z,x',x,y',y) = (\$5,\$12,\$30,\$34,\$96)$

UCI Experiment

Below, we use the same parameters used in the UCI condition given in (3) in the main text.

$(r,p,q) = (.50,.25,.25)$:
Choice 1: $(x',x,y',y,z') = (\$11,\$52,\$56,\$97,\$108)$
Choice 2: $(x',x,y',y,z') = (\$10,\$48,\$52,\$98,\$107)$
Choice 3: $(x',x,y',y,z') = (\$11,\$45,\$49,\$97,\$107)$
Choice 4: $(x',x,y',y,z') = (\$10,\$40,\$44,\$98,\$110)$
Choice 5: $(x',x,y',y,z') = (\$11,\$35,\$39,\$97,\$111)$
Choice 6: $(x',x,y',y,z') = (\$12,\$30,\$34,\$96,\$110)$

$(r,p,q) = (.80,.10,.10)$:
Choice 7: $(x',x,y',y,z') = (\$11,\$52,\$56,\$97,\$108)$
Choice 8: $(x',x,y',y,z') = (\$10,\$48,\$52,\$98,\$107)$
Choice 9: $(x',x,y',y,z') = (\$11,\$45,\$49,\$97,\$107)$
Choice 10: $(x',x,y',y,z') = (\$10,\$40,\$44,\$98,\$110)$
Choice 11: $(x',x,y',y,z') = (\$11,\$35,\$39,\$97,\$111)$
Choice 12: $(x',x,y',y,z') = (\$12,\$30,\$34,\$96,\$110)$

$(r,p,q) = (.60,.30,.10)$:
Choice 13: $(x',x,y',y,z') = (\$11,\$52,\$56,\$97,\$108)$
Choice 14: $(x',x,y',y,z') = (\$10,\$48,\$52,\$98,\$107)$
Choice 15: $(x',x,y',y,z') = (\$11,\$45,\$49,\$97,\$107)$
Choice 16: $(x',x,y',y,z') = (\$10,\$40,\$44,\$98,\$110)$
Choice 17: $(x',x,y',y,z') = (\$11,\$35,\$39,\$97,\$111)$
Choice 18: $(x',x,y',y,z') = (\$12,\$30,\$34,\$96,\$110)$
Choice 19: $(x',x,y',y,z') = (\$10,\$25,\$29,\$98,\$109)$

$(r,p,q) = (.60,.10,.30)$:
Choice 20: $(x',x,y',y,z') = (\$10,\$61,\$65,\$98,\$108)$
Choice 21: $(x',x,y',y,z') = (\$12,\$56,\$60,\$96,\$107)$
Choice 22: $(x',x,y',y,z') = (\$11,\$52,\$56,\$97,\$108)$
Choice 23: $(x',x,y',y,z') = (\$10,\$48,\$52,\$98,\$107)$
Choice 24: $(x',x,y',y,z') = (\$11,\$45,\$49,\$97,\$107)$
Choice 25: $(x',x,y',y,z') = (\$10,\$40,\$44,\$98,\$110)$
Choice 26: $(x',x,y',y,z') = (\$11,\$35,\$39,\$97,\$111)$
Choice 27: $(x',x,y',y,z') = (\$12,\$30,\$34,\$96,\$110)$

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