Hints and the Aha-Accuracy Effect in Insight Problem Solving

Taylor Strickland (tstric3@uic.edu)
Department of Psychology, 1007 W. Harrison St.
Chicago, IL 60607 USA

Jennifer Wiley (jwiley@uic.edu)
Department of Psychology, 1007 W. Harrison St.
Chicago, IL 60607 USA

Stellan Ohlsson (stellan@uic.edu)
Department of Psychology, 1007 W. Harrison St.
Chicago, IL 60607 USA

Abstract

The *Aha-Accuracy effect* refers to the finding that experiencing an Aha! moment is associated with reaching correct solutions on insight problems. Because this effect has generally been demonstrated with verbal problems, this study tested for this effect on spatial problems (matchstick arithmetic). In addition, this study also explored the effect of hints on the Aha! experience and the *Aha-Accuracy effect*. Overall, there was no *Aha-Accuracy effect* in the no-hint control condition. There was an *Aha-Accuracy effect* in the hint condition, but it was limited to problems with solutions that were not directly cued by the hint. When the hint was directly relevant for solution, then many participants were able to reach a correct solution without an Aha! experience. These findings provide evidence that providing hints may not simply increase the likelihood of reaching a solution, but it may also alter the Aha! experience.

Keywords: Aha!; restructuring; insight problem solving; hints; matchstick arithmetic

Introduction

Since the Gestalt movement, there has been interest in a particular form of problem solving that is thought to require more "productive" as opposed to "reproductive" thinking and involving insight as opposed to more routine solution processes (Maier, 1931; Ohlsson, 1992; Wertheimer, 1945). Insight problems share the common characteristic that they appear simple to solve but typically cue incorrect initial representations. Misleading initial representations can be prompted by the way a problem is presented or can come from the activation of prior knowledge or experience that is not relevant to the problem at hand. Incorrect representations will lead to a search space that does not contain a path to solution, and the solver will experience fixation or mental set (Duncker, 1945; Luchins, 1942; Wiley, 1998).

Once the solver has exhausted the search space, or sees failure as likely, they reach a state of impasse where they feel unable to progress further (Ash & Wiley, 2006; MacGregor, Ormerod, & Chronicle, 2001; Moss, Kotovsky, & Cagan, 2011). The Gestaltists proposed that in order to reach a solution, the solver must restructure their understanding of the problem. Consistent with this earlier theorizing, Representation Change Theory (RCT) has proposed two mechanisms, constraint relaxation and chunk decomposition that may result in restructuring (Knoblich et al., 1999). In addition to restructuring as a defining feature of the insight process, the Gestaltists were also intrigued by the Aha! experience that seemed to be a hallmark of insight (Davidson, 1995; Gick & Lockhart, 1995). The Aha! experience refers to the sudden feeling of pleasure or joy of discovery that is often associated with solutions to insight problems.

Since the Aha! moment is an inherently subjective experience, many attempts have been made to measure it through self-reports starting with Metcalfe and Wiebe (1987) and her feelings-of-warmth measure. Participants indicated how close they felt to solution with 0 being "cold" (no idea how to solve) and 10 being "hot" (certain of solution). As participants solved a set of problems that typically prompt an incorrect initial representation (insight problems) and another set of more straightforward math or logic problems (non-insight problems), they were asked to make this warmth rating every 10 seconds. Metcalfe and Wiebe (1987) found that insight problems that were solved correctly were associated with a sudden increase in these warmth ratings immediately before solution. In contrast, the warmth ratings increased more gradually before correct solution on the non-insight problems. The abrupt emergence of the solution seen for insight problems in this study mapped well onto the idea of a discontinuous, sudden solution process.

Another finding that has received attention is that Aha! experiences are more often associated with correct solutions to insight problems than incorrect ones (Danek & Salvi, 2020; Zedelius & Schooler, 2015). In studies where Aha! moments are identified categorically, the *Aha-Accuracy effect* is demonstrated by using Aha! as an independent variable and the likelihood of correct solution serves as the dependent variable. It has been found that solution rates are higher for problems accompanied by an Aha! experience than for problems that are not accompanied by an Aha! experience. (When Aha! ratings are collected on continuous scales, the *Aha-Correctness effect* can be seen by comparing average Aha! ratings for correct and incorrect solutions.) Both effects have been suggested to provide evidence of the "tight coupling" between insightful solutions and the Aha! experience (Threadgold, Marsh, & Ball, 2018).

In J. Culbertson, A. Perfors, H. Rabagliati & V. Ramenzoni (Eds.), *Proceedings of the 44th Annual Conference of the Cognitive Science Society.*

Table 1: Studies showing the Aha-Accuracy Effect

Study	Task	Items	N	Aha Measure	Solution Rates (%) Aha (*M, SD*)	Solution Rates (%) No Aha (*M, SD*)	Aha -No Aha Difference	Cohen's *d*
Danek et al. (2014) E1	Magic	34	48	Binary	77 (44)	56 (29)	21	0.56
Zedelius & Schooler (2015) E1	CRA ENG	30	70	Likert-C	63 (38)	25 (26)	18	1.17
Hedne et al. (2016)	Magic	32	51	Binary	57 (50)	37 (48)	20	0.41
Salvi et al. (2016) E1	CRA ENG	120	38	Binary	94 (17)	78 (17)	16	0.90
Salvi et al. (2016) E2	Anagram	180	51	Binary	98 (09)	92 (09)	6	0.68
Salvi et al. (2016) E3	Rebus	88	110	Binary	79 (15)	63 (15)	16	1.05
Shen et al. (2018)	CRA CHI	48	26	Binary	86 (15)	40 (20)	46	2.63
Threadgold et al. (2018)	Rebus	42	170	Likert-S	65 (27)	54 (27)	11	0.41
Ellis et al. (2021)	CRA ENG	30	459	Likert-S	64 (30)	38 (28)	26	0.70
Laukkonen et al. (2021) E1	Riddles	10	60	Binary	72 (45)	38 (49)	34	0.72
Laukkonen et al. (2021) E2	CRA ENG	10	60	Binary	90 (30)	30 (46)	60	1.57

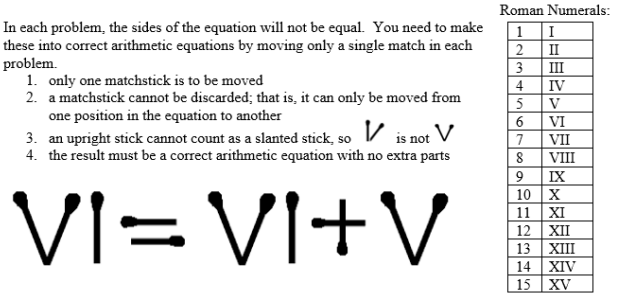
Note: CRA ENG (Compound Remote Associates in English), CRA CHI (Compound Remote Associates in Chinese), Likert-C (rated on a 1-4 Likert Scale, but only Categories 1 and 4 are analyzed), Likert-S (rated on a continuous scale, median split to analyze as Aha and No-Aha). All Cohen's *d*s are computed from reported descriptive statistics.

Table 1 summarizes several studies that have tested for the *Aha-Accuracy effect* and shows that it has been obtained across a range of problem types. Solutions marked by an Aha! experience are more likely to be correct when attempting to solve Compound Remote Associate Problems (CRA, finding a fourth word that forms a good phrase with three other words; Laukkonen et al., 2021; Salvi et al., 2016; Shen et al., 2018; Zedelius & Schooler, 2015). It has also been demonstrated with verbal riddles (Why did the coin collector call the police when offered a coin dated 46 BC?; Laukkonen et al., 2021), solving rebus puzzles (DECI SION, split decision, Salvi et al., 2016; Threadgold et al., 2018), and solving anagrams (Salvi et al., 2016).

All of these are verbal tasks, and what makes these problems a challenge is generally that the solution requires thinking about words or letters in unusual ways, including activating less frequent interpretations or meanings of words, and less frequent combinations of letters. It is important that the *Aha-Accuracy effect* has also been extended to figuring out magic tricks where a solver is initially misdirected by the magician (Danek et al., 2014; Hedne et al., 2016). But what is notably absent is a demonstration of the effect using more spatial, object-based problems, such as matchstick arithmetic problems (Knoblich et al., 1999).

Comparing Verbal and Spatial Problems

As shown in Figure 1, matchstick arithmetic problems present an incorrect mathematical statement using images of matchsticks and ask the solver to move an object (a single match) to turn the incorrect statement into a correct one. Studies have shown that there are sometimes important differences between verbal and spatial insight problems, how they are solved, and what predicts likelihood of solutions (Dow & Mayer, 2004; Gilhooly & Murphy, 2005).



Problem	Type	Process	Solution
VI = VII + I	Practice 1	ST	VII = VI + I
XI = XII + I	Practice 2	ST	XII = XI + I
I = II + II	Test 1	CR1	I = III – II
V = III – II	Test 2	CR2	V – III = II
XI = III + III	Test 3	CD	VI = III + III
VI = VI + I	Test 4	CR1	VI = VII – I
VIII = VI – II	Test 5	CR2	VIII – VI = II
VI = VI + V	Test 6	CD	XI = VI + V

Figure 1: Matchstick Problems and Instructions

Thus, the first purpose of this study was to use matchstick arithmetic problems to test whether the *Aha-Accuracy effect* would also be seen using a spatial insight problem.

Insight Problem Solving and Hints

In addition to testing for the presence of the *Aha-Accuracy effect* on matchstick problems, this study also tested the effect of hints on the Aha! experience. Hints are commonly used in studies on insight problem solving to test theories about the key obstacles to solution, including which constraints or assumptions might need to be relaxed, or

which elements might be perceived in a way (as incorrect chunks or interpretations) that might bias the solver against a solution. For example, researchers studying the nine-dot (see Figure 2) problem have given the hint that solvers need to think outside the box and extend lines past the dots to be able to connect all the dots by drawing only 4 straight lines (Chein, et al, 2010; Maier & Casselman, 1970; Weisberg & Alba, 1981). And Moss et al. (2011) provided hints to the solution of CRAs as part of a separate anagram solving task. It has been shown that providing hints can help to improve solution rates, but it is less clear what effect the hints might have on the Aha! experience.

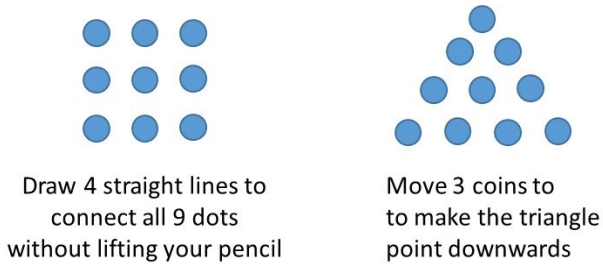


Figure 2: Nine Dot and Triangle of Coins Problems

One study (Cushen & Wiley, 2012) attempted to look at the relation between hints, solution processes, and the Aha! experience. In this study, participants had 10 minutes to solve the Triangle of Coins problem (see Figure 2) where 10 coins are arranged in a triangle. The task is to move only 3 coins to invert the triangle. To provide a hint toward solution, participants made importance-to-solution ratings every minute for either all 10 coins or a particular subset of only 3 coins. The subset provided a hint because 2 of those 3 coins were part of the solution. In the whole-rating condition, they found evidence for sudden changes in importance ratings before solution, reminiscent of the patterns shown by Metcalfe and Wiebe (1987). In contrast, fewer of these sudden-change solution patterns were seen in the subset-rating, hinted condition. When they looked at Aha! ratings, they did not see a difference in the magnitude of the Aha! effect for correct solutions across the two conditions. However, they did not test for differences in the *Aha-Accuracy effect* – that is, whether the presence of a hint would alter whether problems solved with an Aha! would be more likely to be correct. Thus, the second purpose of this study was to test whether giving a hint would alter the *Aha-Accuracy effect*.

In summary, to better understand the relation between the Aha! experience and insight problem solving, the current study tested for the presence of the *Aha-Accuracy effect* on a set of matchstick arithmetic problems. The present study used an explicit hint manipulation based on the chunk decomposition component of RCT (Knoblich et al., 1999), instructing participants that some matchstick arithmetic problems could be solved by turning a V into a X or a X into a V. It was hypothesized that this hint would prompt chunk decomposition, which should result in higher solution rates. The hypotheses for the *Aha-Accuracy effect* are less obvious. If there is a tight coupling between insight solutions and Aha! experiences, then the *Aha-Accuracy effect* should be seen in the control condition (where no hints are given) even with a spatial insight problem. If the presence of the hint alters the solution process to make it more analytic or routine, then the feeling of Aha! may no longer serve as a marker for correct solutions.

Methods

Participants

There were 118 participants recruited using Cloud Research. All participants were paid $3 for participating in the 40-minute long Qualtrics survey. The sample included 64 men and 37 women, with an average age around 40 (M = 38.50, SD = 11.22). Participants were excluded from the study if they did not get at least one of the two practice questions correct. This led to the exclusion of 17 participants (7 from the hint condition and 10 from the control condition) for a total N of 101. There were 46 participants in the control condition and 55 in the hint condition.

Measures

Matchstick Problems. Matchstick arithmetic problems are a type of insight problem that require a single stick to be moved to turn an incorrect statement into a correct one. A few rules must be followed as shown in Figure 1. First, only one stick is to be moved. Second, sticks cannot be added or deleted. Third, a slanted stick cannot serve as a vertical stick. Fourth, the result will be a correct arithmetic statement with no extra parts (an operator such as a plus sign cannot be left hanging at the end of the statement).

Matchstick arithmetic problems have been categorized into two types based on whether they require chunk decomposition or constraint relaxation for restructuring (Knoblich et al., 1999; Öllinger et al., 2008). Chunk decomposition (CD) problems involve changing a V to a X, or vice versa. Constraint relaxation (CR) problems involve relaxing the constraint that operators cannot be manipulated. CR problems can involve turning a + into a – and adding the remaining stick to another numeral (CR1) or moving a stick from the = to the – to turn the – into an = (CR2). Additionally, there are simple standard type (ST) problems that require moving an upright stick from one numeral to another. These do not require restructuring and were given as practice problems. Participants were asked to solve the 2 practice problems and 6 test problems presented in Figure 1.

Hint Manipulation. Participants in the hint condition were given this additional instruction: "One thing that will make them easier to solve is to remember that any stick can be moved even those that are stuck together like the numerals V and X. Remembering that any stick can be moved including a V into a X and a X into a V will help you solve these problems." They were also shown how to solve this example, moving one stick in the V from VI = VIII + III to change the statement into XI = VIII + III. Because this hint

is intended to prompt chunk decomposition it is referred to as the CD Hint.

Aha! Experiences. Self-reports of participants' Aha! experiences were collected following completion of each matchstick arithmetic problem. If a solution was given, participants were asked to respond to the Aha! prompt adapted from Jung-Beeman et al. (2004) by Danek et al. (2014).

> "We would also like to know whether you experienced a feeling of insight when you solved each task: A feeling of insight is a kind of "Aha!" characterized by suddenness and obviousness (and often relief!)—like a revelation. You are relatively confident that your solution is correct without having to check it. In contrast, you experienced no Aha! if the solution occurs to you slowly and stepwise. As an example, imagine a light bulb that is switched on all at once in contrast to slowly dimming it up. We ask for your subjective rating whether it felt like an Aha! experience or not, there is no right or wrong answer. Just follow your intuition."

After each solution attempt, participants responded either yes or no if they had experienced an Aha! moment.

Procedure

The study was conducted entirely online with participants receiving a Qualtrics survey containing all study materials through Cloud Research, a subset of Amazon Mechanical Turk (Litman et al., 2014). All participants watched a 2 minute, 22 second general instruction video narrating the instructions given in Figure 1. Using Qualtrics randomization, participants were then randomly sorted into the hint and control conditions.

Once the participants completed the instruction section, they were given the 2 practice problems. If a solution was provided, then they were asked to reply to the Aha! prompt with yes or no. If participants could not solve either practice problem, then they were excluded from analysis.

After completing the practice problems, participants were given the 6 test problems in the order they are presented in Table 1. Each problem was timed for 4 minutes, and the survey moved on after the time had elapsed. Participants were not forced to enter a response or to guess. No feedback was provided for any of the problems. If a response was given, participants were asked to respond to the Aha! prompt.

Following the completion of the matchstick tasks, participants completed some demographic questions (gender and age) before being debriefed and thanked for their participation.

Results

In the data set there were 101 participants who attempted 6 problems each, resulting in 606 observations. Of these 156 solutions were left blank, leaving 450 solutions with Aha! responses. As shown in Table 2, participants reported experiencing Aha! on roughly half their solution attempts (52%) in both conditions. This result is similar to the lack of difference in Aha! ratings due to hinting in Cushen and Wiley (2012).

Table 2: Raw Frequency (and Proportion) of Attempted Solutions by Aha! Experiences and Condition

Condition	Aha!	No Aha!	No Solution
Control	98 (52%)	90 (48%)	88 (-)
CD Hint	137 (52%)	125 (48%)	68 (-)

Aha-Accuracy Effect

One of the main research questions of this study involved testing the strength of the *Aha-Accuracy effect* using a spatial insight problem, and for differences between the control and CD hint conditions. To test this, a generalized mixed effects model was run using logit and simple effects coding for the fixed effects (Aha! experience and condition), and a random effect for participants, to predict correct solution on each problem. (Due to the small number of problems, the model failed to converge when the random effect for items was added.)

The overall solution rate for the 450 problems where a solution was attempted was 75.10%. Solution rates for responses associated with an Aha! experience and not associated with an Aha! experience are presented by condition in Table 3 and the results of the analysis are shown in the top portion of Table 4. Overall, the significant effect for Aha was because problems accompanied by an Aha! moment were more likely to be solved correctly than those with no Aha!. The significant effect for the CD hint manipulation showed that problems were more likely to be solved correctly with the hint (M= 81%, SD= 38) than without it (M = 68%, SD = 47). Further, there was a significant interaction between these two effects. The presence of an Aha! experience did not significantly predict correct solutions in the control condition where participants did not receive the CD hint (Z = 1.25, p = .22). In contrast, the *Aha-Accuracy effect* was significant in the CD hint condition (Z = 4.47, p < .001).

Effect of Problem Type on Solution Rates

Because the hint given in this study was specific to chunk decomposition (CD) solutions, it was also of interest to test if different patterns might be seen for CD versus CR problems by adding problem type (CD vs. CR) to the logistic mixed effects model.

Table 3: Average Rates of Correct Solution by Condition

	Solution Rates (%)			
	Aha	No Aha	Aha	Cohen's
Condition	*M (SD)*	*M (SD)*	- No Aha	*d*
Control	76 (43)	59 (50)	17	.37
CD Hint	91 (24)	70 (46)	21	.60
Overall	84 (37)	65 (48)	18	

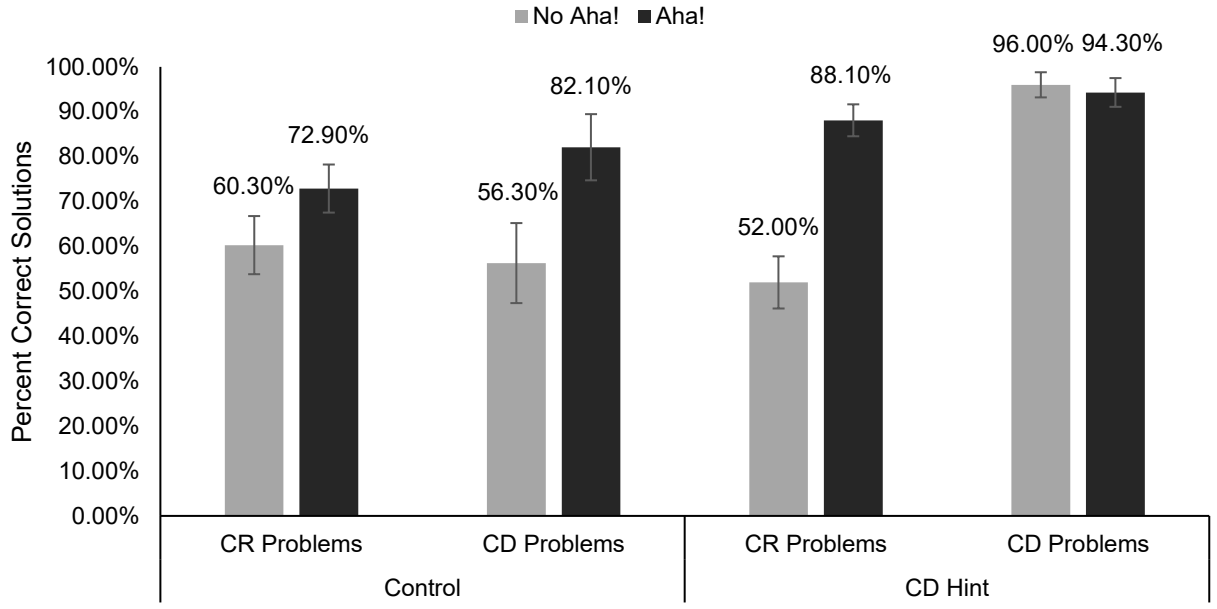


Figure 3: Solution Rates by Hint Condition, Presence of Aha! Experience and Problem Type
(Error bars represent *SE*s)

Table 4: Logistic Mixed Effects Models

Predictors	Odds Ratios Exp(B)	*z*	*p*
Model 1 (R^2 = .52)			
Intercept	5.04	6.27	<.001
Aha	3.70	4.05	<.001
CD Hint	3.10	2.43	.015
Aha * CD Hint	4.30	2.23	.026
Model 2 (R^2 = .60)			
Intercept	6.63	6.54	<.001
Aha	3.10	2.96	.003
CD Hint	5.17	3.12	.002
Problem Type	3.59	3.50	<.001
Aha * CD Hint	4.10	2.07	.038
Aha * Type	0.48	-1.02	.308
Hint * Type	11.74	3.39	<.001

(No differences were seen between CR1 and CR2 items when these analyses were run using three levels for problem type, so the simpler model is reported with just two levels for problem type.) This second model included 3 fixed effects and all 3 two-way interactions (again using simple effects coding and including participants as a random effect). When the three way-interaction was added in a final model it was not significant, Exp(B) = .057, Z= -1.93, *p* =. 054, and failed to improve fit over Model 2 (R^2 = .59). Thus, Model 2 provides the best fit as shown in Table 4.

Problem type also affected the likelihood of correct solutions. The significant effect of problem type was because CD problems were more likely to be solved (*M*= 85%, *SD*= 36) than CR problems (*M* = 69%, *SD* = 46). In addition, as shown in Figure 3, the presence of the CD Hint interacted with problem type with the CD hint leading to more correct solutions specifically on CD problems. As can be seen by comparing the first two pairs of bars, the *Aha-Accuracy effect* was similar for both problem types in the control (no CD hint) condition. As can be seen by comparing the last two pairs of bars, the strongest *Aha-Accuracy effect* was seen on CR problems when participants were given the CD hint, whereas there was no *Aha-Accuracy effect* on CD problems when participants were given the CD hint. The high solution rate for CD problems solved without an Aha! in the final pair of bars suggests that the CD hint may be altering the way that solutions are reached in this condition.

Discussion

This study provides preliminary evidence that giving a hint when solving matchstick problems may alter both the Aha! experience and the solution process. First, there was no significant *Aha-Accuracy effect* when participants were not given a hint. Second, although the likelihood of reporting an Aha! experience was not affected by the presence or absence of a hint, there was evidence that having a hint led to differences in the *Aha-Accuracy effect* contingent on whether the hint was directly relevant for solution of the problem type.

The lack of an *Aha-Accuracy effect* in the control condition in this study stands in contrast the findings of prior results presented in Table 1. One salient difference is that the prior work used mainly verbal problems such as CRAs, riddles, anagrams and rebus puzzles to demonstrate the effect. This suggests that the nature of the problem could matter. Another possible difference is that in the current

study solution attempts were not forced, and participants were not encouraged to guess. When forced responses or guesses are included in calculations of *Aha-Accuracy effects* this could produce sizable increases in the size of the effect (because solvers may be much less likely to report an Aha! experience when they know they are just guessing.) At the same time, the problem set used in the current study was quite small. This could have limited the chance to find the effect. Future studies are needed that test for the *Aha-Accuracy effect* using a larger set and a variety of spatial insight problems.

When the effects of hints on Aha! experiences were analyzed in terms of frequencies, no difference was seen between the control and hint group. This replicates a similar finding in Cushen and Wiley (2012). However, the hint made a large difference in the *Aha-Accuracy effect*. When the hint was not directly relevant for the problem type, then the *Aha-Accuracy effect* was observed, and Aha! moments were associated with correct solutions. In contrast, when the hint was relevant for the problem type, having an Aha! moment did not predict correct solution. When the hint was directly relevant for solution, it led to some correct solutions that were achieved without an Aha! experience. In other words, the lack of an Aha! experience during these solutions suggests they are being reached more analytically or incrementally. This potential difference in the way solutions are being reached as the result of a hint is similar to the results seen in Cushen and Wiley (2012). In that study, guiding participants with an implicit hint led to more incremental patterns of representational change during solution of the "Triangle of Coins" problem. Alternatively, the lack of an effect in the CD hint, CD problem condition could be due to the fact that there was no room for an *Aha-accuracy effect* since performance was already near ceiling.

Many studies have shown that problems used to study the insight process can be solved incrementally or insightfully. Although in some studies there appears to be a strong association between experiencing an Aha! moment and reaching a correct solution, the association is not perfect and this offers the possibility for false insights (Danek & Wiley, 2017). A false insight is when someone reports having an Aha! moment but produces an incorrect answer. It is interesting that the hint did not inflate the rate of incorrect solutions reached with an Aha!

Considering the kinds of hints that have been used in studies on insight, there seem to be differences in the extent to which the hint is explicit (directly removing a constraint or decomposing a chunk for a solver) vs. more implicit (directing attention to important elements or possible routes for solution). An interesting future direction could be to examine possible differences between implicit and explicit hints in Aha! experiences and their relation to solutions in the *Aha-Accuracy effect*. The hint used here was very explicit and told participants exactly how to solve one type of matchstick problem. Perhaps a more implicit hint would increase the likelihood of Aha! experiences for incorrect solutions. Or, alternatively, a more implicit approach could heighten the perception of Aha! experiences during correct solution by more subtly prompting the solver to break the numeral chunks and facilitate changes in operators on their own.

Although Aha! experiences are thought to accompany a sudden restructuring process, the results of this study suggest that the perception of an Aha! moment is not so tightly coupled with correct solutions on this set of problems, consistent with prior work that has found a disconnect between cognitive and affective aspects of insight problem solving (Cushen & Wiley, 2012; Danek & Wiley, 2017; Danek, Wiley, & Öllinger, 2016). Including trace measures to detect whether restructuring is occurring during solution, whether a hint prompts them, and whether Aha! experiences co-occur with them, would be an important next step (Öllinger, Jones, & Knoblich, 2008). Only with measures of both restructuring and Aha! experiences can it be tested what the connection between these two presumed features of insightful solutions might be (Danek, Williams, & Wiley, 2020). These results suggest it is still uncertain if they are directly linked, and also highlights how hint manipulations may change the problem-solving experience.

References

Ash, I. K., & Wiley, J. (2006). The nature of restructuring in insight: An individual-differences approach. *Psychonomic Bulletin & Review, 13*, 66-73. https://doi.org/10.3758/BF03193814

Chein, J. M., Weisberg, R. W., Streeter, N. L., & Kwok, S. (2010). Working memory and insight in the nine-dot problem. *Memory & Cognition, 38*, 883-892. https://doi.org/10.3758/MC.38.7.883

Cushen, P. J., & Wiley, J. (2012). Cues to solution, restructuring patterns, and reports of insight in creative problem solving. *Consciousness and Cognition, 21*, 1166-1175. https://doi.org/10.1016/j.concog.2012.03.013

Danek, A. H., Fraps, T., von Müller, A., Grothe, B., & Öllinger, M. (2014). It's a kind of magic—what self-reports can reveal about the phenomenology of insight problem solving. *Frontiers in Psychology, 5*, 1408. https://doi.org/10.3389/fpsyg.2014.01408

Danek, A. H., & Salvi, C. (2020). Moment of truth: Why Aha! Experiences are correct. *The Journal of Creative Behavior, 54*, 484-486. https://doi.org/10.1002/jocb.380

Danek, A. H., Wiley, J., & Öllinger, M. (2016). Solving classical insight problems without Aha! experience: 9 dot, 8 coin, and matchstick arithmetic problems. *Journal of Problem Solving, 9*, 47-57. https://doi.org/10.7771/1932-6246.1183

Danek, A. H., & Wiley, J. (2017). What about false insights? Deconstructing the Aha! experience along its multiple dimensions for correct and incorrect solutions separately. *Frontiers in Psychology, 7*, 2077. https://doi.org/10.3389/fpsyg.2016.02077

Danek, A. H., Williams, J., & Wiley, J. (2020). Closing the gap: Connecting sudden representational change to the subjective Aha! experience in insightful problem solving.

Psychological Research 84, 111-119. https://doi.org/10.1007/s00426-018-0977-8

Davidson, J. E. (1995). The suddenness of insight. In R. J. Sternberg & J. E. Davidson (Eds.), *The nature of insight* (pp. 125-155). The MIT Press.

Dow, G. T., & Mayer, R. E. (2004). Teaching students to solve insight problems: Evidence for domain specificity in creativity training. *Creativity Research Journal, 16*, 389-402. https://doi.org/10.1080/10400410409534550

Duncker, K. (1945). On problem solving. *Psychological Monographs*. https://doi.org/10.1037/h0093599

Ellis, D. M., Robison, M. K., & Brewer, G. A. (2021). The cognitive underpinnings of multiply-constrained problem solving. *Journal of Intelligence*, *9*, 7. https://doi.org/10.3390/jintelligence9010007

Gick, M. L., & Lockhart, R. S. (1995). Cognitive and affective components of insight. In R. J. Sternberg & J. E. Davidson (Eds.), *The nature of insight* (pp. 197-228). The MIT Press.

Gilhooly, K. J., & Murphy, P. (2005). Differentiating insight from non-insight problems. *Thinking & Reasoning, 11*, 279-302. https://doi.org/10.1080/13546780442000187

Hedne, M. R., Norman, E., & Metcalfe, J. (2016). Intuitive feelings of warmth and confidence in insight and noninsight problem solving of magic tricks. *Frontiers in Psychology, 7:*1314. https://doi.org/10.3389/fpsyg.2016.01314

Jung-Beeman, M., Bowden, E.M., Haberman, J., Frymiare, J. L., Arambel-Liu, S., Greenblatt, R., Reber, P.J. & Kounios, J. (2004). Neural activity when people solve verbal problems with insight. *Public Library of Science Biology, 2*, E97. https://doi.org/10.1371/journal.pbio. 0020097

Knoblich, G., Ohlsson, S., Haider, H., & Rhenius, D. (1999). Constraint relaxation and chunk decomposition in insight problem solving. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 25*, 1534-1555. https://doi.org/10.1037/0278-7393.25.6.1534

Laukkonen, R. E., Ingledew, D. J., Grimmer, H. J., Schooler, J. W., & Tangen, J. M. (2021). Getting a grip on insight: Real-time and embodied Aha experiences predict correct solutions. *Cognition and Emotion, 35*, 918-935. https://doi.org/10.1080/02699931.2021.1908230

Litman, L., Robinson, J., & Abberbock, T. (2017). TurkPrime.com: A versatile crowdsourcing data acquisition platform for the behavioral sciences. *Behavior Research Methods, 49*, 433-442. https://doi.org/10.3758/s13428-016-0727-z

Luchins, A.S. (1942). Mechanization in problem solving: The effect of einstellung. *Psychological Monographs*. https://doi.org/10.1037/h0093502

MacGregor, J. N., Ormerod, T. C., & Chronicle, E. P. (2001). Information processing and insight: A process model of performance on the nine-dot and related problems. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 27*, 176-201. https://doi.org/10.1037/0278-7393.27.1.176

Maier, N.R.F. (1931). Reasoning in humans II: The solution of a problem and its appearance in consciousness. *Journal of Comparative Psychology, 10*, 115-143. https://doi.org/10.1037/h0071361

Maier, N. R., & Casselman, G. G. (1970). Locating the difficulty in insight problems: Individual and sex differences. *Psychological Reports, 26*, 103-117. https://doi.org/10.2466/pr0.1970.26.1.103

Metcalfe, J., & Wiebe, D. (1987). Intuition in insight and noninsight problem solving. *Memory & Cognition, 15*, 236-246.

Moss, J., Kotovsky, K., & Cagan, J. (2011). The effect of incidental hints when problems are suspended before, during, or after an impasse. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 37*, 140-148. https://doi.org/10.1037/a0021206

Ohlsson, S. (1992). Information-processing explanations of insight and related phenomena. In Keane, M. & Gilhooly, K. (Eds.), *Advances in the psychology of thinking* (pp. 1-44). London: Harvester-Wheatsheaf.

Öllinger, M., Jones, G., & Knoblich, G. (2008). Investigating the effect of mental set on insight problem solving. *Experimental Psychology, 55*, 269-282. https://doi.org/10.1027/1618-3169.55.4.269

Salvi, C., Bricolo, E., Kounios, J., Bowden, E., & Beeman, M. (2016). Insight solutions are correct more often than analytic solutions. *Thinking & Reasoning, 22*, 443-460. https://doi.org/10.1080/13546783.2016.1141798

Shen, W., Tong, Y., Yuan, Y., Zhan, H., Liu, C., Luo, J., & Cai, H. (2018). Feeling the insight: Uncovering somatic markers of the "aha" experience. *Applied Psychophysiology and Biofeedback, 43*, 13-21. https://doi.org/10.1007/s10484-017-9381-1

Threadgold, E., Marsh, J. E., & Ball, L. J. (2018). Normative data for 84 UK English rebus puzzles. *Frontiers in Psychology, 9*:2513. https://doi.org/10.3389/fpsyg.2018.02513

Weisberg, R. W., & Alba, J. W. (1981). An examination of the alleged role of "fixation" in the solution of several "insight" problems. *Journal of Experimental Psychology: General, 110*, 169-192. https://doi.org/10.1037/0096-3445.110.2.169

Wertheimer, M. (1945). *Productive thinking*. New York, NY: Harper.

Wiley, J. (1998). Expertise as mental set: The effects of domain knowledge in creative problem solving. *Memory & Cognition, 26*, 716-730. https://doi.org/10.3758/BF03211392

Zedelius, C. M., & Schooler, J. W. (2015). Mind wandering "Ahas" versus mindful reasoning: Alternative routes to creative solutions. *Frontiers in Psychology, 6:*834. https://doi.org/10.3389/fpsyg.2015.0083