Title

Estimating the Unknown by the Hot Hand Belief

Journal

Proceedings of the Annual Meeting of the Cognitive Science Society, 33(33)

ISSN

1069-7977

Authors

Sun, Yanlong
Wang, Hongbin

Publication Date

2011

Peer reviewed

Estimating the Unknown by the Hot Hand Belief

Yanlong Sun (Yanlong.Sun@uth.tmc.edu)
Hongbin Wang (Hongbin.Wang@uth.tmc.edu)

School of Biomedical Informatics, University of Texas Health Science Center
Houston, TX 77030 USA

Abstract

Compared to the gambler's fallacy in which one makes predictions negatively dependent on the past information, in the hot hand belief, one makes predictions positively dependent on the past information. Both phenomena have been attributed to people's misperception of randomness. The present study examines an alternative explanation that the positive dependency in the hot hand belief may be due to people's effort to reduce uncertainty by estimating the unknown probability (common probability estimation), a result known as the Laplace's rule of succession. We report an experiment to demonstrate that the dependency on the history can be reversed from negative to positive by manipulating the participants' assumptions about the unknown probability.

Keywords: hot hand belief; gambler's fallacy; common probability estimation; probability matching.

Introduction

When faced with a series of events, people often attempt to predict what is to occur next based on the history of the previous outcomes, even when the underlying process governing those events is independent and stationary (or, statistically indistinguishable, for example, the same fair or biased coin is tossed repeatedly). As the independence and stationarity assumptions are usually characteristics of a random process, such tendency has often been labeled as misperception of randomness (for a recent review see, Oskarsson, Van Boven, McClelland, & Hastie, 2009). Among those documented, *the gambler's fallacy* has the longest history, even older than the history of experimental psychology (see, Ayton & Fischer, 2004). When a fair coin is tossed repeatedly, a person with the gambler's fallacy will predict a tail after a streak of heads. On the other hand, the same kind of past information can sometimes invoke an opposite prediction. A person with *the hot hand belief* will predict that a basketball player who has just scored several shots in a row is more likely to score again. In the actual shooting sequences, however, little statistical evidence has been found to reject the independence and stationarity hypotheses (Gilovich, Vallone, & Tversky, 1985; Tversky & Gilovich, 1989). (For a comprehensive review on the hot hand studies, see Bar-Eli, Avugos, & Raab, 2006.)

The contrast between the gambler's fallacy and the hot hand belief has received much attention (Ayton & Fischer, 2004; Burns & Corpus, 2004; Caruso, Waytz, & Epley, 2010; Croson & Sundali, 2005; Rabin, 2002; Sundali & Croson, 2006). Most notably, the same representativeness heuristic has been used to account for both the gambler's fallacy and the hot hand belief (Gilovich, et al., 1985; Tversky & Kahneman, 1971). By this account, people's perception of random events are governed by a "law of small numbers" such that a local sample should resemble the underlying population and chance is perceived as "a self-correcting process in which a deviation in one direction induces a deviation in the opposite direction to restore the equilibrium" (Tversky & Kahneman, 1974, p. 1125). Thus, in the gambler's fallacy, a tail is "due" to reverse a streak of heads. In the hot hand belief, a streak of successes would make the observer to reject the randomness of the process and believe that a "hot hand" will make another shot (see also Tversky & Gilovich, 1989).

Nevertheless, the representativeness account has been criticized for its incompleteness. Ayton and Fischer (2004) suggest that the gambler's fallacy arises from the experience of negative recency in sequences of natural events such as sampling without replacement and the hot hand belief arises from the experience of positive recency in serial fluctuations in human performance such as in sports. Burns and Corpus (2004) show that subjects assume negative recency for scenarios they rated as "random" and positive recency for forecasting scenarios they rated as "nonrandom." Moreover, it has been proposed that the hot hand belief may arise as an inference to the properties of other processes based on the outcomes of a random process. For example, people may infer the ability of a mutual fund manager from the fluctuations of the portfolio performance (Rabin, 2002), or, infer a person's luck from the outcomes of a roulette game (Croson & Sundali, 2005; Sundali & Croson, 2006). More recently, Caruso, et al. (2010) report that when people perceive an intentional mind in the underlying process, they are more likely to show the hot hand belief than the gambler's fallacy.

Whereas the perception of randomness has often been implicated in explaining the gambler's fallacy and the hot hand belief, the notion of randomness is highly debated even among mathematicians and philosophers. It has been suggested that the concept has to be broken down into more fundamental properties in order to be of any practical use (e.g., Blinder & Oppenheimer, 2008; Lopes & Oden, 1987). In particular, the gambler's fallacy and the hot hand belief have been considered as misperceptions of randomness since they violate either one or both of the independence and the stationarity assumptions (see, Bar-Eli, et al., 2006; Gilovich, et al., 1985). In the present paper, we examine an alternative factor that may contribute to the contrast between the gambler's fallacy and the hot hand belief, without the need to reject either the independence or the stationarity assumption. We argue that in these two seemingly opposite dispositions, a crucial distinction lies in the assumption about the "common parameter" of the underlying process. That is, people manifest the gambler's fallacy when they perceive the underlying process with a fixed probability of success, for example, the probability of heads is ½ for a fair coin. It has been documented that people tend to match the proportion of a certain outcome in their predictions to the expected value instead of optimizing their predictions by always predicting the outcome with the highest probability, and this tendency of "probability matching" has been used to account for the gambler's fallacy (e.g, Gaissmaier & Schooler, 2008; Koehler & James, 2009; Morrison & Ordeshook, 1975; Shanks, Tunney, & McCarthy, 2002; Wolford, Newman, Miller, & Wig, 2004). In the extreme situation where the previous outcomes consist of only one type of outcomes, e.g., a streak of heads when tossing a coin, people would predict a tail so that the proportion of tails in the predicted sequence will match more closely to the expected value—a manifestation of the previously mentioned "law of small numbers."

On the other hand, in a basketball game (as well as in many real world scenarios), it is often the case that the probability of success is initially unknown. Then, the history of the previous outcomes can provide some information about the parameter. As a consequence, the prediction of the next success, which depends on an estimate of the parameter, will show positive dependency on the number of previous successes. Most notably, such prediction does not require rejecting either the independence or the stationarity assumption of the underlying process. It can still be assumed that the probability of success remains stationary except that it is initially unknown.

The positive dependency in such prediction is a result known as the Laplace's rule of succession (Laplace, 1814)[1]. Suppose that r independent trials, each of which is a success with the same probability p, are performed. When the probability p is a free parameter chosen uniformly on (0, 1), given a total of k successes in the first r trials, the probability that the $(r+1)$st trial will be a success can be computed by:

$$P\{(r+1)\text{st trial is a success} \mid k \text{ successes in first } r\}$$
$$= (r+1)\int_0^1 \binom{r}{k} p^{k+1}(1-p)^{r-k}\,dp \qquad (1)$$
$$= \frac{k+1}{r+2}$$

[1] Interestingly, Laplace (1796/1951) also provided the first documented account of the gambler's fallacy (see, Ayton & Fischer, 2004). Zabell (1989) provides a philosophical discussion on the rule of succession. Here we only describe the relevant result according to Ross (2007, pp. 147-149).

Thus, the prediction of a success on the $(r+1)$st trial is positively dependent on the number of successes in the first r trials (k). Note that the calculation above assumes a uniform prior for p on (0, 1). In special cases where p can only take a limited number of values (e.g., randomly chosen from two values 0.50 and 0.75), by Bayes' Theorem, the positive dependency in the predictions still holds. Based on this result, it is possible that the positive dependency in the predictions by the hot hand belief is a consequence from people's effort of reducing the uncertainty by estimating the unknown probability of successes from the past information.

In the following, we test this hypothesis empirically. Specifically, we predict that when people are provided with a fixed probability of success for the underlying process, they tend to manifest the gambler's fallacy or the behavior of probability matching so that their predictions would be negatively dependent on the past information. In contrast, when the probability of success is not explicitly provided, people would have to guess it in order to make a prediction. As a consequence, their predictions would tend be positively dependent on the past information.

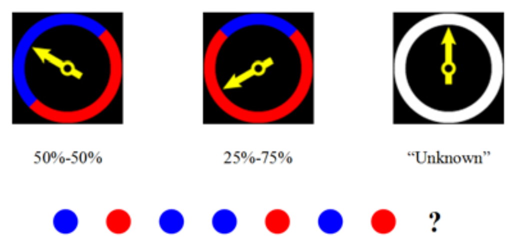

Figure 1. Three clocks in the experiment. The "50%-50%" clock is divided by blue and red colors equally along the 45 degree angle. The "25%-75%" clock has the top 25% in blue and the bottom 75% in red. The "Unknown" clock is all white. In each trial, a needle was spun seven times to generate a sequence of seven dots in corresponding colors, then, participants were instructed to predict the outcome of the eighth spin.

Method

Participants

Eleven college students and graduate students in the Houston medical center area were paid to participate in the experiment.

Procedure

The experiment was programmed in E-Prime and conducted on a computer with a 20 inch LCD monitor. Participants were instructed that they were about to observe one of 3 different clocks (see Figure 1). The circumference of a "50%-50%" clock was divided by blue and red in equal proportions; a "25%-75%" clock was 25% in blue and 75%

in red; and an "Unknown" clock was divided by blue and red with an unknown ratio and was covered in white. Within each clock, a spinning needle will generate a sequence of blue and red dots and the color of each dot is determined by whether the needle stopped at the blue or red portion of the clock after each spin. Participants' task was to predict the color of the 8^{th} spin after observing the outcomes of 7 spins for a given clock.

Three conditions were compared in 3 blocks of trials and each block used a different clock (within-subjects). The order of 3 blocks was randomized across participants. Each block consisted of 128 trials, and the binary sequence of 7 dots displayed in each trial was predetermined by randomly sampling once without replacement from all possible 128 combinations. Thus, all 3 blocks used the same 128 binary sequences so that participants would experience the same probability distribution under each condition. Once a binary sequence was sampled for a trial, the stopping position of the spinning needle after each spin was determined but randomly varied within the arc of the corresponding color. For example, in the "50%-50%" condition, if the program sampled a "red" outcome, the needle would randomly stop at one of the six positions of the red arc (2, 3, 4, 5, 6, and 7 o'clock). The stopping position in the "Unknown" condition always pointed to the 12 o'clock position.

At the beginning of each trial, a clock was presented in the center of the computer screen and the needle initially pointed up (12 o'clock position). Then, the needle was spun seven times and there was an approximately 1 second pause after each spin. After each spin, either a blue or a red dot was shown beneath the clock depending on whether the needle ended pointing to the blue or red portion of the clock. After seven spins, seven dots would line up beneath the clock and a question mark was shown at the eighth position. Participants used mouse buttons to predict the eighth spin (left button for blue and right button for red). At the end of each trial, an instruction screen was displayed to prompt participants to "proceed to the next clock" by clicking a mouse button.

Results

We first examined how participants' predictions were influenced by the percentage information in the history. Table 1 shows the total number of predictions on blue and red outcomes depending on the number of the blue outcomes in the previous 7 spins (aggregated across all participants). It appears that participants' predictions followed different trends under different conditions. For example, when there were 4 blue outcomes in the previous 7 spins, participants predicted blue in 6 trials and red in 379 trials under the "25%-75%" condition, and predicted blue in 92 trials and red in 293 trials under the "50%-50%" condition. Both conditions showed biases towards the red outcomes (more biased in the 25%-75% condition). In contrast, when the underlying color ratio was unknown, the bias was reversed: participants predicted blue in 200 trials and red in 185 trials.

Table 1. Participants' predictions on the 8^{th} spin based on the number of blue outcomes in the sequence of previous 7 spins. For example, in the "Unknown" condition, when there were 3 blue outcomes (fewer blue than red), participants predicted blue in 122 trials and red in 263 trials (385 trials in total); when there were 4 blue outcomes (more blue than red), participants predicted blue in 200 trials and red in 185 trials.

Number of Blue outcomes		0	1	2	3	4	5	6	7
Number of Trials		11	77	231	385	385	231	77	11
"25-75"	Blue	9	54	34	9	6	25	14	2
	Red	2	23	197	376	379	206	63	9
"50-50"	Blue	9	65	175	268	92	38	8	2
	Red	2	12	56	117	293	193	69	9
Unknown	Blue	5	52	59	122	200	130	28	5
	Red	6	25	172	263	185	101	49	6

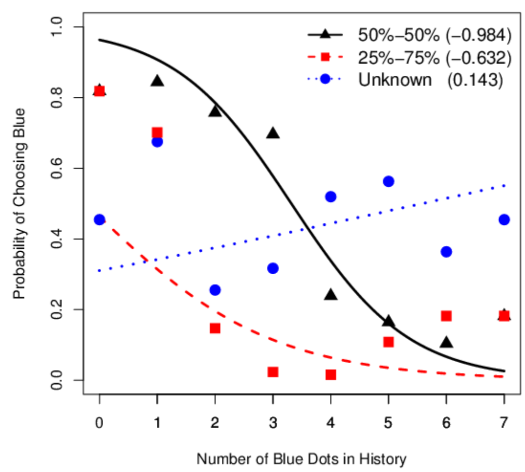

Figure 2: Logistic regression of the probability of predicting blue on the percentage of blue outcomes in the history. Regression coefficients are listed in the parentheses in the legend.

To confirm our observation in Table 1, we conducted a logistic regression to find the line of best fit for the trend under each condition, namely, to examine how the probability of predicting blue was determined by the percentage of blue outcomes in the history. The regression results are shown in Figure 2. All three regression coefficients were statistically significant ($p < .001$). Most notably, the negative trends in the 50%-50% and 25%-75% conditions were reversed to positive in the Unknown condition. This result confirmed our hypothesis that the unknown common probability would have an effect on participants' predictions: they were negatively dependent on the past information when the blue-to-red ratio was provided, but positively dependent on the past information when participants had to guess the underlying probability.

However, Table 1 and Figure 2 also show that participants' tendency of choosing blue in the Unknown condition was not as strong as indicated by the rule of succession (Equation 1). For example, when all previous 7 spins were blue, Equation 1 would predict the probability of blue at the eighth spin as $8/9 \approx 88.9\%$, whereas only 5 out of 11 predictions (45.5%) were made on blue. Besides the small sample size, one possible explanation would be that Equation 1 is based on a uniform distribution of the blue-to-red ratio from 0 to 1, and participants might have not considered all possible values of the ratio in the Unknown condition since they have only encountered two other types of clocks with a 50%-50% and a 25%-75% ratio. Another possibility was that besides the percentage information, there were other patterns (such as streaks) influencing the predictions. To test the effect of streak information on predictions, we conducted a second round of logistic regressions of the probability of predicting blue on the length of the last run ("LLR"). The values of LLR were counted as the number of blue or red outcomes in the last run of the sequence (positive values for blue and negative values for red). For example, LLR = 3 in the sequence (B, R, R, R, B, B, B), and LLR = – 2 in the sequence (B, B, R, B, B, R, R). (This example also shows that the same percentage of blue outcomes in the history can have different values of LLR).

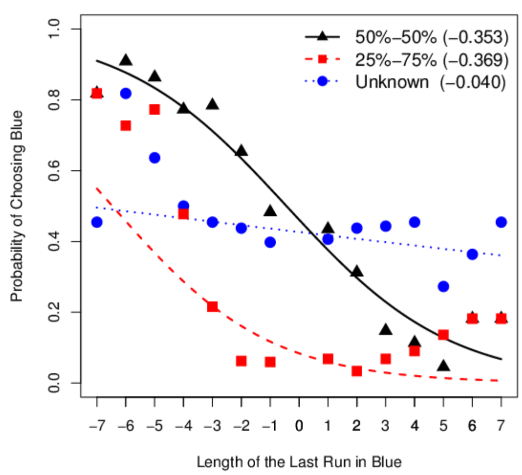

Figure 3. Logistic regression of the probability of predicting blue on the length of the last run (LLR). Positive values represent the lengths of the blue streaks and negative values represent the lengths of the red streaks. Regression coefficients are listed in the parentheses in the legend.

Figure 3 shows that the regression coefficients were significantly different from zero only in the "50%-50%" and "25%-75%" conditions (both $p < .001$) but was not significant in the "unknown" condition ($p \approx .07$). In all three conditions, participants' predictions were negatively dependent on the length of the last run. That is, participants showed a tendency to avoid long streaks at the end of the sequence in all three conditions. Compared with the predictions based on the percentage (Figure 2), such tendency was not surprising in the 50%-50% and 25%-75% conditions, but seemed to contradict the positive dependency in the Unknown condition. We can speculate on three possibilities to this tendency and they are not necessarily exclusive of each other. First, compared to the percentage, LLR only contained partial information of the past. Second, it was a manifestation of the probability matching behavior to reverse the last run. Since the sequences were drawn without replacement from all possible combinations, probability matching was actually a valid strategy for predictions. Third, participants have only experienced three kinds of clocks through the entire experiment. It was possible that in the Unknown condition, participants were guessing the underlying ratio as either 50%-50% or 25%-75%, and in some of the trials they were matching their predictions to the corresponding ratios.

Table 2. Three different strategies compared with participants' actual predictions. For example, under the 50%-50% condition, 76.8% of the predictions were consistent with the "Match to 50%-50%" strategy, and 23.2% of the predictions were consistent with the "Hot Hand" strategy.

	Strategy		
Condition	Match 50%-50%	Match 25%-75%	Hot Hand
50%-50%	76.8%	57.6%	23.2%
25%-75%	54.2%	91.8%	45.8%
Unknown	41.1%	59.2%	58.9%

To test the hypothesis of probability matching, we considered three possible strategies that might have been utilized by participants, two strategies of probability matching ("match to 50%-50%" and "match to 25%-75%") and one strategy by the hot hand belief ("hot hand"). The strategies of "match to 50%-50%" and "match to 25%-75%" would predict the color to make the final sequence more closely matched to the corresponding color ratio, 50%-50% or, 25%-75%, respectively. The strategy of "hot hand" would predict the color that appeared most frequently in the previous 7 spins. Note that "match to 50%-50%" and "hot hand" are completely opposite of each other, and both partially overlap with "match to 25%-75%." We then compared these three strategies with participants' actual predictions. Table 2 shows the percentages in which each strategy was consistent with the actual predictions in each condition. It appears that the "match to 50%-50%" strategy was the most dominant in the 50%-50% condition (with a 76.8% consistency), and the "match to 25%-75%" strategy was the most dominant in the 25%-75% condition (91.8%

consistency). However, in the Unknown condition, all three strategies were about equally dominant but the “match to 50%-50%” strategy had the lowest accuracy level in describing the actual predictions (41.1%).

We made two particular observations in Table 2. First, there was a strategy shift across three conditions. Probability matching was a dominant strategy when the color ratio of the clock was given at either 50%-50% or 25%-75%, but its dominance was greatly reduced when the color ratio was unknown. By contrast, predictions by the “hot hand” strategy were at minimum when the color ratio was given but became substantial when the color ratio was unknown. The strategy shift observed in Table 2 was consistent with dependence reversal shown in Figure 2. Second, both probability matching (50%-50% and 25%-75%) and “hot hand” strategies were present in the Unknown condition. This observation appears to be consistent with our speculation that in the Unknown condition participants were mainly guessing between the 50%-50% and 25%-75% clocks. Moreover, it also indicated that probability matching was a strong tendency that could be reduced but hard to eliminate (e.g., Koehler & James, 2009).

Discussion

Compared with previous studies on the hot hand belief, the present study did not directly examine participants' perception of the underlying process regarding its randomness (i.e., the independence and stationarity assumptions), the intentionality of the process, or, whether it is about human performance or a natural process (e.g., Ayton & Fischer, 2004; Burns & Corpus, 2004; Caruso, et al., 2010; Croson & Sundali, 2005; Sundali & Croson, 2006). Instead, we presented participants with the same clock-and-needle mechanism, under the same underlying probability distributions, across all conditions. The only factor we manipulated was the perceived common probability: either explicitly provided (50%-50% and 25%-75% conditions) or not (Unknown condition). That is, participants did not have to reject either the independence or the stationarity assumption of randomness in order to make predictions. Yet, compared to the 50%-50% and 25%-75% conditions, the overall dependency of the predictions on the past percentage information (entire history) in the Unknown condition was reversed from negative to positive.

It is possible that participants would actually reject randomness in some of the trials. However, rejecting randomness may not be responsible for the difference we found across conditions, especially the positive dependency in the Unknown condition. As we mentioned before, perception of randomness may not be a concept as a whole that influences people's behavior (e.g., Blinder & Oppenheimer, 2008; Lopes & Oden, 1987). Regarding the independence and stationarity assumptions, all three conditions in our experiment were presented in the same way except the color ratio, and in each trial, participants were always presented with the same clock. Furthermore, rejections of randomness, if they did have any effect, would be more pronounced in presence of a given color ratio where the discrepancy between the given ratio and the displayed binary outcomes would be more obvious. For example, when the color ratio was 50%-50% but the displayed 7 outcomes were all blue, participants might have suspected something was wrong then would tend to conclude that the underlying process was not what has been presented (namely, to reject the hypothesis of randomness). On the other hand, such suspicion would be less likely to arise in the Unknown condition as there was no clear contrast between the displayed outcomes and an unknown probability.

The findings that participants' prediction was positively dependent on the percentage (entire history) but negatively dependent on the length of the last run (partial history) indicates that there were at least two mental processes involved. On one hand, when the underlying probability was initially unknown, participants used the past information to estimate the probability. As a consequence, their predictions were positively dependent on the past percentage information (Figure 2). On the other hand, the negative dependency on the length of the last run (Figure 3) indicated that probability matching was a strong tendency that could be reduced but hard to eliminate (e.g., Koehler & James, 2009). Nevertheless, the tendency of probability matching was greatly reduced when the underlying probability initially was unknown and had to be estimated (Table 2). This result was consistent with the findings in some of the studies that associate probability matching with pattern search (e.g., streaks in our experiment). For example, Wolford, et al. (2004) report that distracting people with a secondary verbal working memory task prevents the pattern search and results in less probability matching behavior (also see, Gaissmaier & Schooler, 2008; Wolford, Miller, & Gazzaniga, 2000). Koehler and James (2009) suggest that probability matching is an intuition that can be overridden by deliberate consideration of alternative choice strategies.

It should be noted that the focus of the present study is to demonstrate the common probability estimation as a factor that may contribute to the opposite predictions by the gambler's fallacy and the hot hand belief. We found the reversal of the dependency when the independent variable was the percentage of one outcome in the entire history but not the length of the most recent streaks. Although a prediction positively dependent on the percentage information is more likely to prolong the most recent streak, our experiment did not exactly capture the “streakiness” in the hot hand belief. It is reasonable to conclude that common probability estimation alone may not be able to account for the hot hand belief, and it must be combined with other factors, such as the humanness or intentionality of the process (e.g., Ayton & Fischer, 2004; Caruso, et al., 2010). In addition, a limitation in our experiment is that participants have experienced only three types of clocks and the Unknown condition might have been affected by the other two conditions. A between-subject design, or, adding more levels to the provided color ratio, might be able to make participants estimate the unknown parameter in a wider range thus the positive dependency might be more pronounced. Moreover, the positive dependency observed in our experiment was based on the percentage information (or proportion), and there are usually more than one ways to construct a pattern in a binary sequence, such as alternation or symmetry (e.g., Rapoport & Budescu, 1997) or sequential dependency (e.g., West & Lebiere, 2001). One possibility to extend our experiment would be to vary the length of the presented binary sequences so that the effects of other patterns could be dissociated from the effect of percentage information. Another possible extension would be that the sequences serving as the history are actually generated based on various probability distributions instead of being drawn from a pre-determined sample pool without replacement. In this way, participants' actual experience would be more consistent with the provided color ratio or their estimation. We will leave these possibilities to future studies.

References

Ayton, P., & Fischer, I. (2004). The hot hand fallacy and the gambler's fallacy: two faces of subjective randomness? *Memory & Cognition, 32*(8), 1369-1378.

Bar-Eli, M., Avugos, S., & Raab, M. (2006). Twenty years of "hot hand" research: Review and critique. *Psychology of Sport & Exercise, 7*(6), 525-553.

Blinder, D. S., & Oppenheimer, D. M. (2008). Beliefs about what types of mechanisms produce random sequences. *Journal of Behavioral Decision Making, 21*(4), 414-427.

Burns, B. D., & Corpus, B. (2004). Randomness and inductions from streaks: "Gambler's fallacy" versus "hot hand". *Psychonomic Bulletin & Review, 11*(1), 179-184.

Caruso, E. M., Waytz, A., & Epley, N. (2010). The intentional mind and the hot hand: Perceiving intentions makes streaks seem likely to continue. *Cognition, 116*(1), 149-153.

Croson, R., & Sundali, J. (2005). The gambler's fallacy and the hot hand: Empirical data from casinos. *Journal of Risk and Uncertainty, 30*(3), 195–209.

Gaissmaier, W., & Schooler, L. J. (2008). The smart potential behind probability matching. *Cognition, 109*(3), 416-422.

Gilovich, T., Vallone, R., & Tversky, A. (1985). The hot hand in basketball: On the misperception of random sequences. *Cognitive Psychology, 17*, 295-314.

Koehler, D. J., & James, G. (2009). Probability matching in choice under uncertainty: Intuition versus deliberation. *Cognition, 113*(1), 123-127.

Laplace, P. (1796/1951). *A philosophical essay on probabilities*. New York: Dover.

Laplace, P. (1814). *Essai philosophique sur les probabilities*. Paris.

Lopes, L. L., & Oden, G. C. (1987). Distinguishing between random and nonrandom events. *Journal of Experimental Psychology: Learning Memory and Cognition, 13*(3), 392-400.

Morrison, R. S., & Ordeshook, P. C. (1975). Rational choice, light guessing and the gambler's fallacy. *Public Choice, 22*(1), 79-89.

Oskarsson, A. T., Van Boven, L., McClelland, G. H., & Hastie, R. (2009). What's next? Judging sequences of binary events. *Psychological Bulletin, 135*(2), 262-285.

Rabin, M. (2002). Inference by believers in the law of small numbers. *The Quarterly Journal of Economics, 117*(3), 775-816.

Rapoport, A., & Budescu, D. V. (1997). Randomization in individual choice behavior. *Psychological Review, 104*(3), 603-617.

Ross, S. M. (2007). *Introduction of probability models* (9th ed.). San Diego, CA: Academic Press.

Shanks, D. R., Tunney, R. J., & McCarthy, J. D. (2002). A re-examination of probability matching and rational choice. *Journal of Behavioral Decision Making, 15*(3), 233-250.

Sundali, J., & Croson, R. (2006). Biases in casino betting: The hot hand and the gambler's fallacy. *Judgment and Decision Making, 1*(1), 1-12.

Tversky, A., & Gilovich, T. (1989). The cold facts about the "hot hand" in basketball. *Chance, 2*(1), 16-21.

Tversky, A., & Kahneman, D. (1971). Belief in the law of small numbers. *Psychological Bulletin, 76*(2), 105-110.

Tversky, A., & Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. *Science, 185*, 1124–1131.

West, R. L., & Lebiere, C. (2001). Simple games as dynamic, coupled systems: randomness and other emergent properties. *Cognitive Systems Research, 1*(4), 221-239.

Wolford, G., Miller, M. B., & Gazzaniga, M. (2000). The left hemisphere's role in hypothesis formation. *Journal of Neuroscience, 20*(RC64), 1-4.

Wolford, G., Newman, S. E., Miller, M. B., & Wig, G. S. (2004). Searching for patterns in random sequences. *Canadian Journal of Experimental Psychology, 58*(4), 221-228.

Zabell, S. L. (1989). The Rule of succession. *Erkenntnis, 31*(2/3), 283-321.