Title
Understanding of Principles of Arithmetic with Positive and Negative Numbers

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Understanding of Principles of Arithmetic with Positive and Negative Numbers

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Many models of problem solving include intuitive knowledge components such as principles. Principles are general rules that capture regularities within a domain. Principles reflect conceptual understanding of the underlying structure of a problem domain. For example, in arithmetic, when adding two positive numbers (A + B = X) the answer (X) will always be greater than both operands (A and B) (Dixon, Deets & Bangert, 2001). Problem solvers have been shown to use principles in a variety of problem domains, including counting and arithmetic.

Dixon et al. (2001) investigated participants' understanding of principles that apply to arithmetic operations involving positive numbers. In their study, participants viewed sets of sample problems that had been solved by hypothetical students, and rated the level of understanding that each hypothetical student appeared to have. The analysis compared participants' ratings of problem sets that violated principles and sets that did not violate principles.

The present study built on Dixon et al.'s prior work to investigate participants' understanding of arithmetic operations involving negative numbers. Problem sets were created to test participants' understanding of principles that apply to addition and subtraction with a positive and a negative number, as well as addition and subtraction with positive numbers. The specific principles tested were: (1) *Relationship to Operands*, which specifies the magnitude of the sum or difference relative to the operands, (2*) Direction of Effect*, which specifies how the magnitude of the sum or difference changes as the magnitude of one of the operands is changed, and (3) *Sign*, which specifies the sign of the sum or difference as a function of the relationship between the magnitudes of the operands. As in Dixon et al.'s study, participants rated problem sets that violated principles and sets that did not violate principles. Participants used a scale ranging from 1 (very bad) to 7 (pretty good) to rate the sets. In each case, the relevant analysis compares participants' ratings of violation and nonviolation sets.

As seen in Table 1, participants represented the Direction of Effect principle for operations involving positive numbers and for operations involving negative numbers. Participants represented the Relationship to Operands principle only for addition with positive numbers.

Table 1:
Mean Ratings Provided for Problem Sets
with and without Principle Violations
for Each Principle, Operation, and Number Type

Principle	Operation	No. Type	M Non	M Vio	T
RO	Addition	Positive	3.56	2.89	3.50**
RO	Addition	Negative	3.31	3.15	0.72
RO	Subtraction	Positive	3.11	2.92	0.82
RO	Subtraction	Negative	3.60	3.58	0.10
DE	Addition	Positive	3.89	2.34	6.26**
DE	Addition	Negative	3.06	2.63	2.46*
DE	Subtraction	Positive	3.69	2.53	4.66**
DE	Subtraction	Negative	3.68	2.79	4.75**
Sign	Addition	Negative	2.79	2.81	0.07
Sign	Subtraction	Positive	2.79	2.56	1.17

RO = Relationship to Operands, DE = Direction of Effect, Vio = Violation, Non = Non-violation
* $p < .05$, ** $p < .01$.

The work of Dixon et al (2001) laid a solid foundation for investigating the principles governing arithmetic operations. Our findings replicate some of Dixon et al.'s results, and expand this line of inquiry to negative numbers. Our findings suggest that adults' representations of operations with negative numbers are not as well-established as their representations of operations with positive numbers.

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Reference

Dixon J.A., Deets, J.K., & Bangert, A. (2001). The representation of the arithmetic operations include functional relationships. *Memory & Cognition, 29*, 462-477.