UC Merced

Proceedings of the Annual Meeting of the Cognitive Science Society

Title

A quantum probability perspective on the nature of psychological uncertainty

Permalink

https://escholarship.org/uc/item/7hn8200k

Journal

Proceedings of the Annual Meeting of the Cognitive Science Society, 35(35)

ISSN

1069-7977

Authors

White, Lee
Pothos, Emmanuel
Busemeyer, Jerome

Publication Date

2013

Peer reviewed

A quantum probability perspective on the nature of psychological uncertainty

Lee C. White (l.c.white.517813@swansea.ac.uk)
Department of Psychology, University of Swansea, Singleton Park,
Swansea, SA2 8PP UK.

Emmanuel M. Pothos (e.m.pothos@gmail.com)
Department of Psychology, City University London
London, EC1V 0HB UK.

Jerome R. Busemeyer (jbusemey@indiana.edu)
Department of Psychological and Brain Sciences, Indiana University,
Bloomington 47468 Indiana, USA.

Abstract

Making a choice between alternatives can influence our subsequent evaluation of the selected option (e.g. Sharot, Velasquez & Dolan, 2010). Thus, in resolving psychological uncertainty, the act of making a judgment itself appears to have a constructive role in subsequent related decisions. This study focuses on emotional ambivalence and the development of affective evaluations over two stages, such that (just) making an intermediate evaluation in the first stage is shown to influence the overall affective evaluation in the second stage. Models based on classical probability theory, which assume that an intermediate evaluation simply reads off an existing internal state, cannot accommodate this result in a natural way. An explanation is offered with a quantum probability model, which, under specific circumstances, requires the measurement of an internal state to have a constructive role. The predictions of the quantum probability model were supported by the empirical results.

Keywords: Quantum probability; Interference effects; Affective uncertainty.

Introduction

One basic fact about cognition is that it reflects uncertainty. In fact cognition appears to involve several kinds of uncertainty. As well as uncertainty regarding future events, there is uncertainty about internal states, an inevitable consequence of the fact that life events are often agglomerations of pleasant and unpleasant components. For example, consider 'emotional ambivalence', the apparent ability of the cognitive system to concurrently represent positive and negative affect. Emotional ambivalence is reflected in e.g., students' thoughts about graduation day or advertisements with mixed emotional appeals (Larsen, McGraw & Cacioppo, 2001; Williams & Aaker, 2002).

Understanding how the cognitive system resolves affective uncertainty presents challenges (e.g. Brehm & Miron, 2006). For example, is positive and negative affect experienced sequentially or simultaneously? What happens when people are asked to make a judgment about their affective state whilst experiencing affective uncertainty? Does this judgment resolve uncertainty or does the act of making the judgment itself influence their affect in the same way that choice has been shown to have a constructive influence on preference (e.g. Sharot, Velasquez & Dolan, 2010)? Our objective in this paper is to propose an ambitious new perspective on this question, based on quantum probability (QP) theory (note that in this work by QP theory we simply mean the rules for how to assign probabilities to events from quantum theory; for more specific proposals see Aerts, 2009, or Atmanspacher, Romer & Wallach, 2006).

We can acquire some preliminary intuition from models for response times in choice problems, such as random walk models (e.g., Ashby, 2000; Busemeyer & Townsend, 1993; Ratcliff & Smith, 2004; Usher & McClelland, 2001). In this influential class of models, discriminating between two options involves an accumulation of evidence, so that, on every step, the weight for a particular option is increased. Crucially, at any time point, the system is assumed to be in a specific state. This state may reflect large or little weight for a particular option, but, regardless, it has to be at a specific state. Classical approaches must assume that the system is always at a particular state, even if knowledge of this state is uncertain. Such an assumption seems straightforward. How else could we build a model?

Yet, there is an alternative, intriguing possibility, which emerges from the recent uses of QP theory in cognitive modeling (Busemeyer & Bruza, 2012; Pothos & Busemeyer, in press). QP theory is a framework for assigning probabilities to observables and, therefore, potentially relevant wherever there is a need to formalize uncertainty. QP cognitive models often have the same intentions (Griffiths et al., 2010; Oaksford & Chater, 2007) as classical probability models. But, classical and QP frameworks are founded on different axioms. QP models incorporate certain unique features, such as superposition and the capacity for interference, and there has been growing interest in exploring the relevance of such features for cognitive modeling (e.g., Aerts, 2009; Atmanspacher, Filk & Romer, 2004; Blutner, 2009; Busemeyer, Pothos, Franco & Trueblood, 2011; Khrennikov, 2010; Pothos & Busemeyer, 2009; Wang et al, in press).

In QP theory, a superposition state has amplitude (weight) across more than one possibility. Suppose we are interested in representing whether a stimulus induces a positive or negative affect. Classically, the situation is straightforward: if we are uncertain about a person's state, we assign probabilities to the person having a positive or negative affect. Perhaps there is a dynamic process which evolves (reshuffles) the person's state, until a final state is reached. But, the person is always assumed to be at a particular state. The situation with a QP approach is markedly different: as long as there is weight for both possibilities, the person is in a superposition of possibilities, and it is impossible to interpret the person as being at a particular state, rather, there is a potentiality for each possibility. That this has to be the case is not obvious and it is the result of the famous Kochen-Specker theorem in QP theory. The key implication, which is fundamental to QP theory, is that a transition from a superposition to a definite state must have a constructive role.

The QP perspective enables a simple, but surprising, empirical prediction. The relevant difference between the classical and the QP approach is that in the former the system is always assumed to be at a particular state, while in the latter there is a distinction between particular states and superposition states. Therefore, consider a situation in which affective evaluation is developed over two steps, such that each step involves a stimulus presentation. Classically, it should not matter whether the person is asked to provide an affective evaluation just after the second step, or after the first step as well. In the latter case, the intermediate evaluation would simply 'read off' the existing state and so this should not affect the overall outcome of the affective evaluation. However, in the QP model, an action of affective evaluation (a "measurement") can have a profound impact on the state of the system and, therefore, the intermediate evaluation influences the eventual outcome of the second evaluation. Note that a classical model could incorporate the possibility that an evaluation (or rating etc.) has a constructive role, but this could only be done with additional assumptions, which are not part of classical probability theory.

In the current study stimuli were hypothetical advertisements, appearing as static images. In the positive-negative (PN) condition, a single positively valenced 'positive image' was presented, followed by a mixed advert, including the same positive image presented concurrently with a 'negative image', and vice versa for the negative-positive condition (NP). In the 'single rating' condition, participants viewed the single image advert and then provided an overall affective evaluation for the mixed advert. In the 'double rating' condition the same participants provided an intermediate rating to the single advert, before viewing the mixed advert and rating it. Note that the relative order of the images is likely to impact on the final evaluation. Moore (2002) demonstrated order effects in Gallup poll questionnaires (see also Bergus et al., 1998; McKenzie, Lee, & Chen, 2002). Relatedly, Vlaev et al.

(2009) argued that pain perception depends on recent pain experiences.

Hogarth & Einhorn's (1992) research on order effects in belief updating has obvious similarities with the current research, although as we shall see there are also some important differences. Their *belief-adjustment model* describes order effects as arising from the interaction of key variables including the complexity of stimuli, length of the sequence of items and whether participants respond using a *Step-by-Step* (SbS) procedure, where they report their judgment after integrating each piece of evidence, or an *End-of-Sequence* (EoS) procedure where they report their judgment only after they have viewed all stimuli in the sequence.

In a review of previous research as well as their own experiments they argue that in the case of short sequences (i.e. between 2 and 12 items) requiring simple judgments (i.e. a single item for each stimulus in the sequence) the majority of studies employing a SbS procedure result in a recency effect whereas the majority of studies using an EoS procedure result in a primacy effect. The belief-adjustment model describes a sequential anchoring-and-adjustment process in which the current belief is adjusted by the subsequent pieces of evidence.

Although there are similarities between our research and that described by Hogarth & Einhorn, there are also several important differences. Most results described in Hogarth & Einhorn's paper, including their own experiments, involved 3 or more items in a sequence, whereas the current experiment is concerned with two items. The effects they described were concerned with items that were related to each other whereas in the current experiment items were chosen to be unrelated. In our experiment participants are required to evaluate each individual item in its own right whereas in the studies described in the Hogarth & Einhorn paper each subsequent piece of evidence is evaluated with respect to an overall judgment about a person or object (e.g. trait adjectives used to make social judgments about someone's "likeableness").

However, our objective here is not to demonstrate order effects in affective evaluation, but rather to understand the potential role of an intermediate evaluation on the final one. In Hogarth & Einhorn's terms, whether there is a difference between evaluations produced using EoS and SbS procedures for the same items viewed in the same order. For this, we need to consider some elementary QP principles.

The states of a system are represented by vectors, ψ, within a multidimensional space. Different subspaces represent possibilities for ψ and a projection of ψ onto a subspace involves laying ψ onto the subspace. The squared length of this projection gives the probability that ψ is the possibility represented by the subspace (cf. Sloman, 1993). Finally, the angles between subspaces correspond to the relation between the corresponding possibilities. This can be easily understood by noting that, if a state vector is

consistent with one possibility, then we want it to have large projections on related possibilities.
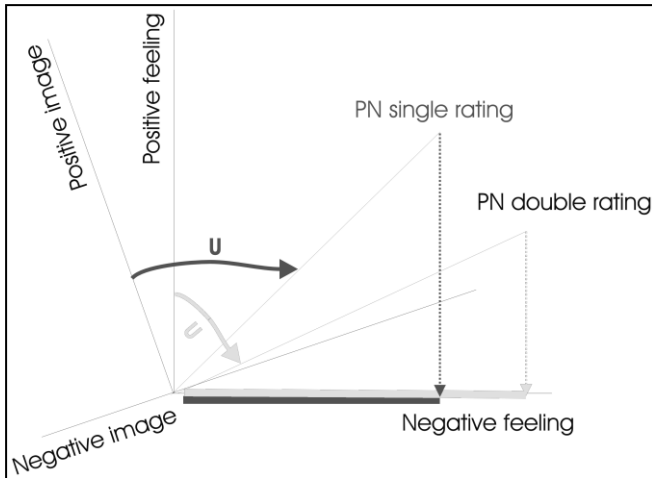

Figure 1: An illustration of how the state for the mixed image is created in the PN condition.

In Figure 1 we represent various possibilities for the affective state of the participant in the PN condition. The positive and negative affect subspaces correspond to purely positive and negative affect respectively; they are orthogonal, since a state in the positive affect subspace must have a zero projection onto the negative affect one. The positive and negative image subspaces represent the affective impact of seeing a positive and negative image during the experiment, respectively. These two subspaces are also nearly orthogonal since the images were chosen to be unrelated. Note that, in this example, the positive image subspace is close to the positive affect one, since perceiving a positive image is more likely to lead to a positive affect (the state vector created as a result of perceiving the positive image has a large projection to the positive affect subspace).

In the single rating condition, after perceiving the positive image, the state vector is aligned with the positive image subspace. The impact of introducing the negative image is represented by a rotation of the state vector, denoted as U, which leads to the state labeled as 'PN single rating'. The subsequent projection to the negative affect subspace is a measure of how negative we expect the resulting rating to be (the thick line along the bad feeling subspace). Specifically, the squared length of the projection of the mixed image state onto the negative affect subspace determines the probability of a negative rating; it is natural to assume that the higher this probability, the more negative the rating for the mixed image.

In the double rating condition, after perceiving the positive image, the intermediate rating is assumed to lead to a transition to the positive affect subspace. This is the critical difference between the single rating and the double rating conditions, which can lead to a prediction about behavioral differences depending on the presence of the intermediate rating or not. In the double rating case, once

the rating for the first image is completed, as before, the impact of introducing the negative image leads to the same angle rotation[1]. But, in this case the starting state is different (it is aligned with the positive affect subspace), therefore the resulting state corresponding to the mixed image is different too (labeled as 'PN double rating'). The resulting state is now closer to the negative affect subspace, which predicts a more negative rating.

Finally, in both the single and double rating PN conditions the final state is assumed to be closer to the negative affect subspace, than to the positive affect one, to reflect a recency effect in the importance of the negative, final image on the affective state (Trueblood & Busemeyer, 2011).

For the NP case (Figure 2), we are led to the converse prediction, namely that the final evaluation in the double rating condition will be more positive than the one in the single rating condition. Thus, a quantum approach predicts a striking interaction in the final affective evaluation in the PN vs. NP conditions, depending on whether single ratings or double ratings are solicited, only on the basis of the role of measurement in QP theory.
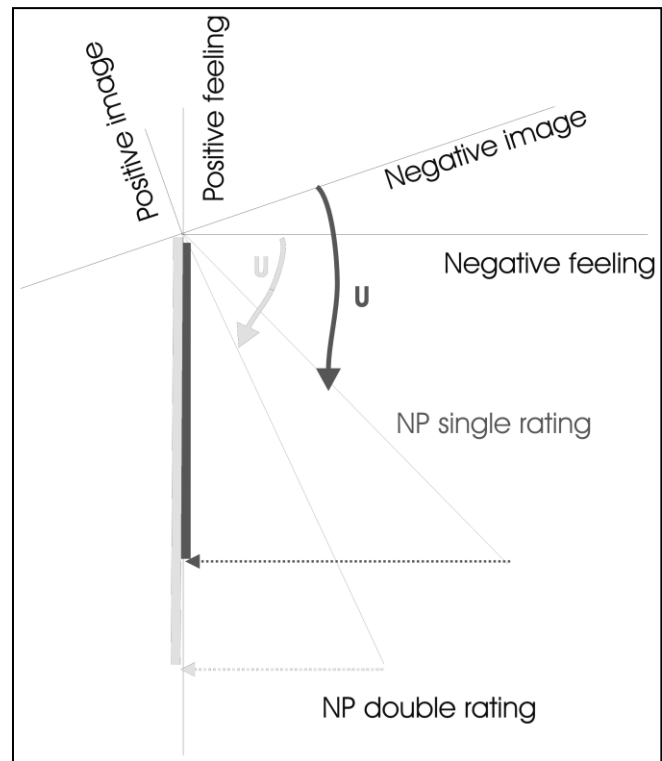

Figure 2: An illustration of how the state for the mixed image is created in the NP condition.

---

[1] A consistency consideration determines the direction of rotation. In the double rating PN condition, in transforming the state vector from the positive affect subspace to the state corresponding to the mixed image, an intermediate state cannot be aligned with the positive image subspace.

Method

Participants

Fifty-four Swansea University students participated in the experiment for course credit (45 women, 9 men, average age 21.74 years).

Stimuli

Realistic-looking adverts were created, so that the positive and negative versions would make sense together. Different advertised products were used for the PN condition (insurance; see Figure 3 for example) and the NP one (smartphone), so as to avoid interference between conditions. For the PN set there were three positive images individually presented, and three mixed images with each of the positive images joined with a negative one, and analogously for the NP set. All images were piloted to confirm their intended affective response and their unrelatedness. The images were randomly presented with 24 adverts for a camera, which were included for a different study and acted as fillers in the current experiment.
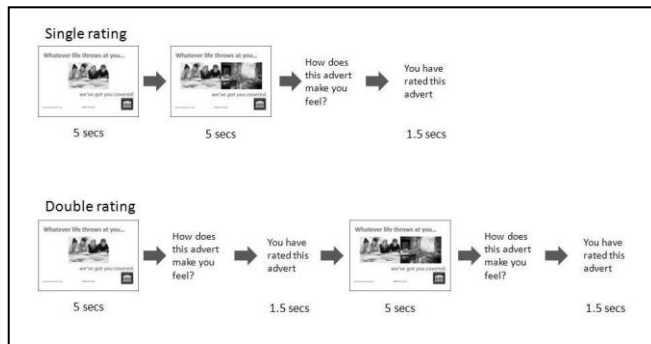

Figure 3: Sample advert used in PN condition and procedure for presentation of single and double rated adverts.

Procedure

Participants first completed a six-item current mood questionnaire. They were then told that they would see several adverts and that for each advert, when asked, they should answer the question 'how does this advert make you feel?', responding on a nine-point scale, with anchors "very unhappy/very happy". Each trial involved the presentation of a single image, followed by a request for rating (double rating condition) or not (single rating condition), followed by the mixed image and a final request for rating (Figure 3). Trials were organized into two blocks. One block contained the six single rating PN adverts and six double rating NP ones, together with 12 filler adverts (which were also rated). The other block contained the same adverts, but switching the requirement for single vs. double rating. Block order and trial order within blocks were randomized across participants.

Results

As we had previously established the valence of the images (with the pilot study), we excluded four participants because their ratings for the single image adverts were over one standard deviation below (for positive adverts) or above (for negative adverts) the mean.

We conducted a two (advert type condition: PN vs. NP) × two (rating condition: single vs. double) repeated measures ANOVA on the participants' ratings for the mixed adverts. There was a main effect of advert type ($F(1,49)=7.98$, $p=.007$), but not of rating condition ($F(1,49)=0.04$, n.s.). Crucially, the advert type x rating condition interaction was significant ($F(1,49)=10.96$, p=.002). Paired samples t-tests further showed that double rated PN adverts ($M=4.04$, $SD=1.17$) were rated significantly lower (i.e. unhappier), compared to single rated adverts ($M=4.34$, $SD=1.43$; $t(49)=2.18$, $p=.02$, two tailed; $d=.31$). For the NP adverts, double rated adverts ($M=4.94$, $SD=1.21$) were rated significantly higher (i.e. happier) than single rated adverts ($M=4.60$, $SD=1.22$; $t(49)=-2.39$, $p=.01$, two tailed; $d=.34$).
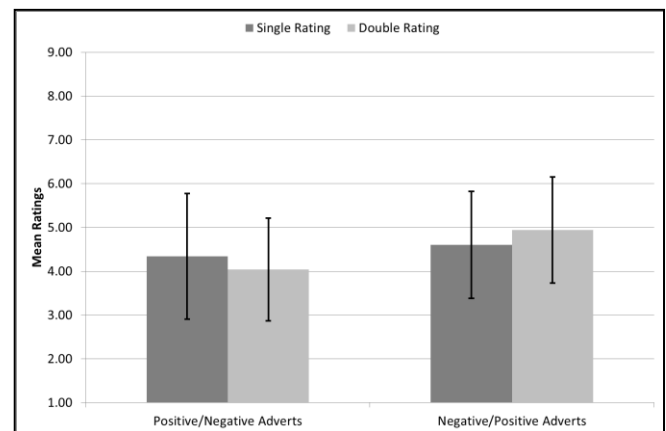

Figure 4: Mean participant ratings of single and double rated PN and NP adverts (error bars represent standard deviations).

We also considered a plausible alternative explanation, that it is the availability of a rating after the first advert, rather than the act of measurement as such, drives the observed result, a possibility consistent with anchoring (Tversky & Kahneman, 1974). In the double rating condition, the more readily accessible rating for the first advert is perhaps a reference point, against which the rating for the second advert is computed. However, there was no evidence for such an anchoring effect, as there were low, non-significant correlations between participant ratings for the first and second advert in the PN ($r=.26$,n.s.) and NP ($r=.18$,n.s) conditions.

Discussion

The results of the experiment confirmed our predictions. For the NP mixed adverts, an intermediate rating led to a higher final evaluation and, for the PN mixed adverts, an intermediate rating led to a lower final evaluation. Such a finding is difficult to reconcile with a classical probability perspective, without additional assumptions, since an intermediate rating should simply read off an existing internal state. However, the QP approach can predict a change of the state of the system, as a result of a measurement, and so is able to predict how an intermediate evaluation could affect the final one. Our finding resonates with the uncertainty intensification hypothesis (Bar-Anan, Wilson & Gilbert, 2009), according to which uncertainty about an event will prolong and intensify how people feel about it. In single rating trials the impact of the first image on the final evaluation is higher, than in the double rating ones (e.g., in PN trials with a single rating, the final evaluation is more positive than with double rating). Perhaps uncertainty about the internal state, after viewing the first image, intensifies its effect on the final rating, but in the double rating condition, reducing this uncertainty results in a greater impact on the final rating from the second image (that is, the component of the mixed image which is novel). The QP model can be seen as a formalization of such ideas.

We can elaborate on the intuition of why the QP approach works. The critical point concerns the state prior to the second, final rating. In, e.g., the PN condition, the single rating case, the state prior to the second rating reflects the impact of seeing the mixed image, that is, the original positive image, together with the new negative one. The impact of this mixed imaged would be slightly weighted in favor of the negative image, since this is shown last. In the double rating case, the intermediate evaluation (which produces a result of positive affect) can be understood as a process of abstracting away some information from the first image, but emphasizing its positive affective qualities. Therefore, this makes the introduction of the negative image produce a more contrasting affective impression. In other words, accepting that the first image is positive, creates a 'perspective' of positive affect for processing the subsequent negative image, which makes it look, well, more negative. The result of the intermediate rating is thus a larger negative final rating in the PN condition and exactly vice versa in the NP one.

The intuition that a measurement is not simply a record of an existing state, but rather it creates a state, is not alien to psychology. Notably, Shafer and Tversky (1985, p.337) proposed "A probability judgment depends not just on the evidence on which it is based, but also on the process of exploring that evidence." Quantum theory provides a formal framework within which to express this intuition.

There are plenty of possible extensions to the present work. First, it would be worth exploring more the putative role of reference points in such experiments, perhaps with the provision of an external rating. Second, we would like to explore this paradigm with changes in procedure and materials, so as to establish its robustness. Finally, there have been recent interesting analyses on the putative constructive role of decisions (e.g. Sharot et al., 2010) and it would be worth exploring further the conceptual links between these ideas and the quantum approach.

References

Aerts, D. (2009). Quantum structure in cognition. *Journal of Mathematical Psychology*, 53(5), 314-348.

Ashby, F. G. (2000). A stochastic version of general recognition theory. *Journal of Mathematical Psychology*, 44(2), 310-329.

Atmanspacher, H., Filk, T. & Romer, H. (2004). Quantum Zeno features of bistable perception. *Biological Cybernetics*, 90(1), 33-40.

Atmanspacher, H., Romer, H., & Wallach, H. (2006). Weak quantum theory: formal framework and selected applications. Weak quantum theory: complementarity and entanglement in physics and beyond. *Foundations of Physics*, 32, 379-406.

Bar-Anan, Y., Wilson, T.D. & Gilbert, D.T. (2009). The feeling of uncertainty intensifies affective reactions. *Emotion*, 9(1), 123-127.

Bergus, G. R., Chapman, G. B., Levy, B. T., Ely, J. W., & Oppliger, R. A. (1998). *Clinical diagnosis and the order information. Medical Decision Making*, 18(4), 412-417.

Blutner, R. (2009). Concepts and bounded rationality: An application of Niestegge's approach to conditional quantum probabilities. In L. Accardi, G. Adenier, C. Fuchs, G. Jaeger, A.Y. Khrennikov, J. Larsson, & S. Stenholm (Eds.), *Foundations of probability and physics-5* (Vol. 1101).

Brehm, J.W. & Miron, A.M. (2006). Can the simultaneous experience of opposing emotions really occur? *Motivation and Emotion*, 30(1), 13-30.

Busemeyer, J. R. & Townsend, J. T. (1993). Decision field theory: A dynamic cognition approach to decision making. *Psychological Review*, 100(3), 432-459.

Busemeyer, J. R. & Bruza, P. (2011). *Quantum models of cognition and decision making*. Cambridge, UK: Cambridge University Press.

Busemeyer, J. R., Pothos, E., Franco, R., & Trueblood, J. S. (2011). A quantum theoretical explanation for probability judgment errors. *Psychological Review*, 118(2), 193-218.

Griffiths, T. L., Chater, N., Kemp, C., Perfors, A., & Tenenbaum, J. B. (2010). Probabilistic models of cognition: exploring representations and inductive biases. *Trends in Cognitive Sciences*, 14(8), 357-364.

Hogarth, R.M., & Einhorn, H.J. (1992). Order effects in belief updating: The belief-adjustment model. *Cognitive Psychology*, 24, 1-55.

Khrennikov, A. Y. (2010). *Ubiquitous quantum structure: From psychology to finance*. Berlin: Springer-Verlag.

Larsen, J.T., McGraw, A.P. & Cacioppo, J.T. (2001). Can people feel happy and sad at the same time? *Journal of Personality and Social Psychology*, 81(4), 684-696.

McKenzie, C. R. M., Lee, S. M., & Chen, K. K. (2002).When negative evidence increases confidence: change in belief after hearing two sides of a dispute. *Journal of Behavioral Decision Making*, 15(1), 1-18.

Moore, D. W. (2002). Measuring new types of question-order effects. *Public Opinion Quarterly*, 66(1), 80-91.

Oaksford, M. & Chater, N. (2007). *Bayesian rationality: The probabilistic approach to human reasoning*. Oxford: Oxford University Press.

Pothos, E. M. & Busemeyer, J. R. (2009). A quantum probability explanation for violations of 'rational' decision theory. *Proceedings of the Royal Society B: Biological Sciences*, 276(1665), 2171-2178.

Pothos, E. M. & Busemeyer, J. R. (in press). Can quantum probability provide a new direction for cognitive modeling? *Behavioral & Brain Sciences*.

Ratcliff, R. & Smith, P. L. (2004). A comparison of sequential sampling models for two-choice reaction time. *Psychological Review*, 111(2), 333-367.

Shafer, G., & Tversky, A. (1985). Languages and designs for probability judgment, *Cognitive Science*, 9, 309-339.

Sharot, T., Velasquez, C.M., & Dolan, R.J. (2010). Do decisions shape preference? : Evidence from blind choice, *Psychological Science*, 21(9) 1231–1235.

Sloman, S. A. (1993). Feature-based induction. *Cognitive Psychology*, 25(2), 231-280.

Trueblood, J. S. & Busemeyer, J. R. (2011). A quantum probability account of order effects in inference. *Cognitive Science*, 35(8), 1518-1552.

Tversky, A., & Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. *Science*, 185, 1124–1130.

Usher, M. & McClelland, J. L. (2001). The time course of perceptual choice: The leaky, competing accumulator model. *Psychological Review*, 108(3), 550-592.

Wang, Z., Busemeyer, J. R., Atmanspacher, H., & Pothos, E. (in press). The potential of using quantum theory to build models of cognition. *Topics in Cognitive Science*.

Williams, P. & Aaker, J.L. (2002). Can mixed emotions peacefully coexist? *Journal of Consumer Research*, 28(4), 636-649.

Vlaev, I., Seymour, B., Dolan, R.J. & Chater, N. (2009). The price of pain and the value of suffering. *Psychological Science*, 20(3), 309-317.