A Metacognitive Stopping Rule for Problem Solving

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Abstract

Although people expect to improve by investing effort in solving a problem, several studies have found negative time-confidence correlations in various problem-solving tasks. The present study employed the metacognitive approach to illuminate why, despite lengthy thinking, people provide solutions in which they have only low confidence. According to the proposed Diminishing Criterion Model (DCM), as people invest longer in a problem, their confidence in their solution increases in a goal-driven manner, in accordance with the common belief. Nevertheless, the process ends up with a negative time-confidence correlation, because people tend to find lower confidence levels as satisfactory as they invest longer in solving a problem, reflecting a compromise in their stopping criterion. The hypotheses derived from the DCM were supported with two problem types. Even when the participants were allowed to submit a “don’t know” response, they still provided low confidence solutions after lengthy thinking, suggesting that they found these low confidence solutions to be satisfactory. The study offers reconciliation between beliefs and empirical findings and explains why people end up offering solutions with low confidence rather than continuing attempts to improve or admitting failure (via the “don’t know” option).

Keywords: Metacognition; problem solving; dual-process theory; stopping rule; time allocation.

Introduction

Solving a problem requires representations of the relevant components, rules (or constraints), and goal, followed by a sequence of inferences or calculations. However, beyond the cognitive process per se, solving a problem also involves regulation of the cognitive effort. Regulation of effort is at the heart of metacognitive theories. According to this approach, to achieve a cognitive goal, people constantly judge, or monitor, the state of their performance relative to the goal they pursue and decide whether to continue to invest further effort or cease (Koriat, Ma'ayan, & Nussinson, 2006; Nelson & Narens, 1990). The metacognitive approach is commonly used for learning research, mainly memorizing, but as yet is rarely employed for problem solving. Although metacognitive considerations have been mentioned in some discussions (e.g., Payne & Duggan, 2011), establishing metacognitive monitoring as a causal link in regulating problem solving has only started to emerge, in particular with regard to dual-process theories.

According to the dual-process theories (Kahneman, 2003), System 1 or Type 1 processes (T1) are responsible for suggesting a quick solution that comes to mind based on default procedures. System 2 or Type 2 processes (T2) execute more deliberate and lengthy analytic reasoning. However, Evans (2009) identified a third type of processes (T3). These T3 processes are responsible for (a) identifying the need for T2 intervention, and (b) examining whether a given model is satisfactory (see also the reflective mind suggested by Stanovich, 2009a). Thompson (2009) proposed that metacognitive processes underlie identifying the need for T2 intervention. Indeed, Thompson and her colleagues (Thompson, Prowse Turner, & Pennycook, 2011; Thompson et al., in press) asked participants to provide an initial answer and their Feeling of Rightness (FOR) about it. Subsequently, they were allowed to reconsider their answer. As expected, lower FORs were associated with more reconsideration time and with a higher likelihood of providing an alternative for the initial answer. These findings support the role of metacognitive monitoring as bridging T1 and T2 processes. This relates to the first aspect mentioned by Evans (2009). The present paper deals with the second aspect he mentioned; namely, examining whether a given model is satisfactory for deciding whether to stop investing effort in a particular problem.

Metacognitive Stopping Rules

In the experimental examinations of metacognitive regulation of memorizing, the fact that people invest more time in studying the more difficult items has led to the development of the discrepancy reduction model (Nelson & Narens, 1990). According to this model, people set a target level according to their motivation for the given scenario and study each item until they consider their knowledge to be satisfactory. This process seems to also be applied to a problem-solving task: people set a criterion for their confidence level, and continue to search for better solutions until they judge their chance for success to be satisfactory (Evans, 2006). Thus, for both memorizing words and solving problems, the discrepancy reduction model suggests that lengthy processing positively correlates with the chance for success. In line with this model, Koriat et al. (2006) associated goal-driven effort with a positive time-judgment correlation.

Considering the final form of time-judgment correlation, if people progress in their goal pursue until they reach the judgment level they consider as satisfactory, we would expect to find no correlation between time and judgment. This is because people are expected to stop investing effort when their judgment passes the preset goal regardless of the time it takes to reach this perceived knowledge level. However, studies of both memorizing and problem solving repeatedly found that negative correlations dominated time-judgment relationships (e.g., Begg, Duft, Lalonde, Melnick, & Sanvito, 1989; Koriat et al., 2006). In particular, in problem solving, this negative time-confidence correlation was found even when reaction times were not valid as predictors of success (Ackerman & Zalmanov, 2012; Topolinski & Reber, 2010). In these studies the participants were more confident when they provided the solutions quickly than when they provided the solutions after lengthy thinking, regardless of the actual chance for success. These findings are puzzling: Why do people stop investing effort when they knowingly provide solutions with low confidence? Is investment of time perceived as a waste of time, with no progress in the assessed chance for success?

Previous explanations for the consistent negative time-judgment correlation were based on bottom-up fluency (Koriat, Ackerman, Adiv, Lockl, & Schneider, in press; Koriat et al., 2006), Ackerman and Zalmanov (2012) included. In light of the goal-oriented nature of problem-solving tasks (Evans, 2006), the present study offers a top-down explanation for the findings. According to the proposed *Diminishing Criterion Model* (DCM), as people invest longer in a problem, their confidence in the currently considered solution option increases in a goal-driven manner, aiming to improve the chance for success, as is also derived from the discrepancy reduction model. However, the stopping criterion does not remain constant along the solving process, but diminishes, reflecting an increased compromise as more time is invested. That is, if an immediate solution option comes to mind with high confidence, higher than the initial stopping criterion, this solution would be provided. If the confidence regarding the initial solution is lower than the criterion, more consideration time is invested, until reaching a satisfying level of confidence. Importantly, a confidence level which may not satisfy the solver regarding a quickly produced solution may satisfy him or her after lengthy consideration.

But what if the confidence after lengthy thinking remains very low and no way to find a better solution is found? In this case, people may prefer to respond with “don’t know”, stemming from their desire to provide solutions with reasonable confidence. In this case, a “don’t know” response may be more socially acceptable than a solution accompanied by a very low confidence (Ackerman & Goldsmith, 2008). The question is whether the “don’t know” option would eliminate the negative time-confidence slope and lead respondents to provide only high confidence solutions. The prediction by the DCM is that it would not, because if a great deal of time is already invested, people compromise and refer to quite low confidence levels as satisfactory.

Two hypotheses derived from the DCM were examined in two experiments, one with regular problems and the other with misleading problems often used in studies of the dual-process approach. The first hypothesis was that judgments are positively correlated with time while the final time-confidence correlation is negative. The second hypothesis was that the opportunity to respond with “don’t know” would not eliminate this pattern of results.

Experiment 1

The task in Experiment 1 was the Compound Remote Associate (CRA) test. In this test, participants are presented with a word triplet and their task is to find a fourth word that forms a compound word or two-word phrase with each cue word separately. In an attempt to solve these problems, immediate associations for each word are expected to come up (Wiley, 1998). However, an association that fits only one or two of the cue words does not satisfy the requirements. For example, for the triplet PINE/CRAB/SAUCE, the word PINE might initially elicit PINECONE rather than the correct PINEAPPLE. Recognition that the initial solution option does not fit should trigger a search for a better solution (Thompson et al., 2011; Thompson et al., in press). In Ackerman and Zalmanov (2012) these problems yielded a strong negative time-confidence correlation.

In the present study there were two groups. The intermediate ratings group was asked to provide ongoing confidence ratings regarding the solution options they considered at each point in time (see also Ackerman & Goldsmith, 2011; Metcalfe & Weibe, 1987; Vernon & Usher, 2003). The “don’t know” group provided intermediate ratings as well, but also had the option to respond with “don’t know”.

Method

Participants. Forty-four undergraduates participated in the experiment for course credit or for payment (M_{age} = 24.8; 50% females). They were randomly assigned to working with or without the “don’t know” option, with 22 participants in each group.

Materials. Thirty-four CRA problems were used. Two problems were used for demonstration and two for self-practice. Pretesting verified that all problems were solvable by the target population.

Procedure. The experiment was conducted on two to eight participants in parallel, in a small computers lab. The instruction booklet detailed the procedure, explained what constituted a valid solution, and illustrated the procedure using two problems. Pressing the “Start” button brought up a problem. Respondents had to type the solution and press the “Continue” button. Response time was measured from when participants pressed “Start” to when they pressed “Continue”. This exposed the question, “How confident are you that your answer is correct?”, and a horizontal scale (0% - 100%). Pressing the “Next” button cleared the screen for the next problem.

The participants were asked to report on intermediate confidence ratings interspersed with solving each problem. The ends of each scale were marked as “I still have no idea”, and “I’m sure I found it”. The first scale, appeared three seconds after the problem's presentation. Later on, an additional scale appeared every 15 seconds and the previous scale became inactive, even if no rating was entered. This way the repeated request to enter a rating was clearly noticeable. The screen could present up to five intermediate scales. The participants could enter the answer at any time, rate their final confidence, and move on to the next problem. The times for entering the intermediate confidence ratings were documented. The only difference for the "don't know" group was that adjacent to the space for answer entry, there was a "don't know" button. Pressing this button deactivated the confidence rating scale.

After demonstration with two problems, the two other practice problems appeared first, and the rest were randomly ordered for each participant. The session lasted 30 minutes.

Results and Discussion

The participants provided meaningful solution words (rather than "XXX", for example) for 97% of the problems. Overall, the results were highly similar to those of the group reported in Ackerman and Zalmanov (2012), which was drawn from the same population and solved the problems without intermediate ratings. In the "don't know" group, 19 participants of 22 used the "don't know" option. Percent correct (with "don't know": M = 56.5%, SD = 20.3; without a "don't know" option: M = 48.0%, SD = 16.8) and confidence ratings were somewhat higher with the "don't know" option than without it, but the differences were not significant; both were ps > .13. The mean response time was shorter with the "don't know" option (with "don't know": M = 29.0 sec., SD = 13.1; without a "don't know" option: M = 41.6 sec., SD = 12.6), $t(42) = 3.27$, $p < .01$. This finding may indicate that the "don't know" option allowed participants to avoid providing the results of their lengthy solving processes. Indeed, the "don't know" responses (M = 56.9, SD = 26.6) were provided after more time than the provided solutions, $t(18) = 5.90$, $p < .0001$. This finding suggests that the participants provided the "don't know" response after deliberation, rather than for moving quickly to the next problem.

To examine the ongoing progress of the confidence ratings, the data was split for each participant for his/her own quarters of final response times, with seven or eight problems in each quarter. The points on the black lines in Figure 1 represent the mean final times and confidence for each quarter, with (panel A) and without (panel B) the "don't know" option. A two-way Analysis of Variance (ANOVA) examining the effects of the Group (2) and Quarter (1-4) on final confidence ratings, revealed only the main effect of the quarter, $F(3, 126) = 136.49$, MSE = 218.32, $p < .0001$. No difference was found among the groups, $F < 1$, and the interaction was not significant, $F(3, 126) = 1.64$, MSE = 218.32, $p = .18$. Thus, confidence ratings were higher for the quickly provided solutions than for the lengthy solutions, and this pattern did not differ among the groups.

The progress of the problem-solving process exposed by the intermediate confidence ratings is also plotted in Figure 1. Because there was no data on all points for all participants, we used the initial confidence (by the first intermediate scale) and final confidence to statistically examine the progress in the ratings. The analysis was based on participants who provided initial confidence under all four quarters (N = 26, 59%). A mixed three-way ANOVA Group (2) × Quarter (1-4) × Rating (initial vs. final confidence) yielded no main effect of the group, $F < 1$. The main effect of the quarter was significant, $F(3, 72) = 90.92$, MSE = 352.64, $p < .0001$, reflecting that the ratings fell from the first to the fourth quarters. The main effect of the rating was also significant, $F(1, 24) = 90.78$, MSE = 404.03, $p < .0001$, supporting the increase from the initial to the final confidence ratings. Importantly, the triple interaction was insignificant, $F < 1$, suggesting a similar pattern of results with and without the "don't know" option.

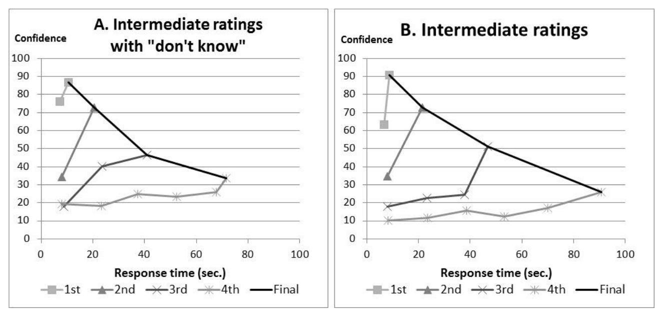

Figure 1: Experiment 1 - The intermediate and final confidence ratings on the timeline of solving the problems, divided by final response time quarters (1-4). Each panel presents the results of one group.

Overall, the results of Experiment 1 support the hypotheses derived from the DCM. There was a positive relationship between the time in which each rating was provided and the progress of the confidence ratings and a negative relationship between the total invested time and final confidence ratings. Importantly, this was the case even with the "don't know" option, which suggests that the participants found low confidence solutions provided after lengthy thinking as satisfactory for that point in time. It is also clear from Figure 1 that confidence levels that were not considered satisfactory in initial stages of the problem-solving process (e.g., the mean of the FOR ratings in the second quarter in panel A, which is 33), were provided if a similar level of confidence was reached after lengthy deliberation (e.g., the mean final confidence rating at the fourth quarter in panel A, which is also 33).

Experiment 2

Misleading problems are commonly used in the literature related to dual-process theories to differentiate between the fast intuitive (T1) solutions and the results of more deliberate processing (T2). For example: "A bat and a ball cost $1.10 in total. The bat costs $1 more than the ball. How much does the ball cost? ____cents" (Kahneman, 2003). The immediate solution that comes to mind is 10 cents, while the correct solution is 5 cents. From a metacognitive point of view, these problems allow dissociation between the confidence and accuracy in their relationship with response time, since the very first solutions tend to be accompanied by high confidence but a low chance for being correct, in particular when presented in an open-ended format (Ackerman & Zalmanov, 2012).

What should we expect with regard to final confidence in solutions provided after lengthy thinking? In cases where the respondent reaches the correct solution, he or she may be aware of the successful processing and be highly confident of the found solution. High confidence after deliberate processing can also be expected to accompany wrong solutions in cases such as over-generalized rules without appropriate exceptions, or investing effort in finding support for the initial and wrong solution (Stanovich, 2009a). Indeed, Ackerman and Zalmanov (2012) found higher confidence ratings regarding lengthy solutions with misleading problems than regarding CRA problems. However, despite this finding, a negative correlation between time and confidence was found even with the misleading problems. This might indicate that even with these problems, people see relatively low confidence levels as satisfying after lengthy thinking, as suggested here. To examine this possibility, Experiment 2 examined whether the "don't know" option allows the participants to avoid the low confidence solutions, with the hypothesis that it will not. Like in Experiment 1, all participants provided intermediate confidence ratings. One group worked with and one without the "don't know" option.

Method

Participants. The 40 participants were drawn from the same population (M_{age} = 25.2; 36% females). The participants were randomly assigned to the "don't know" conditions, with 20 participants in each group.

Materials. The problems used by Ackerman and Zalmanov (2012) were used for this experiment. They included twelve experimental problems and a practice problem for demonstrating the procedure. The experimental problems included the three problems used by Frederick (2005; the bat and ball, water lilies cover half a lake, and machines that produce widgets at a certain rate), the drinks version of Wason's selection task (Beaman, 2002), the A-is-looking-at-B problem (Stanovich, 2009b), and a conditional probability problem (Leron & Hazzan, 2009). The other problems were misleading problems adapted from preparation booklets for the Graduate Management Admission Test (GMAT).

Procedure. The procedure was highly similar to that used in Experiment 1. The practice problem appeared first, and the rest were randomly ordered for each participant.

Results and Discussion

The participants provided meaningful solution words (rather than "XXX", for example) for all the problems. In the "don't know" group, only six participants of 20 utilized the "don't know" option. As in Experiment 1, percent correct (with "don't know" option: M = 47.2%, SD = 16.4; without the "don't know" option: M = 43.9%, SD = 15.5) and confidence ratings were equivalent in both groups, both ts < 1. In this case, no difference was found also for response time, t < 1. Like in Experiment 1, the results were highly similar to those found by Ackerman and Zalmanov (2012), where there were no intermediate confidence ratings.

In this experiment, there were only 12 problems, so they were divided into thirds rather than quarters, with four problems in each third. As can be seen in Figure 2, the overall pattern of results remained with (Panel A) and without (Panel B) the "don't know" option, although confidence levels were higher than in Experiment 1.

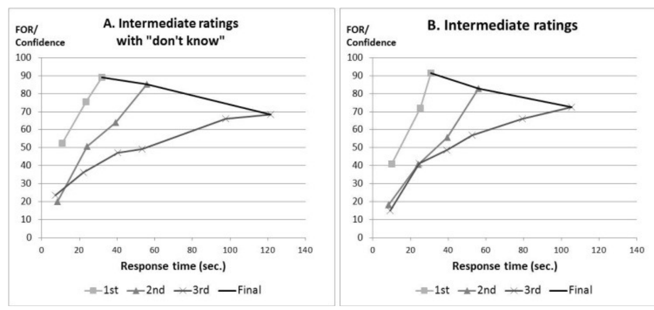

Figure 2: Experiment 2 - The initial feeling of rightness (FOR), and intermediate and final confidence ratings on the timeline of solving the problems, divided by final response latency thirds (1-3). Each panel presents the results of one group.

A two-way ANOVA examining the effects of the Group (2) and Third (1-3) on final confidence ratings, revealed only a main effect of the third, $F(2, 76) = 60.37$, MSE = 67.82, $p < .0001$. No difference was found among the groups, $F < 1$, and no interactive effect, $F(2, 76) = 1.73$, MSE = 67.82, $p = .18$. Thus, confidence ratings were higher for the quickly provided solutions than for the lengthy solutions, but this pattern did not differ among the groups. A mixed three-way ANOVA Group (2) × Third (1-3) × Rating (FOR vs. Final confidence) was based on participants who provided FORs under all thirds (N = 20, 50%). The main effect of the group was not significant, $F(1, 18) = 2.29$, MSE = 1399.72, $p = .15$. The main effect of the third was significant, $F(2, 36) = 29.25$, MSE = 163.62, $p < .0001$, reflecting that the ratings dropped from the quickly provided to the slowly provided solutions. The main effect of the rating was also significant, $F(1, 18) = 95.80$, MSE = 948.31, $p < .0001$, supporting the increase from the initial FORs to the final confidence ratings. The triple interaction was again insignificant, $F(2, 36) = 1.84$, MSE = 180.92, $p = .17$ suggesting a similar pattern of results with and without the "don't know" option.

The results of Experiment 2 support the two hypotheses derived from the DCM as well. This experiment generalizes the results of Experiment 1 with a different type of problems, in which one may expect to find high confidence ratings after lengthy thinking. Even with these problems, the time-confidence relationship is neither positive nor flat. It is consistently negative, even when participants could avoid low confidence solutions by utilizing the "don't know" option.

General Discussion

The motivation for the present study stemmed from the puzzling inconsistency between the goal-driven nature of the problem-solving task — which leads to the expectancy of positive or no correlation between time and confidence — and the empirical findings of persistent negative correlation between them, even when people are free to regulate their solving time (Ackerman & Zalmanov, 2012; Koriat et al., 2006). The proposed DCM suggests that the cognitive process indeed progresses in a goal-driven manner, with a positive correlation between time and confidence. It also suggests that people stop investing effort when their metacognitive monitoring passes their stopping criterion. Until this point the process accords with the well-known discrepancy reduction models (Nelson & Narens, 1990). The unique characteristic of the proposed DCM is the suggestion that the negative correlation stems from the willingness of people to compromise on the satisfactory level of their chance for success. These predictions were supported by the two experiments, with two task types: non-misleading and misleading problems. It was found that although the final time-confidence relationship is negative, the process progresses with a positive correlation between them. The "don't know" procedure was used to ensure that the negative correlation does not stem from the desire to move on to the next problem, even if a satisfactory solution was not yet found. The results suggest that people find the relatively low confidence they experience after lengthy thinking to be satisfactory, even though the same confidence levels were not acceptable if reached earlier in the process.

The present study suggests a difference between the stopping rules for T1 and for T2. Thompson and her colleagues (Thompson et al., 2011; Thompson et al., in press) suggested that FOR is the basis for the stopping rule for T1 and for triggering T2. If FOR is high enough, people provide the first answer that comes to mind. Otherwise they activate T2. The present study extends this idea to the decision to stop T2. While time allocation for T1 was explained to be based on fluency in which the first solution option comes to mind, in a bottom-up manner, the stopping rule for T2 is explained here as stemming from a goal-driven, top-down, effort investment. Importantly, this does not rule out fluency effects on final confidence ratings as well, but suggests that the goal-driven decision dominates the process.

To conclude, metacognitive studies traditionally focus on memorizing word lists. Investigating more complex tasks brings to the fore additional factors that may have broader ecological validity. The present study evolved from considering problem-solving tasks, which are generally understudied from the metacognitive point of view, and which highlight puzzling aspects of time investment and its relationship to metacognitive regulation. By proposing the DCM, this paper aims to shed light on the processes that lead people to end up with low confidence in their success, even when they can potentially avoid it by continuing improvement attempts or admitting failure.

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