Fuzzy Cognitive Quantification

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Abstract

This article presents the results of a two stage investigation about how linguistic quantifiers are used to summarize expressions of quantity. In the first step subjects were asked to give verbal descriptions of arrays of percentages. Then a second group of subjects was asked to reproduce the original array from the verbal description. The second group produced quantities extremely similar to the original percentages even though the verbal descriptions they used did not describe all categories within the arrays. We shall show that quantifiers have implied meaning (e.g., between *a large number* and *most*) and that similar linguistic constructions may refer to amounts that are noticeably different (e.g., *the principal* vs. *principally*). Finally we highlight the importance of the implicit, topic-related meaning in the choice of spoken complements by showing how the concept of fuzzy quantifiers can be applied not only to their modeling, but also to their use in multi-dimensional data searches.

Cognitive semantic approaches of Quantifiers

For Cognitive semantic approaches of Quantifyers have also been quite rare. Just, (1974) defined cognitive traits within three dimensions (Universal-Specific, Large-Small, Negative, Positive) which serve to categorize and give meaning to quantifiers. Quantifiers also project a representation of quantity which influences how information is perceived. The statement " few dots are blue (or red)" with a visual image of two dots of one color and 12 dots of another, will result in attention being focussed on the two dots. If the statement were "a lot of dots are blue (or red)", attention would be focussed on the group of 12 dots (Just & Carpenter, 1971). Quantifiers also carry implied meaning and lead to making inferences about other quantities. This is the case for distinctions based on the positive ("a few of") or negative ("few of") polarity of quantifiers (Paterson, Sanford, Moxey & Dawydiak, 1998). The statement, "there are a few people in the train" designates the amount of people in relation to an empty train, whereas "there are few people in the train" refers to the number of people missing in relation to how many the train should carry.

How much meaning can be inferred from quantifiers? How many bottles of soda should you buy when a friend asks you to pick up "a few"? The analogous approach of Holyoak & Glass, (1978) is based on the idea that there is a direct correspondence between terms and a numerical scale. They showed that confusion arises when the quantifiers are very close in scale, as did Anderson (1981). Furthermore, with their approach it is not possible to evaluate the question when maximum values are unknown (the maximum amount of bottles of soda, for example). The adverbs "generally" and "usually" can be accepted as covering almost all the people or things in consideration. The adverb "often" refers to either the majority of the individuals ("children are often bright") or the majority of occurrences and can thus convey repetition ("demonstrations are often violent"). Continuing in this vein, Hörmann, Cascio & Bass O' Connor, (1974) sought to define how quantifiers would be spread on a scale with class intervals. Unfortunately quantifiers denoted values that varied in relation to what was being quantified. For example "frequently" corresponded to 70% (on average), when referring to how often Miss Sweden was judged attractive, but only to 30% when used to refer to the frequency of airplane crashes (Newstead, 1988). In natural language semantics, where quantifiers denote relations between groups (Geurst, 2003), "a lot" corresponds to the majority. Thus "a lot of A's are B's" can mean that there a more A's that are B's than there are A's that are not B's. This interpretation nevertheless is not valid for the statement "in the last elections a lot of (A: electors) were (B: electors that didn't vote)" which actually refers to the number of non-voters (Barwise & Cooper, 1981). The adverbs "sometimes"' and "rarely" appear to designate respectively a small, but non-negligible frequency, and a quite negligible frequency. Still, it is difficult to say what is "small" or "negligible", as in (1) and (2) where "sometimes" probably is not the same. What a quantifier denotes is therefore dependant on the various elements of the situation being described.

(1) Sometimes I watch the evening news.
(2) Sometimes I go to the movies.

The vast majority of studies have confined themselves to the quantifiers "all", "none" and "some", but even with these most simple cases the process which generates inferences from their meanings remains unclear. In function of given statements 49 to 75% of respondents infer from "all A's are B's" that "all B's are A's" (Newstead, 1988; Chater & Oaksford, 1999). Applying pragmatic linguistics theory,

Grice (1975) similarly found that while "all" should logically encompass "some", it is possible for "some" to be restrictive and exclude "all". Interestingly enough, it has been observed that children do not have the same understanding of quantifiers as adults. Although at 3 years of age children do not differentiate between "all" and "some" (Hollander, Gelman & Star, 2002), by the time they are seven and even though they have not learned all rules of conversation, their answers are logically more valid than adults' are (Smith, 1980).

A question remains as to the information the adult is using to make an inference. Are they saying "some" because they know they cannot include "all" the objects (3), in which case the restrictive interpretation is pragmatically valid, or, are they saying "some" because they have no more information about the other objects (4), in which case a non-restrictive interpretation is more accurate.

(3) There are some broken eggs in this carton

(4) I took some eggs from this carton and they weren't fresh

In contrast to studies which have used ad hoc statements, in this article we have sought to potentially include all the values and expressions that "some - not all" can have in relation to a real situation (results of the 1998 French high school graduation standardized examination, the "baccalauréat"). There were 1,277,282 students in the class of '98. The data from this group are multidimensional; they can be arranged according to many factors: pass rate, gender, age, nationality, region, type of school, presence during the exam, which specialized baccalauréat was taken, which foreign languages were studied, etc.

From the 18 independent dimensions used (each containing 2 to 13 terms), more than 300 million relations are possible when only considering the intersection of the dimensions' terms, because relations may also derive from the fusion of exclusive terms. Our objective was to investigate quantifiers both in terms of their production and their interpretation.

The first step was to observe an initial group of students producing quantifiers (experiment 1). In step two a second group of students was given the opposite task of assigning numerical percentages to the verbal descriptions which were summarized by group 1.

Experiments

In order to assess the wide range of quantifiers used, in experiment 1 we allowed the free production of verbal descriptions for the percentage distributions. In order to assess how often certain quantifiers were used for each distribution, participants then selected terms from a list

Experiment 1: production and choice of quantifiers

Method

Participants. 83 university students in psychology and computer sciences responded to the questionnaires either in writing or via the Internet. In the later case students had to access the research lab's site. All the participants had passed their *baccalauréat* examination one or two years previouslySecond-level headings should be 11 point, initial caps, bold, and flush left. Leave one line space above and 1/4 line space below the heading.

Questionnaires. 18 percentage distributions generated from a database concerning the 1998 baccalauréat, constituted the basic information in the questionnaires. The distributions were selected to include the largest range possible of cases in terms of dimensions, number of variables (from 2 to 13) and values. They were as follows:

D-1: very close values all around a half,
D-2: very different values; one very large, one very small,
D-3: many values; one very large, one small and all others very close and very small,
D-4: many values, but none large (i.e. near a half) and all others very small but not as close as in D-3,
D-5: five not very high values; two quite close, one not very distant and two others quite close and very low,
D-6: three values close to a third and two other very weak ones (almost null),
D-7: data in absolute values (to compare with the first distribution),
D-8: one very strong value, one very weak,
D-9: two strong proportions (above 50%), neither very, nor too distant,
D-10: on two lines, a strong imbalance between columns with a rather large difference between lines with intersections as well,
D-11: two rather close proportions near 50%, but one was 10 points over and the other 10 points below,
D-12 : many very small and close values with two other values close to 20% and a third near 40%,
D-13: two close values, both quite average,
D-14: two high and close values,
D-15: two very weak, close values,
D-16: five values; one near a half, two others near 20%, and two very weak others
D-17: three very weak and close values (two almost equal and the third a little higher)
D-18: two strong, very close values (in comparison to D-14).

*Procedure.*The instructions on page one of booklet one were as follows: "Without using any of the figures in the tables, in a few lines, write what you can say about the table. You have 20 minutes for this task." The second booklet had the same instructions with the constraint to use words from the quantifiers list given: "Without using any of the figures in the tables, in a few lines, use words from the list below to say what you can about the table. If a non-included term seems absolutely necessary you may add it to the list. You have 20 minutes for this task".

Results and Discussion

Quantifier production. The participants' responses fell into two contrasting types of verbal statements summarizing the given distributions. The first type consisted mostly of describing the dominant term(s) and its(their) impact by using quantifiers, like "half". Modifiers like "very" were used extensively. These kinds of responses were particularly common when the distribution table had only two variables and/or when the table had one very high level, very different from the other levels.

The other type of verbal summary was used when there were many variables and when no one term was prominent. The statements first purpose was to organize and structure the data, that is, to make comparisons (especially of near equality) between the different terms, for example by contrasting the girls' results to the boys'. This process can be seen as a relative analysis of the results. Sequencing terms like "first" and "then" were prevalent and no indication in either quantitative or qualitative terms was given for evaluating proportions. When statements of this type set hierarchical relations between variables, they did not on the other hand, establish orders of magnitude. For D-13 and D-14 for example, all the respondents ordered the success rates, but only one gave an indication of the actual values of the scores.

The principal terms found fall into four categories:

Ordering numerical values
- More ___ than ___ and less ___ than ____
- first ... next/then ... the rest ...
- best/better than ...
- many ... few

Quantifiers
- principally, in general, the majority, frequently, often, almost/almost all, a good part
- (very) few of, a minority of
- a third, (nearly) half, more than a million
- about x%

Relations (fuzzy
- about the same, (a little) more often, almost as much, no major difference, lower, raise, constantly, bigger, below, above, …

Modifiers (fuzzy)
- just about, almost, lightly, clearly, noticeably, approximately, (very) (low) minority, large majority, …

It quickly appeared that certain quantifiers were not used as often as would have been assumed. This was the case for the expressions of proportion other than "about half" (i.e. "about a quarter", "about a third") and it was also true for "most of" which wasn't often produced spontaneously, but was chosen when included on the list (see next section). In general the respondents preferred the expression "a lot of". Comparisons and ordering seemed of primary interest and thus, the term "more" was used extensively.

Finally, we observed in most cases that the verbal summary descriptions were incomplete: the respondents only focussed on certain variables in each distribution. It is true that for inter-dimensional variables, orders of magnitude cannot be deduced. However, our distributions were for the most part intra-dimensional. For this reason it was often possible to infer and reconstruct the numerical values of the proportions in the tables (all proportions summed to 100%), from the verbal summaries.

The choice of quantifiers. The quantifiers the respondents chose for summarizing the data are as follows. When more than 75% of the participants chose a certain quantifier, it was then associated with the distribution

If we consider only the data compiled from the Internet questionnaires, only distributions 4 and 10 generated a disparate choice of quantifiers. For the majority of the other cases, three groups of quantifiers stand out; the first being simply "almost all", the second being proportional ("about half", "about a quarter", "about a third", etc.), and the third being composed of quantifiers that describe either the general case ("most of"") or particular cases ("few").

The option "If a non-included term seems absolutely necessary you may add it to the list" was chosen most often to differentiate (and provide a substitute for) descriptions that used the terms "as many/much as", "equals", "similar", "almost equal", "more than", etc., found with test booklet version 1

Items (D-1) and (D-7) presented the same information in numerical values and as a proportion, respectively. For D-7 the participants massively chose the same quantifiers as in the distribution, whereas for the absolute values there was a much greater diversity in the quantifiers selected.

Items (D-14) and (D-18) represent approximately the same numerical data (strong proportions near 75%), but with different dimensions. The temporal dimension is more salient with quantifiers like "constant over time" or "rose by" for D-18, (which described baccaulareat rates by years), than for D-14 (pass rates vs. gender), even though for version 2 the results were quite similar.

Finally, it was noted that although version 2 had different quantifiers added to its list which did not necessarily appear on version 1, the answers concerning the same subject were often very close, for both versions of the questionnaire**.**

Experiment 2: understanding quantifiers

The objective of this second investigation was to determine whether respondents would be able to reconstruct tables of numerical data from the series of verbal summaries containing quantifiers which the subjects in experiment 1 defined.

Method

Participants. As with the first test, the participants were university students in psychology or computer sciences (N=116), who all had passed their baccalauréat one or two years previously. All students used printed questionnaires (none responded via the Internet***).***

Questionnaires. The questionnaire used in experiment 2 contained nine (D-1, D-2, D-3, D-4, D-11, D-13, D-15, D-16 and D-17) of the original 18 distribution tables, without their numeric values. Hereafter they will be designated as D-1 through D-9. Each distribution is associated with a verbal description containing the quantifiers that the majority of the subjects in test one chose. The descriptions are as follows:

D-1: "There are a few more girls than boys",

D-2: "There are very few foreigners enrolled",

D-3: "The test candidates mainly came from public schools, if not, they came principally from state accredited private schools",

D-4: "About half the candidates have no possible area of specialization. Most of the others chose mathematics or a modern foreign language",

D-5: "The majority of test passers were girls",

D-6: "The pass rate was higher (quantifier=more) for the French than for the foreigners and represents the majority",
D-7: "The pass rate was higher (quantifier=more) for the girls than the boys",
D-8: "A quarter of the test takers fail. Half receive no honors. 1/5 receive the honorable mention - good. Very few receive the highest distinction - excellent".
D-9: "The excellent honors rate was very weak for the baccalauréats specialized in economics and social sciences. It is 3 times higher (quantifier=more) for the literary baccalauréats (a little less than double) and it is 4 times higher (quantifier=more) for the scientific baccalauréats".

The tables and their descriptions were on one sheet of paper. On the top of the paper the following instructions appeared: "Use the corresponding summaries to reconstruct tables of numerical data (percentages)".

Procedure. Each participant received the questionnaire sheet with the nine distributions to fill in according to its corresponding verbal summary. They were allotted a maximum of 30 minutes to complete the task.

Some of the tables submitted did not present a total of 100, so they were proportionally corrected to reach 100. In this manner the quantitative values obtained in experiment 2 could be compared to the initial numeric information the subjects used in experiment 1.

Results and Discussion

Item D-7 introduced a comparison with a conditional frequency. The number of boys receiving highest honors was 7.6%; for girls it was 7.8%. This relationship was interpreted by the respondents as being complementary and thus, they furnished values that summed to 100%. Setting aside this distribution, the results demonstrate that the subjects were able to reconstruct numeric tables from the textual summaries. Variation around the average response was quite low in the majority of answers (Figure 1).

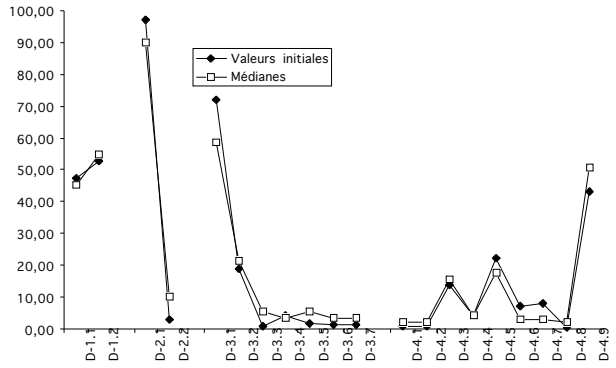

Figure 1: Comparison of the initial values and the reconstructed values.

It is clear that although the verbal descriptions were often incomplete, the participants were capable of filling in the distribution tables with values quite similar to the original ones. Aside from D-7, the averages of the quantities generated by the respondents correlate to the initial values at 0.89 ($p<0.0001$) and the medians correlate at 0.875 ($p<0.0001$). The greatest differences occurred with items D-6 and D-9. In D-6 as in D-7, the participants produced complementary percentages that totaled to 100%. In D-9, they greatly overestimated the percentage of *excellent* honors.

General Discussion

Despite the variances in D-6, D-7 and D-9, the students were able to produce a very good approximation of the initial quantitative data and the question remains as to how they managed this using just the quantifiers in the verbal descriptions.

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